Ratio of Areas of Two Similar Triangles

Objective

To verify **"The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides"** by performing an activity.

Prerequisite Knowledge

1. Concept of parallel lines.
2. Division of a line in a given ratio.

Materials Required

Chart paper, construction box, coloured pens, a pair of scissors, fevicol.

Procedure

1. Take a chart paper and cut a ΔABC with AB = 6cm, BC = 6cm, CA = 6cm.
2. Mark 5 points P_1, P_2, …. , P_5 at a distance of 1cm each on side AB and Q_1, Q_2,…..,Q_5 at a distance of 1cm each on side AC as shown in fig.(i).

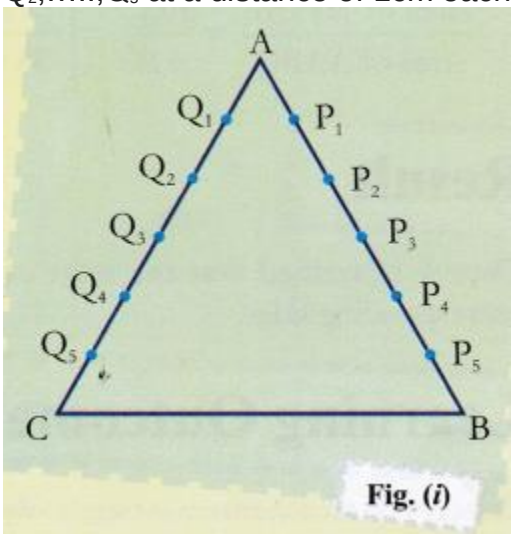


Fig. (*i*)

3. Join P_1Q_1, P_2Q_2, ……, P_5Q_5 as shown in fig. (ii).

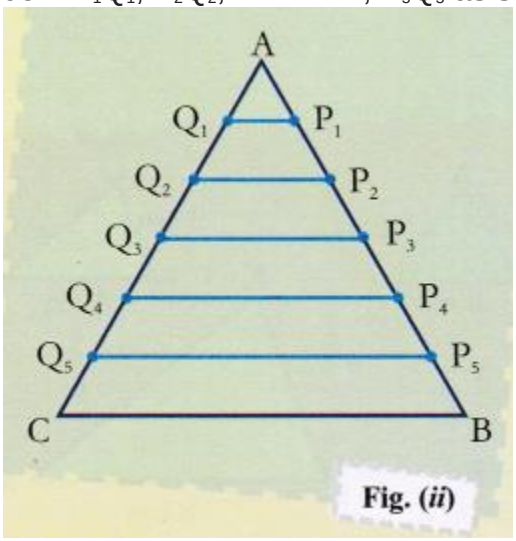


Fig. (*ii*)

4. Draw lines parallel to AC from P_1, P_2, P_5 and also draw lines parallel to AB from the points Q_1, Q_2, ……., Q_5 as shown in fig. (iii).

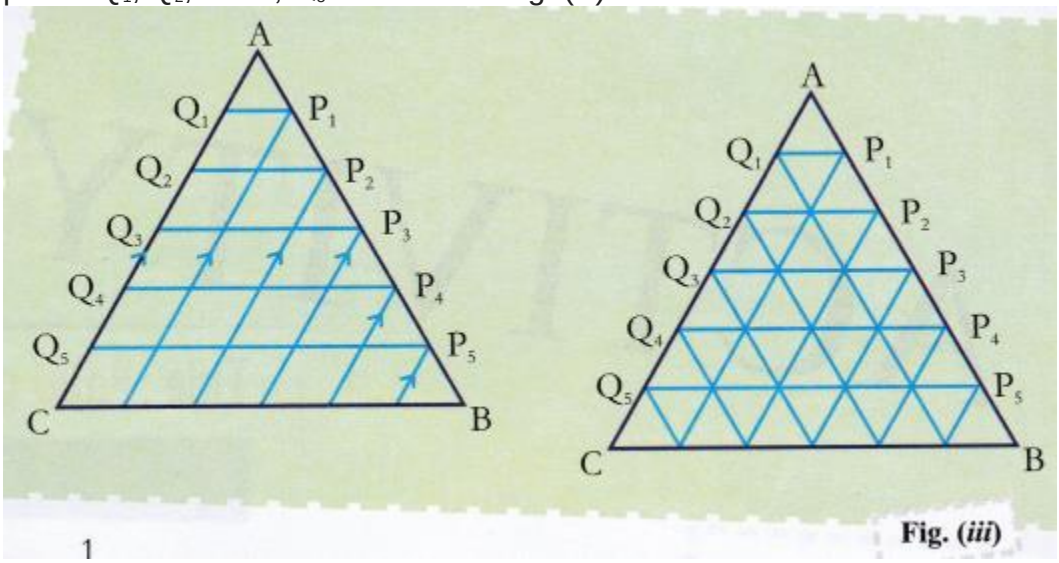
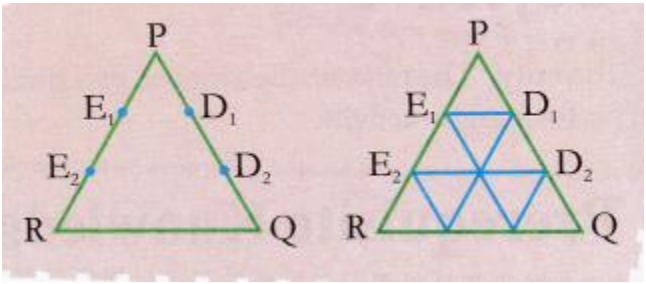


Fig. (*iii*)

1

5. Thus ΔABC is divided into 36 smaller triangles and all are similar to each other and of equal area.
6. Construct a ΔPQR with PQ = $\frac{1}{2}$ of AB, PR = $\frac{1}{2}$ of AC and QR = $\frac{1}{2}$ of BC i.e. 3cm each on another chart paper.
7. Mark D_1, D_2 and E_1, E_2 on sides PQ and PR respectively.
8. Repeat steps 3 and 4.

9. Thus ∆PQR is divided into 9 smaller similar triangles equal in area.



Observation

1. area of ∆ABC = area of 36 smaller ∆'s
2. area of ∆PQR = area of 9 smaller ∆'s
3. $\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2} = \frac{PR}{AC}$
4. $\frac{Area\ \ of\ \ \Delta PQR}{Area\ \ of\ \ \Delta ABC} = \frac{PQ^2}{AB^2}$
   = [9 smaller ∆'s/36 smaller ∆'s = $\frac{1}{4}$ = $(1/2)^2$
   (because ∆ABC ~ ∆PQR)

Result

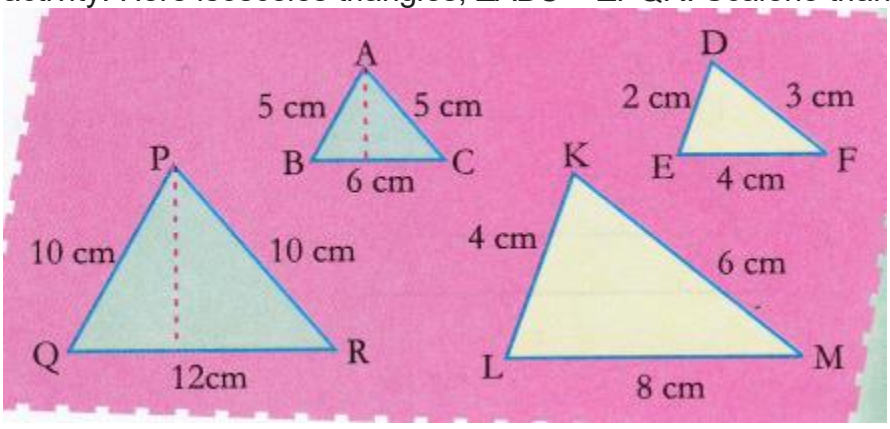
Thus it is verified that the ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

Learning Outcome

Concept of area theorem is clear to the students through this activity.

Activity Time

1. Take isosceles similar triangles and scalene similar triangles and try to verify this activity. Here isosceles triangles, ∆ABC ~ ∆PQR. Scalene triangle ∆DEF ~ ∆KLM



Viva Voce

Question 1.
What are the criteria for two triangles to be similar ?
Answer:
Two triangles are said to be similar, if

- their corresponding angles are equal.
- their corresponding sides are in proportion

Question 2.
$\Delta ABC \sim \Delta DEF$ and their areas are respectively 64 cm^2 and 121 cm^2. If EF = 15.4 cm, then findBC.
Answer:
11.2 cm

Question 3.
Is it true, if the areas of two similar triangles are equal, then they are congruent ?
Answer:
Yes

Question 4.
What is the ratio of the area of an equilateral triangle described on one side of a square to the area of an equilateral triangle described on one of its diagonal ?
Answer:
1:2

Question 5.
Are a square and a rhombus of side 3 cm similar ?
Answer:
No

Question 6.
Is a rhombus of side 3 cm congruent to another rhombus of side 4 cm ?
Answer:
No

Question 7.
Is the ratio of the areas of two similar triangles equal to the square of the ratio of their corresponding medians ?
Answer:
Yes

Multiple Choice Questions

Question 1.
ABC and BDE are two equilateral triangles, such that D is the mid-point of BC. The ratio of the areas of ΔABC and ΔBDE is
(a) 2:1
(b) 1:2
(c) 4:1
(d) 1:4

Question 2.
Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio
(a) 2:3
(b) 4:9
(c) 81:16
(d) 16:81

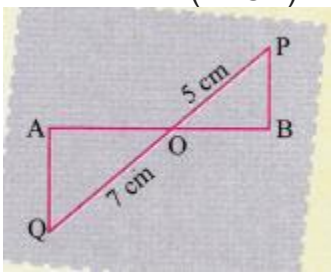
Question 3.
If in two similar triangles PQR and LMN, if QR =15 cm and MN = 10 cm, then the ratio of the areas of triangles is
(a) 3:2
(b) 9:4
(c) 5:4
(d) 7:4

Question 4.
Two isosceles triangles have equal vertical angles and their areas are in the ratio 16 : 25. Then the ratio of their corresponding heights is
(a) 16 : 25
(b) 256 : 625
(c) 4 : 5
(d) none of these

Question 5.
ΔABC ~ ΔDEF. If AC = 19 cm and DF = 8 cm, then the ratio of the areas of the two triangles is
(a) 361 : 64
(b) 19 : 8
(c) 19 : 4
(d) none of these

Question 6.
In the given figure, PB and QA are perpendicular to segment AB. If PO = 5 cm, QO = 7

cm and area (∆POB) = 150 cm^2, then area of ∆QOA is



(a) 254 cm^2
(b) 294 cm^2
(c) 244 cm^2
(d) 49 cm^2

Question 7.
Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.
(a) 4:1
(b) 1:4
(c) 4:4
(d) 2:4

Question 8.
ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, then the ratio of area (∆ABC) to area (∆DBC) is
(a) $\frac{AO}{DO}$
(b) $\frac{AO}{DB}$
(c) $\frac{AC}{DO}$
(d) None of these

Question 9.
Area (∆ABC) : Area (∆DEF) = 25 : 36. Then AB : DE is
(a) 625 : 1296
(b) 25 : 36
(c) 6 : 5
(d) 5 : 6

Question 10.
∆DEF ~ ∆ABC; If DE : AB = 2 : 3 and area ∆DEF is equal to 44 square units, then area (∆ABC) is
(a) 120 sq. units
(b) 99 sq. units
(c) 66 sq. units
(d) none of these

Answers

1. (c)
2. (d)
3. (b)
4. (c)
5. (a)
6. (b)
7. (a)
8. (a)
9. (d)
10. (b)