

NUMBER SYSTEM

$$a \times (b+c) = (a \times b) + (a \times c)$$



Learning objectives

- To understand the concept of addition and subtraction of integers.
- To understand the concept of multiplication and division of integers.
- To understand the properties of four fundamental operations applied to integers.
- To solve applied problems using the four fundamental operations on integers.

Recap

Integers are the collection of natural numbers, zero and negative numbers.

We denote this collection by $\mathbb{Z}$.

Negative integers are represented on the number line to the left of zero and positive integers to the right of zero.



Fig. 1.1

Every integer on this number line is placed in an increasing order from left to right.

The integers at each point A, B, C, D in the figure given below are $A = +3$, $B = +7$, $D = -1$ and $C = -5$.



Fig. 1.2

1. Write the following integers in ascending order:

-5, 0, 2, 4, -6, 10, -10

2. If the integers -15, 12, -17, 5, -1, -5, 6 are marked on the number line then the integer on the extreme left is _____.

3. Complete the following pattern:

____, -40, ____, ____, -10, 0, ____, 20, 30, ____, 50.

4. Compare the given numbers and write "<", ">" or "=" in the boxes.

(a) -65 65 (b) 0 1000 (c) -2018 -2018

5. Write the given integers in descending order :

-27, 19, 0, 12, -4, -22, 47, 3, -9, -35



Try these

1.1 Introduction

In class VI, we studied how to compare and arrange integers. Now, let us try to add and subtract integers.

We know $7-5 = 2$. But, what is $5-7$? Let us try to take away 7 from 5 using the number line. As the integers on the number line increases from left to right, we should move to the left of zero to do this subtraction.

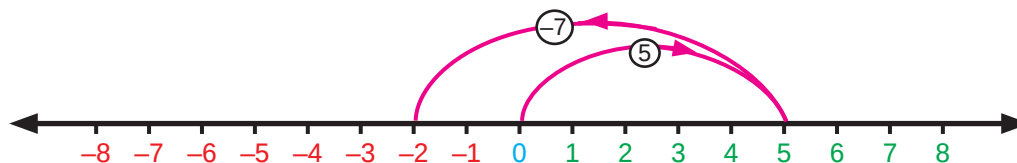
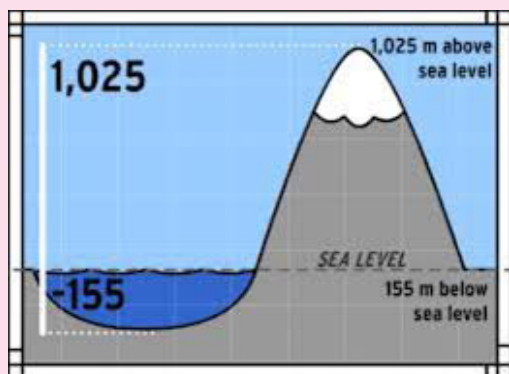


Fig. 1.3

Is it really necessary to do this? Have you come across any such situations in life? Yes, there are situations like increase or decrease in temperature, amount deposited and withdrawn from an account, profit and loss in a business are all instances where integers are involved.

MATHEMATICS ALIVE – NUMBER SYSTEM IN REAL LIFE



Mountain, below and above sea level.



Temperature is $< 0^{\circ}\text{C}$ in Antarctica and Arctic, $> 0^{\circ}\text{C}$ in Equator.

1.2 Addition of Integers

The number line is a simple tool to visualise addition of integers. Let us do an activity with the number line.

Imagine that the number line is a road with markers on it. We are allowed to step forward or backward on this road. One step taken is equal to one unit of number. Initially we start at zero and face positive direction. We step forward for positive integers and backward for negative integers. We maintain the same positive direction for addition operation.

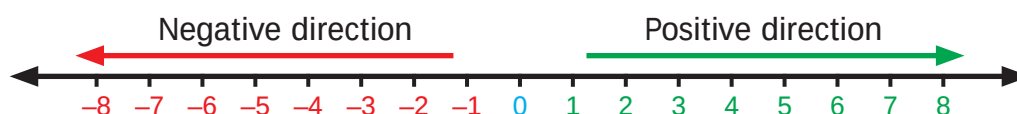


Fig. 1.4

To add $(+5)$ and (-3) , We start at zero facing positive direction and move five steps forward to represent $(+5)$. Since the operation is addition we maintain the same direction and move three units backward to represent (-3) . We land at $+2$. So, $(+5)+(-3)=2$. This is shown in Fig.1.5.

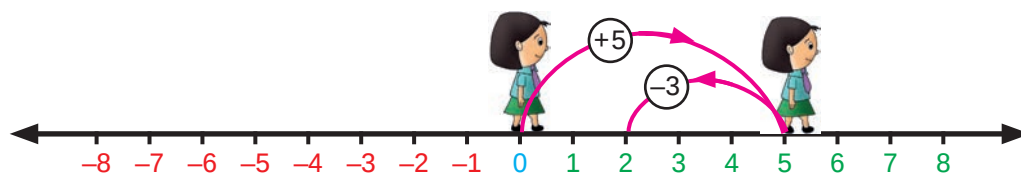


Fig. 1.5

Proceeding in the same way let us try another example. Find the sum of (-6) and (-4) . We start at zero facing positive direction continuing in the same direction and move 6 units backward to represent (-6) and in the same direction move 4 units backward to represent (-4) . We land at (-10) .

Therefore, $(-6)+(-4)=-10$

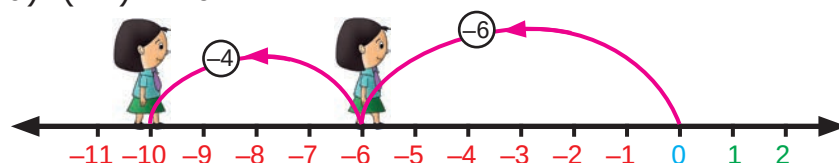


Fig. 1.6



Try this

Find the value of the following using the number line activity:

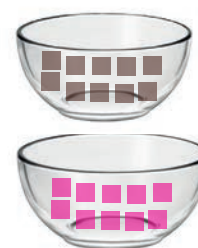
- (i) $(-4) + (+3)$ (ii) $(-4) + (-3)$ (iii) $(+4) + (-3)$



Activity

There are two bowls with tokens of two different colours brown and pink. Let us denote one brown token by $(+1)$ and one pink token by (-1) . A pair of tokens, brown $(+1)$ and pink (-1) , will denote zero $[1+(-1)=0]$

To add two integers, we pick the required number of tokens and form possible zero pairs. The remaining number of tokens left after pairing is the sum of the two integers.



$$\begin{array}{c}
 \text{5 pink tokens} \quad \text{5 brown tokens} \quad \text{2 pink tokens} \\
 (-7) \\
 (+5) \\
 \hline
 -2
 \end{array}$$

To add (-7) and $(+5)$ we pick 7 pink tokens and 5 brown tokens. We form zero pairs from the tokens as above. We can form 5 zero pairs. Then we are left with 2 pink tokens. Hence, $(-7)+(+5)=-2$.



$$\begin{array}{c} \text{■} \text{■} \text{■} \\ (-3) \end{array} (+) \begin{array}{c} \text{■} \text{■} \text{■} \text{■} \\ (-4) \end{array} = -7$$

To add (-3) and (-4) we pick 3 pink tokens first and 4 pink tokens later. The total number of tokens is 7 pink tokens. There are no zero pairs. So, the sum of (-3) and (-4) is (-7) . Teacher can give different integers and ask them to add using tokens.



1. When we add two integers of the same sign the sum will also be an integer of the same sign. When we add two integers of different sign, the sum will be the difference between the two integers and have the sign of the integer with greater value.
2. The integer without sign represents positive integer.

Example 1.1

Add the following integers using number line (i) 10 and -15 (ii) -7 and -9

Solution Let us add the integers using number line

(i) 10 and -15

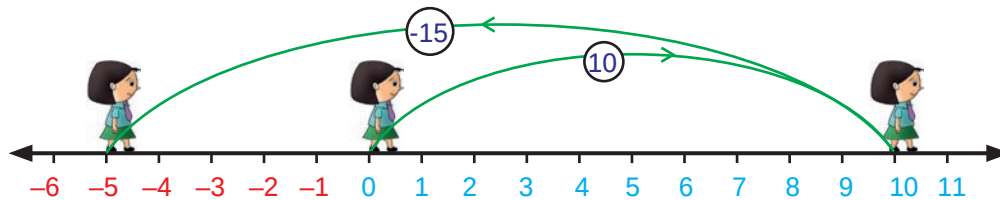


Fig. 1.7

On the number line we first start at zero facing positive direction and move 10 steps forward, reaching 10. Then we move 15 steps backward to represent -15 and reach at -5 . Thus, we get $10 + (-15) = -5$.

(ii) -7 and -9

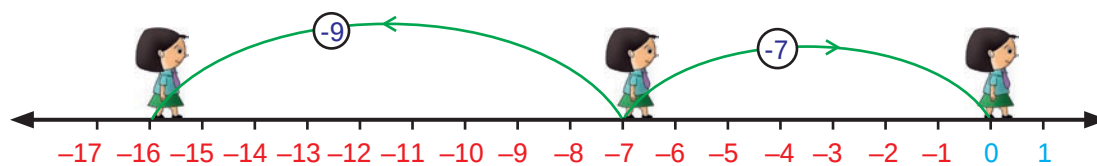


Fig. 1.8



On the number line we first start at zero facing positive direction and move 7 steps backward, reaching -7 . Then we move 9 steps backward to represent -9 and reach at -16 . Thus, we get $(-7) + (-9) = -16$.

Example 1.2

Add (i) (-40) and (30) (ii) 60 and (-50)

Solution

(i) (-40) and (30)

$$-40 + 30 = -10$$

(ii) 60 and (-50)

$$60 + (-50) = 60 - 50 = 10$$

Example 1.3

Add: (i) (-70) and (-12) (ii) 103 and 39 .

Solution

(i) $(-70) + (-12) = -70 - 12 = -82$

(ii) $103 + 39 = 142$

Example 1.4

A submarine is at 32 feet below the sea level. Then it moves up 8 feet. Find the depth of the submarine.

Solution

A submarine is 32 feet below sea level.

Therefore, it is represented by -32

Next it moves up 8 feet.

Moves above is represented as $+8$

The depth of the submarine $= -32 + 8 = -24$

Therefore, the submarine is located at 24 feet below the sea level.

Example 1.5

Sita saved ₹ 225.00 and she has spent ₹ 400 on credit basis for the purchase of stationery. Find her due amount.

Solution

The amount Sita has ₹ 225

The amount spent for stationery on credit = ₹ 400

$$\text{The due amount to be paid} = 225 - 400 = -175$$

Therefore, Sita has to pay ₹ 175

Example 1.6

From the ground floor a man went up six floors and came down six floors. In which floor is he now?

Solution

Starting point = Ground floor

Number of floors climbed up = $+6$

Number of floors climbed down = -6

$$\text{Now the landing point} = +6 - 6 = 0 \text{ (ground floor)}$$

1.2.1 Properties of Addition



In class VI, we have studied that the collection of whole numbers is closed under the addition operation. The sum of two whole numbers is always a whole number. Does this property hold for the collection of integers also?

Complete the given table and check whether the sum of two integers is an integer or not?

(i) $7 + (-5) =$	(ii) $(-6) + (-13) =$	(iii) $25 + 9 =$
(iv) $(-12) + 4 =$	(v) $41 + 32 =$	(vi) $(-19) + (-15) =$
(vii) $52 + (-15) =$	(viii) $(-7) + 0 =$	(ix) $0 + 12 =$
(x) $14 + 0 =$	(xi) $(-6) + (-6) =$	(xii) $(-27) + 0 =$

We observe that in all the above cases the sum of two integers is an integer. Since addition of integers is an integer, we say that integers are closed under addition. This property is known as "closure property" of integers on addition.

Therefore, for any two integers a, b ; $a + b$ is also an integer.

We observe one more property of integers. The order in which we add two integers does not matter. For example, $12 + (-13)$ is the same as $(-13) + (12)$. Also $(-7) + (-5)$ is the same as $(-5) + (-7)$.

This property is known as "commutative property" of integers.

Therefore, for any two integers a, b ; $a + b = b + a$.

What happens when we add three integers? For example, will $(-7) + (-2) + (-9)$ give the same value when they are added in any way of grouping.

Let us check by grouping the integers as $[(-7) + (-2)] + (-9)$ and $(-7) + [(-2) + (-9)]$. First let us find the value of $[(-7) + (-2)] + (-9)$.

$$[(-7) + (-2)] + (-9) = (-9) + (-9) = -9 - 9 = -18$$

Let us illustrate this with the number line : $[(-7) + (-2)] + (-9)$ can be represented as

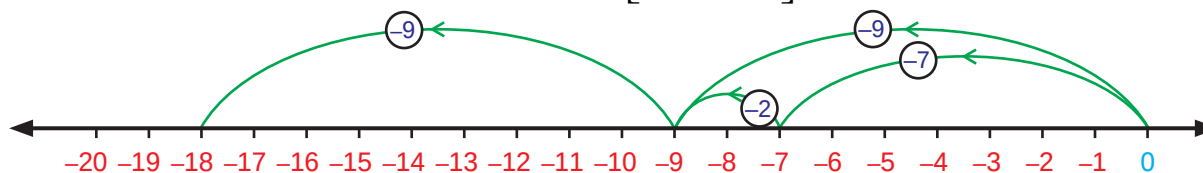


Fig. 1.9

$$[(-7) + (-2)] + (-9) = -18$$

Then we will find the value of $(-7) + [(-2) + (-9)]$

$$(-7) + [(-2) + (-9)] = (-7) + (-11) = -7 - 11 = -18$$

$(-7) + [(-2) + (-9)]$ can be represented on the number line as below.

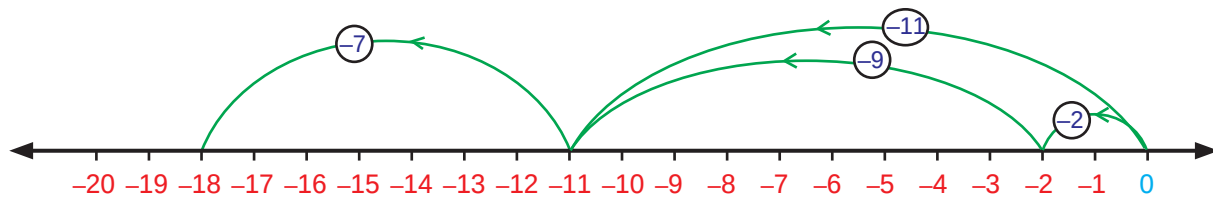


Fig. 1.10

$$(-7) + [(-2) + (-9)] = (-18)$$

We reach the same number -18 in both the cases. Hence, regrouping of integers does not change the value of the sum. This property is known as “**associative property**” under addition.

Therefore, for any three integers a, b, c ; $a + (b + c) = (a + b) + c$

The collection of integers has positive, negative integers and zero. Have you noticed that zero is neither positive nor negative integer. What happens when we add zero to an integer?

For example, we observe that $7 + 0 = 7$, $-3 + 0 = -3$, $-27 + 0 = -27$,
 $-79 + 0 = -79$, $0 + (-69) = -69$, $0 + (-85) = -85$.

From the above it is clear that whenever zero is added to an integer, we get the same integer. Due to this property, zero is called the identity with respect to addition or “**additive identity**” of the collection of integers.

Therefore, for any integers a , $a + 0 = a = 0 + a$

The additive identity zero separates the number line into positive and negative integers. We have $+1$ and -1 , $+5$ and -5 , -15 and $+15$, etc. on opposite sides of the number line that are equidistant from zero. Such integers on either side of zero are called “opposites” of each other. In fact, we find that the “opposites” added together always give the value zero.

For example, $(-15) + 15 = 0$, $21 + (-21) = 0$. This property of integers is named as “**additive inverse**”. (-15) is the additive inverse of 15 because their sum is zero. In the same way, 21 is the additive inverse of -21 . Either of the pair of opposites is known as the “**additive inverse**” of the other.

Therefore, for any integer a , $-a$ is the additive inverse.

$$a + (-a) = 0 = (-a) + a$$



Try these

1. Fill in the blanks:

(i) $20 + (-11) = \underline{\quad} + 20$ (ii) $(-5) + (-8) = (-8) + \underline{\quad}$ (iii) $(-3) + 12 = \underline{\quad} + (-3)$

2. Say true or false.

(i) $(-11) + (-8) = (-8) + (-11)$ (ii) $-7 + 2 = 2 + (-7)$ (iii) $(-33) + 8 = 8 + (-33)$

3. Verify the following:

(i) $[(-2) + (-9)] + 6 = (-2) + [(-9) + 6]$ (ii) $[7 + (-8)] + (-5) = 7 + [(-8) + (-5)]$

(iii) $[(-11) + 5] + (-14) = (-11) + [5 + (-14)]$

(iv) $(-5) + [(-32) + (-2)] = [(-5) + (-32)] + (-2)$

4. Find the missing integers:

(i) $0 + (-95) = \underline{\hspace{2cm}}$

(ii) $-611 + \underline{\hspace{2cm}} = -611$

(iii) $\underline{\hspace{2cm}} + 0 = \underline{\hspace{2cm}}$

(iv) $0 + (-140) = \underline{\hspace{2cm}}$

5. Complete the following:

(i) $-603 + 603 = \underline{\hspace{2cm}}$

(ii) $9847 + (-9847) = \underline{\hspace{2cm}}$ (iii) $1652 + \underline{\hspace{2cm}} = 0$

(iv) $-777 + \underline{\hspace{2cm}} = 0$

(v) $\underline{\hspace{2cm}} + 5281 = 0$

Example 1.7 (i) Are $120 + 51$ and $51 + 120$ equal?

(ii) Are $(-5) + [(-4) + (-3)]$ and $[(-5) + (-4)] + (-3)$ equal?

Solution

(i) When we add, $120 + 51 = 171$; $51 + 120 = 171$

In both the cases we get same answer. This means that integers can be added in any order. Hence, addition of integers is commutative.

(ii) $(-5) + [(-4) + (-3)]$ and $[(-5) + (-4)] + (-3)$

In $(-5) + [(-4) + (-3)]$, (-4) and (-3) are added first and their result is then added with (-5) .

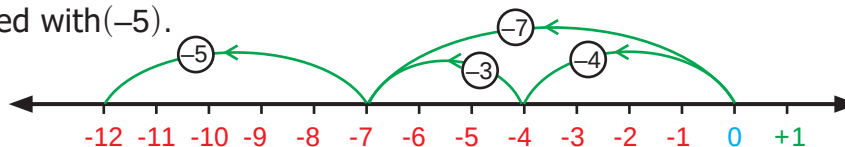


Fig. 1.11

$$(-5) + [(-4) + (-3)] = -12$$

Whereas in $[(-5) + (-4)] + (-3)$, (-4) and (-3) are added first and then the result is added with (-5)

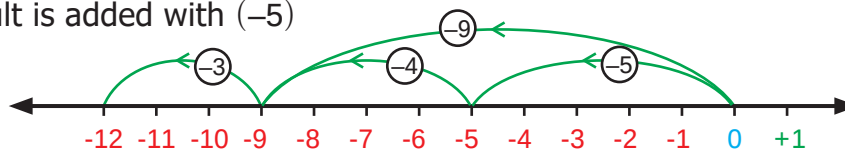


Fig. 1.12

$$[(-5) + (-4)] + (-3) = -12$$

In both the cases, we get -12

$$\text{That is } (-5) + [(-4) + (-3)] = [(-5) + (-4)] + (-3)$$

So, addition is associative.

Example 1.8 Find the missing integers (i) $0 + (-2345) = \underline{\hspace{2cm}}$ (ii) $23479 + \underline{\hspace{2cm}} = 0$

Solution

(i) $0 + (-2345) = -2345$

(ii) $23479 + (-23479) = 0$

Therefore, additive inverse of 23479 is -23479

Example 1.9

Mention the property for the following equations:

(i) $(-45) + (-12) = -57$

(ii) $(-15) + 7 = (7) + (-15)$

(iii) $-10 + 3 = -7$

(iv) $(-7) + (-5) = (-5) + (-7)$

(v) $(-7) + [(-4) + (-3)] = [(-7) + (-4)] + (-3)$

(vi) $0 + (-7245) = -7245$

Solution

(i) Closure Property

(ii) Commutative Property

(iii) Closure Property

(iv) Commutative Property

(v) Associative Property

(vi) Additive Identity

Exercise 1.1

1. Fill in the blanks

(i) $(-30) + \underline{\hspace{1cm}} = 60$

(ii) $(-5) + \underline{\hspace{1cm}} = -100$

(iii) $(-52) + (-52) = \underline{\hspace{1cm}}$

(iv) $\underline{\hspace{1cm}} + (-22) = 0$

(v) $\underline{\hspace{1cm}} + (-70) = 70$

(vi) $20 + 80 + \underline{\hspace{1cm}} = 0$

(vii) $75 + (-25) = \underline{\hspace{1cm}}$

(viii) $171 + \underline{\hspace{1cm}} = 0$

(ix) $[(-3) + (-12)] + (-77) = \underline{\hspace{1cm}} + [(-12) + (-77)]$

(x) $(-42) + [\underline{\hspace{1cm}} + (-23)] = [\underline{\hspace{1cm}} + 15] + \underline{\hspace{1cm}}$

2. Say true or false.

(i) The additive inverse of (-32) is (-32)

(ii) $(-90) + (-30) = 60$

(iii) $(-125) + 25 = -100$

3. Add the following

(i) 8 and -12 using number line

(ii) (-3) and (-5) using number line

(iii) $(-100) + (-10)$

(iv) $20 + (-72)$

(v) $82 + (-75)$

(vi) $-48 + (-15)$

(vii) $-225 + (-63)$

4. Thenmalar appeared for competitive exam which has negative scoring of 1 mark for each incorrect answer. In paper I she answered 25 questions incorrectly and in paper II, 13 questions incorrectly. Find the total reduction of marks.

5. In a quiz competition, Team A scored $+30, -20, 0$ and team B scored $-20, 0, +30$ in three successive rounds. Which team will win? Can we say that we can add integers in any order?

6. Are $(11+7)+10$ and $11+(7+10)$ equal? Mention the property.

7. Find 5 pairs of integers that add up to 2.

Objective type questions

8. The temperature at 12 noon at a certain place was 18° above zero. If it decreases at the rate of 3° per hour at what time it would be 12° below zero?
- (i) 12 mid night (ii) 12 noon
(iii) 10 am (iv) 10 pm
9. Identify the problem with negative numbers as its answer:
- (i) $-9 + (-5) + 6$ (ii) $18 + (-12) - 6$
(iii) $-4 + 2 + 10$ (iv) $10 + (-4) + 8$
10. $(-10) + (+7) = \underline{\hspace{2cm}}$
- (i) +3 (ii) -3 (iii) -17 (iv) +17
11. $(-8) + 10 + (-2) = \underline{\hspace{2cm}}$
- (i) 2 (ii) 8 (iii) 0 (iv) 20
12. $20 + (-9) + 9 = \underline{\hspace{2cm}}$
- (i) 20 (ii) 29 (iii) 11 (iv) 38

1.3 Subtraction of Integers



Let us learn subtraction of integers using number line.

Let us try to subtract integers using the number line activity we studied earlier. We should follow the same instructions as before but whenever we need to subtract, we turn towards the negative side.

To subtract $(+4)$ from $(+7)$

We start at zero facing positive direction. Moves 7 units forward to represent $+7$ then turn towards the negative side for the operation of subtraction and move $+4$ units forward to represent $+4$. We reach the integer $+3$. So, $(+7) - (+4) = +3$.

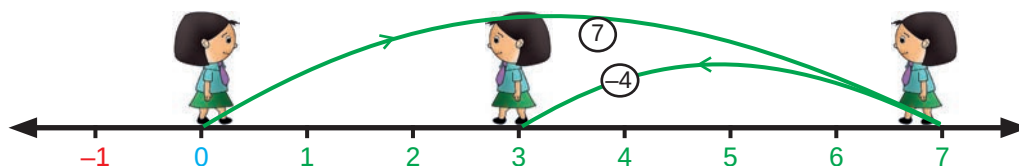


Fig. 1.13

Let us find $(-8) - (-5)$.

We start at zero facing positive direction. Move 8 units backward to represent (-8) . Then turn towards the negative side and move 5 units backwards. We reach -3 . We have $(-8) - (-5) = (-3)$.

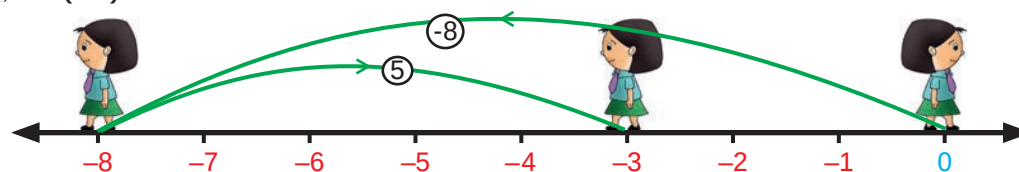


Fig. 1.14



Now, let us learn subtraction of integers in another way.

Observe the following patterns :

$$7 - 2 = 5; \quad 7 - 1 = 6; \quad 7 - 0 = 7$$

What will happen if we extend this to negative integers?

$$7 - (-1) = 8; \quad 7 - (-2) = 9; \quad 7 - (-3) = 10$$

We shall see some more patterns.

$$20 - 2 = 18; \quad 20 - 1 = 19; \quad 20 - 0 = 20; \quad 20 - (-1) = 21; \quad 20 - (-2) = 22$$

We can see from the above patterns that while subtracting consecutive negative integers from 7 and 20 the difference increase consecutively.

It is clear that subtraction of negative integers gives increase in the difference. For example $7 - (-2) = 9$. Hence, subtraction of -2 is equivalent to addition of 2, which is the additive inverse of -2 . That is, $7 + 2 = 9$.

So, to subtract a negative integer from an integer we add the additive inverse of the integer which is to be subtracted.

For example, subtract (-5) from 7.

$$7 - (-5)$$

To subtract (-5) we can add additive inverse of (-5) that is 5 with 7

$$\text{Therefore, } 7 - (-5) = 12$$



Try these

1. Do the following by using number line.
(i) $(-4) - (+3)$ (ii) $(-4) - (-3)$
2. Find the values and compare the answers.
(i) $(-6) - (-2)$ and $(-6) + 2$
(ii) $35 - (-7)$ and $35 + 7$
(iii) $26 - (+10)$ and $26 + (-10)$
3. Put the suitable symbol $<$, $>$ or $=$ in the boxes.
(i) $-10 - 8$ $-10 + 8$ (ii) $(-20) + 10$ $(-20) - (-10)$
(iii) $(-70) - (-50)$ $(-70) - 50$ (iv) $100 - (+100)$ $100 - (-100)$
(v) $-50 - 30$ $-100 + 20$



Every subtraction statement has a corresponding addition statement. For example, $8 - 5 = 3$ is a subtraction statement. This can be seen as the addition statement $3 + 5 = 8$. In the same way, $(-8) - (-5) = -3$ is a subtraction statement which can be written as the addition statement $(-8) = (-3) + (-5)$.

Example 1.10 Subtract the following using the number line.

- (i) $-3 - (-2)$ (ii) $+6 - (-5)$

Solution

- (i) $-3 - (-2)$

To subtract -2 from -3 using number line,

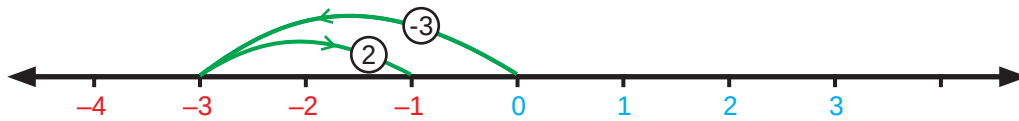


Fig. 1.15

Therefore, $-3 - (-2) = -3 + 2 = -1$

- (ii) $+6 - (-5)$

To subtract -5 from 6 using number line,

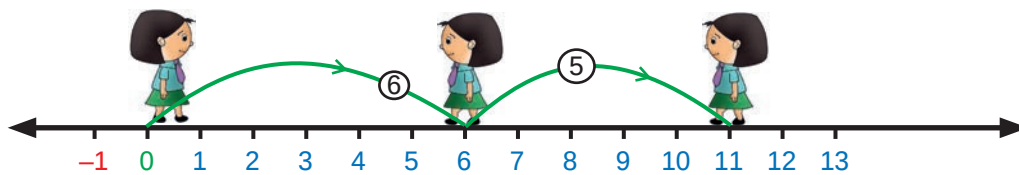


Fig. 1.16

Therefore, $+6 - (-5) = +6 + 5 = 11$.

Now, let us see how to subtract negative integers using additive inverse.

Example 1.11

- (i) Subtract (-40) from 70 (ii) Subtract (-12) from (-20)

Solution

- (i) $70 - (-40)$

$$\begin{aligned} &= 70 + (\text{additive inverse of } -40) \\ &= 70 + 40 \\ &= 110. \end{aligned}$$

- (ii) $(-20) - (-12)$

$$\begin{aligned} &= (-20) + (\text{additive inverse of } (-12)) \\ &= (-20) + 12 \\ &= -8 \end{aligned}$$

Example 1.12

- Find the value of : (i) $(-11) - (-33)$ (ii) $(-90) - (-50)$

**Solution**

$$\begin{aligned}\text{(i)} \quad & (-11) - (-33) \\ &= (-11) + (+33) \\ &= 22\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad & (-90) - (-50) \\ &= -90 - (-50) \\ &= -90 + 50 \\ &= -40\end{aligned}$$

Example 1.13

Chitra has ₹ 150. She wanted to buy a bag which costs ₹ 225. How much money does she need to borrow from her friend?

Solution

$$\begin{aligned}\text{Amount with Chitra} &= ₹ 150 \\ \text{Cost of bag} &= ₹ 225 \\ \text{Amount to be borrowed} &= 225 - 150 \\ &= ₹ 75\end{aligned}$$

Example 1.14

What is the balance in Chezhiyan's account as a result of a purchase for ₹ 1079, if he had an opening balance of ₹ 5000 in his account?

Solution

$$\begin{aligned}\text{Opening balance} &= ₹ 5000 \\ \text{Debit amount} &= ₹ 1079 (-) \\ \text{Balance amount} &= ₹ 3921\end{aligned}$$

Example 1.15

The temperature at Srinagar was -3°C on Friday. If the temperature decreases by 1°C next day, then what is the temperature on that day?

Solution

The temperature at Srinagar was -3°C on Friday. If the temperature decreases by 1°C then, temperature on the next day = $-3^{\circ}\text{C} - 1^{\circ}\text{C} = -4^{\circ}\text{C}$

Example 1.16

A submarine is at 300 feet below the sea level. If it ascends to 175 feet, what is its new position?

Solution

$$\begin{aligned}\text{Initial position of submarine} &= 300 \text{ feet below} \\ &= -300 \text{ feet} \\ \text{Distance ascended by submarine} &= 175 \text{ feet} \\ &= +175 \text{ feet} \\ \text{New position of submarine} &= (-300) + (+175) \\ &= -125\end{aligned}$$



Fig. 1.17

That is, the submarine is 125 feet below the sea level.

1.3.1 Properties of Subtraction

We can test whether all the properties of integers that are true for addition still hold for subtraction or not.

Recall that subtraction of two whole numbers did not always result in a whole number. However, this extended form of integers is enough to make sure that the difference of two integers is also an integer. For example, $(-7) - (-2)$ is an integer, $(-5) + 14$ is an integer and $0 - (-8)$ is an integer. From these examples we observe that the collection of integers is “closed”, that is, the difference of two integers is always an integer.

Therefore, for any two integers a, b ; $a - b$ is also an integer.

What about the other properties? Can you see that $(-2) - (-5) = 3$ but $(-5) - (-2) = -3$. Also, $10 - (-5) = 15$ but $(-5) - 10 = -15$. Therefore, changing the order of integers in subtraction will not give the same value. Hence, the commutative property does not hold for subtraction of integers.

Therefore, for any two integers a, b ; $a - b \neq b - a$.



Try these

1. Fill in the blanks.

(i) $(-7) - (-15) = \underline{\hspace{2cm}}$

(ii) $12 - \underline{\hspace{2cm}} = 19$

(iii) $\underline{\hspace{2cm}} - (-5) = 1$

2. Find the values and compare the answers.

(i) $15 - 12$ and $12 - 15$

(ii) $-21 - 32$ and $-32 - (-21)$



Think

Is associative property true for subtraction of integers?

Take any three examples and check.

Exercise 1.2

1. Fill in the blanks

(i) $-44 + \underline{\hspace{2cm}} = -88$

(ii) $\underline{\hspace{2cm}} - 75 = -45$

(iii) $\underline{\hspace{2cm}} - (+50) = -80$

2. Say true or false

(i) $(-675) - (-400) = -1075$

(ii) $15 - (-18)$ is the same as $15 + 18$

(iii) $(-45) - (-8) = (-8) - (-45)$

3. Find the value of the following

(i) $-3 - (-4)$ using number line

(ii) $7 - (-10)$ using number line

(iii) $35 - (-64)$

(iv) $-200 - (+100)$

4. Kabilan was having 10 pencils with him. He gave 2 pencils to Senthil and 3 to Karthik. Next day his father gave him 6 more pencils, from that he gave 8 to his sister. How many pencils are left with him?

5. A lift is on the ground floor. If it goes 5 floors down and then moves up to 10 floors from there. Then in which floor will the lift be?
6. When Kala woke up, her body temperature was 102°F . She took medicine for fever. After 2 hours it was 2°F lower. What was her temperature then?
7. What number should be added to (-17) to get (-19) ?
8. A student was asked to subtract (-12) from (-47) . He got -30 . Is he correct? Justify.

Objective type questions

9. $(-5) - (-18) = \underline{\hspace{2cm}}$
 (i) 23 (ii) -13 (iii) 13 (iv) -23
10. $(-100) - 0 + 100 = \underline{\hspace{2cm}}$
 (i) 200 (ii) 0 (iii) 100 (iv) -200

1.4. Multiplication of Integers

Situation 1

Ramani and Ravi are playing with pebbles in a heap. Ramani is adding some pebbles to the heap while Ravi removes some pebbles. First, Ramani adds 3 pebbles. Then she adds 3 more pebbles. She repeats this 2 more times. Can you say how many pebbles she added in all? Since adding pebbles is positive we can write $(+3) + (+3) + (+3) + (+3) = +12$ or $4 \times (+3) = 12$

The total number of pebbles added is 12.

Ravi takes out 5 pebbles three times. If we represent "taking out" as a negative integer then $(-5) + (-5) + (-5) = -15$ or we can simply put it as $(-5) \times 3 = -15$. Ravi has removed 15 pebbles from the heap. This illustrates that multiplication of negative integers is also repeated addition just like positive integers or whole numbers.

We can draw a number line and visualise the above multiplication of integers as repeated addition.

$4 \times 3 = 12$ (3 is added four times).



Fig. 1.18

$(-5) \times 3 = -15$ (-5 is added three times).

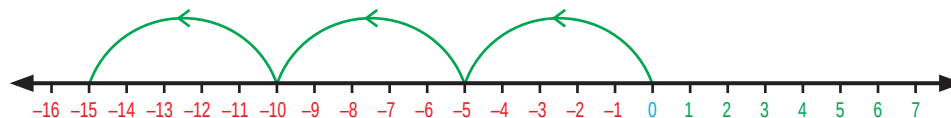


Fig. 1.19

We have no difficulty to understand that a positive integer like $(+7)$ multiplied by another positive integer like $(+8)$ is positive $(+56)$. Also, a positive integer like $+7$ multiplied by a negative integer like -5 is -35 and $+5 \times -7 = -35$. But what about a negative integer

(-3) multiplied with another negative integer -5 ? To understand this, let us observe the following pattern.

$$\begin{aligned}(-5) \times 3 &= -15 \\(-5) \times 2 &= -10 \\(-5) \times 1 &= -5 \\(-5) \times 0 &= 0 \\(-5) \times (-1) &= +5 \\(-5) \times (-2) &= +10 \\(-5) \times (-3) &= +15\end{aligned}$$

Note that how in each step the number increase by 5 from -15 to -10 , from -10 to -5 , from -5 to 0 . Therefore, the next number in the pattern will be $+5$ only and not -5 . Similarly $(-5) \times (-2)$ is a positive integer 10 . So, the product of two negative integers is always a positive integer.



Try these

1. Find the product of the following

$$\begin{aligned}\text{(i)} \quad (-20) \times (-45) &= \underline{\hspace{2cm}} & \text{(ii)} \quad (-9) \times (-8) &= \underline{\hspace{2cm}} \\ \text{(iii)} \quad (-30) \times 40 \times (-1) &= \underline{\hspace{2cm}} & \text{(iv)} \quad (+50) \times 2 \times (-10) &= \underline{\hspace{2cm}}\end{aligned}$$

2. Complete the following table by multiplying the integers in the corresponding row and column headers.

x	-3	-2	-1	0	1	2	3
-3							
-2							
-1							
0							
1							
2							
3							

3. Which of the following is incorrect?

- (i) $(-55) \times (-22) \times (-33) < 0$
- (ii) $(-1521) \times 2511 < 0$
- (iii) $2512 - 1252 < 0$
- (iv) $(+1981) \times (+2000) < 0$



Think

Can you express the product 15×16 as sum or difference of integers?

Yes, here are 4 ways to compute $15 \times 16 = +240$.

- (i) $15 \times 16 = (10 + 5) \times (10 + 6) = 100 + 60 + 50 + 30 = 240$
- (ii) $15 \times 16 = (20 - 5) \times (10 + 6) = 200 + 120 + (-50) + (-30) = 240$
- (iii) $15 \times 16 = (10 + 5) \times (20 - 4) = 200 + (-40) + 100 + (-20) = 240$
- (iv) $15 \times 16 = (20 - 5) \times (20 - 4) = 400 + (-80) + (-100) + \boxed{20} = 240$

From the above pattern one can determine the product of two positive integers, two negative integers, one positive integer with one negative integer.

Example 1.17 Find the value of :

- (i) $(-35) \times (-11)$
- (ii) $96 \times (-20)$
- (iii) $(-5) \times 12$
- (iv) 15×5
- (v) 999×0

Solution

$$(i) (-35) \times (-11) = 385$$

$$(ii) 96 \times (-20) = -1920$$

$$(iii) (-5) \times 12 = -60$$

$$(iv) 15 \times 5 = 75$$

$$(v) 999 \times 0 = 0$$

Example 1.18 A fruit seller sold 5kg of mangoes at a profit of ₹ 15 per kg and 3kg of apples at a loss of ₹ 30 per kg. Find whether it is a profit or loss.

Solution

$$\text{Profit of 1kg mangoes} = ₹ 15$$

$$\text{Profit of 5kg mangoes} = 15 \times 5$$

$$= ₹ 75$$

$$\text{Loss of 1kg apples} = ₹ 30$$

$$\text{Loss of 3kg apples} = 30 \times 3$$

$$= ₹ 90$$

$$\text{Loss} = 90 - 75$$

$$= ₹ 15$$

Example 1.19 Browsing rates in an internet centre is ₹ 15 per hour. Nila works on the internet for 2 hours in a day for 5 days in a week. How much does she pay?

Solution

$$\text{Number of hours spent on an internet for a day} = 2 \text{ hrs}$$

$$\text{Therefore, number of hours spent on the internet for 5 days} = 5 \times 2$$

$$= 10 \text{ hrs}$$

$$\text{Cost of browsing per hour} = ₹ 15$$

$$\text{Cost of browsing for 10 hours} = 15 \times 10$$

$$\text{Therefore, the amount paid by Nila for 5 days} = ₹ 150$$

1.4.1 Properties of Multiplication of Integers

Recall that multiplication of whole numbers had the closure property. If we test this property for integers we find $(-7) \times (-2) = +14$, $(-6) \times 5 = -30$, $4 \times (-9) = -36$. Thus, product of any two integers is always an integer. "Closure property" holds for integers.



Therefore, for any two integers a, b ; $a \times b$ is also an integer.

Consider the examples, $21 \times (-5) = -105$ is the same as $(-5) \times 21 = -105$. The product $(-9) \times (-8) = +72$, $(-8) \times (-9) = 72$. In both the cases the product is the same integer. So, changing the order of multiplication does not change the value of the product. Hence, we conclude that multiplication of integers are "commutative".

Therefore, for any two integers a, b ; $a \times b = b \times a$.

To verify that multiplication of integers is associative, let us check whether $(-5) \times [(-9) \times (-12)]$ and $[(-5) \times (-9)] \times (-12)$ are equal.

$$(-5) \times [(-9) \times (-12)] = (-540) \text{ and } [(-5) \times (-9)] \times (-12) = (-540).$$

Hence, "associative" property is true for multiplication of integers.

Therefore, for any three integers a, b, c ; $(a \times b) \times c = a \times (b \times c)$.

Just as zero added to a integer leaves it unchanged, integer 1 multiplied with any integer leaves the integer unaltered. For example, $57 \times 1 = 57$, and $1 \times (-62) = -62$. We say that the integer 1 is the identity for multiplication of integers or "multiplicative identity".

Therefore, for any integer a ; $a \times 1 = 1 \times a = a$.



Try these

1. Find the product and check for equality :

- (i) $18 \times (-5)$ and $(-5) \times 18$
- (ii) $31 \times (-6)$ and $(-6) \times 31$
- (iii) 4×51 and 51×4

2. Prove the following :

- (i) $(-20) \times (13 \times 4) = [(-20) \times 13] \times 4$
- (ii) $[(-50) \times (-2)] \times (-3) = (-50) \times [(-2) \times (-3)]$
- (iii) $[(-4) \times (-3)] \times (-5) = (-4) \times [(-3) \times (-5)]$



Note

Consider an example, $(-7) \times (-6) \times (-5) \times (-4)$.

Let us try to do the above multiplication of integer,

$$\begin{aligned} (-7) \times (-6) \times (-5) \times (-4) &= [(-7) \times (-6)] \times [(-5) \times (-4)] \\ &= (+42) \times (+20) \\ &= + 840 \end{aligned}$$

From the above example, we see that the product of four negative integers is positive. What will happen if we multiply odd number of negative integers.

Let us consider another example, $(-7) \times (-3) \times (-2)$.

Multiplying the above integers, we get

$$\begin{aligned} (-7) \times (-3) \times (-2) &= [(-7) \times (-3)] \times (-2) \\ &= (+21) \times (-2) \\ &= - 42 \end{aligned}$$

From the above example, we see that the product of three negative integers is negative.

In general, if the negative integers are multiplied even number of times, the product is a positive integer, whereas negative integers are multiplied odd number of times, the product is a negative integer.

1.4.2 Distributive Property of Multiplication over Addition

We have already studied that multiplication distributes over addition on whole numbers. Let us check the property for integers.

Take for example $(-2) \times (4+5) = [(-2) \times 4] + [(-2) \times 5]$

$$\begin{aligned}\text{LHS} &= (-2) \times (4+5) \\ &= (-2) \times 9 \\ &= (-18) \\ &= -18\end{aligned}$$

$$\begin{aligned}\text{RHS} &= [(-2) \times 4] + [(-2) \times 5] \\ &= (-8) + (-10) \\ &= -8 - 10 \\ &= -18\end{aligned}$$

From the above, we can observe that $(-2) \times (4+5) = [(-2) \times 4] + [(-2) \times 5]$

Hence, "distributive property of multiplication over addition" is true for integers.

Therefore, for any three integers a, b, c ; $a \times (b+c) = (a \times b) + (a \times c)$

Try these

1. Find the values of the following and check for equality:

(i) $(-6) \times (4+(-5))$ and $((-6) \times 4) + ((-6) \times (-5))$

(ii) $(-3) \times [2+(-8)]$ and $[(-3) \times 2] + [(-3) \times 8]$

2. Prove the following:

(i) $(-5) \times [(-76)+8] = [(-5) \times (-76)] + [(-5) \times 8]$

(ii) $42 \times [7+(-3)] = (42 \times 7) + [42 \times (-3)]$

(iii) $(-3) \times [(-4)+(-5)] = ((-3) \times (-4)) + [(-3) \times (-5)]$

(iv) $103 \times 25 = (100+3) \times 25 = (100 \times 25) + (3 \times 25)$

Example 1.20

Prove that $(-7) \times (+8)$ is an integer and mention the property.

Solution

$$(-7) \times (+8) = (-56)$$

Hence, -56 is an integer.

Therefore, $(-7) \times (+8)$ is closed under multiplication.

Example 1.21

Are $(-42) \times (-7)$ and $(-7) \times (-42)$ equal? Mention the property.

Solution

Consider, $(-42) \times (-7)$,

$$(-42) \times (-7) = +294$$

Consider, $(-7) \times (-42)$,

$$(-7) \times (-42) = +294$$

Therefore, $(-42) \times (-7)$ and $(-7) \times (-42)$ are equal.

It is commutative.

Example 1.22

Prove that $[(-2) \times 3] \times (-4) = (-2) \times [3 \times (-4)]$.

Solution

In the first case (-2) and (3) are grouped together and in the second case (3) and (-4) are grouped together

$$\text{L.H.S} = [(-2) \times 3] \times (-4)$$

$$= (-6) \times (-4) = 24$$

$$\text{R.H.S} = (-2) \times [3 \times (-4)]$$

$$= (-2) \times (-12) = 24$$

Therefore, L.H.S. = R.H.S.

$$[(-2) \times 3] \times (-4) = (-2) \times [3 \times (-4)] \text{ Hence it is proved.}$$

Example 1.23

Are $(-81) \times [5 \times (-2)]$ and $[(-81) \times 5] \times (-2)$ equal? Mention the property.

Solution

Consider, $(-81) \times [5 \times (-2)]$,

$$(-81) \times [5 \times (-2)] = (-81) \times (-10) = 810$$

Consider, $[(-81) \times 5] \times (-2)$,

$$[(-81) \times 5] \times (-2) = (-405) \times (-2) = 810$$

Therefore, $(-81) \times [5 \times (-2)]$ and $[(-81) \times 5] \times (-2)$ are equal.

It is associative.

Example 1.24

Are $3 \times [(-4)+6]$ and $[3 \times (-4)]+(3 \times 6)$ equal? Mention the property.

Solution

Consider, $3 \times [(-4)+6]$,

$$3 \times [(-4)+6] = 3 \times 2 = 6$$

Consider, $[3 \times (-4)]+[3 \times 6]$,

$$[3 \times (-4)]+[3 \times 6] = -12+18 = 6$$

Therefore, $3 \times [(-4)+6]$ and $[3 \times (-4)]+3 \times 6$ are equal.

It is the distributive property of multiplication over addition.



Exercise 1.3

1. Fill in the blanks
 - (i) $-80 \times \underline{\hspace{1cm}} = -80$
 - (ii) $(-10) \times \underline{\hspace{1cm}} = 20$
 - (iii) $(100) \times \underline{\hspace{1cm}} = -500$
 - (iv) $\underline{\hspace{1cm}} \times (-9) = -45$
 - (v) $\underline{\hspace{1cm}} \times 75 = 0$
2. Say true or false
 - (i) $(-15) \times 5 = 75$
 - (ii) $(-100) \times 0 \times 20 = 0$
 - (iii) $8 \times (-4) = 32$
3. What will be the sign of the product of the following.
 - (i) 16 times of negative integer.
 - (ii) 29 times of negative integer.
4. Find the product of
 - (i) $(-35) \times 22$
 - (ii) $(-10) \times 12 \times (-9)$
 - (iii) $(-9) \times (-8) \times (-7) \times (-6)$
 - (iv) $(-25) \times 0 \times 45 \times 90$
 - (v) $(-2) \times (+50) \times (-25) \times 4$
5. Check the following for equality and if they are equal, mention the property.
 - (i) $(8-13) \times 7$ and $8-(13 \times 7)$
 - (ii) $[(-6)-(+8)] \times (-4)$ and $(-6)-[8 \times (-4)]$
 - (iii) $3 \times [(-4)+(-10)]$ and $[3 \times (-4)+3 \times (-10)]$
6. During summer, the level of the water in a pond decreases by 2 inches every week due to evaporation. What is the change in the level of the water over a period of 6 weeks?
7. Find all possible pairs of integers that give a product of -50 .



Objective type questions

8. Which of the following expressions is equal to -30 .
 - (i) $-20 - (-5 \times 2)$
 - (ii) $(6 \times 10) - (6 \times 5)$
 - (iii) $(2 \times 5) + (4 \times 5)$
 - (iv) $(-6) \times (+5)$
9. Which property is illustrated by the equation: $(5 \times 2) + (5 \times 5) = 5 \times (2 + 5)$
 - (i) commutative
 - (ii) closure
 - (iii) distributive
 - (iv) associative

10. $11 \times (-1) = \underline{\hspace{2cm}}$

(i) -1

(ii) 0

(iii) +1

(iv) -11

11. $(-12) \times (-9) = \underline{\hspace{2cm}}$

(i) 108

(ii) -108

(iii) +1

(iv) -1

1.5 Division of Integers



We have already seen that each multiplication statement has two division statements in whole numbers. Can we try the same for integers also; let us play with this number converter machine and see how division works.

Number Converter

Input	Multiplication Statement	Division Statement	Output
(i) Product of positive integer and positive integer	$8 \times 9 = 72$	$72 \div 9 = 8,$ $72 \div 8 = 9$	Positive integer divided by Positive integer is Positive
(ii) Product of positive integer and negative integer	$-10 \times 7 = -70$	$(-70) \div 7 = -10,$ $(-70) \div (-10) = 7$	Negative integer divided by Positive integer is Negative Negative integer divided by Negative integer is Positive
(iii) Product of negative integer and negative integer	$(-5) \times (-9) = 45$	$45 \div (-9) = -5,$ $45 \div (-5) = -9$	Positive integer divided by Negative integer is Negative

From the above table, we infer that the division of two integers with the same sign is a positive integer.

Division of two integers with opposite signs gives a negative integer.



Try these

(i) $(-32) \div 4 = \underline{\hspace{2cm}}$

(ii) $(-50) \div 50 = \underline{\hspace{2cm}}$

(iii) $30 \div 15 = \underline{\hspace{2cm}}$

(iv) $-200 \div 10 = \underline{\hspace{2cm}}$

(v) $-48 \div 6 = \underline{\hspace{2cm}}$

Example 1.25 Divide: (i) (-85) by 5 (ii) (-250) by (-25) (iii) 120 by (-6) (iv) 182 by (-2)

Solution

(i) $(-85) \div 5 = -17$

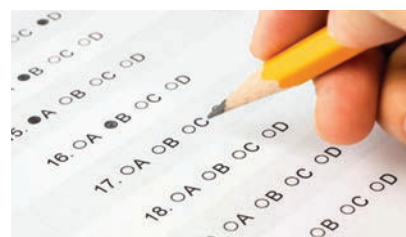
(ii) $(-250) \div (-25) = +10$

(iii) $120 \div (-6) = -20$

(iv) $182 \div (-2) = -91$

Example 1.26

In a competitive exam 4 marks are given for every correct answer and (-2) marks are given for every incorrect answer, kalaivizhi attended the exam and answered all the questions and scored 20 marks only even though she got 10 correct answers. How many questions did she answer incorrectly?



Solution

$$\text{Marks given for one correct answer} = 4$$

$$\begin{aligned}\text{Marks given for 10 correct answers} &= 10 \times 4 \\ &= 40\end{aligned}$$

$$\text{Kalaivizhi's final score} = 20$$

$$\begin{aligned}\text{Marks reduced for incorrect answers} &= 40 - 20 \\ &= 20\end{aligned}$$

$$\text{Therefore, number of questions answered incorrectly} = 20 \div 2 = 10$$

Example 1.27

A shopkeeper earns a profit of ₹ 5 by selling one notebook and incurs a loss of ₹ 2 per pen while selling of his old stock. In a particular day he earns neither profit nor loss. If he sold 20 notebooks, how many pens did he sell?

Solution

Since neither profit nor loss

$$\text{Profit} + \text{Loss} = 0$$

$$\text{(ie) profit} = -\text{Loss}$$

$$\text{Profit earned from 1 notebook} = ₹ 5$$

$$\begin{aligned}\text{Profit earned from 20 notebooks} &= 20 \times ₹ 5 \\ &= ₹ 100\end{aligned}$$

$$\begin{aligned}\text{Loss incurred by selling pens} &= ₹ 100 \text{ Loss} \\ &= -100\end{aligned}$$

$$\text{Loss for 1 pen} = ₹ 2$$

$$\text{loss} = -2$$

$$\begin{aligned}\text{Total number of pens sold} &= (-100) \div (-2) \\ &= 50 \text{ pens.}\end{aligned}$$



Activity

Students may be divided into two groups and do the activity using "In-out box". This box takes any number as input and apply the rule to the given number and gives the answer. One group will carry out the role of finding output applying the corresponding rule. One is done in each rule for your reference.

Table (I)	Table (II)	Table (III)	Table (IV)																																																
Rule : Add -7	Rule : Subtract -10	Rule : Multiply by -5	Rule : Divide by -3																																																
<table><tr><th>in</th><th>out</th></tr><tr><td>-10</td><td>-17</td></tr><tr><td>-7</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>16</td><td></td></tr><tr><td>4</td><td></td></tr></table>	in	out	-10	-17	-7		5		16		4		<table><tr><th>in</th><th>out</th></tr><tr><td>-20</td><td>-10</td></tr><tr><td>-13</td><td></td></tr><tr><td>7</td><td></td></tr><tr><td>10</td><td></td></tr><tr><td>15</td><td></td></tr></table>	in	out	-20	-10	-13		7		10		15		<table><tr><th>in</th><th>out</th></tr><tr><td>-7</td><td>$+35$</td></tr><tr><td>-12</td><td></td></tr><tr><td>$+15$</td><td></td></tr><tr><td>18</td><td></td></tr><tr><td>-5</td><td></td></tr></table>	in	out	-7	$+35$	-12		$+15$		18		-5		<table><tr><th>in</th><th>out</th></tr><tr><td>-18</td><td>$+6$</td></tr><tr><td>27</td><td></td></tr><tr><td>-99</td><td></td></tr><tr><td>-273</td><td></td></tr><tr><td>-35</td><td></td></tr></table>	in	out	-18	$+6$	27		-99		-273		-35	
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1.5.1 Properties of Division

Can you find the value when (-5) is divided by 3? The value is not an integer. Clearly, the collection of integers is not 'closed' under division. Test it by taking 5 more examples

To test the commutative property of division on integers, let us take -5 and 3.

$$(-5) \div 3 \neq 3 \div (-5). \text{ Also } 5 \div (-3) \neq (-3) \div 5 \text{ and } (-5) \div (-3) \neq (-3) \div (-5)$$

Therefore, division of integers are **not commutative**. Verify this with 5 more examples.

Since the commutative property is not true, associative property does not hold.

Let us take integers, -7 and 1.

$$(-7) \div 1 = -7 \text{ but } 1 \div (-7) \neq -7.$$

Let us take $(-6) \div (-1)$ and $6 \div (-1)$.

$$(-6) \div (-1) = 6 \text{ and } 6 \div (-1) = -6.$$

Therefore, when we divide an integer by -1 , we will not get the same integer.

Hence, there does not exist an identity for division of integers.

Take any five integers and verify the above.



Note

An integer divided by zero is not defined. But zero divided by a non-zero integer is zero.

Exercise 1.4

- Fill in the blanks.
 - $(-40) \div \underline{\hspace{1cm}} = 40$
 - $25 \div \underline{\hspace{1cm}} = -5$
 - $\underline{\hspace{1cm}} \div (-4) = 9$
 - $(-62) \div (-62) = \underline{\hspace{1cm}}$
- Say true or false.
 - $(-30) \div (-6) = -6$
 - $(-64) \div (-64)$ is 0
- Find the values of the following.
 - $(-75) \div 5$
 - $(-100) \div (-20)$
 - $45 \div (-9)$
 - $(-82) \div 82$
- The product of two integers is -135 . If one number is -15 , Find the other integer.
- In 8 hours duration, with uniform decrease in temperature, the temperature dropped 24° . How many degrees did the temperature drop each hour?
- An elevator descends into a mine shaft at the rate of 5 m/min . If the descent starts from 15 m above the ground level, how long will it take to reach -250 m ?
- A person lost 4800 calories in 30 days. If the calory loss is uniform, calculate the loss of calory per day.
- Given $168 \times 32 = 5376$ then, find $(-5376) \div (-32)$.



9. How many (-4) 's are there in (-20) ?
10. (-400) divided into 10 equal parts gives ____.

Objective type questions

11. Which of the following does not represent an Integer?
- (i) $0 \div (-7)$ (ii) $20 \div (-4)$ (iii) $(-9) \div 3$ (iv) $(12) \div 5$
12. $(-16) \div 4$ is the same as
- (i) $-(-16 \div 4)$ (ii) $-(16) \div (-4)$ (iii) $16 \div (-4)$ (iv) $-4 \div 16$
13. $(-200) \div 10$ is
- (i) 20 (ii) -20 (iii) -190 (iv) 210
14. The set of integers is not closed under
- (i) Addition (ii) Subtraction (iii) Multiplication (iv) Division

One of the earliest exposition on negative numbers is by the 7th century Indian Mathematician and Astronomer Brahmagupta, who in his famous text "Brahmasphutasiddhanta" gave a very clear understanding of the concept of zero and consequently the negative integers. His texts were in the form of poetry which when translated reads something like this...

Rule for dealing with positive (fortune) and negative (debt) numbers.

- A debt minus zero is a debt
- A fortune minus zero is a fortune.
- Zero minus zero is a zero.
- A debt subtracted from zero is a fortune.
- A fortune subtracted from zero is a debt.
- The Product of zero by a debt or fortune is zero.
- The Product of zero by zero is zero.
- The Product of quotient of two fortunes is one fortune.
- The Product of quotient of a debt and a fortune is a debt.
- The Product or quotient of a fortune and a debt is a debt.

- The story of mathematics

1.6 Statement Problems on Integers using all Fundamental Operations.

We have learned how to add, subtract, multiply and divide integers. This section will review the rules learned for operations with integers.

All the mathematical problems are life oriented problems.

Situation 1

If a person's initial balance is ₹ 530 in a particular month. In the same month if he deposits ₹ 230 withdraws ₹ 150, again a withdrawal of ₹ 200 and a deposit of ₹ 99.

How will you find the answer for this?

Situation 2

If a person buys 8 pens for 80 rupees and he sells 4 pens with a profit of ₹ 3 per pen and 3 pens with a loss of ₹ 2 per pen and one pen at the buying cost, find the total loss or profit of him.

Do you have any idea to approach this problem?

To solve all the statement problem, the following steps may be followed.

1. Read the problem thoroughly.
2. Write down what is given.
3. Find out what they are asking.
4. Use the required formulae or easy way to attain the answer.
5. Apply it.
6. Solve it.
7. Arrive the answer.
8. Check your answer.

Let us approach the above two situations as given below:

S.No.	Steps	Situation 1	Situation 2
1	Read the problem	If a person's initial balance is ₹ 530 in a particular month. In the same month if he deposits ₹ 230 withdraws ₹ 150, again a withdrawal of ₹ 200 and a deposit of ₹ 99. How will you find the answer for this?	If a person buys 8 pens for 80 rupees and he sells 4 pens with a profit of ₹ 3 per pen and 3 pens with a loss of ₹ 2 per pen and one pen at the buying cost, find the total loss or profit of him.
2	Write down what is given	Initial balance = ₹ 530 Deposit 1 = ₹ 230 Deposit 2 = ₹ 99 Withdrawal 1 = ₹ 150 Withdrawal 2 = ₹ 200	Buying 8 pens for ₹ 80 Sells 4 pens at a profit of ₹ 3 per pen. Sells 3 pens at a loss of ₹ 2 per pen. Sells 1 pen at the cost price.
3	What is asked	Final balance	Amount of loss or profit
4	Use formulae	Add and subtract	Find the cost of each pen. Selling price of each pen.

5.	Apply it	$530 + 230 + 99 - (150 + 200)$	Cost of each pen = $\frac{80}{8} = 10$
6.	Solve it	$530 + 230 + 99 - (150 + 200)$ = ₹ 509	Cost of each pen = $\frac{80}{8} = 10$ S.P. of 4 pens = $13 \times 4 = 52$ S.P. of 3 pens = $8 \times 3 = 24$ S.P. of 1 pen = $1 \times 10 = 10$ Total S.P. = $52 + 24 + 10 = 86$
7.	Arrive the answer	Final balance = ₹ 509	S.P. (86) > C.P. (80) Therefore, profit = ₹ 6

Example 1.28

Ferozkhan collects ₹1150 at the rate of ₹ 25 per head from his classmates on account of the 'Flag Day' in his school and returns ₹ 8 to each one of them, as instructed by his teacher. Find the amount handed over by him to his teacher.

Solution

Ferozkhan collects ₹1150 at the rate of ₹ 25 per head from his classmates on account of the 'Flag Day'

Total amount collected = ₹1150

Amount per head = ₹ 25

Number of students = $1150 \div 25 = 46$

Amount returned to each student is ₹ 8

Amount returned to 46 students = $46 \times 8 = ₹ 368$

Amount handed over to the class teacher = ₹ 1150

₹ 368 (–)

₹ 782

Amount handed over to the class teacher = ₹ 782

$$\begin{array}{r} 46 \\ 25 \overline{) 1150} \\ \underline{100} \downarrow \\ 150 \\ \underline{150} \\ 0 \end{array}$$

Example 1.29

Each day, the workers drill down 22 feet further until they hit a pool of water. If the water is at 110 feet, on which day will they hit the pool of water?

Solution

Depth drilled in one day = –22 feet

Depth of water = –110 feet

Number of days required = $-110 \div -22 = 5$

Hence the workers will reach resource in 5 days.

$$\begin{array}{r} 5 \\ 22 \overline{) 110} \\ \underline{110} \\ 0 \end{array}$$

Example 1.30

How many years are between 323 BC(BCE) and 1687 AD(CE)?

Solution

Years in AD(CE) are taken as positive integers and BC(BCE) as negative integers.

Therefore, the difference is

$$= 1687 - (-323)$$

$$= 1687 + 323 = 2010 \text{ years}$$



Exercise 1.5

- One night in Kashmir, the temperature is -5°C . Next day the temperature is 9°C . What is the increase in temperature?
- An atom can contain protons which have a positive charge (+) and electrons which have a negative charge (-). When an electron and a proton pair up, they become neutral (0) and cancel the charge out. Now, Determine the net charge:
 - 5 electrons and 3 protons $\rightarrow -5 + 3 = -2$ that is 2 electrons $\ominus \ominus$
 - 6 Protons and 6 electrons $\rightarrow$
 - 9 protons and 12 electrons $\rightarrow$
 - 4 protons and 8 electrons $\rightarrow$
 - 7 protons and 6 electrons $\rightarrow$
- Scientists use the Kelvin Scale (K) as an alternative temperature scale to degrees Celsius ($^{\circ}\text{C}$) by the relation $T^{\circ}\text{C} = (T + 273)\text{K}$.
Convert the following to kelvin:
 - -275°C
 - 45°C
 - -400°C
 - -273°C
- Find the amount that is left in the student's bank account, if he has made the following transaction in a month. His initial balance is ₹ 690.
 - Deposit (+) of ₹ 485
 - Withdrawal (-) of ₹ 500
 - Withdrawal (-) of ₹ 350
 - Deposit (+) of ₹ 89
 - If another ₹ 300 was withdrawn, what would the balance be?
- A poet Tamizh Nambi lost 35 pages of his 'lyrics' when his file had got wet in the rain. Use integers, to determine the following:
 - If Tamizh Nambi wrote 5 page per day, how many day's work did he lose?
 - If four pages contained 1800 characters, (letters) how many characters were lost?
 - If Tamizh Nambi is paid ₹250 for each page produced, how much money did he lose?



- (iv) If Kavimaan helps Tamizh Nambi and they are able to produce 7 pages per day, how many days will it take to recreate the work lost?
- (v) Tamizh Nambi pays kavimaan ₹ 100 per page for his help. How much money does kavimaan receive?
6. Add 2 to me. Then multiply by 5 and subtract 10 and divide now by 4 and I will give you 15! Who am I?
7. Kamatchi, a fruit vendor sells 30 apples and 50 pomegranates. If she makes a profit of ₹ 8 per apple and loss ₹ 5 per pomegranate, what will be her overall profit(or)loss?
8. During a drought, the water level in a dam fell 3 inches per week for 6 consecutive weeks. What was the change in the water level in the dam at the end of this period?
9. Buddha was born in 563 BC(BCE) and died in 483 BC(BCE). Was he alive in 500 BC(BCE)? and find his life time. (Source: Compton's Encyclopedia)

Exercise 1.6

Miscellaneous Practice problems



- What Should be added to -1 to get 10?
- $-70 + 20 = \square - 10$
- Subtract 94860 from (-86945)
- Find the value of $(-25) + 60 + (-95) + (-385)$
- Find the sum of (-9999) , (-2001) and (-5999)
- Find the product of $(-30) \times (-70) \times 15$
- Divide (-72) by 8
- Find two pairs of integers whose product is $+15$.
- Check the following for equality
 - $(11+7)+10$ and $11+(7+10)$
 - $(8-13) \times 7$ and $8-(13 \times 7)$
 - $[(-6)-(+8)] \times (-4)$ and $(-6)-[8 \times (-4)]$
 - $3 \times [(-4)+(-10)]$ and $[3 \times (-4)+3 \times (-10)]$
- Kalaivani had ₹ 5000 in her bank account on 01.01.2018. She deposited ₹ 2000 in January and withdrew ₹ 700 in February. What was Kalaivani's bank balance on 01.04.2018, if she deposited ₹ 1000 and withdrew ₹ 500 in March?
- The price of an item x increases by ₹ 10 every year and an item y decreases by ₹ 15 every year. If in 2018, the price of x is ₹ 50 and y is ₹ 90, then which item will be costlier in the year 2020.



12. Match the statements in column A and column B

S.No.	A	B
1.	For any two integers 72 and 108, $72+108$ is an also an integer.	(a) Distributive property of multiplication over addition.
2.	For any three integers 68, 25 and 99 $68 \times (25 + 99) = (68 \times 25) + (68 \times 99)$	(b) Multiplicative identity.
3.	$0 + (-138) = (-138) = (-138)+0$	(c) Commutative property under multiplication.
4.	For any two integers (-5) and 10 $(-5) \times 10 = 10 \times (-5)$	(d) Closed under addition.
5.	$1 \times (-1098) = (-1098) = (-1098) \times 1$	(e) Additive identity.

Challenge Problems

13. Say true or false.

- (i) The sum of a positive integer and a negative integer is always a positive integer.
- (ii) The sum of two integers can never be zero.
- (iii) The product of two negative integers is a positive integer.
- (iv) The quotient of two integers having opposite sign is a negative integer.
- (v) The smallest negative integer is -1 .

14. An integer divided by 7 gives a quotient -3 . What is that integer?

15. Replace the question mark with suitable integer in the equation

$$72 + (-5) - \boxed{?} = 72.$$

16. Can you give 5 pairs of single digit integers whose sum is zero?

17. If $P = -15$ and $Q = 5$ find $(P - Q) \div (P + Q)$

18. If the letters in the English alphabets A to M represent the number from 1 to 13 respectively and N represents 0 and the letters O to Z correspond from -1 to -12 , find the sum of integers for the names given below.

For example,

$$\text{MATH} \rightarrow \text{sum} \rightarrow 13 + 1 - 6 + 8 = 16$$

- (i) YOUR NAME
 - (ii) SUCCESS
19. From a water tank 100 litres of water is used every day. After 10 days there is 2000 litres of water in the tank. How much water was there in the tank before 10 days?
20. A dog is climbing down in to a well to drink water. In each jump it goes down 4 steps. The water level is in 20^{th} step. How many jumps does the dog take to reach the water level?

21. Kannan has a fruit shop. He sells 1 dozen banana at a loss of Rs.2 each because it may get rotten next day. What is his loss?
22. A submarine was situated at 650 feet below the sea level. If it descends 200 feet, what is its new position?
23. In a magic square given below each row, column and diagonal should have the same sum, Find the values of x , y and z .

1	-10	x
y	-3	-2
-6	4	z

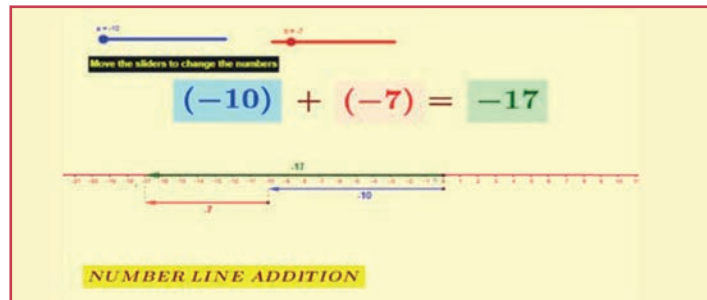
Summary

- Integers are the collection of natural numbers, zero and negative numbers.
- The number line gives a visual representation of the set of all integers with positive integers to the right of zero and negative integers to the left of zero.
- The sum of the two positive integers is positive and two negative integers is negative.
- The sum of a positive and a negative integer is the difference of the two numbers in value and has the sign of the greater integer.
- The addition of integers has the closure, commutative and associative properties.
- The product of two positive integers and two negative integers are positive.
- The product of two integers with opposite signs is negative.
- The multiplication of integers has the closure, commutative and associative properties.
- The integer 0 is the additive identity for integers.
- The integer 1 is the multiplicative identity for integers.



ICT Corner

Expected Result is shown
in this picture



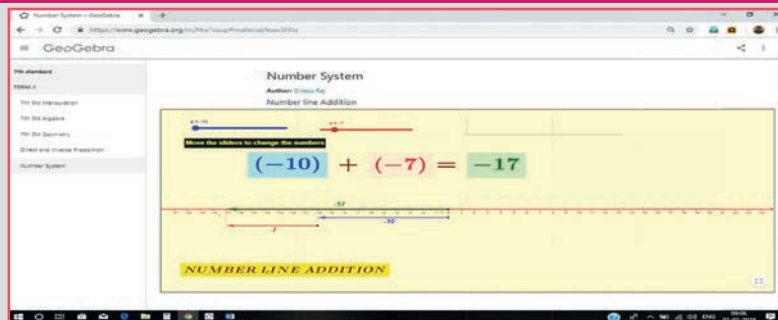
Step - 1 :

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named “Number System” will open. Select Term-1.

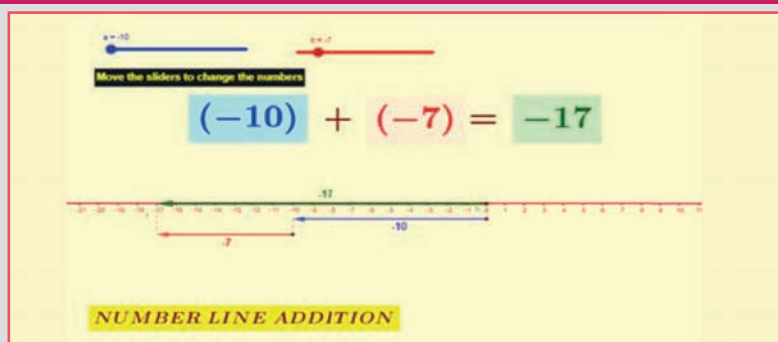
Step - 2 :

There are several work sheets for each chapter. Solect the worksheet “Number System”. In this Number line addition is given. Move the sliders to change the numbers. You can see the answer in green colour arrow.

Step 1



Step 2



Browse in the link

Number system : <https://ggbm.at/f4w7csup>
or Scan the QR Code.

