Sample Question Paper 2020-21

Max. Marks: 80 Duration:3 hours

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:

1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:

1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

PART-A

Section-I

1. Which of the following has a terminating decimal expansion?

   A. $\frac{32}{91}$ B. $\frac{19}{80}$

   C. $\frac{23}{45}$ D. $\frac{25}{42}$

2. Show that the x = -3 is a solution of $x^2 + 6x + 9 = 0$

3. If the points (2, 1) and (1,-2) are equidistant from the point (x, y), show that x + 3y = 0.

   OR

   If R (5, 6) is the midpoint of the line segment joining the points A (6, 5) and B (4, y) then find the value of 'y'.

4. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

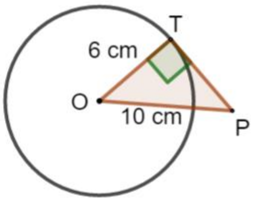
5. The line segment joining the midpoints of the adjacent sides of a quadrilateral form a _________ .

6. If $\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$ then x =?

OR

If A and B are acute angles such that sin A = cos B, prove that A + B = 90°.

7. In the given figure, PT is a tangent to the circle with centre O. If OT = 6 cm and OP = 10 cm, then length of tangent PT is _______ .



8. A man goes 80 m due east and 150 m due north. How far is he from the starting point?

9. The diameter of a sphere is 14 cm. Calculate its volume.

10. Which of the following cannot be the probability of an event?

A. 1.5	B. 0.6
C. 25%	D. 0.3

11. If -2 and 3 are the zeros of the quadratic polynomial $x^2 + (a + 1)x + b$ then a = ____ and b = _______.

12. The distance of the point P (-6, 8) from the origin is ____ units.

13. The first three terms of an AP are (3y − 1), (3y + 5) and (5y + 1), the value of y is ______.

Solution:

Let three terms are a_1, a_2 and a_3.

Then, $a_2 - a_1 = a_3 - a_2$

Substituting the values, we get

$\Rightarrow 3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$

$\Rightarrow 6 = 2y - 4$

$\Rightarrow y = 5$

The value of y is 5.

OR

If 18, a, (b − 3) are in AP, then value of (2a − b) is _____.

14. tan 10° tan 15° tan 75° tan 80° = ______.

15. PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA ⊥ PB, then length of each tangent is ________.

16. Find the values of k for which the system of equations kx – y = 2, 6x – 2y = 3 has a unique solution.

Section II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark

17. At a DIWALI mela, a stall keeper in one of the food stalls has a large cylindrical vessel filled with orange juice and small cylindrical glasses.

(i). If the small cylindrical glass having base radius 2r and height h is open at the top, then total surface area is

(a) 2πr(r + h)

(b) 3πr(r + h)

(c) 4πr(r + h)

(d) None of the above

(ii). Volume of the large cylindrical vessel of radius R and height H is

(a) πR^2H

(b) $1/3\pi R^2H$

(c) $4/3\pi R^3$

(d) $2/3\pi R^3$

(iii). If large cylindrical vessel has base radius = 15 cm, height = 32 cm and small cylindrical vessel has base radius = 3 cm, height = 8 cm.

Then the number of small cylindrical vessel filled with juice out of a large cylindrical vessel is

(a) 500

(b) 100

(c) 150

(d) 200

(iv). If the stall keeper sold small glass of juice for Rs 3 each, amount received by the stall keeper by selling the juice completely is

(a) ₹ 250

(b) ₹ 275

(c) ₹ 300

(d) ₹ 325

18. Study the following graphs carefully and answer the following questions

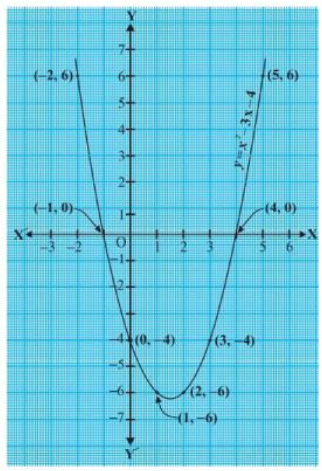


Fig-A

$x^3 + 4$.

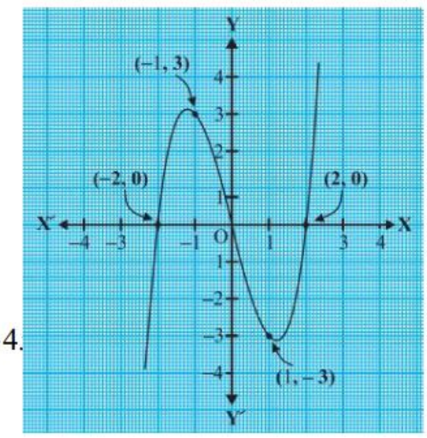


Fig-B

(i). Number of Zeros of a polynomial is defined as the number of times the curve cut the

(a) X – axis

(b) Y – axis

(c) Both x and y axis

(d) None of the above

(ii). Number of zeroes of the polynomial in fig-A and fig-B

(a) 1, 2

(b) 2, 3

(c) 2, 1

(d) 3,2

(iii). Polynomial of the curve in fig-A

(a) $x^2 + 3x - 4$

(b) $x^2 + 3x + 4$

(c) $x^2 - 3x - 4$

(d) $x^2 - 3x + 4$

(iv). Polynomial of the curve in fig-B

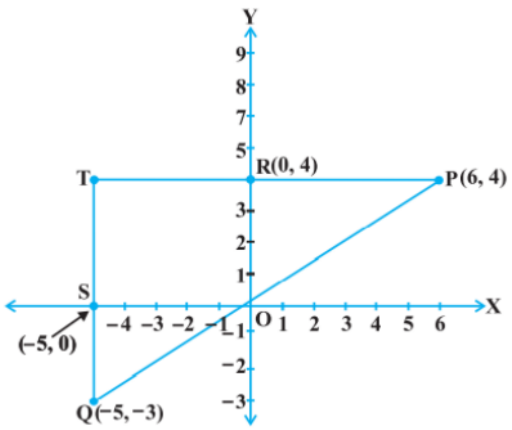
(a) $2x^3 - 4x$

(b) $3x^3 - 4x$

(c) $x^3 + 4x$

(d) $x^3 - 4x$

19. Consider the points P(6, 4) and Q(−5, −3). Draw QS perpendicular to the x-axis. Also draw a perpendicular PT from the point P on QS (extended) to meet y-axis at the point R.



(i). Coordinates of point T is

(a) (-5, 5)

(b) (-5, 4)

(c) (-4, 5)

(d) (4, -5)

(ii). Equation of line PT is

(a) $x = 4$

(b) $x + y = 0$

(c) $y = 4$

(d) $x - y = 0$

(iii). Distance between points P and Q is

(a) 14 units

(b) 15 units

(c) 13.5 units

(d) None of the above

(iv). If S be any point in the 4th quadrant such that PTQS is the rectangle, then coordinates of point S is

(a) (6, -3)

(b) (5, -4)

(c) (6, -4)

(d) None of the above

20. A survey regarding the heights (in cm) of 50 girls of a class was conducted and the following data was obtained

Heights (in cm)	120-130	130-140	140-150	150-160	160-170	Total
No. of girls	2	8	12	20	8	50

(i). Mean height (in cm) of the girls is

(a) 146.8

(b) 147.8

(c) 148.8

(d) 149.8

(ii). The commutative frequency table is useful in determining the

(a) Mean

(b) Median

(c) Mode

(d) All of these

(iii). Lower limit of the median class is

(a) 130

(b) 140

(c) 150

(d) None of the above

(iv). Sum of lower limit of median class and modal class is

(a) 270

(b) 290

(c) 310

(d) None of these

PART–B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Show that $(2 + \sqrt{3})$ is an irrational number.

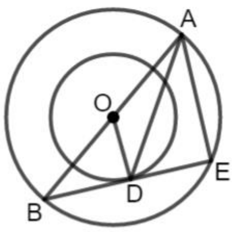
22. The line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal area. Prove that $AX : XB = (\sqrt{2} - 1) : 1$.

OR

In a trapezium ABCD, O is the point of intersection of AC and BD, AB || CD and AB = 2 × CD. If the area of ΔAOB is 84 cm^2, find the area of ΔCOD.

23. If a 1.5 m tall girl stands at a distance of 3 m from a lamp-post casts a shadow of length 4.5 m on the ground then find the height of the lamp-post.

24. In the given figure, the radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D. Find the length of AD.



25. Divide a line segment of length 9 cm internally in the ratio 4 : 3.

26. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing

(i) an ace (ii) a red king (iii) a diamond

OR

Two different dice are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7 (ii) is a doublet of odd numbers

PART–B

All questions are compulsory. In case of internal choices, attempt anyone.

27. An electric device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 11:00 AM. At what time will they beep together again?

28. If α and β are the zeroes of the polynomial $f(x) = x^2 + x - 2$, find the value of $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)$.

29. A bookseller buys a number of books for ₹ 1760. If he had bought 4 more books for the same amount, each book would have cost ₹ 22 less. How many books did he buy?

OR

A two-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

30. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2.5 m apart. What is the length of the wood required for the rungs?

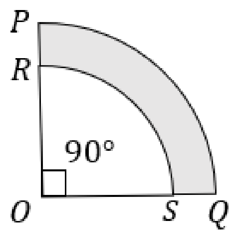
31. If R(x, y) is a point on the line segment joining the point P(a, b) and Q(b, a) then prove that x + y = a + b.

32. If x = cot A + cos A and y = cot A − cos A, prove that $\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$

OR

If a cos θ − b sin θ = c, prove that $a \sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

33. In the given figure, PQSR represents a flower bed. If OP = 21 m and OR = 14 m, find the area of the flower bed.



PART–B

All questions are compulsory. In case of internal choices, attempt anyone.

34. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

35. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

OR

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth are 1.4 cm. Find the volume of wood in the entire stand.

36. If the median of the following data is 32.5, find the missing frequencies.

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency:	f_1	5	9	12	f_2	3	2	40

***

Hints & Solutions

1. **Solution:** A rational number $\frac{p}{q}$ has a terminating decimal expansion, if the prime factorization of q is of the form $2^n \times 5^m$, where n and m are non-negative integers.

   For option b, we can write 80 as $2^4 \times 5$.

   Hence, b is the correct answer.

2. **Solution:** $x^2 + 6x + 9 = 0$

   We can write this equation as

   $\Rightarrow (x+3)^2 = 0 \quad \Rightarrow x = -3, -3$

   Hence, it is proved that x = -3 is a solution of the given equation.

3. **Solution:** Distance from point (2, 1) = Distance from point (1, -2)

   $$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$$

   Square roots are cancelled, therefore

   $$(x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$$

   $$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 + 4y + 4$$

   $$-4x + 2x - 2y - 4y = 0$$

   $$x + 3y = 0$$

OR

Solution:

Coordinates of midpoint of line joining (x_1, y_1) and (x_2, y_2) are

$\Rightarrow$ midpoints $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\Rightarrow$ midpoints of line segment joining A and B $= \left(\frac{6+4}{2}, \frac{5+y}{2}\right)$

To find the value of y, we only need y coordinate of midpoint.

$\Rightarrow \frac{5+y}{2} = 6$

$\Rightarrow 5 + y = 12$

$\Rightarrow y = 7$

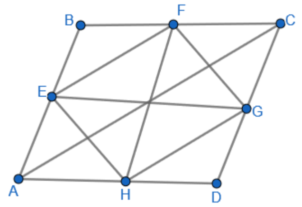
Hence, the value of y is 7.

4. **Solution:**

   We know that LCM × HCF = Product of the numbers

Therefore, Other Number = $\frac{LCM \times HCF}{One\ number}$ = $\frac{2175 \times 145}{725}$ = 435

5. **Solution:** Consider the below diagram.



ADCB is the quadrilateral. E, F, G, H are the midpoints of the corresponding sides. Join EF, FG, GH, HE, FH, EG and AC.

Midpoint theorem states that in a triangle, the line segment joining the mid points of any two sides of a triangle is parallel to the third side and is half of the third side.

Using midpoint theorem, In triangle BAC

⇒ EF || AC, EF = $\frac{1}{2}$AC

Using midpoint theorem, In triangle ACD

⇒ HG || AC, HG = $\frac{1}{2}$AC

From these two equations, we conclude that

⇒ EF || HG and EF = HG

∴ EFGH is a parallelogram.

6. **Solution:** Substituting the values, we get

$\Rightarrow 1^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{3}{4} = \frac{1}{2}x$

$\Rightarrow x = \frac{1}{2}$

OR

Solution: A and B are acute angles.

$\Rightarrow \sin A = \cos B$

$\Rightarrow \sin A = \sin(90 - B)$ As $\sin(90 - \theta) = \cos\theta$

⇒ A = 90 – B

⇒ A + B = 90°

Hence proved.

7. **Solution:** We know, tangent at a point is perpendicular to the radius through point of contact

$\therefore$ OT $\perp$ PT

In the given figure, In right angled triangle PTO

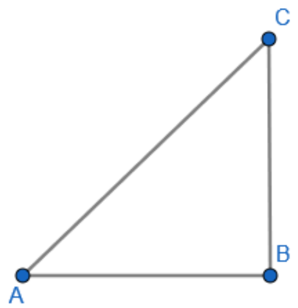
Applying Pythagoras theorem,

$\Rightarrow \mathrm{PO}^2 = \mathrm{TP}^2 + \mathrm{TO}^2$

$\Rightarrow 10^2 = \mathrm{TP}^2 + 6^2 \quad \Rightarrow \mathrm{TP} = 8\ \mathrm{cm}$

8. **Solution:** Diagram is shown below,

Initially, man is standing at A. He goes 80 m due east and reaches B. Then he goes 150 m due north and reaches C.



AB = 80 m and BC = 150 m

Applying Pythagoras theorem, we get

$\Rightarrow \mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$

$\Rightarrow \mathrm{AC}^2 = 6400 + 22500$

$\Rightarrow \mathrm{AC}^2 = 28900$

$\Rightarrow \mathrm{AC} = 17\mathrm{m}$

Hence, required answer is 17m.

9. **Solution:** diameter = 14 cm

$\Rightarrow$ radius = 7 cm

$\Rightarrow$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 7^3$

$\Rightarrow \frac{4}{3} \times 22 \times 49 = 1437\frac{1}{3}\mathrm{cm}^3$

10. **Solution:** We know that probability of any event can't exceed 1.

Hence, A can't be the probability of an event.

11. **Solution:** a = -2, b = -6

Sum of roots = $-\frac{\text{coeff of x}}{\text{coeff of x}^2}$

$\Rightarrow -\frac{(a+1)}{1} = 3 - 2$

$\Rightarrow a = -2$

Again, product of roots = $\frac{\text{constant}}{\text{coeff of x}^2}$

$\Rightarrow \frac{b}{1} = -6$

$\Rightarrow b = -6$

12. **Solution:** Using distance formula,

The required distance = $\sqrt{(-6-0)^2+(8-0)^2}$

$\Rightarrow \sqrt{36+64}$

$\Rightarrow \sqrt{100}$

∴ Required distance is 10 units.

13. **Solution:** Let three terms are a_1, a_2 and a_3.

Then, $a_2 - a_1 = a_3 - a_2$

Substituting the values, we get

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow y=5$

The value of y is 5.

OR

Solution:

Let three terms are a_1, a_2 and a_3.

Then, $a_2 - a_1 = a_3 - a_2$

Substituting the values, we get

$\Rightarrow a-18=b-3-a$

$\Rightarrow 2a-b= -3+18$

$\Rightarrow 2a-b=15$

The required value is 15.

14. **Solution:** We know that, $\tan(90-\theta)=\cot\theta$

$\Rightarrow \tan 10° \tan 15° \tan 75° \tan 80°$

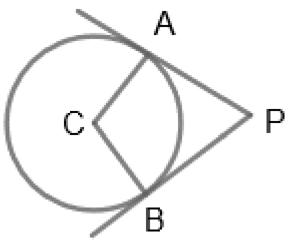
$\Rightarrow \tan 10° \tan 15° \tan(90°-15°) \tan(90°-10°)$

$\Rightarrow \tan 10° \tan 15° \cot 15° \cot 10°$

$\Rightarrow 1\times 1=1$ as $\tan\theta \times \cot\theta = 1$

∴ The value of the given expression is 1.

15. **Solution:** The diagram is given below.



It is given that, ∠APB = 90° [PA ⊥ PC]

After joining AC and BC, we find that PACB is a square.

$\angle$CAB and $\angle$CBP are both 90° as radii of a circle is perpendicular to the tangent at the point of contact.

Hence, PA = PB = 4cm.

16. **Solution:** For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to have a unique solution, it must satisfy $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ $kx - y - 2$ and $6x - 2y - 3 = 0$

$\Rightarrow$ $a_1 = k$, $b_1 = -1$, $a_1 = 6$, $b_2 = -2$

$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2}$

$\Rightarrow k \neq 3$

Hence, for all values expect k = 3, the above pair of equations have a unique solution.

17. (i). Answer: 4πr(h+r)

(ii). Answer: πR^2H

(iii). Answer: 100 vessels

(iv). Answer: ₹ 300

18. (i). Answer: X-axis

(ii). Answer: 2,3

(iii). Answer: $x^2 - 3x - 4$

(iv). Answer: $x^3 - 4x$

19. (i). Answer: (-5, 4)

(ii). Answer: y = 4

(iii). Answer: None of the above

(iv). Answer: (6, -3)

20. (i). Answer: 149.8

(ii). Answer: Median

(iii). Answer: 140

(iv). Answer: 290

21. **Solution:** Let's assume that $2 + \sqrt{3}$ is a rational number.

We know that, rational number is represented in the form of $\frac{p}{q}$ where p and q are integers and q ≠ 0.

$\Rightarrow 2 + \sqrt{3} = \frac{a}{b}$, where a and b are coprime and b ≠ 0

$\Rightarrow 2 - \frac{a}{b} = \sqrt{3}$

$\Rightarrow \sqrt{3} = \frac{2b-a}{b}$

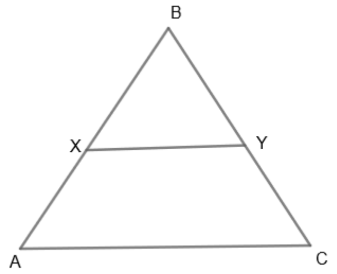
Here, a and b are integers, therefore $\frac{2b-a}{b}$ must also be integer.

Right hand side of the above equation is integer, hence left side must also be integer.

But, above statement contradicts the fact that $\sqrt{3}$ is irrational. This contradiction arises because of our wrong initial assumption that $2+\sqrt{3}$ is a rational number.

Hence, $2+\sqrt{3}$ is an irrational number.

22. **Solution:** The diagram is shown below.



In this diagram, area (BXY) = area (XYAC) = $\frac{1}{2}$ area (ABC)

⇒ ∠BXY = ∠BAC and ∠BYX = ∠BCA as they are corresponding angles.

⇒ ΔBXY ~ ΔBAC

Using theorem, the ratio of areas of two similar triangles is equal to the ratio of square of the corresponding sides.

$\Rightarrow \frac{\text{ar(BXY)}}{\text{ar(ABC)}} = \left(\frac{\text{BX}}{\text{AB}}\right)^2$

$\Rightarrow \frac{1}{2} = \left(\frac{\text{BX}}{\text{AB}}\right)^2$

$\Rightarrow \frac{\text{BX}}{\text{AB}} = \frac{1}{\sqrt{2}}$

To find, $\frac{\text{AX}}{\text{XB}} = \frac{\text{AB}-\text{XB}}{\text{XB}}$

$= \frac{\text{AB}}{\text{XB}} - 1$

$= \sqrt{2} - 1$

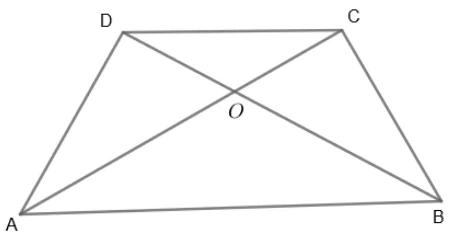
∴ ratio is (√2 − 1) : 1

Hence, proved.

OR

Solution:

The diagram is shown below.



⇒ ∠AOB = ∠DOC as they are vertically opposite angles

$\Rightarrow \angle ODC = \angle OBA$ as they are alternate angles

$\Rightarrow \Delta COD \sim \Delta AOB$ [By AA similarity criterion]

Using theorem, the ratio of areas of two similar triangles is equal to the ratio of square of the corresponding sides.

$\Rightarrow \frac{\text{ar(DCO)}}{\text{ar(AOB)}} = \left(\frac{\text{DC}}{\text{AB}}\right)^2$

But AB = 2DC

$\Rightarrow \frac{\text{ar(DCO)}}{84} = \frac{1}{4}$

$\therefore$ area (DCO) = 21 cm^2

23. **Solution:** The diagram is shown below.

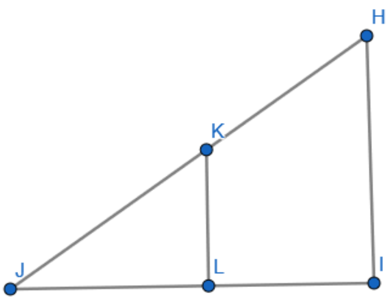
Height of the girl, KL = 1.5m

Distance of girl from lamp post, LI = 3 m

Shadow of the girl, JL = 4.5 m

Height of the lamp post, HI, to be calculated.

$\Rightarrow$ JI = JL + LI = 4.5 + 3 = 7.5 m



From the above figure, $\angle HJI = \angle KJL$

$\Rightarrow \tan\angle HJI = \tan\angle KJL$

$\Rightarrow \frac{HI}{JI} = \frac{KL}{JL}$

$\Rightarrow \frac{HI}{7.5} = \frac{1.5}{4.5}$

$\therefore$ HI = 2.5 m

Hence, the height of the lamp post is 2.5 m.

24. **Solution:** In the given diagram,

AB = 13 cm, OD = 8 cm

$\Rightarrow OD \perp BE$ [Radii of a circle is perpendicular to the tangent at the point of contact]

Using Thales theorem, if three points A, B, E lies on a circle, and AB is the diameter, then $\angle AEB = 90°$

$\Rightarrow \angle AEB = \angle ODB = 90°$

$\Rightarrow$ O is the midpoint of AB and D is the midpoint of BE.

$\Rightarrow$ AE = 2 × OD = 16 cm.

In triangle, OBD

Using Pythagoras theorem,

$\Rightarrow OB^2 = OD^2 + BD^2$

$\Rightarrow 13^2 = 8^2 + BD^2$

$\Rightarrow BD = \sqrt{105}$

But BD = DE

Again, using Pythagoras theorem in ADE

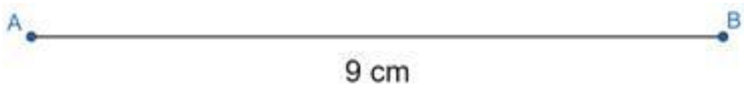
$\Rightarrow AD^2 = AE^2 + DE^2$

$\Rightarrow AD^2 = 16^2 + 105$

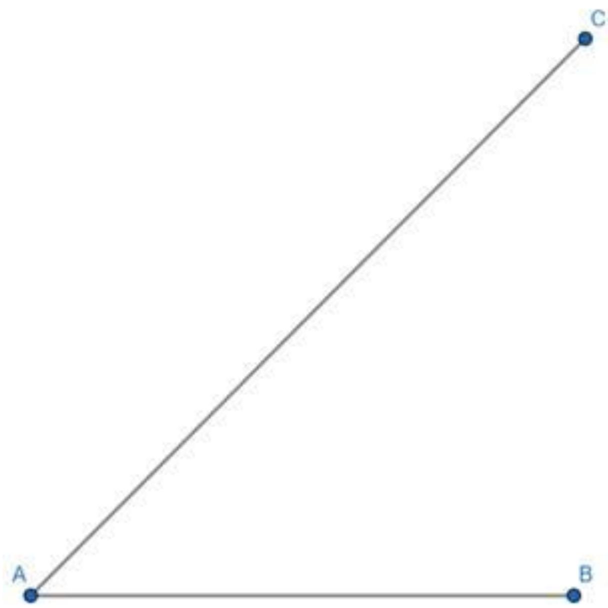
$\Rightarrow AD = 19 \text{ cm}.$

Hence, the required answer is 19 cm.

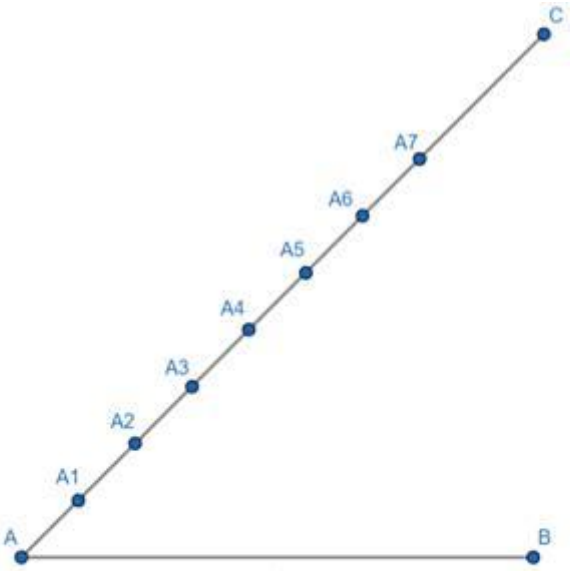
25. **Solution:**



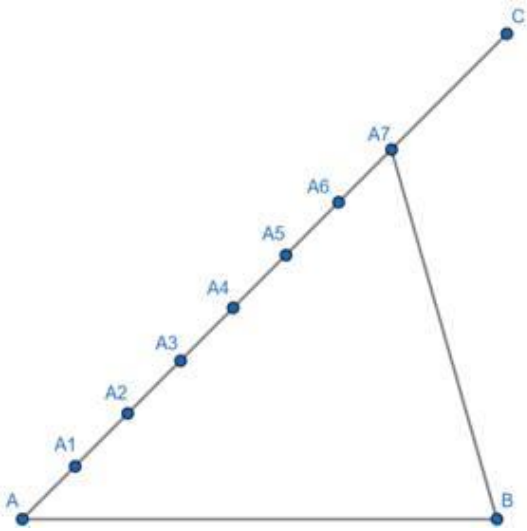
We need to divide this line segment AB of length 9 cm internally in the ratio 4 : 3.



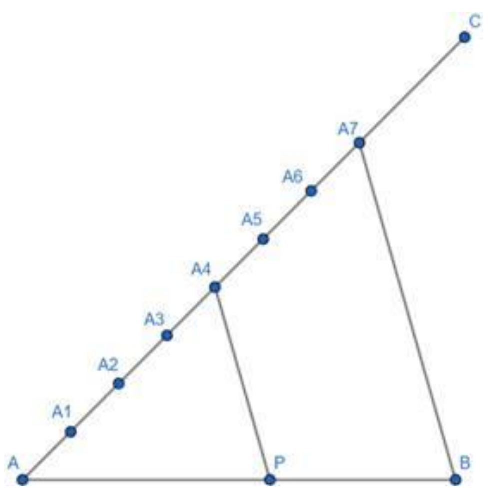
Step 1: Draw a line segment AC of arbitrary length and at an any angle to AB such that ∠CAB is acute.



<u>Step 2:</u> We plot (4 + 3 =) 7 points A_1, A_2, A_3, A_4, A_5, A_6, and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$



<u>Step 3:</u> We join points A_7 and B.



Step 4: We draw line segment A_4P such that $A_4P \parallel A_7B$ and P is the point of intersection of this line segment with AB.
Point P divides AB in the ratio 4 : 3.

26. **Solution:**

Total number of cards = 52

We know, Probability of an event E is

$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$

(i) Total number of possible outcomes = 52

The favourable outcomes = No. of aces in a deck = 4

P(an ace) $= \frac{4}{52} = \frac{1}{13}$

(ii) Total number of possible outcomes = 52

The favourable outcomes = No. of red kings = 2

P(a red king) $= \frac{2}{52} = \frac{1}{26}$

(iii) Total number of possible outcomes = 52

The favourable outcomes = No. of diamonds = 13

P(a diamond) $= \frac{13}{52} = \frac{1}{4}$

OR

Solution: When two dices are tossed together, possible outcomes are

{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(i) Total number of possible outcomes = 36

Number of favourable outcomes = 15 [(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)]

P (have a sum less than 7) $= \frac{15}{36} = \frac{5}{12}$

(ii) Total number of possible outcomes = 36

Number of favourable outcomes = 9 [(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)]

P (doublet of odd numbers) $= \frac{9}{36} = \frac{1}{4}$

27. **Solution:** We need to find LCM of 60 and 62.

LCM (60, 62) = 1860 sec = 31 min [1 min = 60 sec]

∴ They will beep together again at = 11:00 AM + 31 min = 11:31 AM.

Hence, correct answer is 11:31 AM

28. **Solution:**

We know that, if α and β are two zeroes of a quadratic polynomial $ax^2 + bx + c$ then

$$\alpha + \beta = -\frac{b}{a}$$

and $\alpha\beta = \frac{c}{a}$

here, α and β are the zeroes of the given polynomial f(x)

$\Rightarrow \alpha + \beta = -1$ and $\alpha\beta = -2$

$\Rightarrow (\beta - \alpha)^2 = \beta^2 + \alpha^2 - 2\alpha\beta$

$\Rightarrow (\beta - \alpha)^2 = \beta^2 + \alpha^2 + 2\alpha\beta - 4\alpha\beta$

$\Rightarrow (\beta - \alpha)^2 = (\beta + \alpha)^2 - 4\alpha\beta$

$\Rightarrow (\beta - \alpha)^2 = 1 + 8$

$\Rightarrow \beta - \alpha = 3, -3$

To find, $\frac{1}{\alpha} - \frac{1}{\beta}$

$$= \frac{\beta - \alpha}{\alpha\beta}$$

Substituting the value of β - α, we get

$$= \frac{\beta - \alpha}{\alpha\beta} = -\frac{3}{2} \text{or} \frac{3}{2}$$

29. **Solution:** Let the number of books bought initially is y.

Let the cost of each book bought initially is Rs x.

Initial condition,

$\Rightarrow \frac{1760}{y} = x$

Final condition,

$\Rightarrow \frac{1760}{y+4} = x - 22$

Subtracting above two equations, we get

$\Rightarrow 1760\left(\frac{1}{y} - \frac{1}{y+4}\right) = 22$

$\Rightarrow \frac{y+4-y}{(y^2+4y)} = \frac{22}{1760}$

$\Rightarrow y^2 + 4y - 320 = 0$

$\Rightarrow y^2 + 20y - 16y - 320 = 0$

$\Rightarrow y(y + 20) - 16(y + 20) = 0$

$\Rightarrow (y - 16)(y + 20) = 0$

$\Rightarrow y = 16 \text{ or } y = -20$

Number of books can't be negative.

Hence, number of books = 16.

OR

Solution:

Let the one's place digit is y and digit at ten's place is x.

It is given that, $x \times y = 18$

The number can also be represented as, $10x + y$

$\Rightarrow 10x + y - 63 = 10y + x$

$\Rightarrow 9x - 9y = 63$

$\Rightarrow x - y = 7$

Substituting $x = \frac{18}{y}$ in this equation.

$\Rightarrow \frac{18}{y} - y = 7$

$\Rightarrow y^2 + 7y - 18 = 0$

$\Rightarrow y^2 + 9y - 2y - 18 = 0 \quad \Rightarrow y(y+9) - 2(y+9) = 0$

$\Rightarrow (y-2)(y+9) = 0$

$\Rightarrow y = 2, y = -9$

The value of y can't be negative. So, y = 2 and x = 9

The required number is 92.

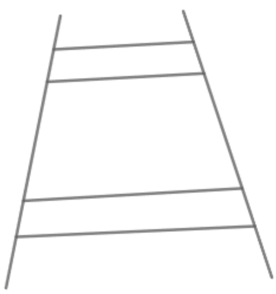
30. **Solution:**

The diagram is shown below.

It is given that,

The distance between two adjacent rungs = 25 cm.

The distance between topmost and bottommost rungs = 2.5m = 250 cm.



$\Rightarrow$ Total number of rungs = $\frac{250}{25} + 1 = 10 + 1 = 11$

Length of the bottom most rung = 45 cm and,

Length of the top most rung = 25 cm

And the length decreases uniformly from top to bottom. It is forming a finite series AP.

$\Rightarrow$ Required answer = $\frac{n}{2}[a + l]$, a = first term, l = last term

$\Rightarrow \frac{11}{2}[45 + 25]$

$\Rightarrow \frac{11}{2}[70]$

$\Rightarrow$ 385 cm = 3.85 m

Hence, length of wood required is 3.85 m.

31. **Solution:** Let R divides PQ in the ratio k:1

Then we have,

$R(X, Y) = \left(\frac{kx_1 + x_2}{k+1}, \frac{ky_1 + y_2}{k+1}\right)$

Here, $x_1 = a, y_1 = b,\ x_2 = b, y_2 = a$

Then $P(x, y) = \left(\frac{bk + a}{k+1}, \frac{ak + b}{k+1}\right)$

$\Rightarrow x = \frac{bk + a}{k+1}$ and $y = \left(\frac{ak + b}{k+1}\right)$

$\Rightarrow kx + x = bk + a$ and $yk + y = ak + b$

$\Rightarrow k(x - b) = a - x$ $\Rightarrow k(y - a) = b - y$

$\Rightarrow k = \frac{a - x}{x - b}$ ---(i) $\Rightarrow k = \left(\frac{b - y}{y - a}\right)$ ---(ii)

from (i) and (ii)

$\frac{a - x}{x - b} = \frac{b - y}{y - a}$

$\Rightarrow ay - a^2 - xy + ax = bx - b^2 + by - xy$

$\Rightarrow (a - b)y + (a - b)x - (a^2 - b^2) = 0$

$\Rightarrow (a - b)[y + x - (a + b)] = 0$

$\Rightarrow x + y - (a + b) = 0$

$\Rightarrow x + y = a + b$

Hence proved

32. **Solution:**

$\Rightarrow x - y = \cot A + \cos A - \cot A + \cos A$

$\Rightarrow x - y = 2\cos A$

$\Rightarrow x + y = \cot A + \cos A + \cot A - \cos A$

$\Rightarrow x + y = 2\cot A$

Substituting above values in the equation, we get

$\Rightarrow \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$

$\Rightarrow \left(\frac{\cos A}{\cot A}\right)^2 + (\cos A)^2$

$\Rightarrow \left(\frac{(\cos A \times \sin A)}{\cos A}\right)^2 + (\cos A)^2$

$\Rightarrow \sin^2 A + \cos^2 A = 1$

Hence, Proved.

OR

Solution:

$\Rightarrow (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2 + b^2$

$\therefore (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2 = a^2 + b^2$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - (a\cos\theta - b\sin\theta)^2$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow (a\sin\theta + b\cos\theta) = \pm\sqrt{a^2 + b^2 - c^2}$

Hence, proved

33. **Solution:** area of flower bed = area of quadrant (POQ) – area of quadrant (ROS)

$\Rightarrow$ Area of quadrant = $\frac{1}{4}\pi r^2$

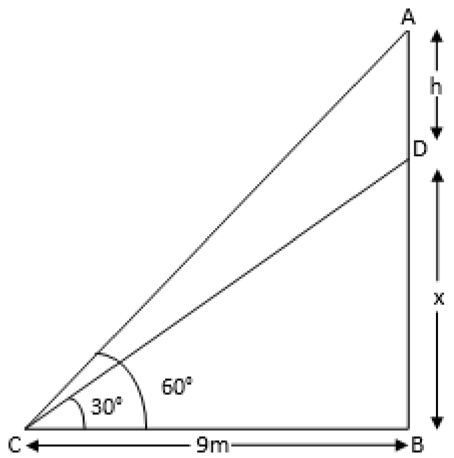
$\Rightarrow$ Area of flower bed = $\frac{1}{4}\pi(OP)^2 - \frac{1}{4}\pi(OR)^2$

$\Rightarrow$ Area of flower bed = $\frac{1}{4}\pi[21^2 - 14^2]$

$\Rightarrow$ Area of flower bed = $\frac{1}{4} \times \frac{22}{7} \times 245 = 192.5\ m^2$

34. **Solution:** Let the height of the Flag-pole = h(m)

And height of tower = x (m)



In ΔABC,

tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{h+x}{9}$

h+ x = 9√3 -------(1)

In ΔDBC

$\tan 30° = \frac{DB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{x}{9}$

$\sqrt{3}\, x = 9$

$x = \frac{9}{\sqrt{3}} \Rightarrow \frac{9\sqrt{3}}{3}$

$x = 3\sqrt{3}$ -------(2)

Now substituting value of x in eqn. (1)

$h + 3\sqrt{3} = 9\sqrt{3}$

$h = 9\sqrt{3} - 3\sqrt{3}$

$h = 6\sqrt{3}$ m

Therefore, height of tower is $3\sqrt{3}$ m. and height of flag pole is $6\sqrt{3}$ m.

35. **Solution:** Radius of cylindrical and hemispherical part = 1.4 cm
Length of hemispherical part = 1.4 cm
Length of cylindrical part, h = 5 – (2 × 1.4) = 2.2 cm
Gulab jamun is shown below.

⇒ Volume of gulab jamun = volume of cylinder + 2 × volume of hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3} \pi \times r^3$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$

$= 13.552 + 11.50 = 25.052 \text{ cm}^3$

⇒ Volume of 45 gulab jamuns = $45 \times 25.052 = 1127.34 \text{ cm}^3$

⇒ Syrup required = 30 % of above volume = $1127.34 \times 0.3 = 338.202 \text{ cm}^3$

OR

Solution: Required volume = volume of cuboid – 4 × volume of cone

⇒ Required volume = $\mathrm{lbh} - 4 \times \frac{1}{3}\pi r^2 h$

⇒ $15 \times 10 \times 3.5 - \frac{4}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

⇒ $525 - 1.467$

⇒ $523.533\ \mathrm{cm}^3$

Hence, volume of wood in the entire sand is $523.533\ \mathrm{cm}^3$.

36. **Solution:**

Class interval	Frequency	Cumulative frequency
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$ (F)
30-40	12 (f)	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	N = 40	

Given, Median = 32.5
Then median class = 30-40

$l = 30$, $h = 10$, $f = 12$, $F = 14 + f_1$

Median = $l + \frac{\frac{N}{2} - F}{f}$

$32.5 = 30 + \frac{20-(14+f1)}{12} * 10$

$2.5 = \frac{6-f1}{6} * 5$

$15 = (6 - f_1)\ 5$

$3 = 6 - f_1$

$f_1 = 3$

Given, sum of frequencies = 40

$= 3 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$= 34 + f_2 = 40$

$= f_2 = 6$

Therefore, $f_1 = 3$ and $f_2 = 6$

***