Binomial Theorem

- The coefficients of the expansions of a binomial are arranged in an array. This array is called Pascal's triangle. It can be written as

Index	Coefficient(s)
0	$^0C_0$ $(=1)$
1	$^1C_0$ $(=1)$ $^1C_1$ $(=1)$
2	$^2C_0$ $(=1)$ $^2C_1$ $(=2)$ $^2C_2$ $(=1)$
3	$^3C_0$ $(=1)$ $^3C_1$ $(=3)$ $^3C_2$ $(=3)$ $^3C_3$ $(=1)$
4	$^4C_0$ $(=1)$ $^4C_1$ $(=4)$ $^4C_2$ $(=6)$ $^4C_3$ $(=4)$ $^4C_4$ $(=1)$
5	

- **General Term:** The $(r + 1)^{th}$ term (denoted by T_{r+1}) is known as the general term of the expansion $(a + b)^n$ and it is given by $T_{r+1} = {}^nC_r\, a^{n-r} b^r$

Example 1: In the expansion of $(5x - 7y)^9$, find the general term?

Solution: $T_{r+1} = {}^9C_r\,(5x)^{9-r}(-7y)^r = (-1)^r\,{}^9C_r\,(5x)^{9-r}(7y)^r$

- **Middle term in the expansion of $(a + b)^n$:**
  - If n is even, then the number of terms in the expansion will be $n + 1$. Since n is even, $n + 1$ is odd. Therefore, the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.
  - If n is odd, then $n + 1$ is even. So, there will be two middle terms in the expansion. They are $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.

- In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, where $x \neq 0$, the middle term is $\left(\frac{2n}{2}+1\right)^{th}$, i.e., $(n + 1)^{th}$ term [since $2n$ is even].

It is given by $^{2n}C_n x^n\left(\frac{1}{x}\right)^n = {}^{2n}C_n$ which is a constant.

This term is called the term independent of x or the constant term.

Note: In the expansion of $(a + b)^n$, r^{th} term from the end = $(n - r + 2)^{th}$ term from the beginning

Example 2: In the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$, find the middle term and find the term which is independent of x.

Solution: As 4 is even, the middle term in the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$ is the $\left(\frac{4}{2}+1\right)^{th}$ term, i.e., 3^{rd} term, which is given by

$$T_3 = T_{2+1} = {}^4C_2\left(\frac{x^3}{4}\right)^2\left(\frac{-12}{x}\right)^2$$

$$= 6 \times \frac{x^6}{16} \times \frac{144}{x^2}$$

$$= 54x^4$$

Now, we will find the term in the expansion which is independent of x. Suppose $(r + 1)^{th}$ term is independent of x.

The $(r + 1)^{th}$ term in the expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

Hence, the $(r + 1)^{th}$ term in the expansion of $\left(\frac{x^3}{4}-\frac{12}{x}\right)^4$ is given by