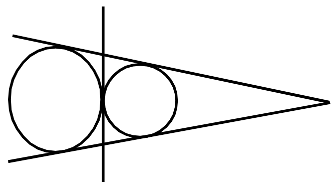
SYSTEM OF CIRCLES

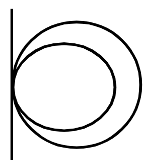
SYNOPSIS

- Let C, D be the centres and r_1, r_2 be the radii of two circles.

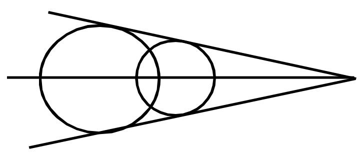
  a) If CD = $r_1 + r_2$ then the two circles touch each other externally. Then we can draw three common tangents to the circles.



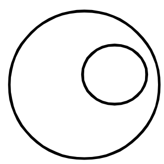
  b) If CD = $|r_1 - r_2|$ then the two circles touch each other internally. Then we can draw one common tangent to the circles.



  c) If $|r_1 - r_2| <$ CD $< r_1 + r_2$ then the two circles intersect each other. Then we can draw two common tangents to the circles.

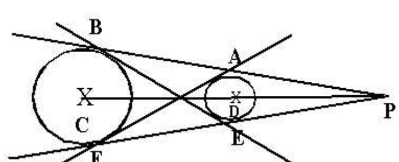


  d) If CD $< |r_1 - r_2|$ then one circle entirely lies inside the other circle. Then we cannot draw common tangent.



  e) If CD > $r_1 + r_2$ then the two circles do not intersect.

  Then we can draw four common tangents to the circles.



- The concurrect point of two direct common tagents and line of thier centres to two intersecting and non- intersecting circles, is called external centre of similitude.
- s= 0 and $s' = 0$ be two circles with centres c_1, c_2 and radii "r_1" and "r_2". The point which divides c_1c_2 in the ratio $r_1 : r_2$ internally is called the internal centre of similitude and the point which diivides externally is called the external centre of similitude.
- Length of the direct common tangent of the circle is $\sqrt{d^2 - (r_1 - r_2)^2}$.
- Length of the transverse common tangent of the circle circle is $\sqrt{d^2 - (r_1 + r_2)^2}$.
- If d is distance between the centres, r_1, r_2 are the raddi of two intersecting circles then the angle between the circles is $\cos^{-1}\left(\frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}\right)$.
- If $d^2 = r_1^2 + r_2^2$ then the angle betwee the two circles is 90^0.
- If θ is angle between the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ then

$$\cos\theta = \frac{(c + c_1) - 2(gg_1 + ff_1)}{2\sqrt{g^2 + f^2 - c}\sqrt{g_1^2 + f_1^2 - c_1}} .$$

- If the circles cut each other orthogonally then $2gg_1 + 2ff_1 = c + c_1$.
- Two circles with centres c_1, c_2 and radius "r" cut each other orthogonally. Then $r = \frac{c_1c_2}{\sqrt{2}}$.
- Two circles of radii r_1, r_2 cut orthogonally then area included between the circles is $r_1^2 \tan^{-1}(r_1/r_2) + r_2^2 \tan^{-1}(r_1/r_2) - r_1r_2$.
- If P and Q are conjugate points with respect to the circle s = 0 then the circle on PQ as diameter cuts the circle s = 0 orthogonally.
- The lengths of the tangents from P to two circles are equal then the locus of P is called the radical axis of the two circles. The radial axis of two circles is a straight line perpendicular to the line joining to the centres of the two circles.
- The equation of the radical axis of the circles s = 0 and s^1=0 is s - s^1 = 0
- If two circles intersect each other then their common chord is radical axis of the two circles.
- If two circles touch each other then their radical axis is the common tangent at the point of contact.

- The locus of the centre of a circle which cuts the given 2 circles orthogonally is the radical axis of the given two circles.
- The point of intersection of the radical axes of three circles whose centres are not collinear is called the radical centre of the three circles.
- The lengths of the tangents from the radical centre of 3 circles to the 3 circles are equal.
- If P is the radical centre of three circles and PA is the length of the tangent from P to one of the three circles then the circle whose centre is P and radius is PA cuts the three circles orthogonally.
- The radical centre of the three circles described on the sides of a triangle as diameters is the orthocentre of the triangle.
- If A, B, C are the centres of three circles which cut each other orthogonally then the radical centre of the three circles is the ortho centre of the triangle ABC.
- If A,B,C are the centres of three circles which touch each other externally then the radical centre of the 3 circles is the in - centre of the triangle ABC.
- In a system of circles if every pair of circles has the same radical axis then that system of circles is called a coaxal system of circles.
- In the coaxal system of circles the radical axis of any two circles is called the common radical axis of the coaxal system.
- In the coaxal system of circles the centres of all the circles are collinear and this line is called the line ofcentres of the circles of the coaxal system.
- In the coaxal system of circles the common radical axis is perpendicular to the line of centres of the circles of the coaxal system.
- The point circles of a coaxal system are called limiting points of the coaxal system.
- The limiting points of the coaxal system lie on the line of centres of the circles of the coaxal system.
- If 2 circles of a coaxal system intersect each other then that coaxal system has no limiting point.
- If two circles of a coaxal system touch other than that coaxal system has only one limiting point.
- If two circles of a coaxal system do not intersect then that coaxal system has two limiting points. These 2 limiting points lie on opposite sides of the common radical axis of the coaxal system.
- If (0,0) is one limiting point and $x^2+y^2+2gx+2fy+c=0$ is one circle of a coaxal system then the other limiting point is $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$
- If (x_1, y_1) is a limiting centres $C1 \& C2$ and radius a cut each other orthogonally. Then $a=\frac{C1\,C2}{\sqrt{2}}$.
- The circle with common tangent of two circles of a coaxial system as diamater passes through the limiting points of the coaxial system (i.e). The common tangent of any two circles of a coaxial system subtends a right angle at each of the limiting points of the coaxial system.
- If a coaxal system has two limiting points then those two points are inverse points with respect to every circle belonging to the coaxal system.
- In a coaxal system if we take the line of centres of the circles as x - axis and the common radical axis as y - axis then the equation of the coaxal system in the simplest form is $x^2+y^2+2\lambda x+c=0$ where λ is a variable. The limiting points of the above coaxal system are $(\pm\sqrt{c}, 0)$.
- The equation of the circle which cuts every circle of the coaxal system $x^2+y^2+2\lambda\ \mathrm{x}+\mathrm{c}=0$ (λ is a variable) orthogonally is $x^2+y^2+2fy-c=0$, (f is variable)
- The equation $x^2+y^2+2fy-c=0$ where f is a variable represents a coaxal system of circles. Its line of centres is the y - axis and common radical axis is the x - axis. Its limiting points are $(0, \pm\sqrt{-c})$
- In two coaxal systems if every circle of one coaxal system cuts every circle of the other coaxal system orthogonally then those two coaxal systems are called orthogolal coaxal systems or conjugate coaxal systems.

- If $s = 0$ and $s^1=0$ are two circles of a coaxal system then the equation of the coaxal system is $S+\lambda S^1=0$ where λ is a variable.
- If $c < 0$ then the circles of the coaxal system $x^2+y^2+2\lambda x+c=0$ intersect each other.
- If $c = 0$ then the circles of the coaxal system $x^2+y^2+2\lambda x+c=0$ touch at the origin.
- If $c > 0$ then the circles of the coaxal system $x^2+y^2+2\lambda x+c=0$ do no intersect.
- The equation of a tangent to the circle $x^2+y^2+2gx+2fy+c=0$ and having slope 'm' is $y+f=m(x+g)\pm\sqrt{g^2+f^2-c}\ \sqrt{1+m^2}$.
- Two circles whose radii are r_1 and r_2 and whose distance between the centres is 'd' cut each other orthogonally. Then the length of their common chord is $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$.
- If θ is the angle between two circles with centres r_1 and r_2 then length of the common chord is $\frac{2r_1r_2\sin\theta}{\sqrt{r_1^2+r_2^2+2r_1r_2\cos\theta}}$.
- Let L, L^1 are the limiting points of a coaxal system. C be the centre of a circle systen and "r" be the radius of the circle then $LL^1=2\sqrt{CM^2-r^2}$, where CM is the perpendicular distance from C to the radical axis.

CONCEPTUAL QUESTIONS

1. Radical axis exists for
   1) any two circles
   2) any two concentric circles
   3) any two non-concentric circles 4) Can't say
2. If A, B, C are the centres of three circles which touch each other externally then the radical centre of the three circles is .... of the Δ ABC
   1) ortho centre 2) Circum centre
   3) Centriod 4) in-centre
3. The radical centre of three circles described on the sides of a triangle as diameters is .... of the Δ ABC
   1) Ortho centre 2) Centroid
   3) in-centre 4) Circumcentre
4. $S = 0$ and $S^1 = 0$ are the equations of the two circles. If $\lambda+\lambda^I=0$ then the equation $\lambda s+\lambda^1 s^1=0$ represents
   1) the common tangent of the circles $s = 0$ and $s^1=0$
   2) the line perpendicular to the line joining centres of the circles 3) a circle
   4) the line parallel to the line joining the centres of the circles
5. A, B, C are the centres of three circles which cut each other orthogonally. The radical centre of the three circles is .... of the ΔABC
   1) Incentre 2) Centroid
   3) Ortho centre 4) Circum centre
6. If two circles cut a third circle orthogonally then the radical axis of the two circles passes through
   1) Radical centre 2) Origin
   3) Centre of the third circle
   4) cannot be determined
7. If the radical axis of the circles with centres C_1, C_2 bisects $\overline{C_1C_2}$ then their radii are
   1) unequal 2) can not say
   3) equal 4) one is square of other
8. The locus of a point which moves such that the sum of the squares of its distances from three vertices of a triangle ABC is constant is a circle whose centre is at the
   1) centroid of triangle ABC
   2) Circumcentre of triangle ABC
   3) Orthocentre of triangle ABC
   4) incentre of triangle ABC
9. If three circles are mutually orthogonal and their centres are the vertices of a triangle then the radical centre of the three circles is .... of the triangle
   1) circumcentre 2) orthocentre
   3) incentre 4) centroid
10. The polars of a point on the R.A. of a coaxal system meet on
    1) the line of centre 2) at infinity
    3) the R.A
    4) at te point of intersection of radical axis and line of centres.
11. The locus of a point the lengths of the tangents from which to the circles are in a constant ratio is a circle
    1) cuting them orthogonally
    2) touching each other externally
    3) coaxal with them
    4) lying inside the circles

12. The locus of a point such that difference of the squares of the tangents from it to two given circles is constant, is given by
1) A circle
2) A line perpendicular to radical axis
3) A line parallel to radical axis
4) A pair of straight lines

13. $r_1,\ r_2$ are the radii of two non - intersecting circles A, B as centres. If P is the centre of AB then the perpendicular distance of P from their radical axis is
1) $\frac{r_1^2+r_2^2}{2AB}$ 2) $\left|\frac{r_1^2-r_2^2}{2AB}\right|$ 3) $\left|\frac{2AB}{r_1^2-r_2^2}\right|$ 4) AB

14. The locus of the centres of the circles which touch the given two circles externally is
1) Pair of lines 2) Ellipse
3) Hyperbola 4) Circle

15. 'O' is the origin and $A_k\left(x_k,\ y_k\right)$ where k = 1.2 are two points. If the circles are described on OA_1 and OA_2 as diametres, then the length of their common chord is equal to
1) $\left|x_1y_2-x_2y_1\right|$ 2) $\frac{1}{2}\left|x_1y_2-x_2y_1\right|$
3) $\frac{1}{2}A_1A_2$ 4) $\left|x_1y_2-x_2y_1\right|/A_1A_2$

16. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g_1x+2f_1y+c_1=0$ then
1) $2g_1\left(g-g_1\right)+2f_1\left(f-f_1\right)=c-c_1$
2) $2g_1\left(g-g_1\right)+2f_1\left(f-f_1\right)+c-c_1=0$
3) $g_1\left(g-g_1\right)+f_1\left(f-f_1\right)=c-c_1$
4) $2g\left(g-g_1\right)+2f_1\left(f-f_1\right)=c-c_1$

17. If a circle cuts three circles $S=0$, $S^1=0$ and $S^{11}=0$ orthogonally then its equation is $\lambda_1S+\lambda_2S^1+\lambda_3S^{11}=0$
1) $\lambda_1S+\lambda_2S^1+\lambda_3S^{11}=0$
2) $\lambda_1S+\lambda_2S^1=0$
3) $\lambda_1S^1+\lambda_2S^{11}=0$ 4) $\lambda_1S+\lambda_2S^{11}=0$

18. Two circles of radii r and R intersect at an acute angle θ. The length of their common chord is
1) $\frac{2rR\sin\theta}{\sqrt{r^2+R^2-2rR\cos\theta}}$
2) $\frac{2rR\sin\theta}{\sqrt{r^2+R^2}}$
3) $\frac{2rR\sin\theta}{\sqrt{R^2+r^2}}$
4) $\frac{2rR\sin\theta}{\sqrt{r^2+R^2+2rR\cos\theta}}$ ..

KEY

01. 3	02. 4	03.1	04.2	05.3
06.3	07.3	08.1	09.3	10.3
11.3	12.3	13.2	14.3	15.4
16.1	17.1	18.4		

LEVEL-1

1. The length of the transverse common tangent of the circles $x^2+y^2-2x+4y+4=0$ and $x^2+y^2+4x-2y+1=0$ is
1) $\sqrt{17}$ 2) 3 3) 9 4) $\sqrt{15}$

2. If the distance between the centres of two circles of radii 3, 4 units is 25, then the length of their transverse common tangent is
1) 24 2) 12 3) 26 4) 13

3. The length of the direct common tangent of the circles $x^2+y^2-4x-10y+28=0$ and $x^2+y^2+4x-6y+4=0$ is
1) 2 2) 4 3) $\sqrt{20}$ 4) 16

4. The number of common tangents that can be drawn to the circles $x^2+y^2-12x+8y+48=0$ and $x^2+y^2-4x+2y-4=0$ is
1) 0 2) 4 3) 2 4) 3

5. The circles $x^2+y^2+4x-2y+4=0$ and $x^2+y^2-2x-4y-20=0$
1) intersect each other
2) touch each other externally
3) touch each other internally
4) are such that one circle lies entirely inside another circle.

6. If the circles $x^2+y^2-4x-6y-12=0$ and $5(x^2+y^2)-8x-14y-32=0$ touch each other then the point of contact of the circles is
1) (1, -1) 2) (-1,1) 3) (1,2) 4) (-1,-1)

7. If the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other then $1/c =$

1) $a^2 + b^2$ 2) $\frac{1}{a}+\frac{1}{b}$ 3) $\frac{1}{a^2}+\frac{1}{b^2}$ 4) $a + b$

8. If the circles $x^2 + y^2 + kx + y = 0$ and $x^2 + y^2 + 4x - 2y = 0$ touch each other then $k =$

1) -2 2) 2 3) -4 4) $\frac{-1}{2}$

9 If the circles $(x+1)^2 + (y-1)^2 = a^2$ and $x^2 + y^2 - 4x + 6y - 3 = 0$ have three common tangents only then $a^2 - 2a + 1 =$

1) 1 2) 0 3) 64 4) -1

10. If the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 + 2ax = 0$ touch each other then

1) $a^2 = c^2$ 2) $2a^2 = c$ 3) $4c^2 = a$ 4) $4a^2 = c^2$

11 If the circles $x^2 + y^2 + 2ax + 2by + c = 0$ and $x^2 + y^2 + 2bx + 2ay + c = 0$ touch each other then

1) $(a - b)^2 = 2c$ 2) $a + b = 2c$

3) $(a + b)^2 = 2c$ 4) $(a + b)^2 = c$

12 The centre of the circle of radius 2 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at (-1,-1) is

1) $\left(\frac{1}{5},\frac{3}{5}\right)$ 2) $\left(\frac{-1}{5},\frac{3}{5}\right)$ 3) $\left(\frac{-1}{7},\frac{1}{7}\right)$ 4) $\left(\frac{3}{5},\frac{1}{5}\right)$

13. If the circles $x^2 + y^2 + 5(2x + 1) = 0$ and $x^2 + y^2 + 5(y + 1) = 0$ touch each other then the point of contact is

1) (1, 2) 2) (1,-2) 3) (-1,2) 4) (-1, -2)

14. If the circles $x^2 + y^2 + 2x + c = 0$ and $x^2 + y^2 + 2y + c = 0$ touch each other then $c =$

1) 1/2 2) 1/4 3) 2 4) 4

15. The triangle formed by the common tangents of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 - 6x = 0$ is

1) an isosceles triangle 2) an equilateral triangle

3) a scalene triangle 4) a right angled triangle

16. If the number of common tangents of the circles $x^2 + y^2 + 8x + 6y + 21 = 0$, $x^2 + y^2 + 2y - 15 = 0$ are 2 ,then the point of their intersection is

1) (-4, -3) 2) (-8, -5) 3) (8, -5) 4) (8, 5)

17. If the circles $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 + 2x + 2y - 7 = 0$ touch each other, then the equation of the common tangent is

1) $3x + 4y - 8 = 0$ 2) $4x - 3y - 20 = 0$

3) $4x + 3y - 15 = 0$ 4) $3x + 4y - 7 = 0$

18 A is a point on the circle $x^2 + y^2 - 8x - 8y + 28 = 0$, B is a point on the circle $x^2 + y^2 - 2x - 3 = 0$. If the distance between A and B is 'd' then

1) $1 \le d \le 9$ 2) $2 \le d \le 8$3) $1 \le d \le 5$ 4) $3 \le d \le 6$

19 The number of circles passing through the points (0,0), (1,0) and touching the circle $x^2 + y^2 = 9$ is

1) 1 2) 2 3) 3 4) 4

20. The centre of the circle which passes through $(0,0),(1,0)$ and touches the circle $x^2 + y^2 = 9$ is

1) $\left(-2,\sqrt{2}\right)$ 2) $\left(\frac{1}{2},-2\right)$

3) $\left(\frac{1}{2},-\sqrt{2}\right)$ 4) $\left(\frac{1}{2},2\right)$

21. The length of the common chord of the circles $x^2 + y^2 + 6x + 5 = 0$ and $x^2 + y^2 + 4y - 5 = 0$ is

1) $\sqrt{\frac{12}{13}}$ 2) $\frac{12}{\sqrt{13}}$ 3) $\frac{\sqrt{12}}{13}$ 4) $\sqrt{\frac{13}{12}}$

22. The centre of the circle which cuts the three circles $x^2 + y^2 = 9$, $x^2 + y^2 - 2x + 4 = 0$ and $x^2 + y^2 - 4y + 5 = 0$ orthogonally is

1) (-13, -7) 2) (-13/2, -7/2)

3) (13/2, 7/2) 4) (13, 7)

23. The length of the common chord of the circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$ is

1) $\frac{ab}{\sqrt{a^2+b^2}}$ 2) $\frac{2ab}{a^2+b^2}$ 3) $\frac{ab}{a+b}$ 4) $\frac{2ab}{\sqrt{a^2+b^2}}$

24. If 3, 4 are the radii and 5 is the distance between the centres of two intersecting circles then the length of the common chord of the circles is

1) 12/5 2) 24/25 3) 24/5 4) 5/24

25. The length of the common chord of the circles $x^2 + y^2 + px = 0$ and $x^2 + y^2 + qy = 0$ is

1) $\frac{2pq}{\sqrt{p^2+q^2}}$ 2) $\frac{pq}{2\sqrt{p^2+q^2}}$

3) $\frac{pq}{\sqrt{p^2+q^2}}$ 4) $\frac{pq}{p^2+q^2}$

26. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ then the length of the common chord of the circles is

1) $2\sqrt{g_1^2+f_1^2-c_1}$ 2) $\sqrt{g_1^2+f_1^2-c_1}$

3) $\sqrt{g^2+f^2-c}$ 4) $2\sqrt{g^2+f^2-c}$

27. Two circles of radii 3, 4 intersect orthogonally. Then the length of the common chord is

1) 12/5 2) 24/25 3) 24/5 4) 25/24

28. The radius of one circle is twice the radius of another circle whose centres are (2, 0), (1, 2) respectively cutting orthogonally. Then the radius of the first circle is

1) 1 2) 2 3) 3 4) 5

29. The circle $2x^2 + 2y^2 + px + 6y - 10 = 0$ and $3x^2 + 3y^2 + 15x + py + 21 = 0$ are orthogonal then p =

1) 7/8 2) 5/8 3) 8/7 4) 8/5

30. Length of common tangents of the circles $x^2 + y^2 = 6x$, $x^2 + y^2 + 2x = 0$ are

1) $\sqrt{3}$ 2) $\sqrt{3}, 3\sqrt{3}$

3) $2\sqrt{3}$ 4) $2\sqrt{3}, 3\sqrt{3}$

31. If the circles $x^2 + y^2 + 2gx + 2fy = 0$, $x^2 + y^2 + 2g^1x + 2f^1y = 0$ touch each other then

1) $fg = f^1g^1$ 2) $fg^1 = f^1g$

3) $f + g = f^1 + y^1$ 4) $f + f^1 = g + g^1$

32. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points then

1) r < 2 2) r = 2 3) r > 2 4) 2 < r < 8

33. If radii of two circles are 4 and 3 and distance between centres is $\sqrt{37}$ then angle between the circles is

1) 30^0 2) 45^0 3) 60^0 4) 90^0

34. The circle with centre (2, 3) and intersecting $x^2 + y^2 - 4x + 2y - 7 = 0$ orthogonally has the radius

1) 1 2) 2 3) 3 4) 4

35. The equations of two circles are $x^2 + y^2 + 2\lambda x + 5 = 0$ and $x^2 + y^2 + 2\lambda y + 5 = 0$. P is any point on the line $x - y = 0$. If PA and PB are the lengths of the tangents from P to the two circles and PA = 3 then PB = ...

1) 1.6 2) 6 3) 4 4) 3

36. The angle betweenl the circles $x^2 + y^2 - 4x - 6y - 3 = 0, x^2 + y^2 + 8x - 4y + 11 = 0,$

1) $\pi/2$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/12$

37. If the cirlces $x^2 + y^2 + 2a^1x + 2b^1y + c^1 = 0$ and $2x^2 + 2y^2 + 2ax + 2by + c = 0$ intersect orthogonally, then

1) $aa^1 + bb^1 = c + c^1$ 2) $aa^1 + bb^1 = c + \frac{c^1}{2}$

3) $aa^1 + bb^1 = \frac{c}{2} + c^1$ 4) $2(aa^1 + bb^1) = c + c^1$

38. If two circles $a(x^2 + y^2) + bx + cy = 0$ and $A(x^2 + y^2) + Bx + Cy = 0$ touch each other, then

1) aC = cA 2) bC = cB

3) aB = bA 4) aA = bB = cC

39. The length of the common chord of the two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy - c = 0$ is

1) $2\sqrt{\frac{(g^2 - c)(f^2 - c)}{g^2 + f^2}}$

2) $2\sqrt{\frac{(g^2 + c)(f^2 + c)}{g^2 + f^2}}$

3) $2\sqrt{\frac{(g^2 - c)(f^2 + c)}{g^2 + f^2}}$

4) $2\sqrt{\frac{(g^2 + c)(f^2 - c)}{g^2 + f^2}}$

40. Two equal circles with thier centres on x- and y-axis will posses the radical axis in the following form

1) $ax - by - \frac{a^2 + b^2}{4} = 0$

2) $2gx - 2fy + f^2 - g^2 = 0$

3) $g^2x + f^2y - g^4 - f^4 = 0$

4) $2g^2x + 2f^2y - g^4 - f^4 = 0$

41. The number of points such that the tangents from it to three given circles are euqal in length, is

1) 1 2) 2 3) 3 4) 4

42. If $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ are the three circles whose radical centre is the point P, then the lengths l_1, l_2, l_3 of the tangents form P to the three circles are such that

1) $l_1 = 2l_2 = 3l_3$ 2) $l_1 = l_2 = l_3$

3) $l_1 \neq l_2 \neq l_3$ 4) $l_1 = l_2 \neq l_3$

KEY

01. 2	02. 1	03. 2	04. 4	05. 4
06. 4	07. 3	08. 1	09. 2	10. 4
11. 3	12. 1	13. 4	14. 1	15. 2
16. 2	17. 1	18. 1	19. 2	20. 3
21. 2	22. 3	23. 4	24. 3	25. 3
26. 1	27. 3	28. 2	29. 3	30. 3
31. 2	32. 4	33. 3	34. 2	35. 4
36. 3	37. 3	38. 2	39. 3	40. 2
41. 1	42. 2			

HINTS

1. Use the formula $\sqrt{d^2-(r_1+r_2)^2}$
6. Find the radical axis and verify the options
9. Circles are touch externally
12. (-1, -1) divides line joining centres of the given circles externally in the ratio of their radii
15. Ratio of radii is equal to 3:1 therefore tangents form eqilateral triangle
18. D is the distance between the centres of the circles : $D-(r_1+r_2) \leq d \leq D+(r_1+r_2)$
19. Draw diagram of the given circle and represent points (0,0) and (1,0)
20. Radius of required cirlce is 3/2 distances between (0,0) to 3rd opiton and (1,0) to 3rd option is 3/2

LEVEL- 2

1. If the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + K = 0$ touch internally then K =
   1) - 24 2) 24 3) 17 4) 10
2. The two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy - c = 0$ touch each other if g^2 =
   1) c 2) 2c 3) 3c 4) 4c
3. If the circle $x^2 + y^2 + ax + c = 0$ lies inside the circle $x^2 + y^2 + bx + c = 0$ then
   1) ab < 0 and c > 0 2) ab < 0 and c < 0
   3) ab > 0 and c > 0 4) ab > 0 and c < 0
4. The equation to the circle passing through (0, 0) (1,0) and touching the circle $x^2 + y^2 = 9$ is
   1) $x^2 + y^2 - x - 2\sqrt{2}y = 0$
   2) $x^2 + y^2 + x - \sqrt{2}y = 0$
   3) $x^2 + y^2 + x + 2\sqrt{2}y = 0$
   4) $x^2 + y^2 + x - 2\sqrt{2}y = 0$
5. If a circle of radius 3 units rolls inside of the circle $(x - 2)^2 + (y -1)^2 = 25$ then the locus of the centre of that circle is a circle of diameter
   1) 2 2) 8 3) $2\sqrt{2}$ 4) 4
6. If the circles $(x - a)^2 + (y- b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ touch each other then $(a + b)^2$ =
   1) $2c^2 + 4ab$ 2) $4c^2$
   3) $2c^2$ 4) $2c^2 + 2ab$
7. If the circles $(x - a)^2 + (y - b)^2 = r^2$, $(x - b)^2 + (y - a)^2 = r^2$ have three common tangents then
   1) $(a - b)^2 = 2r^2$ 2) $(a + b)^2 = 2r^2$
   3) $a^2 + b^2 = 2r^2$ 4) $a^2 + b^2 = r^2$
8. A circle C of radius 2 units rolls inside the rim of the circle $x^2 + y^2 + 8x - 2y - 19 = 0$. Then the locus of the centre of C is
   1) $x^2 + y^2 + 8x - 2y - 47 = 0$
   2) $x^2 + y^2 + 8x - 2y - 1 = 0$
   3) $x^2 + y^2 + 8x - 2y + 1 = 0$
   4) $x^2 + y^2 - 8x + 2y + 1 = 0$
9. If the locus of the centre of a circle which touches the line $x\cos\alpha + y\sin\alpha = p$ and the circle $(x - a)^2 + (y - b)^2 = c^2$ is $(x - a)^2 + (y-b)^2 = (x\cos\alpha + y\sin\alpha + k)^2$ then k =
   1) p 2) $-p \pm c$ 3) pc 4) - p
10. The circle $(x-2)^2 + (y-5)^2 = a^2$ will be inside the circle $(x - 3)^2 + (y - 6)^2 = b^2$ if
    1) $b > a + \sqrt{2}$ 2) $a - b < \sqrt{2}$
    3) $a < \sqrt{2} - b$ 4) $a + b > \sqrt{2}$
11. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points then
    1) r < 2 2) r = 2 3) r > 2 4) 2 < r < 8
12. The point of contact of $x^2 + y^2 + 4x + 2y - 4 = 0$ and $x^2 + y^2 + 2y = 0$ is
    1) (-1/2, -1) 2) (1, -1)
    3) (2, -2) 4) (1, 1)
13. The centre of similitude of the two circles $x^2 + y^2 + 4x + 2y - 4 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is
    1) (4,1) 2) (1,-4) 3) (1,4) 4) (4, 2)
14. If the circles $x^2 + y^2 + 2ax + 4ay + 3a^2 = 0$ and $x^2 + y^2 - 8ax - 6ay - 7a^2 = 0$ touch each other externally then the point of contact is
    1) $\left(0, \frac{a}{2}\right)$ 2) (0, a) 3) (0, 2a) 4) (0, -a)
15. The internal centre of similitude of the circles $x^2 + y^2 - 2x + 4y + 4 = 0$ and $x^2 + y^2 + 4x-2y+1= 0$ is
    1) (0,1) 2) (-1,0) 3) (0,-1) 4) (1,-1)

16 If the external centre of similitude of the circles $x^2 + y^2 - 2x- 4y + 4 = 0$ and $x^2 + y^2 + 4x- 2y+1 = 0$ is Q then Q =
   1) (-4,-3) 2) (4,3) 3) (4,-3) 4) (-4,3)

17. If $\left(\frac{1}{3},1\right)$ and Q are centres of similtude of two circles whose centres are (1,3) and (0,0) then Q =

1) (1,3) 2) (-1,-3) 3) (1,-3) 4) (-1,3)

18. If $\left(0,\frac{5}{2}\right)$ is a centre of similitude for the circles $x^2 + y^2 + 6x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 6y + 9 = 0$ then the length of the common tangent of the circles through it is

1) 6 2) 3 3) 2 4) 1

19. The circle cutting $x^2+y^2-6x+4y-12=0$ orthogonally and having centre (-1, 2) is

1) $x^2+y^2+2x-4y-2=0$

2) $x^2+y^2+2x-4y+2=0$

3) $x^2+y^2-2x+4y-2=0$

4) $x^2+y^2+2x-4y-4=0$

20. The circle through origin and cutting $x^2+y^2+6x-15=0$, $x^2+y^2-8y+10=0$ orthogonally is

1) $2x^2+2y^2-10x-5y=0$

2) $2x^2+2y^2+10x+5y=0$

3) $x^2+y^2-5x+5y=0$

4) $2x^2+2y^2+10x-5y=0$

21. A circle passes through origin and has its centre on x = y and cuts $x^2+y^2-4x-6y+10=0$ orthogonally, then its equation is

1) $x^2+y^2-x-y=0$

2) $x^2+y^2-4x-4y=0$

3) $x^2+y^2-2x-2y=0$

4) $x^2+y^2-3x-3y=0$

22. The circle through (-2, 5), (0, 0) and intersecting the circle $x^2+y^2-4x+3y-1=0$ orthogonally is

1) $2x^2+2y^2-11x-16y=0$

2) $x^2+y^2-4x-4y=0$

3) $x^2+y^2+2x-5y=0$

4) $x^2+y^2+2x-5y+1=0$

23. The radical axis of the circles $x^2+y^2+4x-6y=12$ and $x^2 + y^2 + 2x -2y - 1 = 0$ divides the line segement joining the centres of the circles in the ratio

1) 27 : 17 2) 3 : 7

3) -27 : 17 4) -3 : 7

24. The locus of the centre of a circle which cuts the circles $2x^2 + 2y^2 - x - 7 = 0$ and $4x^2 + 4y^2 - 3x - y = 0$ orthogonally is a straight line whose slope is

1) -1 2) 1 3) - 2 4) $\frac{-5}{2}$

25. If C_i is the centre of the circle $x^2+y^2+2g_ix+5=0$ and t_i is the length of the tangent from any point to this circle, $i=1, 2, 3$; then the points $\left(g_1, t_1^2\right)$, $\left(g_2, t_2^2\right)$ and $\left(g_3, t_3^2\right)$ are

1) Collinear 2) not collinear

3) either collinear or non collinear

4) not defined

26. The perpendicular distance from the origin to the radical axis of the circles $2x^2 + 2y^2 - 3x - y + 3 = 0$ and $3x^2 + 3y^2 - x + y - 1 = 0$ is

1) $\sqrt{2}$ 2) $\frac{11}{\sqrt{74}}$ 3) $\frac{\sqrt{5}}{2}$ 4) $\sqrt{\frac{5}{2}}$

27. If the circles $x^2 + y^2 - 10x + 2y + 10 = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ touch each other then the slope of the common tangent at the point of contact of the circles is

1) 3/4 2) 4/3 3) -4/3 4) -3/4

28. The slope of the radical axis of the circles $(x + 2)^2 + (y + 3)^2 = 25$ and $(x + 1)^2 + (y - 1)^2 = 25$ is

1) -1/4 2) 1/4 3) - 4 4) -1/2

29. (a, c) and (b, c) are the centres of two circles whose radical axis is the y-axis. If the radius of first circle is r then the diameter of the other circle is

1) $2\sqrt{r^2-b^2+a^2}$ 2) $\sqrt{r^2-a^2+b^2}$

3) $2(r^2-b^2+a^2)$ 4) $2\sqrt{(r^2-a^2+b^2)}$

30. The radii of two circles are 2 units and 3 units. If the radical axis of the circles cuts one of the common tangents of the circle in P then ratio in which P divides the common tangent is

1) 2 : 3 2) 3 : 2 3) 4 : 9 4) 1 : 1

31. The distance of the point (1, 2) from the common chord of the circles $x^2 + y^2 + 6x - 16 = 0$ and $x^2 + y^2 - 2x - 6y - 6 = 0$ is

1) 1 2) 1/5 3) 5 4) 2

32. If $ax + by + c = 0$ is the equation of the common radical axis of the coaxial system $(3x^2 + 3y^2 - 2x - 2y - 1) + \lambda(x^2 + y^2 - x - 2y - 3) = 0$ then $ab + bc + ca =$
1) 44 2) 13 3) 40 4) 12

33. The distance of (1, -2) from the common chord of $x^2 + y^2 - 5x + 4y - 2 = 0$ and $x^2 + y^2 - 2x + 8y + 3 = 0$ is
1) 2 2) 1 3) 0 4) 3

34. The equation of the circle which passes through the point (2a, 0) and whose radical axis with the circle $x^2 + y^2 = a^2$ is the line $x = a/2$ is
1) $x^2 + y^2 - 2ax = 0$ 2) $x^2 + y^2 + 2ax = 0$
3) $x^2 + y^2 - ax = 0$ 4) $x^2 + y^2 + ax = 0$

35. A and B are two points on the circle $x^2 + y^2 = 1$. If the x co-ordinates of A and B are the roots of the equation $x^2 + ax + b = 0$ and the y- coordinates of A and B are the roots of the equation $y^2 + by + a = 0$ then the equation of the line AB is
1) $ax + by = 0$ 2) $ax + by + 1 = 0$
3) $bx + ay + a + b = 0$ 4) $ax + by + a + b + 1 = 0$

36. If (1,2),(4,3) are the limiting points of coaxal system,then the equation of the circle in its conjugate system having minimum area is
1) $x^2 + y^2 - 5x - 5y + 10 = 0$
2) $x^2 + y^2 - 8x - 6y + 25 = 0$
3) $x^2 + y^2 - 2x - 2y + 10 = 0$
4) $x^2 + y^2 + 5x + 5y - 10 = 0$

37. If the length of the radical axis of two circles $x^2 + y^2 + 8x + 1 = 0$ and $x^2 + y^2 + 2\mu y - 1 = 0$ is $2\sqrt{6}$ then $\mu =$.......
1) ± 2 2) ± 4 3) ± 8 4) ± 3

38. From the point O (2, 3), tangents OP, OQ are drawn to circle $x^2 + y^2 = 1$. Equation to the line joining the midpoint of OP and OQ is........
1) $2x + 3y = 7$ 2) $3x + 2y = 7$
3) $x + 2y = 3$ 4) $2x + y = 3$

39. B and C are points on the circle $x^2 + y^2 = a^2$. A point A(b, c) lies on that circle such that $AB = AC = d$. The equation to BC is.........
1) $bx + ay = a^2 - d^2$ 2) $bx + ay = d^2 - a^2$
3) $bx + cy = 2a^2 - d^2$ 4) $2(bx + cy) = 2a^2 - d^2$

40. The locus of the centre of the circles which intersects the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x + y = 0$ orthogonally is
1) a line whose equation is $2x - y - 1 = 0$
2) a line whose equation is $2x + y = 1$
3) a circle 4) a pair of lines

41. The locus of the centre of a circle which cuts orthogonally the circle $x^2 + y^2 - 20x + 4 = 0$ and touches the line $x = 2$ is
1) $y^2 = 16x + 4$ 2) $x^2 = 16y + 4$
3) $x^2 = 16y$ 4) $y^2 = 16x$

42. The length of the common chord of the circles $(x-a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ is
1) $\sqrt{4c^2 - 2(a-b)^2}$ 2) $\sqrt{2c^2 - (a-b)^2}$
3. $\sqrt{4c^2 - (a-b)^2}$ 4. $\sqrt{4c^2 + 2(a-b)^2}$

43. If the radical centre of the three circles $x^2 + y^2 - 2x - 1 = 0$, $x^2 + y^2 - 3y = 1$ and $2x^2 + 2y^2 - x - 7y - 2 = 0$ is Q then $Q_x + Q_y =$
1) 3 2) 0 3) 1 4) -1

44. The radical centre of the three circles $x^2 + y^2 = 9$, $x^2 + y^2 - 2x - 2y - 5 = 0$ and $x^2 + y^2 + 4x + 6y - 19 = 0$ is
1) (1, -1) 2) (1, 2) 3) (1, 1) 4) (-1, -1)

45. If Q is the radical centre of the three circles $x^2 + y^2 = a^2$, $(x - g)^2 + y^2 = a^2$ and $x^2 + (y - f)^2 = a^2$ then $Q_x + Q_y =$
1) $g + f$ 2) $\frac{g+f}{2}$ 3) $2g + 2f$ 4) $\frac{-g-f}{2}$

46. The point from which the lengths of tangents to the three circles $x^2 + y^2 - 4 = 0$, $x^2 + y^2 - 2x + 3y = 0$ and $x^2 + y^2 + 7y - 18 = 0$ are equal is
1) (2, 5) 2) (3, 4) 3) (4, 3) 4) (5, 2)

47. If the radical centre of $x^2 + y^2 - 4x + 2y + 3 = 0$, $x^2 + y^2 - x + 4y + 4 = 0$ and $x^2 + y^2 + 2gx + 5y + 7 = 0$ is (-1, 1) then $g =$
1) -3 2) 3 3) -3/2 4) 3/2

48. The radical centre of the circles $(x - 1)^2 + (y - 2)^2 = 341$, $(x - 4)^2 + (y - 1)^2 = 341$, $(x - 5)^2 + (y - 4)^2 = 341$ is
1) (3, 3) 2) (4,1) 3) (6, 6) 4) $\left(\frac{10}{3}, \frac{7}{3}\right)$

49. The equation of the circle which cuts the three circles $x^2 + y^2 = a^2$, $(x - g)^2 + y^2 = a^2$ and $x^2 + (y - f)^2 = a^2$ orthogonally is
1) $x^2 + y^2 - 2gx - 2fy + a^2 = 0$
2) $x^2 + y^2 - gx - fy + a^2 = 0$

3) $x^2 + y^2 - fx - gy + a^2 = 0$
4) $x^2 + y^2 + gx + fy - a^2 = 0$

50. $x = 1$ is the radical axis of two circles which cuts each other orthogonally. If $x^2 + y^2 - 8x + 4 = 0$ is the equation of one circle then the radius of the other circle is
1) 4 2) 2 3) 6 4) 3

51. The centre of the circle cutting $x^2 + y^2 - 2x + 4y - 1 = 0$ orthogonally and passing through (0, 0), (2, 0) is
1) (3/2, 1) 2) (1, 3/4) 3) (1, -3/4) 4) (-1, -3)

52. If the line $x \cos\alpha + y \sin\alpha = p$ and the circle $x^2 + y^2 = a^2$ intersect at A and B then the equation of the circle on AB as diameter is $(x^2 + y^2 - a^2) + k(x \cos\alpha + y \sin\alpha - p) = 0$ then k =
1) p 2) -p 3) -4p 4) - 2p

53. The line $2x + 3y = 1$ intersects the circle $x^2 + y^2 = 4$ at A and B. If the equation of the circle on AB as diameter is $x^2 + y^2 + 2gx + 2fy + c = 0$ then c=
1) -50 2) -54/13 3) 50/13 4) -50/13

54. The equation of the circle describes on the common chord of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 + 2y = 0$ as diameter is
1) $x^2 + y^2 - x - y = 0$ 2) $x^2 + y^2 + x - y = 0$
3) $x^2 + y^2 + x + y = 0$ 4) $x^2 + y^2 - x + y = 0$

55. The line $2x + 3y = 1$ cuts the circle $x^2 + y^2 = 4$ in P and Q. Then the equation of the circle on $\overline{PQ}$ as diameter is
1) $13(x^2 + y^2) - 4x - 6y + 50 = 0$
2) $13(x^2 + y^2) - 6y - 50 = 0$
3) $13(x^2 + y^2) - 4x - 6y - 50 = 0$
4) $13(x^2 + y^2) - 4x - 50 = 0$

56. The equation of circle passing through (0, 0) and the points of intersection of $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 + 4x - 2y - 4 = 0$ is
1) $13x^2 + 13y^2 + 20x - 42y = 0$
2) $5x^2 + 5y^2 + 52x + 6y = 0$
3) $x^2 + y^2 + 20x - 42y = 0$
4) $x^2 + y^2 - 20x - 42y = 0$

57. The equation of the circle of least radius belonging to the coaxal system of circles orthogonal to the system $x^2 + y^2 + 2\lambda x + 4 = 0$ is
1) $x^2 + y^2 = 0$ 2) $x^2 + y^2 = 4$
3) $x^2 + y^2 + 2x - 4 = 0$ 4) $x^2 + y^2 + 2\lambda x + 4 = 0$

58. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circles $x^2 + y^2 = 4$; $x^2 + y^2 - 6x - 8y + 10 = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter, then its centre is
1) (2, 3) 2) (-2, 3) 3) (2, -3) 4) (-2, -3)

59. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to
1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) None

60. The circle $x^2 + y^2 + 4x + 4y - 1 = 0$
1) cuts the circle $x^2 + y^2 + 2x - 3 = 0$ orthogonally
2) touches the circle $x^2 + y^2 + 2x - 3 = 0$
3) bisects the circumference of the circle $x^2 + y^2 + 2x - 3 = 0$
4) neither intersects nor touches $x^2 + y^2 + 2x - 3 = 0$

61. If a circle passes through the point (a, b) and cuts the circle orthogonally then the locus of its centre is
1) a circle 2) a parabola
3) an ellipse 4) a straight line

62. If the circle $3x^2 + 3y^2 + 10x + y - 27 = 0$ bisects the circumference of the circle $x^2 + y^2 = k$ then $k^2 - 1 =$
1) 27 2) 728 3) 9 4) 80

63. If the circle $x^2 + y^2 - 2x - 2y - 1 = 0$ bisects the circumference of the circle $x^2 + y^2 = 1$ then the length of the common chord of the circles is
1) 1 2) 2 3) $\sqrt{3}$ 4) $2\sqrt{3}$

64. The polars of a fixed point w.r.t.a coaxal system of circle are
1) Paralled 2) Per pendicular
3) Con current 4) Coicident

65. The circle $x^2 + y^2 - x - y - 1 = 0$
1) touches the circle $x^2 + y^2 = 1$
2) intersects the circle $x^2 + y^2 = 1$
3) cuts the circle $x^2 + y^2 = 1$ orthogonally
4) bisects the circumference of thecircle$x^2 + y^2 = 1$

66. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$ then c + d =
1) 60 2) 50 3) 40 4) 30

67. If the tangents from P(h, k) to the circles $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 + 6x - 2y + 6 = 0$ are equal then
1) $2h + 10k + 11 = 0$ 2) $2h - 10k + 11 = 0$
3) $10h - 2k + 11 = 0$ 4) $10h + 2k + 11 = 0$

68. The angle between the tangents from a point on $x^2 + y^2 + 2x + 4y - 31 = 0$ to the circle $x^2 + y^2 + 2x$

$+ 4y - 4 = 0$ is

1) $\pi/6$ 2) $\pi/2$ 3) $\pi/4$ 4) $\pi/3$

69. If the circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 3 = 0$ subtend equal angles at P, then locus of P is
1) $2x^2 + 2y^2 - 6x + 5 = 0$
2) $2(x^2 + y^2) - 6y - 1 = 0$
3) $x^2 + y^2 - 2x - 2y + 2 = 0$
4) $x^2 + y^2 - 6x + 5 = 0$

70. A transverse common tangent to the circles $x^2 + y^2 + 4x + 2y = 4$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is
1) $x = 1$ 2) $x = 2$
3) $3x + 4y + 5 = 0$ 4) $2x + 3 = 0$

71. For the given two circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$ the line $4x - 3y - 5 = 0$ is
1) A direct common tangent to the cirlces
2) An indirect common tangent to the circles
3) A tangent to the first circle only
4) A tangent to the second cricle only

72. The locus of the midpoints of chords of the circle $x^2 + y^2 = 6$ making an angle 90^0 at (1, 1) is
1) $2x^2 + 2y^2 - x - y - 4 = 0$
2) $x^2 + y^2 - x - y + 2 = 0$
3) $x^2 + y^2 - x - y - 2 = 0$
4) $4 - 2 [(x + y - 6) - (x^2 + y^2 - 6)] = 0$

73. If (0, 0) is one limiting point and $x^2 + y^2 + gx + fy + c = 0$ is one circle of a coxal system then the other limiting point is
1) $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$
2) $\left(\frac{-2gc}{g^2+f^2}, \frac{-2fc}{g^2+f^2}\right)$
3) $\left(\frac{-fc}{2(g^2+f^2)}, \frac{-fc}{2(g^2+f^2)}\right)$
4) $\left(\frac{2gc}{(g+f)}, \frac{2fc}{(g+f)}\right)$

74. If origin is one limiting point of the coaxal system $x^2 + y^2 + \lambda(2ax + 2by - a^2 - b^2) = 0$ then the other limiting point is
1) (-a, b) 2) (a, -b) 3) (b, a) 4) (a, b)

75. If (0, 0) is one limiting point and $x^2 + y^2 - 6x - 8y + 1 = 0$ is one circle of a coaxal system then the other limiting point is
1) (-2, -4) 2) $\left(\frac{-3}{25}, \frac{-4}{25}\right)$
3) $\left(\frac{3}{25}, \frac{4}{25}\right)$ 4) $\left(\frac{3}{7}, \frac{4}{7}\right)$

76 If $x^2 + y^2 + 2x + 4y + 7 = 0$ and $x^2 + y^2 + 4x + 2y + 5 = 0$ are two circles of a coaxal system then the limiting points of the coaxal systems are
1) (2, 1), (0, 3) 2) (1, 2), (0, -3)
3) (-2, -1), (0, -3) 4) (2, -1), (-3, 0)

77. If $x^2 + y^2 - 6x - 4y - 3 = 0$ is one circle and (-5, -6) is one limiting point of a coaxal system then the other limiting point is
1) (1, 2) 2) (-1, 2) 3) (2, -1) 4) (2, 1)

78. If $x + y + 4 = 0$ is the equation of the common radical axis and (2, 1) is one limiting point of a coaxal system then the other limiting point is
1) (5, 6) 2) $\left(\frac{-3}{2}, \frac{-5}{2}\right)$
3) (-6, -5) 4) (-5, -6)

79. (2, 3) and (-3, 2) are two limiting points of coaxal system. If the equation of the circle belonging to the coaxal system and passing through the point (-1, 3) is $x^2 + y^2 + 2gx + 2fy + c = 0$ then c =
1) 26 2) -13 3) 13 4) -26

80. The limiting points of the coaxal system $x^2 + y^2 + 2fy + 9 = 0$ (f is a variable) are
1) $(\pm 3, 0)$ 2) $(\pm 9, 0)$ 3) $(0, \pm 9)$ 4) $(0, \pm 3)$

81. If (4, 4) and (8, 2) are limiting points of a coaxal system then the equation of the common radical axis of the coaxal system is $ax + by + c = 0$ (a >0). Then a - b + c =
1) 6 2) - 6 3) -8 4) 10

82. If (2, 3) and (-3, 2) are the limiting points of a coaxal system whose equation is $(x^2 + y^2 - 4x - 6y + 13) + \lambda(ax + by + c) = 0$ then a + b - c =
1) 6 2) - 6 3) 4 4) - 4

83. The sum of the y-coordinates of the limiting points of the coaxal system $(x^2 + y^2 - 2x - 4y + 5) + \lambda(x^2 + y^2 - 6x - 8y + 25) = 0$
1) -6 2) 4 3) 6 4) 2

84. If (1, 2) and (3, 4) are limiting points of the given coaxal system then the least circle belonging to the orthogonal coaxal system is $x^2 + y^2 + ax + by + c = 0$. Then (a, c) =
1) (-4, 11) 2) (-6, 11) 3) (4, 11) 4) (4, -11)

85. If x = 1 is the equation of the common radical axis and (2, 3) is one limiting point of a coaxial system then the other limiting point is
1) (3, 0) 2) (0, 3) 3) (0, 1) 4) (0, -3)

86. If the limiting points of the coaxal system $x^2 + y^2$

$-2x+13+\lambda(x+y+4)=0$ where λ is a variable are (-5, -6) and P then $P_x + P_y =$

1) (2, 1) 2) 3 3) 1 4) -3

87. If (1, 2), (3, 4) are limiting points and $x^2 + y^2 - x + ky = 0$ is one circle of a coaxal system then k =

1) 3 2) - 3 3) - 9 4) 9

88. If (1, 2) is one limiting point and $x^2 + y^2 - 6x - 8y + 25 = 0$ is one circle of a coaxal system then the equation of the radical axis of the coaxal system is

1) x - y - 5 = 0 2) x + y + 5 = 0
3) x + y - 20 = 0 4) x + y - 5 = 0

89. If (0, 3) and (0, -3) are the limiting points of a coaxal system then the equation of the coaxal system is

1) $x^2 + y^2 + 2\lambda x + 9 = 0$ 2) $x^2 + y^2 + 2fy + 9 = 0$
3) $x^2 + y^2 + 2\lambda x - 9 = 0$
4) $x^2 + y^2 + 2fy - 9 = 0$

90. The number of real circles belonging to the coaxal system $x^2 + y^2 + 2\lambda x + c = 0$ (λ is a variable) and whose centre lie between the limiting points of the given coaxal system is

1) 1 2) ∞ 3) 2 4) 0

91. If the limiting points of a coaxal system are (0, 0) and (1, 0) then one member of the system is

1) $x^2 + y^2 - 6x + 3 = 0$ 2) $x^2 + y^2 - 2x - 1 = 0$
3) $x^2 + y^2 = 1$ 4) $x^2 + y^2 - 2y + 1 = 0$

92. (-2,3) is the middle point of chord AB of the circle $x^2+y^2=81$. The equation of the circle through the points A,B and (0,1) is

1) $x^2+y^2-16x+24y-23=0$
2) $x^2+y^2+16x-24y+23=0$
3) $x^2+y^2-2y+1=0$
4) $x^2+y^2-16x-24y=0$

93. The limiting points of a coaxal system whose radical axis is x + y - 1 = 0 and having $x^2 + y^2 + 2x + 4y - 1 = 0$ as one member are

1) (2, 1), (2, -3) 2) (2, 1), (0, -1)
3) (2, 1), (4, 3) 4) (2, 1), (0, 1)

94. Limiting points of the coaxal system whose radical axis is x + y - 1 = 0 and having one member as $x^2 + y^2 - 4x - 2y + 5 = 0$ are

1) (2, 1), (0, -1) 2) (2, 1), (4, 3)
3) (2, 1), (0, 1) 4) (2, 1), (-2, -3)

95. The coaxal system $x^2 + y^2 + 2gx - 8 = 0$ will have

1) two real and distinct limiting points
2) coincident limiting points
3) imaginary limiting points
4) one imaginary, one real limiting points

96. In the co-axal system of circles the common tangent of two circles subtends an angles θ_1 *and* θ_2 at the limiting points then the value of $\cos(\theta_1+\theta_2)=$

1) 0 2) 1 3) –1 4) 2

97. P (-2, -1) and Q(0, 3) are the limiting points of a coaxal system of which $C=x^2 + y^2 + 5x+ y+4 = 0$ is a member. The circle $S = x^2 + y^2 - 4x - 2y - 15 = 0$ is orthogonal to the circle C=0. The point where the polar of P with respect to C=0 cuts the circle S=0 is

1) (3, 6) 2) (-3, 6) 3) (-6, 3) 4) (6, 3)

98. The number of limiting points of orthogonal coaxal system of the coaxal system $x^2 + y^2 + 2\lambda x - 5=0$ are

1) 0 2) 1 3) 2 4) 4

99. The centre of any circle belonging to the coaxal system $x^2 + y^2 - 20x + 10y + 9 + \lambda(7x - 3y + 2) = 0$ does not lie between

1) (4, 1) (3, -2) 2) (-4, 1), (3, 2)
3) (-11, 4), (3, -2) 4) (-4, 1), (3, -2)

100. The equation of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at the point (5,5) is

1) $x^2+y^2+18x+16y-120=0$
2) $x^2+y^2-18x-16y+120=0$
3) $x^2+y^2-18x-16y-120=0$
4) $x^2+y^2+18x+16y+120=0$

101. The equation of the coaxal system which is orthogonal to the coaxal system having limiting points $(\pm a, 0)$ is

1) $x^2 + y^2 + 2\mu y - a^2 = 0$
2) $x^2 + y^2 + 2\lambda x - a^2 = 0$
3) $x^2 + y^2 + 2\mu y + a^2 = 0$
4) $x^2 + y^2 + 2\lambda x + a^2 = 0$

102. x = 1 is the radical axis of two circles which cut each other orthogonally. If $x^2 + y^2 = 9$ is the equation of one circle then the equation of the other circle is

1) $x^2 + y^2 - 9x + 9 = 0$ 2) $x^2 + y^2 + 18x - 9 = 0$
3) $x^2 + y^2 - 18x + 9 = 0$ 4) $x^2 + y^2 + 9x + 9 = 0$

103 The equation of the common radical axis of the orthogonal coaxal system of the coaxal system $(x^2 + y^2 - 4x - 6y + 5) + \lambda(2x + 3y + 4) = 0$ is

1) 3x - 2y = 0 2) 3x - 2y + 1 = 0
3) 3x + 2y - 12 = 0 4) 3x - 2y - 1 = 0

104. The equation of the line of centers of the orthogonal

coaxal system

$(4x^2 + 4y^2 - 12x + 6y - 3) + \lambda(x + 2y - 6) = 0$ is

1) $8x - 4y - 15 = 0$ 2) $x + 2y - 6 = 0$

3) $x + 2y = 0$ 4) $x - 2y = 0$

105 If (2, -1) and (-6, -7) are the limiting points of a coaxal system then the diameter of the smallest circle belonging to the conjugate coaxal system is

1) 5 2) 20 3) 10 4) 5/2

106. (-2, -1) is one limiting point and $x^2 + y^2 + 2x + 4y + 7 = 0$ is one circle of the coaxal system. The centre of the circle which passes through (-2, -1) and cuts the given circle Orthogonally lies on the line

1) $x - y - 1 = 0$ 2) $x + y - 1 = 0$

3) $x - y - 2 = 0$ 4) $x - y + 1 = 0$

107. The system of circles orthogonal to $x^2 + y^2 + 2gx + 10 = 0$ is

1) $x^2 + y^2 - 2gx - 10 = 0$

2) $x^2 + y^2 - 2fy + 10 = 0$

3) $x^2 + y^2 + 2gx + 2fy + 10 = 0$

4) $x^2 + y^2 + 2fy - 10 = 0$

108. (-2, -1) is a limiting point of a coaxal system of which $x^2 + y^2 + 2x + 4y + 7 = 0$ is a member. The equation of the orthogonal system is

1) $x^2 + y^2 + x + 3y + C/3\ (x + y + 3) = 0$

2) $x^2 + y^2 + x + 3y + C/2\ (x + y + 3) = 0$

3) $x^2 + y^2 + x + 3y + C/4\ (x + y + 3) = 0$

4) $x^2 + y^2 + x + 3y + C(x + y + 3) = 0$

109. The limiting points of the system of coaxal circles $x^2 + y^2 + 2\lambda y - 25 = 0$ are

1) $(0, \pm 25)$ 2) $(0, \pm 5)$

3) $(\pm 5, 0)$ 4) Not existing

110. The coaxal system $x^2 + y^2 + 8x - 2y + 3 + \lambda(7x - y - 21) = 0$ is

1) Intersecting 2) non-intersecting

3) touching 4) cannot be determined

111. If (1, 2), (4, 3) are the limiting points of a coaxal system, then the equation of the circle in its conjugate system having minimum area is

1) $x^2 + y^2 - 2x - 4y + 5 = 0$

2) $x^2 + y^2 - 8x - 6y + 25 = 0$

3) $x^2 + y^2 - 5x - 5y + 10 = 0$

4) $x^2 + y^2 + 5x + 5y - 10 = 0$

112. For the co-axal system $x^2 + y^2 + 4x + 2y + 1 + \lambda(x + y - 2) = 0$ line of centres is

1) $x - y + 1 = 0$ 2) $x + y + 3 = 0$

3) $2x + y - 1 = 0$ 4) $x + y = 2$

113. For the the co-axal system $x^2 + y^2 + 2\lambda x + c = 0$ lline of centres is

1) x= 0 2) y = 0 3) y = c 4) x = c

114. For a co-axal system $x^2 + y^2 + 2\mu y + c = 0$ radical axis is

1) x = 0 2) y = 0 3) y = c 4) y + c = 0

115. If A(2, 1), B(0, 5) are limiting points of a co-axal system then the radical axis of the conjugate co-axal system is

1) $2x + y - 5 = 0$ 2) $2x - y - 3 = 0$

3) $x - 2y + 5 = 0$ 4) $x + 2y - 7 = 0$

116. The equations of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$. The radius of a circle touching all the four circles externally is

1) $2\sqrt{2a}$ 2) $(\sqrt{2} + 1)a$

3) $(\sqrt{2} - 1)a$ 4) $(2 + \sqrt{2})a$

117. There are two circles whose equations are $x^2 + y^2 = 9$ and $x^2 + y^2 - 8x - 6y + n^2 = 0$, $n \in Z$. If the two circles have exactly two common tangents then the number of possible values of n is

1) 2 2) 8 3) 9 4) 5

118. The locus of a point the lengths of the tangents from which to two circles are in a constant ratio is a circle

1) cutting them orthogonally

2) touching each other externally

3) coaxal with them

4) lying inside the circles

KEY

001. 1	002. 1	003. 3	004. 1	005.4
006. 1	007. 1	008. 3	009. 2	010. 1
011. 4	012. 2	013. 4	014. 4	015. 3
016. 2	017. 2	018. 3	019. 1	020. 1
021. 3	022. 1	023. 3	024. 1	025. 1
026. 2	027. 1	028. 1	029. 4	030. 4
031. 1	032. 1	033. 3	034. 1	035. 4
036. 1	037. 4	038. 1	039. 4	040. 1
041. 4	042. 1	043. 2	044. 3	045. 2
046. 4	047. 4	048. 1	049. 2	050. 2
051. 2	052. 4	053. 4	054. 3	055. 3
056. 1	057. 2	058. 1	059. 4	060. 3
061. 4	062. 4	063. 2	064. 3	065. 4
066. 2	067. 3	068. 4	069. 4	070. 1
071. 1	072. 3	073. 2	074. 4	075. 3
076. 3	077. 4	078. 4	079. 3	080. 4
081. 2	082. 1	083. 3	084. 1	085. 2

086. 2	087. 2	088. 4	089. 2	090. 4
091. 1	092. 2	093. 2	094. 1	095. 3
096. 3	097. 4	098. 3	099. 4	100. 2
101. 1	102. 3	103. 1	104. 2	105. 3
106. 1	107. 4	108. 1	109. 4	110. 2
111. 3	112. 1	113. 2	114. 2	115. 1
116.3	117. 3	118.3		

HINTS

10. $C_1C_2 < |r_1 - r_2|$
12. Find the radical axis and verify the answer.
24. Find the radical axis of the given circles.
29. Let the radius of the second circle be R. Find the radical axis of the circles and compare with x=0.
35. Add the quadratic equations and subtract the circle equation from it.
41. Use the formula $\frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$
46. Find the radical centre of the circles.
56. Find the radical axis and verify the options.
55. Take $S + \lambda L = 0$ and substitute the centre.
59. Find radical axis and use $\cos\frac{\theta}{2} = \frac{d}{r}$
61. Find the radical axis and substitute the centre of the second circle.
68. Use the formula $Sin\frac{\theta}{2} = \left(\frac{r}{R}\right)$
76. Find the radical axis of the given circles and verify the options.
83. Radii of the given circles of the system are equal to 0.
84. Limiting points are extremities of diameter of the given circle.
92. verification
93. One limiting point is image of the other limiting point with respect to radical axis.
99. No centre lies between the limiting points.
108. Substitute the limiting points in the circle and verify the radical axis if necessary.
110. d = r touching
     d < r intersecting
     d > r Neither touching nor intersecting
111. The circle with given points as the ends of diameter.

LEVEL-3

1. In $n(n \geq 3)$ circles the centres of no three circles are collinear. If the number of the radical axes of the circles is equal to the number of the radical centres of the circles then $n^2 - 4n - 5 =$
   1) 5 2) 0 3) 50 4) 7
2. The length of the tangent from the radical centre of the three circles $x^2 + y^2 + a_r x + b_r y + c = 0$ $(r = 1, 2, 3 \text{ and } c > 0)$ to one of the three circles is
   1) $\sqrt{a_1 + b_1 + c}$ 2) c
   3) $\sqrt{c}$ 4) $\sqrt{a_1a_2a_3 + b_1b_2b_3}$
3. If the equation of the circle whose one diameter is the common chord of the cirlces $x^2 + y^2 + 4x + 2y - 4 = 0$ and $x^2 + y^2 - 2x - 4y - 4 = 0$ is $x^2 + y^2 + ax + by + c$ then $a + b - c =$
   1) 4 2) -4 3) -8 4) -2
4. The orthocentre of the triangle formed by the parametric points α, β and γ on the circle $x^2 + y^2 = a^2$ is
   1) (0, 0)
   2) $\left(\Sigma\frac{a}{3}\cos\alpha, \Sigma\frac{a}{3}\sin\alpha\right)$
   3) $(\Sigma a\cos\alpha, \Sigma a\sin\alpha)$
   4) $\left(\Sigma a\cos\frac{\alpha}{2}, \Sigma a\sin\frac{\alpha}{2}\right)$
5. Let A, B, C be the centres of three coaxal circles if t_1, t_2, t_3 are the lengths of the tangents to them from any point then $BC.\ t_1^2 + CA.\ t_2^2 + AB.\ t_3^2 =$
   1) 0 2) $t_1t_2t_3$ 3) 1 4) 2
6. The equation of the orthogonal system of the coaxal system
   $x^2 + y^2 + 2x - 4y - 5 + \lambda(x - 3y + 7) = 0$ is
   1) $x^2 + y^2 + 14x + 19 + \mu(3x + y + 1) = 0$
   2) $x^2 + y^2 + 14x + 19 + \mu(x - 3y + 7) = 0$
   3) $x^2 + y^2 + 14x - 19 + \mu(3x + y + 1) = 0$
   4) $x^2 + y^2 + 14x + 19 + \mu(3x + y - 1) = 0$
7. $A = (x_1, y_1)$, $B = (x_2, y_2)$, $C = (x_3, y_3)$ then the radical centre of the circles $(x - x_1)^2 + (y - y_1)^2 = a^2$, $(x - x_2)^2 + (y - y_2)^2 = a^2$, $(x - x_3)^2 + (y - y_3)^2 = a^2$ is the
   1) Centroid of ΔABC
   2) Orthocentre of ΔABC
   3) Incentre of ΔABC
   4) Circumcentre of ΔABC
8. Let $x^2 + y^2 + g_i x + c = 0$, $i = 1, 2, 3$ be three coaxal circles whose radii are r_1, r_2, r_3 respectively. Let t_1, t_2, t_3 be lengths of the tangents to these

circles from an outside point. Then $(g_2 - g_3)t_1^2 + (g_3 - g_1)t_2^2 + (g_1 - g_2)t_3^2 =$
1) 0 2) 1 3) 2 4) 3

KEY

1) 2	2) 3	3) 1	4) 3
5) 1	6) 1	7) 4	8) 1

HINTS

1. $nc_2 = nc_3 \Rightarrow n=5$
2. All the radical axes passes through origin
3. Find the radical axis and then the circle
4. Ortho centre is equal to $\left(\sum x_1, \sum y_1\right)$
5. Take the random point as(0,0) and the circles in simplest form of coaxal system
6. $r_1:r_2 = r_2:r_3 = r_3:r_1 = 1:1$ the radical axis bisects perpendicularly the sides of the triangle.
7. Use the hint of the 6[th] problem.

LEVEL- 4

1. I : The condition that the circles $(x-\alpha)^2 + (y-\beta)^2 = r^2, (x-\beta)^2 + (y-\alpha)^2 = r^2$ may touch each other is $(\alpha - \beta)^2 = 2r^2$
II : The condition that the circles $x^2 + y^2 + 2ax + 2by + c = 0$, $x^2 + y^2 + 2bx + 2ay + c = 0$ touch each other is $(a+b)^2 = 2c$.
1) Only I is true 2) Only II is true
3) both I & II are true 4) neither I nor II true

2. I : The equation of the circle cutting othogonally the circles $x^2 + y^2 - 8x - 2y + 16 = 0$, $x^2 + y^2 - 4x - 4y - 1 = 0$ and passing through the point (1, 1) is $3x^2 + 3y^2 - 14x + 23y - 15 = 0$.
II : The equation of the circle which cuts orthogonally the three circles $x^2 + y^2 + 2x + 17y + 4 = 0$, $x^2 + y^2 + 7x + 6y + 11 = 0$, $x^2 + y^2 - x + 22y + 3 = 0$ is $x^2 + y^2 - 6x - 4y - 44 = 0$
1) Only I is true 2) Only II is true
3) both I & II are true 4) neither I nor II true

3. I : The equations to the direct common tangents to the circles $x^2 + y^2 + 6x + 4y + 4 = 0$, $x^2 + y^2 - 2x = 0$ $y - 1 = 0$, $4x - 3y - 9 = 0$
II : The equations to the transverse common tangents to the circles $x^2 + y^2 - 4x - 10y + 28 = 0$, $x^2 + y^2 + 4x - 6y + 4 = 0$ are x -1 = 0, 3x + 4y- 21 = 0
1) Only I is true 2) Only II is true
3) both I & II are true 4) neither I nor II true

4. I : Let $x^2 + y^2 + 2g_i x + c = 0$, $i = 1, 2, 3$ be three coaxal circles whose radii r_1, r_2 *and* r_3 are respectively and t_1, t_2, t_3 the lengths of the tangents to the circles from an outside point then $(g_2 - g_3)t_1^2 + (g_3 - g_1)t_2^2 + (g_1 - g_2)t_3^2 = 0$
II : If p, q, r are the the powers of a point for three circles whose entres are A, B, C respectively, then p. BC + q. CA + r. AB = 0
1) Only I is true 2) Only II is true
3) both I & II are true 4) neither I nor II true

5. I : The limiting points of the co-axal system of which containing the two circles $x^2 + y^2 + 2x - 2y + 2 = 0$ and $25(x^2 + y^2) - 10x - 80y + 65 = 0$ are (-1, 1), (1/5, 8/5)
II : The equation of the circle belonging to the coaxal system of which (1, 2), (4, 3) are the limiting points and passing through the origin is $2x^2 + 2y^2 - x - 7y = 0$
1) Only I is true 2) Only II is true
3) both I & II are true 4) neither I nor II true

6. If the locus of the centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y- axis is $y^2 + ax + by + c = 0$, then the descending order of a, b, c is
1) a, b, c 2) b, c, a 3) c, a, b 4) c, b, a

7. If the locus of the centre of the circle which cuts the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is $ax + by + c = 0$, then the ascending order of a, b, c is
1) a, b, c 2) b, c, a 3) c, a, b 4) a, c, b

8. If the equation of the circle passing through the origin and the points of intersection of the two circles $x^2 + y^2 - 4x - 6y - 3 = 0$, $x^2 + y^2 + 4x - 2y - 4 = 0$ is

$x^2+y^2+2ax+2by+c=0$ then the ascending order of a, b, c is

1) a, b, c 2) b, c, a 3) c, a, b 4) a, c, b

9. Match the following :

I: If $x^2+y^2-6x-8y+12=0$, a) 1 cut orthogonally $x^2+y^2-4x+6y+k=0$ then k=

II : If $x^2+y^2-2x+3y+k=0$, b) -10 cut $x^2+y^2+8x-6y-7=0$ orthogonally then k=

III: If $x^2+y^2+2x-2y+4=0$, c) -24 cut $x^2+y^2+4x-2ky+2=0$ orthogonally then k=

1) a, b, c 2) b, c, a 3) c, b, a 4) a, c, b

10. Match the following :

	Circles	Number of common tangents
I:	$x^2+y^2=4$, $x^2+y^2-8x+12=0$	a) 0
II :	$x^2+y^2=1$, $x^2+y^2-2x-6y+6=0$	b) 1
III :	$x^2+y^2=16$, $x^2+y^2-8x+6y-56=0$	c) 2
IV.	$x^2+y^2-2x-6y+9=0$, $x^2+y^2+6x-2y+1=0$	d) 3
		e) 4

1) a, b, c, d 2) d, e, b, a 3) c, b, e, d 4) a, c, b, d

11. Match the following :

	Cirlces	Radical centre
I.	$x^2+y^2=1$, $x^2+y^2-2x=1$, $x^2+y^2-2y=1$	a) (0, 0)
II.	$x^2+y^2-x+3y-3=0$, $x^2+y^2-2x+2y+2=0$, $x^2+y^2+2x+3y-9=0$	b) (2,3)
III.	$x^2+y^2-8x+40=0$, $x^2+y^2-5x+16=0$, $x^2+y^2-8x+16y+160=0$	c) (8, -15/2)

1) a, b, c 2) b, c, a 3) c, a, b 4) a, c, b

12. Match the following :

	Cirlces	Limiting Points
	$x^2+y^2+2x+4y+7=0$, $x^2+y^2+4x+2y+5=0$	a) (-1, 1), (1/5, 8/5)
II.	$x^2+y^2-6x-8y+5=0$, $x^2+y^2-8x-10y+5=0$	b) (1, 2), (3, 1)
III.	$x^2+y^2+2x-6y=0$, $2(x^2+y^2)-10y+5=0$	c) (1, 2), (-2, -1)
IV.	$x^2+y^2+2x-2y+2=0$, $25(x^2+y^2)-10x-80y+65=0$	d) (-2, -1), (0, -3)

1) a, b, c, d 2) b, a, c, d 3) c, d, a, b 4) d, c, b, a

13. A: If the circles $x^2+y^2=a^2$, $x^2+y^2-6x-8y+9=0$ touch externally then a = 1

R: Two circles with centres C_1, C_2 and radii r_1, r_2 respectively touch externally iff $C_1C_2=r_1+r_2$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not the correct explanation of A
3) A is true but R is false 4) A is false but R is true

14. A: If $x^2+y^2-2x+3y+k=0$, $x^2+y^2+8x-6y-7=0$, cut each other orthogonally then k = 10

R: The circles $x^2+y^2+2gx+2fy+c=0$, $x^2+y^2+2g'x+2f'y+c'=0$ cut each other orthogonally iff $2gg'+2ff'=c+c'$.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not the correct explanation of A
3) A is true but R is false 4) A is false but R is true

15. A: The radical centre of the circles $x^2+y^2=4, x^2+y^2-3x=4, x^2+y^2-4y=4$ is (0, 0)

R: Radical centre of three circles is the point of concurrence of the radical axes of the circles taken in pairs.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not the correct explanation of A
3) A is true but R is false 4) A is false but R is true

16. A : If origin is a limiting point of a coaxal system of which $x^2+y^2-6x-8y+1=0$ is a member then the other limiting point is (3/25, 4/25)

R: If origin is a limiting point of the coaxal system containing the circle $x^2+y^2+2gx+2fy+c=0$, then the other limiting point is $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not the correct explanation of A
3) A is true but R is false 4) A is false but R is true

17. i) The circles $x^2+y^2-8x+6y+21=0$,

$x^2+y^2+4x-10y-115=0$ touch externally.

ii) The circles $x^2+y^2-4x-6y-12=0$, $x^2+y^2+6x-2y+1=0$ intersect each other.

Which of the statement is correct.

(1) Only i (2) Only ii

(3) Both i & ii (4) Neither i nor ii

18. i) The coaxal system $x^2+y^2+2\lambda x+5=0$ is a non intersecting system.

ii) The coaxal system $x^2+y^2+4\lambda x-3=0$ is an intersecting system.

which of above statement is correct.

(1) Only i (2) Only ii

(3) Both i & ii (4) Neither i nor ii

19. Observe the following statements:

I. The lengths of the tangents from any point on the line 2x + 3y=5 to the circles $x^2+y^2=9$ and $x^2+y^2+4x+6y=19$ are equal in length.

II. There is only one point such that the tangents from it to the three given circles are equal in length. Then the correct statement is:

(1) Only I (2) Only II

(3) Both I & II (4) Neither I nor II

20. Observe the following statements:

I. If two circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g^1x+2f^1y=0$ touch each other, then $gf^1=fg^1$.

II. There are 4 circles of radius ‘a’ which touch both the axes and have their centres on the line y = x.

Then the correct statement is:

(1) Only I (2) Only II

(3) Both I & II (4) Neither I nor II

21. Observe the lists:

List-I	List II
A) The radical axis of two circles	1) is the square of the distance between their centers
B) The common tangent to two	2) is perpendicular to the line joining the centres inter secting circles of equal radii
C) The common chord of two	3) is parallel to the line joining the centres intersecting circles
D) The sum of squares of the radii	4) is bisected by the line joining the centres of two circles intersecting orthogonally.

The correct match is

	A	B	C	D		A	B	C	D
1)	a	b	c	d	2)	b	c	d	a
3)	b	c	a	d	4)	a	b	d	c

22. Observe the lists:

List I	List II
A) The circles $x^2+y^2+2x+c=0$ and $x^2+y^2+2y+c=0$ touch each other	1) If c = 1
B) The circles $x^2+y^2+2x+3y+c=0$ and $x^2+y^2-x+2y+c=0$ intersect orthogonally.	2) If c < 2
C) The circle $x^2+y^2=9$ contains the circle $x^2+y^2-2x+1-c^2=0$	3) If c=1/2
D) The circle $x^2+y^2=9$ contained in the circle	4) If c> 8

The correct match is:

	A	B	C	D		A	B	C	D
1)	a	b	c	d	2)	c	a	b	d
3)	c	b	a	d	4)	d	a	b	c

23. Assertion (A):

$S_1 : x^2+y^2+4x-2y+3=0$

$S_2 : x^2+y^2-x-3y+2=0$

$S_3 : x^2+y^2+14x+5=0$

are members of a coaxial system

Reason (R): In a coaxial system of circles every pair of circles has the same radical axis.

The correct answer is

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true and R is not the correct explanation of A.

3) A is true but R is false (4) A is false but R is true

24. Assertion(A): The circles $S=0, S^1=0$ intersect each other, then the radical axis is $S-S^1=0$.

Reason (R): The radical axis is perpendicular to the line of centres.

The correct answer is

(1) Both A and R are true and R is the correct

explanation of A.

(2) Both A and R are true and R is not the correct explanation of A.

(3) A is true but R is false

(4) A is false but R is true

25. Observe the following statements:

Assertion(A): The equation $x^2+y^2+2\lambda x+4=0$ represents a real circle for all $\lambda \in R$

Reason (R): The radical axis of any two circles of the family represented by $x^2+y^2+2\lambda x+4=0$ is the x-axis

The correct statement among the following is:

(1) A is true, (R) is false

(2) (A) is false, (R) is true

(3) (A) is true, (R) is true

(4) (A) is false, (R) is false

KEY

01. 3	02. 3	03. 3	04. 3	05. 3
06. 4	07. 2	08.1	09. 3	10. 2
11. 1	12. 4	13. 1	14. 4	15. 1
16. 1	17. 2	18. 3	19. 3	20. 1
21. 2	22. 2	23. 1	24.2	25.4

LEVEL-5

Q 1: The radical axis of two non concentric circles id the locus of a point,which moves so that its power w.r.to the two circles are

equal,and the point of concurrence of the radical axes of 3 circles, whose centres are non collinear taken in pairs is the radical centre.

1) The perpendicular distance of radical axis determind by the circles

$x^2+y^2+2x+4y-7=0$ and

$x^2+y^2-6x+2y-5=0$ from the origin is

1) $\frac{1}{\sqrt{17}}$ 2) $\frac{1}{4}$ 3) $\frac{1}{5}$ 4) $\frac{2}{17}$

2) The radical axis of two circles divides the lines segment joining the centre of circles in the ratio of their

1) areas 2) radii 3) 1:1 4) none

3) Circles with radical centre as centre and radius equals to length of tangent from radical centre to any of the three circles will

1) Bisects the circumference of all the three circles

2) Bisects the circumference of at least one of the circle

3) Orthogonal to all the three circles

4) Orthogonal to at least one of the circle

4) From any point P tangents of length t_1 and t_2 are drawn to two circles with centre A,B and if PN is the perpendicular from P to the radical axis then $t_1^2-t_2^2=K$. PN.AB then K =

1) 0 2) 1 3) 2 4) $\frac{1}{2}$

Q 2: The system of circles is called a coaxal system of circles if any two members of the system have the same radical axis.The point circles belongs to the coaxal system are called its limiting points.

1) If A,B,C are centres and r_1,r_2,r_3 are the radii of three circles belongs to the coaxal system then the value of $r_1^2BC+r_2^2CA+r_3^2 AB+AB.BC.CA=$

1) $(r_1+r_2+r_3)^2$ 2) $(AB+BC+CA)^2$

3) $r_1r_2+r_2r_3+r_3r_1$ 4) zero

2) One of the limiting point of the coaxal system determined by the circles $x^2+y^2+2x-6y=0$ and $2x^2+2y^2-10y+5=0$ is

1) (1,2) 2) (3,-1) 3) (-3,1) 4) (-3,-1)

3) Every circle through the limiting points

1) belongs to the coaxal system

2) is bisected by the radical axis

3) orthogonal to every circle belongs to the system

4) none of these

4) Equation of circle passing through (0,0) and belongs to the system having limiting points (1,2) and (4,3) is

1) $2x^2+2y^2-x-7y=0$

2) $x^2+y^2-2x-7y=0$

3) $2x^2+2y^2+2x-7y=0$

4) $x^2+y^2+2x-7y=0$

KEY

Q 1:

1) 1 2)2 3) 3 4) 3

Q 2:

1) 4 2) 1 3) 3 4) 1

PREVIOUS EAMCET

01. If $x-y+1=0$ meets the circles $x^2+y^2+y-1=0$ at A,B, then the equation of the circle with AB as diameter is

$(Eamcet-2005)$

1) $2\left(x^2+y^2\right)+3x-y+1=0$

2) $2\left(x^2+y^2\right)+3x-y+2=0$

3) $2\left(x^2+y^2\right)+3x-y+3=0$

4) $x^2+y^2+3x-y+1=0$

02. The equation of the circle whose diameter is the common chord of the circles $x^2+y^2+2x+3y+2=0$ $x^2+y^2+2x-3y-4=0$ is $\left(Eamcet-2005\right)$

1) $x^2+y^2+2x+2y+2=0$

2) $x^2+y^2+2x+2y-1=0$

3) $x^2+y^2+2x+2y+1=0$

4) $x^2+y^2+2x+2y+3=0$

03. The equation of the radical axis of the two circles $7x^2+7y^2-7x+14y+18=0$ and $4x^2+4y^2-7x+8y+20=0$ is given by: (REE-1989)

(1) $3x^2+3y^2-6y-2=0$
(2) 21x -68 = 0
(3) x - 2y - 5 = 0
(4) None

04. The two circles $x^2+y^2-2x-4y=0$ and $x^2+y^2-8y-4=0$ (REE-1990)

(1) Intersect each other
(2) Touch each other internally
(3) Touch each other externally
(4) None of these

05 If the circle $x^2+y^2+6x-2y+k=0$ bisects the circumference of the circle $x^2+y^2+2x-6y-15=0$ then k= (2003)

(1) 21 (2) -21 (3) 23 (4)-23

06. The radical axis of circles $x^2+y^2+3x+4y-5=0$ and $x^2+y^2-5x+5y-6=0$ is (EAMCET 2001)

1) 8x +y + 1 = 0 2) 8x - y + 1 = 0
3) 8x - 8y + 1 = 0 4) -8x + y + 1 = 0

07. The limiting points of a coaxal system containing the two circles $x^2+y^2+2x-2y+2=0$ and $25x^2+25y^2-10x-80y+65=0$ are (EAMCET 2001)

1) (1, -1), (-5, -40) 2) (1, -1), (-1/5, -8/5)
3) (-1, 1), (1/5, 8/5) 4) (-1, 1), (-1/5, 8/5)

08. If (1, 2) is a limiting point of a coaxal system of circles containing the circle $x^2+y^2+x-5y+9=0$, then the equation of radical axis is (EAMCET 91)(2000)

1) x + 3y + 9 = 0 2) 3x - y + 4 = 0
3) x + 9y - 4 = 0 4) 3x - y - 1 = 0

09. The number of common tangents that can be drawn to the circles $x^2+y^2=1$ and $x^2+y^2-2x-6y+6=0$ is (EAMCET 2000)

1) 1 2) 2 3) 3 4) 4

10. Two circles of equal radius 'r' cut orthogonally. If their centres are (2, 3) and (5, 6), then r = (EAMCET 2000)

1) 1 2) 2 3) 3 4) 4

11. The slope of the radical axis of the circles $x^2+y^2+3x+4y-5=0$ and $x^2+y^2-5x+5y-6=0$ is (EAMCET 99)

1) 1 2) 3 3) 5 4) 8

12. If the circle $x^2+y^2+2x-2y+4=0$ cuts the circle $x^2+y^2+4x+2fy+2=0$ orthogonally, then f= (EAMCET 99)

1) 1 2) 2 3) -1 4) -2

13. The radical axis of the circles $x^2+y^2-6x-4y-44=0$ and $x^2+y^2-14x-5y-24=0$ is (EAMCET 98)

1) 8x + y - 30 = 0 2) 8x + y + 20 = 0
3) 8x + 3y - 20 = 0 4) 8x + y - 20 = 0

14. The radical axis of the coaxal system having the limiting points (1, 2) and (4, 3) is (EAMCET 97)

1) 3x - y + 10 = 0 2) 3x + y - 10 = 0
3) 3x + y + 10 = 0 4) x + 3y - 10 = 0

15. If (0, 0) is one limiting point of the coaxal system with radical axis x + y = 1, then the other limitingpoint is (EAMCET 96)

1) (-1, 1) 2) (1, -1) 3) (-1, -1) 4) (1, 1)

16. If the circles of same radius 'a' and centres at (2, 3), (5, 6) cut orthogonally then a= (EAMCET 96)

1) 4 2) $4\sqrt{2}$ 3) $3\sqrt{2}$ 4) 3

17. Limiting points of the coaxal system determined by the circles $x^2+y^2+14x-8y-5=0$, $x^2+y^2+4x+2y+5=0$ are (EAMCET 96)

1) (0, -3), (2, 1) 2) (-2, -1), (0, -3)
3) (-2, -1), (0, 3) 4) (2, 1), (0, -3)

18. The distance of (1, -2) from the common chord of $x^2+y^2-5x+4y-2=0$ and

$x^2 + y^2 - 2x + 8y + 3 = 0$ is (EAMCET 96)

1) 2 2) 1 3) 0 4) 3

19. Radical centre of $x^2 + y^2 - x + 3y - 3 = 0$, $x^2 + y^2 - 2x + 2y + 2 = 0$ and $x^2 + y^2 + 2x + 3y - 9 = 0$ is (EAMCET 96)

1) (2, 3) 2) (3, 2) 3) (-2, 3) 4) (-3, -2)

20. If the chords of contact of points on $x^2 + y^2 = a^2$ with respect to the circle $x^2 + y^2 = b^2$ touch the circle $x^2 + y^2 = c^2$ then a, b, c are in (EAMCET 95)

1) AP 2) GP 3) HP 4) AGP

21. Number of common tangents to $x^2 + y^2 - x = 0$ and $x^2 + y^2 + x = 0$ is (EAMCET 94)

1) 2 2) 1 3) 4 4) 3

22. The circles $x^2 + y^2 - 4x + 6y + 8 = 0$ and $x^2 + y^2 - 10x - 6y + 14 = 0$ (EAMCET 91)

1) touch externally 2) touch internally
3) intersect 4) do not meet

23. The equation of the circle passing through the origin and the points of intersection of the circles $x^2 + y^2 - 4x - 6y - 3 = 0$, $x^2 + y^2 + 4x - 2y - 4 = 0$ (EAMCET 91)

1) $x^2 + y^2 + 28x + 18y = 0$
2) $x^2 + y^2 - 18x - 28y = 0$
3) $x^2 + y^2 - 28x + 18y = 0$
4) $x^2 + y^2 - 28x - 18y = 0$

24. Equation to the circle whose one of the diameters is the common chord of $(x - a)^2 + y^2 = a^2$, $x^2 + (y - b)^2 = b^2$ is (EAMCET 89)

1) $(a^2 + b^2)(x^2 + y^2) = 2ab(bx + ay)$
2) $(a^2 + b^2)(x^2 + y^2) = 2ab(ax + by)$
3) $x^2 + y^2 = 2ab / (a^2 + b^2)(ax - by)$
4) $x^2 + y^2 = ab / (a^2 + b^2)(ax + by)$

25. If the circles of equal radius and centres at (2, 3), (5, 6) cut orthogonally, then the radius of one of the circles is (EAMCET 88)

1) 3 2) $3\sqrt{2}$ 3) 6 4) 4

26. The angle at whcih the circles $x^2 + y^2 + 8x - 2y - 9 = 0$, $x^2 + y^2 - 2x + 8y - 7 = 0$ intersect is (EAMCET 87)

1) obtuse 2) $\pi/6$ 3) $\pi/3$ 4) $\pi/2$

27. The number of common tangents to the circles $x^2 + y^2 + 2x + 8y - 23 = 0$, $x^2 + y^2 - 4x - 10y + 19 = 0$ (EAMCET 87)

1) 4 2) 2 3) 3 4) 1

28. The equation of the circle passing through (0, 0) and cutting the circles $x^2 + y^2 + 6x - 15 = 0$, $x^2 + y^2 - 8y + 10 = 0$ orthogonally is (EAMCET 86)

1) $(x - 5/2)^2 + (y - 5/4)^2 = \frac{125}{16}$
2) $x^2 + y^2 - 5x - 5y = 0$
3) $2(x^2 + y^2) - 10x - 5y = 0$
4) $x^2 + y^2 - 5x + 5y = 0$

29. Number of circles that can be drawn touching all the three lines $x + y - 2 = 0$, $3x + 4y + 7 = 0$ and $2x + 2y - 3 = 0$ (EAMCET 86)

1) 0 2) 1 3) 2 4) 4

30. Number of circles that can be drawn touching all the three lines $x + y - 1 = 0$, $x - y - 1 = 0$ and $y + 1 = 0$ (EAMCET 85)

1) 0 2) 2 3) 3 4) 4

31. If the circles $(x + a)^2 + (y + b)^2 = a^2$, $(x + \alpha)^2 + (y + \beta)^2 = \beta^2$ cut orthogonally then $\alpha^2 + b^2$ (EAMCET 85)

1) $a\alpha + b\beta$ 2) $a^2 + \beta^2$
3) $-2(a\alpha + b\beta)$ 4) $2(a\alpha + b\beta)$

Eamcet-2007

32. The condition for the coaxial system $x^2 + y^2 + 2\lambda x + c = 0$, where λ is a parameter and 'c' is a constant, to have distinct limiting points is **E-2007**

1) c = 0 2) c < 0 3) c = -1 4) c > 0

KEY

01.1	02.3	03.2	04.2	05.4
06.1	07.3	08.2	09.4	10.3
11.4	12.3	13.4	14.2	15.4
16.3	17.2	18.3	19.1	20.2
21.4	22.1	23.4	24.1	25.1
26.4	27.3	28.3	29.3	30.4
31.4	32. 4			