Class: XII

SESSION : 2022-2023

SUBJECT: Mathematics SAMPLE

QUESTION PAPER - 20

with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. $\int \frac{\tan^{-1}x}{1+x^2}dx$ is equal to **[1]**

a) $(\log(1+x^2))\tan^{-1}x + C$ b) $\log\left|\tan^{-1}x\right| + C$

c) $\tan^{-1}x + \sec^{-1}x + C$ d) $\frac{1}{2}(\tan^{-1}x)^2 + C$

2. The 2 vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a $\triangle$ABC. The length of the median through A is **[1]**

a) $\frac{\sqrt{48}}{2}$ b) $\sqrt{18}$

c) $\frac{\sqrt{34}}{2}$ d) None of these

3. The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is **[1]**

a) 45° b) 60°

c) 30° d) 90°

4. The area bounded by the curves $y=\sqrt{x}$, 2y + 3 = x and the x-axis in the first quadrant is **[1]**

a) 36 b) 18

c) 9 d) none of these

5. Two students X and Y appeared in an examination. The probability that X will qualify for the examination is 0.05 and Y will qualify for the examination is 0.10. **[1]**

The probability that both will qualify for the examination is 0.02. What is the probability that only one of them will qualify for the examination?

a) 0.14
b) 0.12
c) 0.11
d) 0.15

6. The area enclosed between the curves y = $\sqrt{x}$, $x = 2y + 3$ and the x-axis is [1]

a) none of these
b) 9
c) 27
d) 18

7. Maximize Z = 50x + 60y , subject to constraints x +2 y ≤ 50 , x +y ≥ 30, x, y ≥ 0. [1]

a) 1600
b) 1547
c) 2500
d) 1525

8. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect: [1]

a) $\vec{b} = \lambda\vec{a}$ for some scalar λ
b) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.
c) $\vec{a} = \pm\vec{b}$
d) the respective components of $\vec{a}\ and\ \vec{b}$ are not proportional

9. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find P(A ∪ B). [1]

a) 0.62
b) 0.58
c) 0.51
d) 0.55

10. What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2$? [1]

a) 3
b) 2
c) 1
d) 4

11. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is [1]

a) 2
b) $\frac{3}{2}$
c) not defined
d) 4

12. General solution of $(1 + x^2)\,dy + 2xy\,dx = \cot x\,dx\ (x \neq 0)$ is [1]

a) $y(1 + x^2) = log|sinx| + c$

b) $y=(1+x)^{-1}\log|\sin x|$
$-C(1+x^2)^{-1}$

c) $y=(1+x)^{-1}\log|\sin x|$
$+C(1-x^2)^{-1}$

d) $y=(1+x)^{-1}\log|\sin x|$
$-C(1-x^2)^{-1}$

13. $\int \frac{\sqrt{16+(\log x)^2}}{x}dx=?$ **[1]**

a) None of these

b) $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+4\log$
$|\log x+\sqrt{16+(\log x)^2}|+C$

c) $\log x\cdot\sqrt{16+(\log x)^2}+16\log$
$|\log x+\sqrt{16+(\log x)^2}|+C$

d) $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+8\log$
$|\log x+\sqrt{16+(\log x)^2}|+C$

14. If f(x) = |x - 3| and g(x) = fof(x), then for x > 10, g'(x) is equal to **[1]**

a) none of these

b) 1

c) 0

d) -1

15. Find the particular solution for $\frac{dy}{dx}-\ \frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0; y=0\ when\ x=1$ **[1]**

a) $\cos\left(\frac{y}{x}\right)=\ \log|2ex|$

b) $\cos\left(\frac{y}{2x}\right)=\ \log|3ex|$

c) $\cos\left(\frac{y}{x}\right)=\ \log|ex|$

d) $\cos\left(\frac{2y}{x}\right)=\ \log|ex|$

16. If A and B are square matrices such that B = - A^{-1} BA, then $(A + B)^2$ = **[1]**

a) O

b) A + B

c) $A^2 + B^2$

d) $A^2 + 2AB + B^2$

17. $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cot^{-1}(3)=$ **[1]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{2}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{6}$

18. The angle between two lines having direction ratios 1, 1, 2 and $(\sqrt{3}-1)$, $(-\sqrt{3}-1)$, 4 is **[1]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{6}$

d) $\frac{\pi}{2}$

19. **Assertion (A):** The maximum value of Z = x + 3y. Such that 2x + y $\leq$ 20, x + 2y $\leq$ 20, x, y $\geq$ 0 is 30. **[1]**

Reason (R): The variables that enter into the problem are called decision variables.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right) = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$. **[1]**

Reason (R): $\frac{d}{dx}$[log (log(x))] = $\frac{1}{x \log x}$, x > 1

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the initial value problem: $x\frac{dy}{dx}$ + 1 = 0; y (-1) = 0 **[2]**

22. Find $\frac{dy}{dx}$, when x = a (1 - cos θ) and y = a (θ + sin θ) at $\theta = \frac{\pi}{2}$ **[2]**

23. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ and 3x - y - 22 + 4 = 0 = 2x + y + z + 1. **[2]**

OR

If a unit vector makes an angle $\frac{\pi}{2}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{K}_p$, then find the value of θ.

24. A fair coin and an unbiased die are tossed. Let A be the event head appear on the coin and B be the event 3 on the die. Check whether A and B are independent events or not. **[2]**

25. Find the principal value of cosec^{-1}(-1). **[2]**

Section C

26. Minimise Z = 13x - 15y, subject to the constraints: **[3]**
$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.

27. Evaluate: $\int$ e^{2x} sin 3x dx **[3]**

OR

Evaluate the Integral: $\int \frac{dx}{(x^{1/2}+x^{1/3})}$

28. Find the shortest distance between the lines L_1 and L_2, given by **[3]**
$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(4i - 2j + 2k)$

OR

Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ are intersecting. Hence, find their point of intersection.

29. Sketch the region lying in the first quadrant and bounded by $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$. Find the area of the region using integration. **[3]**

OR

Find the area of the region bounded by the curves $y^2 = 9x$, $y = 3x$.

30. Draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval [1, 5] . Find the area under the curve and between the lines x = 1 and x = 5. **[3]**

31. Find the derivative of the function given by $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ and hence find $f'(1)$. **[3]**

Section D

Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

OR

32. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

33. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ **[5]**

OR

Solve the system of the linear equations by Cramer's rule:

$2y - 3z = 0$

$x + 3y = -4$

$3x + 4y = 3$

34. Integrate the (rational) function $\frac{5x}{(x+1)(x^2-4)}$ **[5]**

35. Using vectors: Prove that if a, h, c are the lengths of three sides of a triangle, then its area $\triangle$ is given by $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$, where 2s - a + b + c. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(i) Express cost of tank as a function of height(h).

(ii) Verify by second derivative test that cost is minimum at critical point.

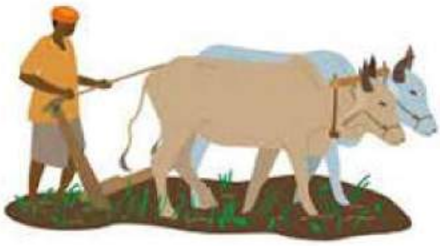
(iii) Find the value of h at which c(h) is minimum.

OR

Find the minimum cost of tank?

37. **Read the text carefully and answer the questions:** **[4]**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

(i) Find the combined sales of Masoor in September and October, for farmer Girish.

(ii) Find the combined sales of Urad in September and October, for farmer Ankit.

(iii) Find a decrease in sales from September to October.

OR

If both the farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October.

38. **Read the text carefully and answer the questions:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.
Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(i) Find the probability that it is due to the appointment of Ajay (A).

(ii) Find the probability that it is due to the appointment of Ramesh (B).

SOLUTION

Section A

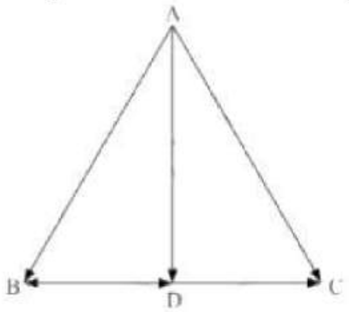
1. **(d)** $\frac{1}{2}(\tan^{-1}x)^2 + C$

Explanation: Subsitute $\tan^{-1}x = t$ then $\frac{1}{1+x^2}dx = dt$

$$\Rightarrow \int t\,dt = \frac{t^2}{2} + C \Rightarrow \frac{(\tan^{-1}x)^2}{2} + C$$

2. **(c)** $\frac{\sqrt{34}}{2}$

Explanation: In $\triangle ABC$,



Using the triangle law of vector addition, we have

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$= (3\hat{i} - \hat{j} + 4\hat{k}) - (\hat{j} + \hat{k})$

$\therefore \overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$ (since AD is the median)

In $\triangle$ABD, using the triangle law of vector addition, we have

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= (\hat{j} + \hat{k}) + \left(\frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}\right)$

$= \frac{3}{2}\hat{i} + 0\hat{j} + \frac{5}{2}\hat{k}$

$\therefore AD = \sqrt{\left(\frac{3}{2}\right)^2 + 0^2 + \left(\frac{5}{2}\right)^2} = \frac{1}{2}\sqrt{34}$

Hence, the length of the median through A is $\frac{1}{2}\sqrt{34}$ units.

3. **(d)** 90°

Explanation: We have,

$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$

The direction ratios of the given lines are proportional to 2, 5, 4 and 1, 2, -3.

The given lines are parallel to the vectors $\vec{b_1} = 2\hat{i} + 5\hat{j} + 4\hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} - 3\hat{k}$

Let θ be the angle between the given lines.

Now,

$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

$= \frac{(2\hat{i}+5\hat{j}+4\hat{k}) \cdot (\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$

$= \frac{2+10-12}{\sqrt{45}\sqrt{14}}$

$= 0$

$\Rightarrow \theta = 90°$

4. **(c)** 9

Explanation: Required area:

$\int_0^9 \sqrt{x}dx - \int_3^9 \left(\frac{x-3}{2}\right)dx = \left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9$ sq.units

5. **(c)** 0.11

Explanation: Let A and B be the events that X and Y qualify the examination,

respectively. We have, P(A) = 0.05, P(B) = 0.10 and P(A ∩ B) = 0.02
Required probability
$= P(A \cap \bar{B}) + P(B \cap \bar{A})$
= P(A) - (A ∩ B) + P(B) - P(A ∩ B)
= P(A) + P(B) - 2P(A ∩ B)
= 0.05 + 0.1 - 2(0.02)
= 0.15 - 0.04 = 0.11

6. **(b)** 9

Explanation: The two curves meet where;

$\sqrt{x} = \frac{x-3}{2}$...(i)

$\Rightarrow 4x = x^2 - 6x + 9$

$\Rightarrow x^2 - 10x + 9 = 0 \Rightarrow x = 9,1.$

Therefore, the two curves meet where x = 9.
Therefore, required area:

$$= \int_0^9 \sqrt{x}dx - \int_3^9 \frac{x-3}{2}dx = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left[\frac{(x-3)^2}{2}\right]_3^9 = 9$$

7. **(c)** 2500

Explanation: Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence, the maximum value is 2500

8. **(b)** both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

Explanation: If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then, they are parallel to the same line irrespective of their magnitudes and directions.

9. **(b)** 0.58

Explanation: Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4
Since the events are independent, P(A ∩ B) = P(A) · P(B)
Therefore P(A ∪ B) = P(A) + P(B) = 0.3 + 0.4 - 0.12 = 0.58

10. **(a)** 3

Explanation: 3

11. **(a)** 2

Explanation: In general terms for a polynomial the degree is the highest power.

The differential equation is $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

Square both the sides

$$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^2\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or .... $\frac{d^ny}{dx^n}$

The given differential equation is polynomial in differentials $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation.

Here the highest derivative is $\frac{d^2y}{dx^2}$ and there is only one term of highest order

derivative in the equation which is $(\frac{d^2y}{dx^2})^2$ whose power is 2 hence degree is 2.

12. **(a)** $y(1+x^2)=log|sinx|+c$

Explanation: $(1+x^2)dy=(cotx-2xy)dx$

$\frac{dy}{dx}=\frac{cotx-2xy}{1+x^2}$

$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{cotx}{1+x^2}$

It is a linear differential equation in y.

Therefore, Solution is

$ye^{\int\frac{2xdx}{1+x^2}}=\int\frac{cotx}{1+x^2}e^{\int\frac{2xdx}{1+x^2}}dx+c$

$y(1+x^2)=\int\frac{cotx}{1+x^2}(1+x^2)dx+c$

$y(1+x^2)=\int cotxdx+c$

$y(1+x^2)=log|sinx|+c$

13. **(d)** $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+8\log|\log x+\sqrt{16+(\log x)^2}|+C$

Explanation: The given integral is $\int\frac{\sqrt{16+(\log x)^2}}{x}dx$

Put log x = t and $\frac{1}{x}dx=dt$. Then,

$$I = \int \sqrt{t^2 + (4)^2} = \frac{t}{2}\sqrt{t^2 + 16} + \frac{16}{2}\log t + \sqrt{t^2 + 16}| + C$$

$$= \frac{1}{2}\log x\sqrt{(\log x)^2 + 16} + 8\log|\log x + \sqrt{16 + (\log x)^2}| + C.$$

14. **(b)** 1

Explanation: f(x) = |x - 3|

g(x) = fof(x)

g(x) = f(|x - 3|)

g(x) = |x - 3| - 3

g(x) = x - 3 - 3 ($\because$ x > 10)

g(x) = x - 6

g'(x) = 1

15. **(c)** $\cos\left(\frac{y}{x}\right) = \log|ex|$

Explanation: Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Question becomes $v + x\frac{dv}{dx} = v - cosecv$

$$x\frac{dv}{dx} = -cosecv$$

$$-\int sinv dv = \int \frac{dx}{x}$$

$$cosv = logx + logc$$

$$cos(\frac{y}{x}) = logx + logc$$

when x = 1 and y = 0

$$cos(\frac{0}{1}) = log1 + logc \ \{ \log c = 1 \}$$

$$c = e$$

$$cos(\frac{y}{x}) = logx + loge$$

$$cos(\frac{y}{x}) = log|ex|$$

16. **(c)** $A^2 + B^2$

Explanation: $B = -A^{-1}BA$

$\Rightarrow$ $AB = -AA^{-1}BA$

$\Rightarrow$ AB = -IBA

$\Rightarrow$ AB = -BA

$\Rightarrow$ AB + BA = O ...(i)

Consider, $(A + B)^2 = A^2 + AB + BA + B^2$ ...($\because$ AB $\neq$ BA)

$(A + B)^2 = A^2 + O + B^2$ ...from (i)

$(A + B)^2 = A^2 + B^2$

17. **(a)** $\frac{\pi}{4}$

Explanation: $sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + cot^{-1}(3)$

$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{3}\right)$because $\frac{1}{2}.\frac{1}{3} < 1$

$$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$$

$\Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

18. **(d)** $\frac{\pi}{2}$

Explanation: Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{6}, |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$

$$\cos\alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot ((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k})}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$\cos\alpha = \frac{1}{2}$

$\alpha = 60^\circ$

19. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation: Both A and R are true but R is not the correct explanation of A.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: **Assertion:** Let $y = \left(e^{\sqrt{x}}\right)^{\frac{1}{2}}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{\frac{1}{2}-1} \frac{d}{dx} e^{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{\frac{1}{2}} \cdot e^{\sqrt{x}} \cdot \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \frac{e^{\sqrt{x}}}{\sqrt{e^{\sqrt{x}}}} \times \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$$

Reason: Let y = log (log x)

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\log(\log x)) = \frac{1}{\log x}\left\{\frac{d}{dx}(\log x)\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x^2}, \ x > 1$$

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Section B

21. We have, $x\frac{dy}{dx} + 1 = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{x}$$

$$\Rightarrow dy = \left(\frac{-1}{x}\right) dx \text{ [separating variables]}$$

Integrating both sides, we get

$$\Rightarrow \int dy = \int \left(\frac{-1}{x}\right) dx$$

$\Rightarrow$ y = -log |x| + C ...(i)

It is given that y(-1) = 0

$\therefore$ 0 = -log |-1| + C

$\Rightarrow$ C = 0

Substituting the value of C in (i), we get

y = -log |x|

Hence, y = -log |x| is the solution to the given differential equation.

22. We have,

x = a (1 - cos θ) and y = a (θ + sin θ)

Differentiating x with respect to θ, we get

$$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}[a(1 - \cos\theta)] = a(\sin\theta)$$

and differentiating y with respect to θ, we get

$$\frac{dy}{d\theta} = \frac{d}{d\theta}[a(\theta + \sin\theta)] = a(1 + \cos\theta)$$

$$\therefore \left[\frac{dy}{dx}\right]_{\theta=\frac{\pi}{2}} = \left[\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right]_{\theta=\frac{\pi}{2}\pi} = \left[\frac{a(1+\cos\theta)}{a(\sin\theta)}\right]_{\theta=\frac{\pi}{2}} = \frac{a(1+0)}{a} = 1$$

23. The given the equation of the plane containing the line 3x - y - 2z + 4 = 0 = 2x + y + z + 1 is

$(3x - y - 2z + 4) + \lambda(2x + y + z + 1) = 0$

or $(3 + 2\lambda)x + (\lambda - 1)y + (\lambda - 2)z + (\lambda + 4) = 0$ ....(i)

If it is parallel to the line

$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$, then

$2(3 + 2\lambda) + 4(\lambda - 1) + (\lambda - 2) = 0$

$\Rightarrow 9\lambda = 0$

$\Rightarrow \lambda = 0$

Substituting

$\lambda = 0$ in (1), we obtain

$3x - y - 2z + 4 = 0$ ....(2)

This is the equation of the plane containing the second line and parallel to the first line.

Now, the line

$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ passes through (1, 3, -2)

$\therefore$ Shortest distance between the given lines

= Length of the perpendicular from (1, 3, -2) to the plane 3x - y - 2z + 4 = 0

$$= \left|\frac{3\times1-3-2\times(-2)+4}{\sqrt{3^2+(-1)^2+(-2)^2}}\right|$$

$$= \left|\frac{3-3+4+4}{\sqrt{9+1+4}}\right|$$

$$= \frac{8}{\sqrt{14}} \text{ units}$$

OR

The direction cosines of the vector are

$$l = \cos\frac{\pi}{3} = \frac{1}{2}$$

$m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

n = $\cos\theta$

We know,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{1}{2} - \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

24. Let A be the sample space of given experiment.

S= { (H, 1), (H, 2) , (H, 3), (H, 4) , (H, 5) , (H, 6) , (T, 1), (T , 2), (T, 3), (T, 4), (T, 5), (T, 6) }

A : Head appear on the coin

B : 3 appear on the dice

A = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

B = {(H, 3), (T, 3)}

$A \cap B = \{(H, 3)\}$

$P(A) = \frac{6}{12} = \frac{1}{2}, p(B) = \frac{2}{12} = \frac{1}{6}$

$P(A \cap B) = \frac{1}{12}$

$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

$= P(A \cap B)$

Hence A and B are independent events.

25. We know that the range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]

Let cosec^{-1} (-1) = θ. Then we have, cosec θ = -1

$\text{cosec } \theta = -1 = -\text{cosec } \frac{\pi}{2} = \text{cosec } \left(\frac{-\pi}{2}\right)$

$\therefore \theta = \frac{-\pi}{2} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - [0]$

Hence, the principal value of cosec^{-1} (-1) is equal to $\frac{-\pi}{2}$

Section C

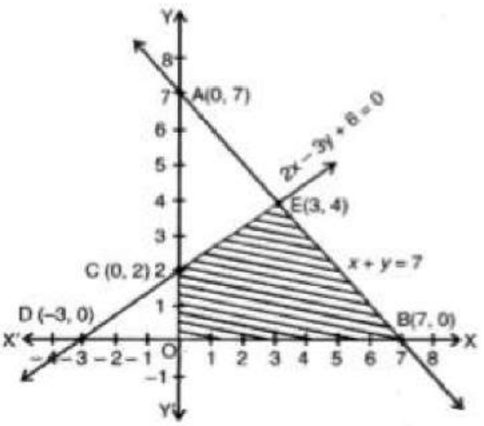
26. Consider $x + y = 7$

When $x = 0$, *then* $y = 7$ and

when $y = 0$, *then* $x = 7$

So, A(0, 7) and B(7, 0) are the points on line

$x + y = 7$



Consider $2x - 3y + 6 = 0$

When x = 0, then y = 2 and when y = 0, then x = - 3, So C(0, 2) and D(-3, 0) are the points on line $2x - 3y + 6 = 0$

Also, we have x > 0 and v > 0.

The feasible region OBEC is bounded, so, minimum value will obtain at a comer point of this feasible region.

Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2)

$Z = 13x - 15y$

At $O(0, 0)$, $Z = 0$

At $B(7, 0)$, $Z = 13(7) - 15(0) = 91$

At $E(3, 4)$, $Z = 13(3) - 15(4) = -21$

At $C(0, 2)$, $Z = 13(0) - 15(2)$

= -30 (minimum)

Hence , the minimum value is -30 at the point (0, 2).

27. Let I = $\int e^{2x} \sin 3x\, dx$

Then, $I = \int e^{2x} I \sin II 3x dx$

Using integration by parts.

$$\Rightarrow I = e^{2x}\left(-\frac{\cos 3x}{3}\right) - \int 2e^{2x}\left(-\frac{\cos 3x}{3}\right)dx$$

$$\Rightarrow I = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x} I \cos II 3x dx$$

$$\Rightarrow I = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\left\{e^{2x}\frac{\sin 3x}{3} - \int 2e^{2x}\frac{\sin 3x}{3}dx\right\}$$

$$\Rightarrow I = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}\int e^{2x}\sin 3x\, dx$$

$$\Rightarrow I = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}I$$

$$\Rightarrow I + \frac{4}{9}I = \frac{e^{2x}}{9}(2\sin 3x - 3\cos 3x)$$

$$\Rightarrow \frac{13}{9}I = \frac{e^{2x}}{9}(2\sin 3x - 3\cos 3x)$$

$$\Rightarrow I = \frac{e^{2x}}{13}(2\sin 3x - 3\cos 3x) + C$$

OR

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x}dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

Now let the given integral be, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$\Rightarrow x^{\frac{1}{6}} = t$

$\Rightarrow 6t^5 dt = dx$

Putting this value in equation (i) we get,

$$I = \int \frac{6t^5 dt}{\left(t^3 + t^2\right)}$$

$$I = \int \frac{6t^5 dt}{t^2 (t+1)}$$

$$I = 6\int \frac{t^3 dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)} dt$$

$$I = 6\int \frac{(t+1)\left(t^2 - t + 1\right)}{(t+1)} dt - \int \frac{1}{(t+1)} dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

28. The given lines are

$L_1 : \vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ ...(i)

$L_2 : \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + 2\mu(2\hat{i} - \hat{j} + \hat{k})$ ...(ii)

These equation are of the form:

$\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + 2\mu\vec{b} = \vec{a_2} + \mu'\vec{b}$, where

$\vec{a_1} = (\hat{i} + \hat{j}), \vec{a_2} = (2\hat{i} + \hat{j} - \hat{k}), \vec{b} = (2\hat{i} - \hat{j} + \hat{k})$ and $\mu' = 2\mu$

Therefore, the given lines are parallel.

Now,we have $(\vec{a_2} - \vec{a_1}) = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) = (\hat{i} - \hat{k})$

$$\therefore [\vec{b} \times (\vec{a_2} - \vec{a_1})] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$= (1 - 0)\hat{i} - (-2 - 1)\hat{j} + (0 + 1)\hat{k}$

$= (\hat{i} + 3\hat{j} + \hat{k})$

$\Rightarrow |\vec{b} \times (\vec{a_2} - \vec{a_1})| = \sqrt{1^2 + 3^2 + 1^2} = \sqrt{11}$

and $|\vec{b}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$\Rightarrow$ shortest distance between L_1 and L_2.

= distance between L_1 and L_2

$$= \frac{|\vec{b} \times (\vec{a_2} - \vec{a_1})|}{|\vec{b}|} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$= \left(\frac{\sqrt{11}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}\right) = \frac{\sqrt{66}}{6} \text{ units}$$

OR

The position vectors of two arbitrary points on the given lines are

$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(i + 2\hat{j} + 2\hat{k}) = (3 + \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (2\lambda - 4)\hat{k}$ and

$5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) = (5 + 3\mu)\hat{i} + (-2 + 2\mu)\hat{j} + 6\mu\hat{k}$

If the lines intersect, then they have a common point. So, for some values of λ and μ we must have,

$3 + \lambda = 5 + 3\mu$ .....(i)

$2 + 2\lambda = -2 + 2\mu$ .....(ii)

$2\lambda - 4 = 6\mu$ .....(iii)

Solving (i) and (ii), we get

$\lambda = -4$

$\mu = -2$

Substituting the values

$\lambda = -4$ and $\mu = -2$ in (iii), we get

LHS = $2\lambda - 4$

= 2(-4) - 4

= -12

RHS = 6μ

= 6(-2)

= -12

$\Rightarrow$ LHS = RHS

Since $\lambda = -4$ and $\mu = -2$ satisfy (iii), the lines intersect.

$\mu = -2$ in the second line, we get

$\vec{r} = 5\hat{i} - 2\hat{j} - 6\hat{i} - 4\hat{j} - 12\hat{k} = -\hat{i} - 6\hat{j} - 12\hat{k}$ as the position vector of the point of intersection.

Thus, the coordinates of the point of intersection are (-1, -6, -12).

29. The equation $y = 9x^2$ represents an upward opening parabola with axis as y-axis and vertex at the origin.

From the below it is Clear that the shaded region is the region lying in the first quadrant and bounded by $y = 9x^2$, x = 0, y = 1 and y = 4.

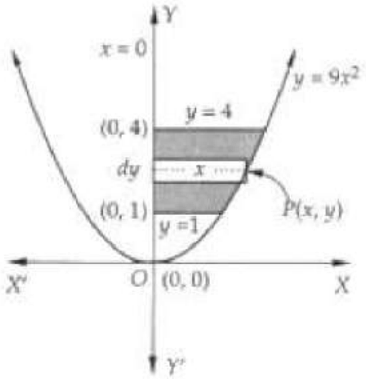
Let us slice this region into horizontal rectangular strips.

The approximating rectangle shown in Fig. has length = $|x|$ and width = dy

thus we have area = $|x|$ dy.

Clearly, it can move vertically between y = 1 and y = 4.

So, the required area denoted by A is given by



$A = \int_1^4 |x|\,dy = \int_1^4 x\,dy$ [$\because x \geq 0 \quad \therefore |x| = x$]

$\Rightarrow \quad A = \int_1^4 \sqrt{\frac{y}{9}}\,dy$ [$\because$ P(x, y) lies on $y = 9x^2$ $\therefore x = \sqrt{\frac{y}{9}}$]

$\Rightarrow \quad A = \frac{1}{3}\int_1^4 \sqrt{y}\,dy$

$\Rightarrow \quad A = \frac{1}{3} \times \frac{2}{3}\left[y^{3/2}\right]_1^4$

$= \frac{2}{9}(8 - 1)$

$= \frac{14}{9}$ sq. units

OR

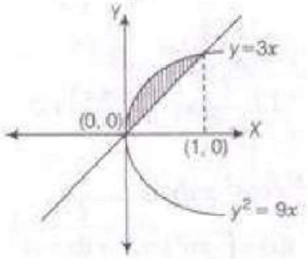
We have $y^2 = 9x$ and $y = 3x$

$\Rightarrow (3x)^2 = 9x$

$\Rightarrow 9x^2 - 9x = 0$

$\Rightarrow 9x(x - 1) = 0$

$\Rightarrow x = 1, 0$



$\therefore$ Required area, $A = \int_0^1 \sqrt{9x}dx - \int_0^1 3xdx$

$= 3\int_0^1 x^{1/2}dx - 3\int_0^1 xdx$

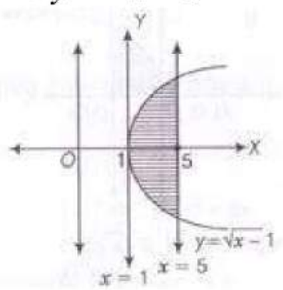
$= 3\left[\frac{x^{3/2}}{3/2}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$

$= 3\left(\frac{2}{3} - 0\right) - 3\left(\frac{1}{2} - 0\right)$

$= 2 - \frac{3}{2} = \frac{1}{2}$ sq units

30. Given equation of the curve is $y = \sqrt{x - 1}$

$\Rightarrow y^2 = x - 1$



$\therefore$ Area of shaded region, $A = \int_1^5 (x - 1)^{1/2}dx = \left[\frac{2.(x-1)^{3/2}}{3}\right]_1^5$

$= \left[\frac{2}{3}.(5 - 1)^{3/2} - 0\right] = \frac{16}{3}$ sq unit

31. Given: $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

Taking log on both sides, we get

$\log f(x) = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \log(1 + x^8)$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}\log f(x) = \frac{d}{dx}\log(1+x) + \frac{d}{dx}\log\left(1+x^2\right) + \frac{d}{dx}\log\left(1+x^4\right) + \frac{d}{dx}\log\left(1+x^8\right)$$

$$\Rightarrow \frac{1}{f(x)} \cdot \frac{d}{dx}[f(x)] =$$

$$\frac{1}{1+x} \cdot \frac{d}{dx}(1+x) + \frac{1}{1+x^2} \cdot \frac{d}{dx}\left(1+x^2\right) + \frac{1}{1+x^4} \cdot \frac{d}{dx}\left(1+x^4\right) +$$

$$\frac{1}{1+x^8} \cdot \frac{d}{dx}\left(1+x^8\right)$$

$$\Rightarrow f'(x) = f(x)\left[\frac{1}{1+x} + \frac{1}{1+x^2} \cdot (2x) + \frac{1}{1+x^4} \cdot \left(4x^3\right) + \frac{1}{1+x^8} \cdot \left(8x^7\right)\right]$$

$$\Rightarrow f'(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

$$\Rightarrow f'(1) = (1+1)\left(1+1^2\right)\left(1+1^4\right)(1+1^8)$$

$$\left[\frac{1}{1+1} + \frac{2(1)}{1+1} + \frac{4(1)^3}{1+(1)^4} + \frac{8(1)^7}{1+(1)^8}\right]$$

$$\Rightarrow f'(1) = (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$

$$\Rightarrow f'(1) = 16\left(\frac{1+2+4+8}{2}\right)$$

$$\Rightarrow f'(1) = 16\left(\frac{15}{2}\right)$$

$$\Rightarrow f'(1) = 120$$

Section D

32. $A = R - \{3\}$ and $B = R - \{1\}$ and $f(x) = \dfrac{x-2}{x-3}$

Let $x_1, x_2 \in A$, then $f(x_1) = \dfrac{x_1-2}{x_1-3}$ and $f(x_2) = \dfrac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one function.

Now $y = \dfrac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \dfrac{3y-2}{y-1}$

$$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{3y-2-3y+3} = y$$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

OR

R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivty

For any $a \in A$, $|a-a|=0$ Which is divisible by 2.

$\therefore$ (a, a) $\in$ r for all a $\in$ A

So ,R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all a, b $\in$ R

|a−b| is divisible by 2

|b−a| is divisible by 2

$(a, b) \in r \Rightarrow (b, a) \in R$

So, R is symmetirc .

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$
|a−b| is divisible by 2
|b−c| is divisible by 2
Two cases :
Case 1:
When b is even
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
$\therefore$ (a, c) $\in$ R

Case 2:
When b is odd
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So R is transitive.
Hence , R is an equivalence relation

33. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

$$\therefore A^{-1} = \frac{1}{|A|} adj. A = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$$\therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$$

$$\therefore B^{-1} = \frac{1}{|B|} adj. B = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Now $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 67(61) - 87(47) = 4087 - 4089 = -2 \neq 0$

Now L.H.S. $= (AB)^{-1} = \frac{1}{|AB|} adj.(AB) = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$....(i)

R.H.S. $= B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$....(ii)

$\therefore$ From eq. (i) and (ii), we get

L.H.S. = R.H.S.

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

OR

Given: $2y - 3z = 0$

$x + 3y = -4$

$3x + 4y = 3$

Let D $= \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$

Expanding along R_1,

$= 0(0) - 2(0) - 3(-5) = 15$

Also $D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$

Expanding along R_1,

$= 0(0) - 2(0) - 3(-25) = 75$

Again $D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$

Expanding along R_1,

$= 0(0) - 0(0) - 3(15) = -45$

Also $D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$

Expanding along R_1,

$= 0(25) - 2(15) + 0(-5) = -30$

Now,

$x = \frac{D_1}{D} = \frac{75}{15} = 5$

$y = \frac{D_2}{D} = \frac{-45}{15} = -3$

$z = \frac{D_3}{D} = \frac{-30}{15} = -2$

Hence x = 5, y = -3, z = -2

34. $\frac{5x}{(x+1)\left(x^2-4\right)}$

$= \frac{5x}{(x+1)(x+2)(x-2)}$

$= \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2}$ ...(i)

$\Rightarrow 5x = A(x + 2) + B(x + 1)(x - 2) + C(x + 1)(x + 2)$

$\Rightarrow 5x = A(x^2 - 4) + B(x^2 - x - 2) + C(x^2 + 3x + 2)$

$\Rightarrow 2x = Ax^2 - 4A + Bx^2 - Bx - 2B + Cx^2 + 3Cx + 2C$

Comparing coefficients of x^2: A + B + C = 0 ....(ii)

Comparing coefficients of x: B + 3C= 5 ....(iii)

Comparing constants: 4A 2B + 2C = 0 ....(iv)

On solving eq. (i), (ii) and (iii), we get $A = \frac{5}{3}, B = \frac{-5}{2}, C = \frac{5}{6}$

Putting the values of A, B and C in eq. (i),

$\frac{5x}{(x+1)\left(x^2-4\right)}$

$= \frac{\frac{5}{3}}{x+1} + \frac{\frac{-5}{2}}{(x+2)} + \frac{\frac{5}{6}}{x-2}$

$$\therefore \int \frac{5x}{(x+1)\left(x^2-4\right)} dx = \frac{5}{3}\int \frac{1}{x+1} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{x-2} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + c$$

35. We have,

$\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$.

Then, $|\overrightarrow{BC}| = |\vec{a}| = a, |\overrightarrow{CA}| = |\vec{b}| = b$ and $|\overrightarrow{AB}| = |\vec{c}| = c$

Using triangle law of addition of vectors, we get

$\overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$

Now, by area of triangle

$\triangle = \frac{1}{2}|\overrightarrow{BC} \times \overrightarrow{BA}|$

$\Rightarrow \triangle = \frac{1}{2}|\vec{b} \times -\vec{c}| = \frac{1}{2}|\vec{b} \times \vec{c}|$

$\Rightarrow 2\triangle = |\vec{b} \times \vec{c}|$

$\Rightarrow 4\triangle^2 = |\vec{b} \times \vec{c}|^2$

$\Rightarrow 16\triangle^2 = 4|\vec{b} \times \vec{c}|^2$

$\Rightarrow 16\triangle^2 = 4\left\{-(\vec{b} \cdot \vec{c})^2 + |\vec{b}|^2|\vec{c}|^2\right\}$ [Using Lagrange's identity]

$\Rightarrow 16\triangle^2 = 4|\vec{b}|^2|\vec{c}|^2 - 4(\vec{b} \cdot \vec{c})^2$

$\Rightarrow 16\triangle^2 = 4|\vec{b}|^2|\vec{c}|^2 - \{-2(\vec{b} \cdot \vec{c})\}^2$

$\Rightarrow 16\triangle^2 = 4|\vec{b}|^2|\vec{c}|^2 - \left\{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{b} + \vec{c}|^2\right\}^2$

$\Rightarrow 16\triangle^2 = 4|\vec{b}|^2|\vec{c}|^2 - \left\{|\vec{b}|^2 + |\vec{c}|^2 - |-\vec{a}|^2\right\}^2$

$[\because \vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{b} + \vec{c} = -\vec{a}]$

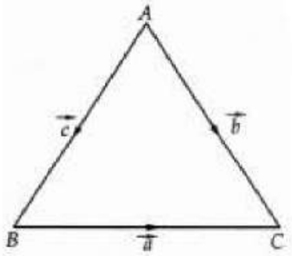
$\Rightarrow 16\triangle^2 = 4|\vec{b}|^2|\vec{c}|^2 - \left\{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2\right\}^2$

$\Rightarrow 16\triangle^2 (2bc)^2 - \{b^2 + c^2 - a^2\}\{a^2 - (b - c)^2\} = (2bc + b^2 + c^2 - a^2)(2bc - b^2 - c^2 + a^2)$

$\Rightarrow 16\triangle^2 \{(b+c)^2 - a^2\}\{a^2 - (b-c)^2\} = (b+c+a)(b+c-a)(a+b-c)(a-b+c)$

$\Rightarrow \triangle^2 = s(s-a)(s-b)(s-c)$

$\Rightarrow \triangle = \sqrt{s(s-a)(s-b)(s-c)}$



Section E

36. **Read the text carefully and answer the questions:**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(i) $c(h) = 100h + 320 + \frac{1600}{h}$

Let l ft be the length and h ft be the height of the tank. Since breadth is equal to 5 ft. (Given)

$\therefore$ Two sides will be 5h sq. feet and two sides will be lh sq. feet. So, the total area of the sides is $(10h + 2lh)ft^2$

Cost of the sides is ₹10 per sq. foot. So, the cost to build the sides is $(10h + 2lh) \times 10$ = ₹$(100h + 20lh)$

Also, cost of base = $(5l) \times 20$ = ₹$100l$

$\therefore$ Total cost of the tank in ₹ is given by $c = 100h + 20lh + 100l$

Since, volume of tank = 80 ft^3

$\therefore 5lh = 80\ ft^3 \therefore l = \frac{80}{5h} = \frac{16}{h}$

$\therefore$ c(h) $= 100h + 20\left(\frac{16}{h}\right)h + 100\left(\frac{16}{h}\right)$

$= 100h + 320 + \frac{1600}{h}$

(ii) $C(h) = 100h + 320 + \frac{1600}{h}$

$\frac{dC(h)}{dh} = 100 - \frac{1600}{h^2}$

$\frac{d^2C(h)}{dh^2} = -\left(\frac{-2}{h^3}\right)1600$

at h = 4

$\frac{d^2C(h)}{dh^2} = 50 > 0$

Hence cost is minimum when h = 4 ft

(iii) To minimize cost, $\frac{dc}{dh} = 0$

$\Rightarrow\ 100 - \frac{1600}{h^2} = 0$

$\Rightarrow\ 100h^2 = 1600\ \Rightarrow\ h^2 = 16\ \Rightarrow\ h = \pm 4$

$\Rightarrow$ h = 4 [$\because$ height can not be negative]

OR

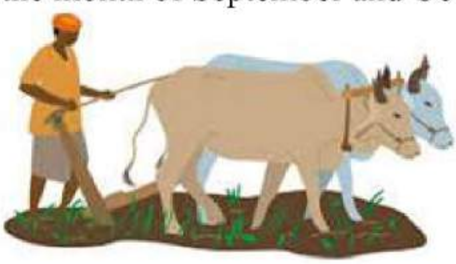
Minimum cost of tank is given by

$c(4) = 400 + 320 + \frac{1600}{4}$

$= 720 + 400 =$ ₹1120

37. **Read the text carefully and answer the questions:**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} Ankit \\ Girish \end{matrix}$$

(i)

$$A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$$

The combined sales of Masoor in September and October, for farmer Girish ₹40000.

(ii)

$$A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$$

The combined sales of Urad in September and October, for farmer Ankit is ₹15000.

(iii)

$$A - B = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} - \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 10,000 - 5000 & 20,000 - 10,000 & 30,000 - 6000 \\ 50,000 - 20,000 & 30,000 - 10,000 & 10,000 - 10,000 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$$

OR

Profit = 2% × sales on october

$$= \frac{2}{100} \times B$$

$$= 0.02 \times \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.02 \times 5000 & 0.02 \times 10,000 & 0.02 \times 6000 \\ 0.02 \times 20,000 & 0.02 \times 10,000 & 0.02 \times 10,000 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$$

38. **Read the text carefully and answer the questions:**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(i) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3,\ P(A/E_2) = 0.8,\ P(A/E_3) = 0.5$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{4}{7} \times 0.3}{\frac{4}{7} \times 0.3 + \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7} + \frac{0.8}{7} + \frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$$

$$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$$

(ii) Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3,\ P(A/E_2) = 0.8,\ P(A/E_3) = 0.5$

$$P(E_2/A) = \frac{P\left(E_1\right) \cdot P\left(A/E_1\right)}{P\left(E_1\right) \cdot P\left(A/E_1\right) + P\left(E_2\right) P\left(A/E_2\right) + P\left(E_3\right) P\left(A/E_3\right)}$$

$$= \frac{\frac{1}{7} \times 0.8}{\frac{4}{7} \times 0.3 + \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7} + \frac{0.8}{7} + \frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$$

$$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$$