Cubes and Cube Roots

MATHEMATICAL REASONING

1. The value of $45^3 - 65^3 - 20^3$ is __.
(a) 175500 (b) – 191500
(c) 170000 (d) – 170000

2. If $\sqrt[3]{-2744} \div \sqrt[3]{0.008} = x$, then the value of x is __.
(a) 70 (b) – 70
(c) 14 (d) – 14

3. If $\sqrt[3]{3\left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)} = 2$, then $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} =$ _____ .
(a) $\frac{10}{3}$ (b) $-\frac{10}{3}$
(c) $\frac{3}{15}$ (d) Both (a) and (b)

4. How many cubes of side 2 cm can be packed in a cubical box with inner side equal to 4 cm?
(a) 6 (b) 4
(c) 8 (d) 2

5. Find the smallest natural number by which 1458 must be divided so that the quotient is a perfect cube.
(a) 4 (b) 2
(c) 6 (d) 8

6. In the five digit number 1b6a3, a is the greatest single digit perfect cube and twice of it exceeds b by 7. Then the sum of the number and its cube root is __.
(a) 18700 (b) 11862
(c) 19710 (d) 25320

7. The value of $\sqrt[3]{\frac{a^6 \times b^3 \times c^{21}}{c^9 \times a^{12}}}$ is _____.
(a) $\frac{-bc^3}{a^2}$ (b) $\frac{bc^4}{a^2}$
(c) $\frac{-ab^4}{c^2}$ (d) $\frac{-bc^4}{a^2}$

8. Three numbers are in the ratio 2 : 3 : 5 to one another. The sum of their cubes is 54880. The numbers are____.
(a) 14, 21, 35
(b) 12, 15, 17
(c) 14, 18, 21
(d) 21, 28, 32

9. The cube of a 2-digit number will contain
(a) 4 digits (b) 5 digits
(c) 6 digits (d) 4, 5 or 6 digits

10. The cube of an odd natural number is always ___.
(a) Even (b) Odd
(c) Even or odd (d) Can't say

11. The length of each side of a cubical box is 2.4 m. Its volume is ___.
(a) $1.3824 \times 10^7 cu.cm$
(b) $13.824 cu.cm$
(c) 1.3824×10^6 cu.cm
(d) 1.3824×10^4 cu.cm

12. The unit's digit of the cube of a number is 9. The unit's digit of its cube root is ___.
(a) 9 (b) 7
(c) 3 (d) 1

13. The cube of a number x is nine times of x, then find x, where $x \neq 0$ and $x \neq -3$.
(a) 8 (b) 2
(c) 4 (d) 3

14. Two cubes have volumes in the ratio 1 : 27. The ratio of the area of the face of one to that of the other is __.
(a) 1 : 3
(b) 1 : 6
(c) 1 : 9
(d) 1 : 18

15. The smallest number by which 392 must be multiplied so that the product is a perfect cube, is ____.
(a) 3
(b) 5
(c) 7
(d) 9

EVERYDAY MATHEMATICS

16. Mohit gave a problem to Samrath.

Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, find the cube root of the larger number.

Help Samrath to answer the question.
(a) 4
(b) 6
(c) 8
(d) 10

17. A tank is in the form of a cube whose volume is 9261000 m^3. Find the length of side of the tank
(a) 230 m
(b) 250 m
(c) 210 m
(d) 180 m

18. AtuI made a cuboid of plasticine. Length, breadth and height of the cuboid are 25 cm. 25 cm and 50 cm. How many minimum such cuboids he needs to make a perfect cube?
(a) 4 (b) 20
(c) 12 (d) 25

19. A rectangular cubical piece of metal of dimensions $2\ cm \times 3\ cm \times 4\ cm$ is melted. Some more of the metal is added and it is made into a cube. The cube has integral measures for its sides. What is the minimum amount of metal that is added and what is the side of this cube?
(a) 10 cm^3, 4 cm (b) 3 cm^3, 3 cm
(c) 11 cm^3, 3 cm (d) 4 cm^3, 3 cm

20. To collect rain water. Mini made a cubical tank which can hold 91125 m^3 water. She uses this water for watering the plants of her garden. What is the height of the tank?
(a) 50 m
(b) 25 m
(c) 45 m
(d) 40 m

ACHIEVERS SECTION (HOTS)

21. Which of the following options is INCORRECT?
(a) Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. The numbers will be 5, 10, 15.
(b) The digit in the units place for the cube of a four digit number of the form $xyz8$ is 2.
(c) The smallest number by which 3600 be divided to make it a perfect cube is 450.
(d) None of these

22. Find the cube root of:
(i) 0.003375 = P .
(ii) 1.331 = Q .
(iii) 4.913= R .
(iv) 15.625= S .

	P	Q	R	S
(a)	0.215	1.31	2.7	2.55
(b)	0.115	1.11	1.17	3.25
(c)	0.15	1.1	1.7	2.5
(d)	0.25	1.21	2.17	4.15

23. Match the following.

Column – I	Column – II
(P) The smallest number that should be subtracted from 130 to make it perfect cube is	(i) 4
(Q) The smallest number that should be subtracted from 9268 to make it perfect cube is	(ii) 3
(R) The smallest number that should be added to 2194 to make it perfect cube is	(iii) 5
(S) The smallest number that should be added to 6855 to make it perfect cube is	(iv) 7

(a) P → (iii); Q → (i); R → (iv); S → (ii)
(b) P → (ii); Q → (iv); R → (i); S → (iii)
(c) P → (iii); Q → (i); R → (ii); S → (iv)
(d) P → (iii); Q → (iv); R → (ii); S → (i)

24. Evaluate the following.

(i) $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt[3]{\frac{0.729}{0.512}} - \frac{1}{3}$

(ii) $\sqrt[3]{343} + \sqrt[3]{0.064} - \sqrt[3]{0.125}$

(iii) $\left[\left(\sqrt[3]{\frac{-216}{42875}} + \sqrt[3]{\frac{64}{125}}\right)\right] \times \sqrt[3]{\frac{343}{1331}}$

	(i)	(ii)	(iii)
(a)	1	6.9	$\frac{2}{5}$
(b)	3	7.1	$\frac{1}{5}$
(c)	4	7.9	$\frac{2}{5}$
(d)	1	6.5	$\frac{1}{5}$

25. Which of the following statements is CORRECT?

Statement - 1: Cube root of 117.649 is a rational number.

Statement - 2: Cube of an odd number may or may not be odd.

(a) Only Statement - 1
(b) Only Statement - 2
(c) Both Statement -1 and Statement - 2
(d) Neither Statement -1 nor Statement – 2

ANSWER KEY									
1.	B	**2.**	B	**3.**	D	**4.**	C	**5.**	B
6.	C	**7.**	D	**8.**	A	**9.**	D	**10.**	B
11.	A	**12.**	A	**13.**	D	**14.**	C	**15.**	C
16.	B	**17.**	C	**18.**	A	**19.**	B	**20.**	C
21.	D	**22.**	C	**23.**	D	**24.**	A	**25.**	A

HINTS & EXPLANATIONS

1. (b) : We have, $45^3 - 65^3 - 20^3$
$= 91125 - 274625 - 8000 = -191500$

2. (b) : We have, $x = \sqrt[3]{-2744} \div \sqrt[3]{0.008}$
$= \sqrt[3]{(-14)\times(-14)\times(-14)} \div \sqrt[3]{0.2\times0.2\times0.2}$
$= -14 \div 0.2 = \frac{-14}{2} \times 10 = -70$

3. (d) : We have $\sqrt[3]{3\left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)} = 2$

Cubing both sides, we get

$3\left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right) = 8 \Rightarrow \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}} = \frac{8}{3}$

Now, squaring both sides, we get

$\left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)^2 = \frac{64}{9}$

Now, $\left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = \left(\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}\right)^2 + 4$

$= \frac{64}{9} + 4 = \frac{64+36}{9}$

$\therefore \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right)^2 = \frac{100}{9}$

$\Rightarrow \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right) = \frac{10}{3} \text{ or } -\frac{10}{3}$

4. (c) : Side of cubical box $= 4\ cm$
$\therefore$ Volume of cubical box $= 4^3 = 64 cm^3$
Side of each cube inside cubical box $= 2\ cm$
$\therefore$ Volume of each cube inside cubical box
$= 2^3 = 8\ cm^3$

$\therefore$ Required number of cubes $= \frac{64}{8} = 8$

5. (b): $1458 = 2 \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$ Here 2 is unpaired. So, 1458 must be divided by 2 to make it a perfect cube.

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

6. (c) : The given five digit number is $1b6a3$.
We know that, the greatest single digit perfect cube is 8,
$\therefore a = 8$ Also, $b = 2 \times (8) - 7 = 9$
So, the five digit number becomes 19683.
Now,
$19683 + \sqrt[3]{19683} = 19683 + 27 = 19710$

7. (d) : We have, $\sqrt[3]{\dfrac{-a^6 \times b^3 \times c^{21}}{c^9 \times a^{12}}} = \sqrt[3]{\dfrac{-b^3 \times c^{12}}{a^6}}$

$$= \left(\frac{(-b) \times (-b) \times (-b) \times c^4 \times c^4 \times c^4}{a^2 \times a^2 \times a^2} \right)^{\frac{1}{3}}$$

$$= \left(\frac{-b \times c^4}{a^2} \right)^{3 \times \frac{1}{3}} = \frac{-bc^4}{a^2}$$

8. (a) : Let the numbers be $2x$, $3x$ and $5x$.
So, $(2x)^3 + (3x)^3 + (5x)^3 = 54880$
$= 8x^3 + 27x^3 + 125x^3 = 54880$
$\Rightarrow 160x^3 = 54880 \Rightarrow x^3 = \dfrac{54880}{160} = 343$
$\Rightarrow x = \sqrt[3]{343} = 7$
So, numbers are 14, 21 and 35

9. (d) :

10. (b) : Cube of an odd number is always odd.

11. (a) : Side of cubical box = 2.4 m
$\therefore$ Volume = (side)^3 = (2.4 m)^3
$= 13.824\ m^3 = 13.824 \times 10^6\ cm^3$
$= 1.3824 \times 10^7\ cm^3$

12. (a) :

13. (d) : It is given that, $x^3 = 9x, x \neq 0, x \neq -3$
Now, $x^3 - 9x = 0 \Rightarrow x(x^2 - 9) = 0$
$\Rightarrow x^2 - 9 = 0 (\because x \neq 0)$
$\Rightarrow x^2 = 9 \Rightarrow x = 3$ or -3
But $x \neq -3$ $\therefore x = 3$

14. (c) : Let sides of two cubes be a_1 and a_2
So, $\dfrac{a_1^3}{a_2^3} = \dfrac{1}{27}$.
Taking cube root, we get $\dfrac{a_1}{a_2} = \dfrac{1}{3}$
Area of face of first cube $= a_1^2$
And area of face of other cube $= a_2^2$
$\therefore$ Required ratio $= \dfrac{a_1^2}{a_2^2} = \left(\dfrac{a_1}{a_2} \right)^2$
$= \left(\dfrac{1}{3} \right)^2 = \dfrac{1}{9} = 1:9$

15. (c) : We have, $392 = 2 \times 2 \times 2 \times 7 \times 7$ So, 392 must be multiplied by 7 to make it perfect cube.

2	392
2	196
2	98
7	49
7	7
	1

16. (b) : Let the larger perfect cube be x^3 and smaller perfect cube be y^3
According to question,
$x^3 - y^3 = 189$ ...(*i*)
Also, $\sqrt[3]{y^3} = 3 \Rightarrow y^3 = 3^3 = 27$
$x^3 - 27 = 189$ [*from* (*i*)]
$\Rightarrow x^3 = 189 + 27 = 216$
$\Rightarrow x = \sqrt[3]{216}$

$216 = \underline{2\times2\times2}\times\underline{3\times3\times3} = (2\times3)^3$

$\Rightarrow \sqrt[3]{216} = 2\times3 = 6$

$\therefore x = \sqrt[3]{216} = 6$

$\because$ Larger Perfect cube is x^3.

$\therefore$ Its cube root is $\sqrt[3]{x^3} = x = 6$

$\therefore$ Its cube root is $\sqrt[3]{x^3} = x = 6$

17. (c) : Volume of cubical tank $= 9261000\ m^3$

$\therefore (Side)^3 = 9261000$

$\Rightarrow$ Side $\sqrt[3]{9261000}$

$\Rightarrow$ Side=210m

18. (a) : Volume of cuboid $= 25\times25\times50$

$= 5\times5\times5\times5\times5\times5\times2$

Since there is only one 2 In the prime factorisation.

So, he needs $2\times2 = 4$, to make it a perfect cube.

$\therefore$ He need 4 such cuboids to make a perfect cube.

19. (b) : Volume of cubical piece of metal

$= 2\times3\times4 = 24cm^3$

To make it a perfect cube, we add $3\ cm^3$ more metal into it.

Volume of new cube $= (24+3)cm^3 = 27cm^3$

$(Side)^3 = 27 \Rightarrow$ Side=$\sqrt[3]{27} = 3cm$

20. (c) : Volume of cubical tank $= 91125\ m^3$

$(Side)^3 = 91125 \Rightarrow Side = \sqrt[3]{91125} = 45m$

$\therefore$ Height of tank= 45 m.

21. (d) : (a) Let the numbers be x, 2x and 3x

Sum of cubes of the numbers = 4500

$\therefore (x)^3 + (2x)^3 + (3x)^3 = 4500$

$\Rightarrow x^3 + 8x^3 + 27x^3 = 4500$

$\Rightarrow 36x^3 = 4500 \Rightarrow x^3 = \dfrac{4500}{36} = 125$

$\Rightarrow x = \sqrt[3]{125}$

Now, $125 = \underline{5\times5\times5} = 5^3$

$\Rightarrow \sqrt[3]{125} = 5$

$\therefore x = \sqrt[3]{125} = 5$

$\therefore$ Required numbers are 5, 10 and 15.

(b) Since the unit's place digit of xyz8 is 8

$\therefore$ Unit's place digit of cube of $xyz8$ is 2.

(c) $3600 = 2^3\times5^2\times3^2\times2$

To make it a perfect cube, it must be divided by $5^2\times3^2 = 2$ i.e., 450.

22. (c)=(i) We have, $\sqrt[3]{0.003375} = \sqrt[3]{\dfrac{3375}{1000000}}$

$= \dfrac{15}{100} = 0.15$

(ii) We have, $\sqrt[3]{1.331} = \sqrt[3]{\dfrac{1331}{1000}} = \dfrac{11}{10} = 1.1$

(iii) We have, $\sqrt[3]{4.913} = \sqrt[3]{\dfrac{4913}{1000}} = \dfrac{17}{10} = 1.7$

(iv) We have,

$\sqrt[3]{15.625} = \sqrt[3]{\dfrac{15625}{1000}} = \dfrac{25}{10} = 2.5$

23. (d) : (P) $5^3 \le 130 \le 6^3$

As perfect cube less than 130 = 125

So, 130 - 125 = 5

$\therefore$ The smallest number that should be subtracted from 130 to make it a perfect cube = 5

(Q) $21^3 \le 9268 \le 22^3$

$\therefore$ Perfect cube less than 9268 is 9261.

So, smallest number that should be subtracted from 9268 to make it a perfect cube = 7

(R) $12^3 \le 2194 \le 13^3$

Perfect cube just greater than 2194 is 2197.

So, smallest number that should be added to 2194 to make it perfect cube is 3.

(S) $18^3 < 6855 < 19^3$

Perfect cube greater than 6855 is 6859.

So, 6859 - 6855 = 4

$\therefore$ Smallest number that should be added to 6855 to make it a perfect cube = 4.

24. (a) : We have,

(i) $\sqrt[3]{\dfrac{0.027}{0.008}} \div \sqrt[3]{\dfrac{0.729}{0.512}} - \dfrac{1}{3}$

$= \sqrt[3]{\dfrac{0.3\times0.3\times0.3}{0.2\times0.2\times0.2}} \div \sqrt[3]{\dfrac{0.9\times0.9\times0.9}{0.8\times0.8\times0.8}} - \dfrac{1}{3}$

$$=\left(\frac{0.3}{0.2}\div\frac{0.9}{0.8}\right)-\frac{1}{3}=\left(\frac{0.3}{0.2}\times\frac{0.8}{0.9}\right)-\frac{1}{3}$$

$$=\frac{4}{3}-\frac{1}{3}=1$$

(ii) We have, $\sqrt[3]{343}+\sqrt[3]{0.064}-\sqrt[3]{0.125}$

$$-\sqrt[3]{0.5\times0.5\times0.5}$$

$$=7+0.4-0.5=7-0.1=6.9$$

(iii) We have,

$$\left[\left(\sqrt[3]{\frac{-216}{42875}}+\sqrt[3]{\frac{64}{125}}\right)\right]\times\sqrt[3]{\frac{343}{1331}}$$

$$=\left[\left(\frac{(-6)\times(-6)\times(-6)}{35\times35\times35}\right)^{\frac{1}{3}}+\left(\frac{4\times4\times4}{5\times5\times5}\right)^{\frac{1}{3}}\right]$$

$$\times\left(\frac{7\times7\times7}{11\times11\times11}\right)^{\frac{1}{3}}$$

$$=\left(\frac{-6}{35}+\frac{4}{5}\right)\times\left(\frac{7}{11}\right)=\left(\frac{-6+28}{35}\right)\times\frac{7}{11}$$

$$=\frac{22}{35}\times\frac{7}{11}=\frac{2}{5}$$

25. (a) : Statement- 1:

$\sqrt[3]{117.649}=4.9$ which is a rational number.

Statement-2:

Cube of an odd number is always odd.