Mathematics

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Question	Very Short Answer (VSA) 1 Mark	Short Answer– I (SA I) 2 Marks	Short Answer– II (SA II) 2 Marks	Long Answer (LA) 5 Marks	Total Marks	% Weightage
1.	**Remembering**	2	2	2	2	20	25%
2.	**Understanding**	2	1	1	4	23	29%
3.	**Application**	2	2	3	1	19	24%
4.	**High Order Thinking Skills**	-	1	4	-	14	17%
5.	**Inferential and Evaluative**	-	-	-	1	4	5%
	Total	6 × 1 = 6	6 × 2 = 12	10 × 3 = 30	8 × 4 = 32	80	100%

Time allowed: 3 hours **Maximum marks: 80**

General Instructions:

(i) All question are compulsory
(ii) The question paper consists of 30 question divided into four section A, B, C and D.
(iii) Section A contains 6 questions of I mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

1. If p, q, r are zeroes of $p(x) = 9x^3 - 3x^2 - 7x + 1$, then find the value of $p^{-1} + q^{-1} + r^{-1}$.
2. Find the value of a, if the distance between the points $A(-3, -14)$ and $B(a, -5)$ is 9 units.
3. Find the median of following distribution :
   3, 5, 7, 4, 2, 4, 3, 1, 4, 7
4. If $\tan(3x + 30°) = 1$, then find the value of x.
5. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.
6. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then find the value of k.

SECTION - B

7. Find the quadratic equation, if $x = \sqrt{5 + \sqrt{5 + \sqrt{5 + ...\infty}}}$ and x is a natural number.
8. Prove that the area of triangle whose vertices are $(t, t - 2)$, $(t + 2, t + 2)$ and $(t + 3, t)$, is independent of t.
9. Find the height of a tower if the angle of elevation of top of tower is 60° and the horizontal distance from eye to the foot of the tower is 100 m.
10. Construct the frequency distribution table for the given data.

Marks	Number of students
Less than 10	14
Less than 20	22
Less than 30	37
Less than 40	58
Less than 50	67
Less than 60	75

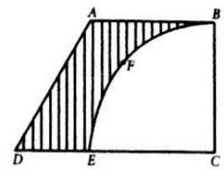
11. Find the 20^{th} term of the sequence 7, 3, – 1, – 5....
12. An hour glass is made using identical double glass cones of diameter 10 cm each. If total height is 24 cm, then find the surface area of the glass used in making it.

SECTION - C

13. Solve graphically, the pair of equations $2x + y = 6$ and $2x - y + 2 = 0$. Find the ratio of the areas of the two triangles formed by the line representing these equations with X-axis and the lines with Y-axis.

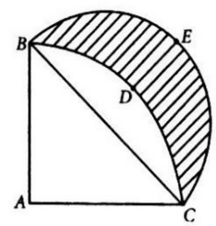
14. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that median AD bisects PQ.

15. From a thin metallic piece, in the shape of a trapezium $ABCD$ in which $AB || CD$ and $\angle BCD = 90°$, a quarter circle $BFEC$ is removed (see figure). Given $AB = BC = 3.5$ cm and $DE = 2$ cm, calculate the area of remaining (shaded) part of the metal sheet. $\left[\text{Use } \pi = \frac{22}{7}\right]$



OR

In the given figure, ABC is a quadrant of a circle of radius 28 cm and a semi-circle is drawn with BC as diameter. Find the area of the shaded region.



16. Find the value of k, if the mean of the following distribution is 20.

x	15	17	19	$20 + k$	23
f	2	3	4	$5k$	6

OR

From the following frequency distribution, prepare the "less than" ogive.

Rainfall (in cm)	5-15	15-25	25-35	35-45	45-55	55-65
Number of days	22	10	8	15	5	6

Prepare a cumulative frequency table of less than type and draw an ogive.

17. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

18. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

19. The vertices of a ΔABC are $A(7, 8)$, $B(4, 2)$ and $C(8, 2)$. The mid-point of the side BC is (6, 2). Show that the median AD divides the ΔABC into two triangles equal in area. Also, find the area of ΔABC to verify your answer.

20. Seven years ago, Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two fifth of Varun's age Find their present ages.

21. Prove that : $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta} = 1 + 2\cot^2\theta + 2\operatorname{cosec}^2\theta\cos\theta$.

OR

Evaluate : $\sin A\cos A - \dfrac{\sin A\cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \dfrac{\cos A\sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)}$

22. The sum of first n, $2n$ and $3n$ terms of an AP are S_1, S_2 and S_3, respectively. Prove that $S_3 = 3(S_2 - S_1)$.

OR

The sum of the first term and the fifth term of an ascending AP is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this AP.

SECTION - D

23. A farmer has field in the form of circle. He wants to fencing the field. The field is to be ploughed at the rate of ₹ 0.75 per m^2. If the cost of fencing of a circular field at the rate of ₹ 25 per m is ₹ 5500, then

(i) find the length of fencing the circular field.

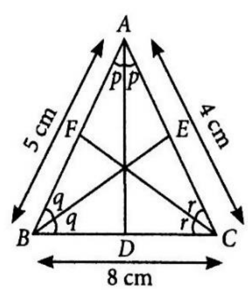
(ii) find the cost of ploughing the field.

(iii) Which value is depicted by the farmer in fencing the field? $\left[\text{take, } \pi = \dfrac{22}{7}\right]$

24. In ΔABC, if AD is the median, then show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$.

OR

D, E and F are the points on sides BC, CA and AB respectively, such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If $AB = 5$ cm, $BC = 8$ cm and $CA = 4$ cm, then determine AF, CE and BD.



25. The angle of elevation of a cliff from a fixed point is θ. After going up a distance k metres towards the top of the cliff at an angle of ϕ, it is found that the angle of elevation is α, show that the height of the cliff is $\dfrac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$.

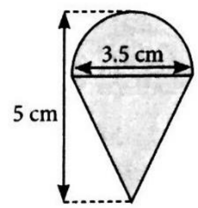
OR

From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is $h(1 + \tan\theta\cot\phi)$

26. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9 of the curved surface of he whole cone, fined the ratio of the line segments into which the altitude of the cone is divided by the plane.

OR

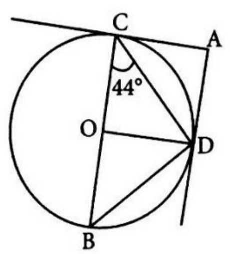
In given figures, the top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area it has to colour. $\left(\text{Take } \pi = \frac{22}{7}\right)$



27. From a pack of 52 cards jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is:

1. a black queen
2. a red card
3. a black jack
4. a picture card

28. AC and AD are tangents at C and D, respectively. If $\angle BCD = 44°$, then find $\angle CAD$, $\angle CBD$ and $\angle ACD$



29. Find the mean age (in years) from the following frequency distribution:

Age (in years)	15 - 19	20 - 24	25 - 29	30 -34	35 - 39	40 - 44	45 - 49	Total
Frequency	3	13	21	15	5	4	2	63

30. If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$.

OR

Prove the following identity :

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

Solution

1. Here, $pq + qr + rp = -\frac{7}{9}$ and $pqr = -\frac{1}{9}$

Now, $p^{-1} + q^{-1} + r^{-1} = \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$

$$= \frac{qr + rp + pq}{pqr} = \frac{-\frac{7}{9}}{-\frac{1}{9}} = 7$$

2. Given, $PA = 12$ cm

We know that, radius is perpendicular to the tangent at the point of contact therefore, $AO \perp AP$.

Now, in right angled ΔPAO,

$OP = \sqrt{(PA)^2 + (AO)^2}$ [By Pythagoras theorem]

$= \sqrt{(12)^2 + (5)^2}$ [$\because$ Radius, $AO = 5$ cm (given)]

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13$ cm

Similarly, in right angled ΔPBO, $PB = \sqrt{OP^2 - OB^2}$

$= \sqrt{(13)^2 - (3)^2} = \sqrt{169 - 9} = \sqrt{160} = 4\sqrt{10}$ cm

[$\because$ Radius, $OB = 3$ cm (given)]

3. We have 3, 5, 7, 4, 2, 4, 3, 1, 4, 7

Arrange in ascending order

1, 2, 3, 3, 4, 4, 4, 5, 7, 7

$n = 10$

$$\text{Median} = \frac{(5^{\text{th}} + 6^{\text{th}})\text{ observation}}{2} = \frac{4 + 4}{2} = 4$$

4. Given, $\tan(3x + 30°) = 1 = \tan 45°$

$\Rightarrow 3x + 30° = 45° \Rightarrow 3x = 15°$

$\Rightarrow x = 5°$

Hence, the value of x is 5°.

5. Total number of possible outcomes = 52

Number of cards that are neither a king nor a queen

= 52 – (4 + 4) = 44

$\therefore$ Total number of favourable outcomes = 44

$\therefore$ Required probability $= \frac{44}{52} = \frac{11}{13}$

6. Since –3 is a zero of the given polynomial.

$\therefore \quad (k - 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9k - 9 - 3k + 1 = 0 \Rightarrow 6k - 8 = 0 \Rightarrow k = \frac{4}{3}$

7. Given, $x = \sqrt{5+\sqrt{5+\sqrt{5+\ldots\infty}}}$

Then, $x = (\sqrt{5+x})$

On squaring both sides, we get $(\sqrt{5+x})^2$

$\Rightarrow x^2 = 5 + x \Rightarrow x^2 - x - 5 = 0$

Hence, the required quadratic equation is

$x^2 - x - 5 = 0$

8. Let coordinates of $A \equiv (x_1, y_1) = (t, t-2)$

Coordinates of $B \equiv (x_2, y_2) = (t+2, t+2)$

and coordinates of $C \equiv (x_3, y_3) = (t+3, t)$.

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|t(t+2-t) + (t+2)\{t-(t-2)\} + (t+3)\{(t-2) - (t+2)\}|$

$= \frac{1}{2}|2t + (t+2)(2) + (t+3)(-4)|$

$= \frac{1}{2}|2t + 2t + 4 - 4t - 12|$

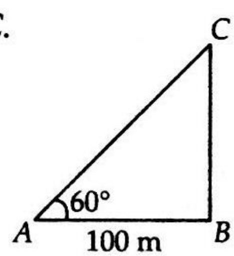
$= \frac{1}{2}|-8| = \frac{1}{2} \times 8 = 4$

$\therefore$ Area of $\Delta ABC = 4$ sq units

Hence, area of ΔABC is independent of t.

9. Let the height of the tower be BC.

Horizontal distance $AB = 100$ m



In ΔABC, $\tan\theta = \frac{BC}{AB}$

$\therefore \tan 60° = \frac{BC}{100}$

$\Rightarrow \sqrt{3} = \frac{BC}{100} \Rightarrow BC = 100\sqrt{3}$ m

Hence, height of the tower is $100\sqrt{3}$ m.

10. Here, we have the cumulative frequency distribution of less than type. We observe that the number of students getting marks less than 10 is 14 and 22 students have marks less than 20.

Therefore, number of students getting marks between 10 and 20 is 22 - 14 = 8. Similarly, the number of students getting marks between 20 and 30 is 37 - 22 = 15 and so on.

Thus, we have the following frequency distribution table

Marks	Number of students
0-10	14
10-20	22 – 14 = 8
20-30	37 – 22 = 15
30-40	58 – 37 = 21
40-50	67 – 58 = 9
50-60	75 – 67 = 8

11. Given, sequence is 7, 3, –1, – 5, ...

Here, $3 - 7 = -4$, $-1 - 3 = -4$, $-5 + 1 = -4$

So, given sequence is an AP in which $a = 7$ and $d = -4$.

Since, n^{th} term, $a_n = a + (n - 1)d$

On putting $n = 20$, we get

$$a_{20} = a + (20 - 1)d$$

$$= 7 + 19(-4) \qquad [\because a = 7, d = -4]$$

$$= 7 - 19 \times 4 = 7 - 76 = -69$$

Hence, 20^{th} term of given sequence is – 69.

12. Let l be the slant height of each cone.

Given, radius of each cone $= \frac{10}{2} = 5$ cm

and height of each cone $= \frac{24}{2} = 12$ cm

Now, we know that,

$$l = \sqrt{r^2 + h^2}$$

$$\Rightarrow \quad l = \sqrt{5^2 + 12^2}$$

$$\Rightarrow \quad l = \sqrt{169} \quad \Rightarrow l = 13 \text{ cm}$$

$\therefore$ Total surface area of one cones $= \pi rl + \pi r^2$

$$= \pi \times 5 \times 13 + \pi \times (5)^2$$

$$= 65\pi + 25\pi = 90\pi \text{ cm}^2$$

Hence, total surface area of two cones $= 2 \times 90\pi$

$$= 180\pi \text{ cm}^2$$

13. Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

Table for equation $2x + y = 6$ or $y = 6 - 2x$ is

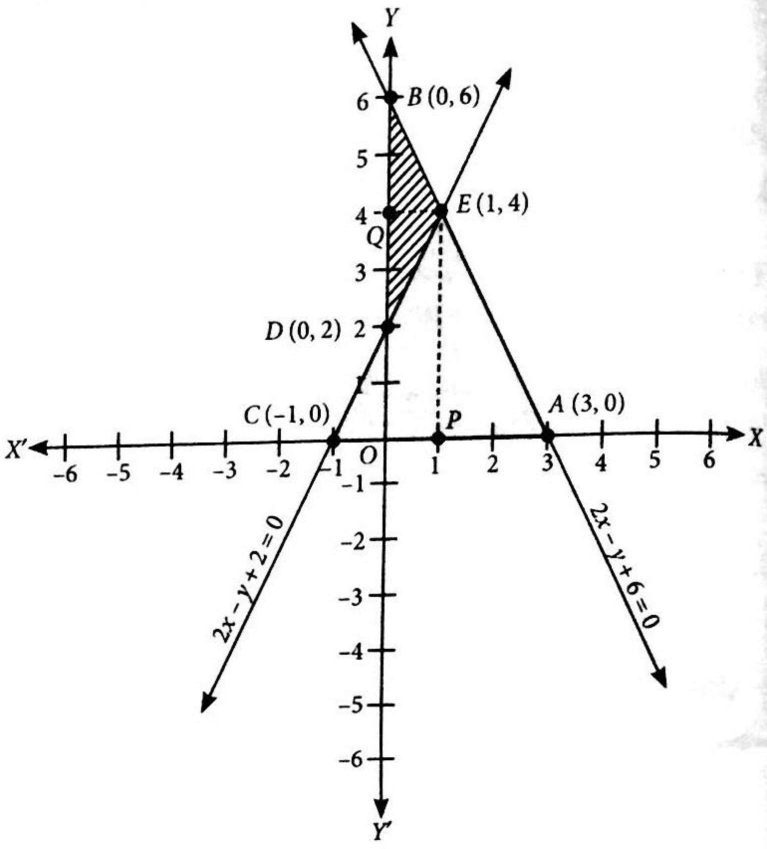
x	0	1	3
$y = 6 - 2x$	6	4	0
Points	$B(0, 6)$	$E(1, 4)$	$A(3, 0)$

Now, plot all these points on a graph paper and join them to get a line AB.

Table for equation $2x - y + 2 = 0$ or $y = 2x + 2$

x	0	−1	1
$y = 2x + 2$	2	0	4
Points	$D(0, 2)$	$C(-1, 0)$	$E(1, 4)$

Now, plot all these points on a graph paper and join them to get a line CE.



It is clear from the graph that pair of equations intersect at point $E(1, 4)$, *i.e.* $x = 1$ and $y = 4$

Thus, from the graph, we get two triangles ΔACE (triangle formed by lines and X-axis) and ΔBDE (triangle formed by the lines and Y-axis).

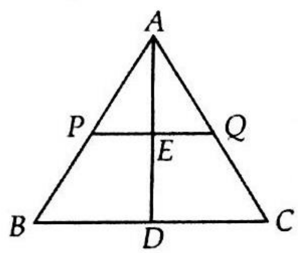
Then, area of $\Delta ACE = \frac{1}{2} \times AC \times PE$

$= \frac{1}{2} \times 4 \times 4 = 8$ sq units

Area of $\Delta BDE = \frac{1}{2} \times BD \times QE = \frac{1}{2} \times 4 \times 1 = 2$ sq units

$\therefore$ Ratio of areas of ΔACE and $\Delta BDE = 8 : 2 = 4 : 1$

14. Suppose the median intersects PQ at E.



$PQ || BC$ (given)

$\Rightarrow \angle APE = \angle B$

and $\angle AQE = \angle C$

(coresponding angles)

So, in Δ's APE and ABD

$\angle APE = \angle ABD$

$\angle PAE \sim \angle BAD$ (common)

$\therefore \Delta APE = \Delta ABD$

$$\Rightarrow \frac{PE}{BD} = \frac{AE}{AD} \quad ...(i)$$

Similarly, $\Delta AQE \sim \Delta ACD$

$$\therefore \frac{QE}{CD} = \frac{AE}{AD} \quad ...(ii)$$

From (i) and (ii)

$$\frac{PE}{BD} = \frac{QE}{CD} \Rightarrow \frac{PE}{BD} = \frac{QE}{BD} \quad [\because BD = CD]$$

$\Rightarrow PE = QE \quad \therefore \quad AD$ bisects PQ.

15. We have, $AB = BC = 3.5$ cm, $DE = 2$ cm,
$\angle BCD = 90°$
$DC = DE + EC = DE + BC = 2 + 3.5 = 5.5$ cm
[$\because EC = BC$ (radii of same quarter circle)]
Height of trapezium, $BC = 3.5$ m
Area of the shaded part of the metal sheet
= Area of trapezium $ABCD$ – Area of quarter circle $BFEC$

$= \frac{1}{2}(AB + CD)BC - \frac{1}{4}\pi(BC)^2$

$= \frac{1}{2}(3.5 + 5.5)3.5 - \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$

$= 15.75 - 9.625 = 6.125 \text{ cm}^2$

OR

Area of shaded region = Area of ΔABC + Area of semi-circle BEC – Area of quadrant $ABDC$ ...(i)

Now, Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$ ...(ii)

In ΔABC, $(AB)^2 + (AC)^2 = (BC)^2$
$\Rightarrow (28)^2 + (28)^2 = (BC)^2$
$\Rightarrow (BC)^2 = 2 \times (28)^2 \Rightarrow BC = 28\sqrt{2}$ cm

Area of semi-circle $BEC = \frac{\pi}{2}\left(\frac{BC}{2}\right)^2 = \frac{\pi}{2} \times \left(\frac{28\sqrt{2}}{2}\right)^2$

$= \frac{22}{7} \times \frac{1}{2} \times 14 \times 14 \times 2 = 616 \text{ cm}^2$ ...(iii)

Area of quadrant $ABDC = \frac{\pi}{4}(AB)^2$

$= \frac{22}{7} \times \frac{1}{4} \times 28 \times 28 = 616 \text{ cm}^2$ ...(iv)

Using (ii), (iii) and (iv) in (i), we get
Area of shaded region = $392 + 616 - 616 = 392 \text{ cm}^2$

16. Table for the given data is

x_i	f_i	$x_i f_i$
15	2	30
17	3	51
19	4	76
$20 + k$	$5k$	$100k + 5k^2$
23	6	138
Total	$\Sigma f_i = 5k + 15$	$\Sigma f_i x_i = 295 + 100k + 5k^2$

Here, $\Sigma f_i = 5k + 15$ and $\Sigma f_i x_i = 295 + 100k + 5k^2$

Given, mean = 20

We know that, Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$\Rightarrow \frac{295 + 100k + 5k^2}{5k + 15} = 20$$

$\Rightarrow 295 + 100k + 5k^2 = 20(5k + 15)$

$\Rightarrow 295 + 100k + 5k^2 = 100k + 300$

$\Rightarrow 295 + 100k + 5k^2 - 100k - 300 = 0$

$\Rightarrow 5k^2 - 5 = 0 \Rightarrow k^2 - 1 = 0$

$\Rightarrow (k + 1)(k - 1) = 0$

If $k + 1 = 0$, then $k = -1$

If $k - 1 = 0$, then $k = 1$

Here, negative value of k is not possible

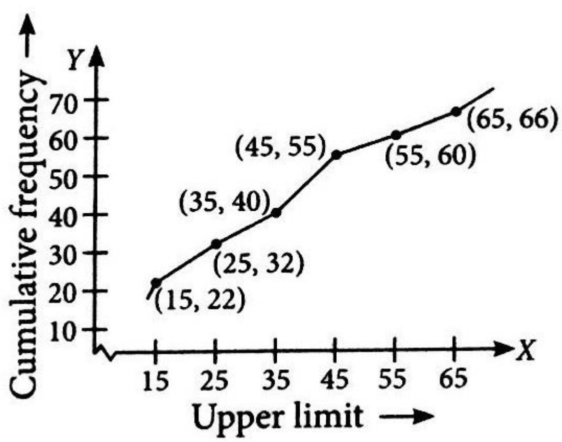
Hence, the required value of k is 1.

OR

Let us first prepare the less than cumulative frequency distribution as given below.

Rainfall (in cm)	Number of days	Rainfall (in cm)	Cumulative frequency
5-15	22	less than 15	22
15-25	10	less than 25	22 + 10 = 32
25-35	8	less than 35	32 + 8 = 40
35-45	15	less than 45	40 + 15 = 55
45-55	5	less than 55	55 + 5 = 60
55-65	6	less than 65	60 + 6 = 66

Now, take upper limits along the X-axis and cumulative frequencies along Y-axis, on graph paper. Then, plot the points (15, 22), (25, 32), (35, 40), (45, 55), (55, 60) and (65, 66), on graph paper and join them by a freehand smooth curve.



17. Steps of construction :

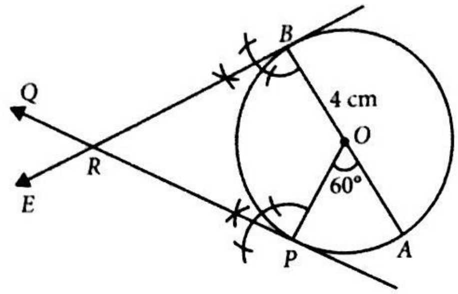
Step 1 : Draw a circle with centre O and radius 4 cm.

Step 2 : Draw any diameter AOB.

Step 3 : Take a point P on the circle such that $\angle AOP = 60°$.

Step 4 : Draw $PQ \perp OP$ and $BE \perp OB$. Let PQ and BE intersect at R.

Hence, RB and RP are the required tangents.



18. Let us assume, to the contrary that $\sqrt{3}+\sqrt{5}$ is a rational number.

$\therefore \quad \sqrt{3}+\sqrt{5}=\frac{p}{q}, q \neq 0$ and $p, q \in Z$

$\Rightarrow \quad \sqrt{3}=\frac{p}{q}-\sqrt{5}$

Squaring both sides, we have

$$3=\frac{p^2}{q^2}+5-2\sqrt{5}\frac{p}{q}$$

$$\Rightarrow \quad 2\sqrt{5}\frac{p}{q}=\frac{p^2}{q^2}+5-3 \Rightarrow 2\sqrt{5}\frac{p}{q}=\frac{p^2}{q^2}+2$$

$$\Rightarrow \quad \sqrt{5}=\frac{q}{2p}\left(\frac{p^2}{q^2}+2\right)=\text{rational}$$

which contradicts the fact that $\sqrt{5}$ is irrational.

Hence, $\sqrt{3}+\sqrt{5}$ is irrational.

19. Given points are $A(7, 8)$, $B(4, 2)$ and $C(8, 2)$ and mid-point of BC is $D(6, 2)$, (say D).

$$\because \text{Area of } \Delta = \frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1) + x_3(y_1-y_2)]|$$

$$\therefore \quad \text{Area of } \Delta ABD = \frac{1}{2}|[7(2-2)+4(2-8)+6(8-2)]|$$

$$=\frac{1}{2}|[0-24+36]| = 6 \text{ sq units}$$

$$\text{Area of } \Delta ADC = \frac{1}{2}|[7(2-2)+6(2-8)+8(8-2)]|$$

$$=\frac{1}{2}|[0-36+48]| = 6 \text{ sq units}$$

$\therefore$ Area of ΔABD = Area of ΔADC

Also, area of $\Delta ABC = \frac{1}{2}|[7(2-2)+4(2-8)+8(8-2)]|$

$= \frac{1}{2}|[0-24+48]| = 12$ sq units

i.e., area of ΔABD = area of $\Delta ADC = \frac{1}{2}$(area ΔABC)

20. Seven years ago, let Swati age be x, then Varun's age = $5x^2$ years

$\therefore$ Swati's present age = $(x + 7)$

and Varun's present age = $(5x^2 + 7)$

Three years hence, we have,

Swati's age = $(x + 7 + 3) = (x + 10)$

Varun's age = $(5x^2 + 7 + 3) = (5x^2 + 10)$

According to the question, $x + 10 = \frac{2}{5}(5x^2 + 10)$

$\Rightarrow x + 10 = 2x^2 + 4$

$\Rightarrow 2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$ [by factorisation]

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (2x + 3)(x - 2) = 0$

$\Rightarrow x - 2 = 0$ or $2x + 3 = 0$

$\Rightarrow x = 2$ or $x = \frac{-3}{2}$

But $x \neq \frac{-3}{2}$ as $x > 0$ [since, age cannot be negative]

$\therefore x = 2$

Hence, Swati's present age = $(2 + 7) = 9$ years

Varun's present age = $[5 \times (2)^2 + (7)] = 27$ years

21. $$\text{L.H.S.} = \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta} = \frac{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{(1+\cos\theta)/\sin\theta}{(1-\cos\theta)/\sin\theta} = \frac{1+\cos\theta}{1-\cos\theta}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$[\because \ (a-b)(a+b) = a^2 - b^2]$

$$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$

$= \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}^2\theta\cos\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}^2\theta\cos\theta$

$[\because \ \operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$= 1 + 2\cot^2\theta + 2\operatorname{cosec}^2\theta\cos\theta =$ R.H.S.

OR

We have, $\sin A\cos A - \frac{\sin A\cos(90^\circ - A)\cos A}{\sec(90^\circ - A)} - \frac{\cos A\sin(90^\circ - A)\sin A}{\operatorname{cosec}(90^\circ - A)}$

$$= \sin A\cos A - \frac{\sin A\sin A\cos A}{\operatorname{cosec} A} - \frac{\cos A\cos A\sin A}{\sec A}$$

$$= \sin A\cos A - \sin A\cos A\left(\frac{\sin A}{\operatorname{cosec} A} + \frac{\cos A}{\sec A}\right)$$

$= \sin A\cos A - \sin A\cos A(\sin^2 A + \cos^2 A)$

$= \sin A\cos A - \sin A\cos A = 0$

22. Let a be the first term and d be the common difference of given AP.

According to the question,

$$S_1 = S_n = \frac{n}{2}[2a+(n-1)d] \quad \text{...(i)}$$

$$S_2 = S_{2n} = \frac{2n}{2}[2a+(2n-1)d] \quad \text{...(ii)}$$

and $$S_3 = S_{3n} = \frac{3n}{2}[2a+(3n-1)d] \quad \text{...(iii)}$$

Now, $$S_2 - S_1 = \frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]$$

$= \frac{n}{2}[2\{2a+(2n-1)d\}-\{2a+(n-1)d\}]$

$= \frac{n}{2}[2a+(3n-1)d]$

$\therefore\ 3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)d]$

$\Rightarrow\ 3(S_2 - S_1) = S_3$ [using eq. (iii)]

or $S_3 = 3(S_2 - S_1)$ Hence proved

OR

Let a and $d > 0$ be the first term and common difference of given AP. According to the given condition, we have

$a_1 + a_5 = 26$

$\Rightarrow\ a + a + (5-1)d = 26$

$\Rightarrow\ 2a + 4d = 26$

$\Rightarrow\ a + 2d = 13$ [dividing both sides by 2] ...(i)

Also, we have $a_2 \times a_4 = 160$

$\Rightarrow\ (a+d) \times (a+3d) = 160$

$\Rightarrow\ (13-2d+d)(13-2d+3d) = 160$ [from eq. (i)]

$\Rightarrow\ (13-d)(13+d) = 160$

$\Rightarrow\ (13)^2 - (d)^2 = 160$

$\Rightarrow\ d^2 = 169 - 160 \quad \Rightarrow \quad d^2 = 9 \quad \Rightarrow \quad d = \pm 3$

But $d > 0 \quad \therefore \quad d = 3$

On putting $d = 3$ in (i), we get

$a + (2 \times 3) = 13$

$\Rightarrow\ a = 13 - 6 = 7$

Now, the sum of first seven terms,

$S_7 = \frac{7}{2}[2 \times a + (7-1)d]$

$= \frac{7}{2}[2 \times 7 + 6 \times 3]$

$= \frac{7}{2}[14+18] = \frac{7 \times 32}{2} = 112$

23. (i) Given, total cost of fencing = ₹ 5500
and rate of fencing per metre = ₹ 25

$$\text{Length of fencing} = \frac{\text{Total cost}}{\text{Rate of fencing per metre}}$$
$$= \frac{5500}{25} = 220 \text{ m}$$

(ii) Circumference of the circular field
= Length of the fence

$\Rightarrow \quad 2\pi r = 220 \Rightarrow 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{2 \times 22} \Rightarrow r = 35$ m

i.e., Radius of the circular field = 35 m

$\therefore$ Area of the circular field = πr^2

$= \frac{22}{7} \times (35)^2 = \frac{22}{7} \times 35 \times 35 = 22 \times 5 \times 35 = 3850 \text{ m}^2$

Now, cost of ploughing at the rate of ₹ 0.75 per m^2
= ₹ 3850 × 0.75 = ₹ 2887.5

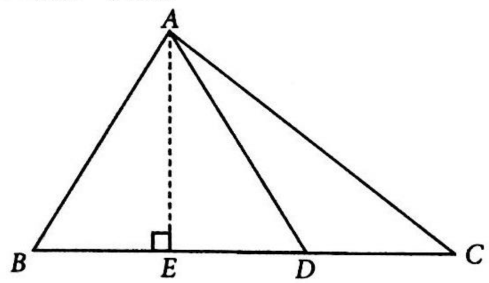
Hence, total cost of ploughing the field is ₹ 2887.5

(iii) Security and separate the boundary of a field. Artist believes in self employment, which gives self respect and dignity of labour.

24. Draw $AE \perp BC$

In right angled ΔAED, using Pythagoras theorem,

$AD^2 = AE^2 + ED^2$ ...(i)



In right angled ΔAEB, using Pythagoras theorem

$AB^2 = AE^2 + BE^2 = AE^2 + (BD - ED)^2$

$[\because BE = BD - ED]$

$= AE^2 + BD^2 + ED^2 - 2BD \cdot ED$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= (AE^2 + ED^2) + BD^2 - 2BD \cdot ED$

$\therefore \quad AB^2 = AD^2 + BD^2 - 2BD \cdot ED$ [From Eq. (i)] ...(ii)

In right angled ΔAEC, using Pythagoras theorem,

$\Rightarrow \quad AC^2 = AE^2 + EC^2 = AE^2 + (ED + DC)^2$

$[\because ED = ED + DC]$

$= AE^2 + ED^2 + CD^2 = 2ED \cdot DC$ ...(iii)

On adding Eqs. (ii) & (iii), we get

$\Rightarrow \; AB^2 + AC^2 = 2\,AD^2 + 2\,BD^2$

$\Rightarrow \; AB^2 + AC^2 = 2\,(AD^2 + BD^2)$ Hence proved.

OR

Given, in ΔABC, CF bisects $\angle C$

We know that the internal bisector of an angle of a triangle divide the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \quad \frac{AF}{FB} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8} \qquad [\because FB = AB - AF = 5 - AF]$$

$$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$$

$\Rightarrow \; 2\,AF = 5 - AF \Rightarrow 2\,AF + AF = 5 \Rightarrow 3\,AF = 5$

$$\therefore \quad AF = \frac{5}{3}\text{ cm}$$

Again, in ΔABC, BE bisects $\angle B$

$$\therefore \quad \frac{AE}{EC} = \frac{AB}{BC} \qquad \text{[by above theorem]}$$

$$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8}$$

$\Rightarrow \; 8(4 - CE) = 5CE \;\Rightarrow\; 32 - 8\,CE = 5CE$

$\Rightarrow \; 32 = 13\,CE$

$$\therefore CE = \frac{32}{13}\text{ cm}$$

Similarly, in ΔABC, AD bisects $\angle A$

$$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{BD}{8-BD} = \frac{5}{4}$$

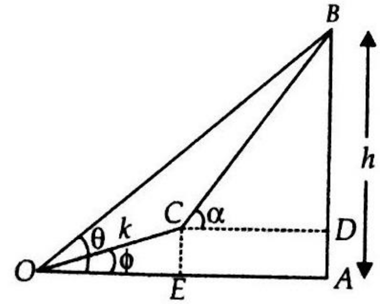
$\Rightarrow \; 4\,BD = 5(8 - BD) \;\Rightarrow\; 4\,BD = 40 - 5BD$

$\Rightarrow \; 9\,BD = 40$

$$\therefore BD = \frac{40}{9}\text{ cm}$$

Hence $AF = \frac{5}{3}$ cm, $CE = \frac{32}{13}$ cm and $BD = \frac{40}{9}$ cm

25. Let AB be the cliff and O be the fixed point such that the angle of elevation of the cliff from O is θ, *i.e.*, $\angle AOB = \theta$. Let $\angle EOC = \phi$ and $OC = k$ metres. From C draw CD and CE perpendiculars on AB and OA respectively. Then, $\angle DCB = \alpha$, let h be the height of the cliff AB.



In ΔOCE, we have

$$\sin\phi = \frac{CE}{OC} \Rightarrow \sin\phi = \frac{CE}{k} \Rightarrow CE = k\sin\phi \quad \text{...(i)}$$

$$\text{and} \quad \cos\phi = \frac{OE}{OC} \Rightarrow \cos\phi = \frac{OE}{k} \Rightarrow OE = k\cos\phi$$

In ΔOAB, we have

$$\tan\theta = \frac{AB}{OA} \Rightarrow \tan\theta = \frac{h}{OA} \Rightarrow OA = \frac{h}{\tan\theta}$$

$$\Rightarrow OA = h\cot\theta \Rightarrow OE + EA = h\cot\theta$$

$$\Rightarrow EA = h\cot\theta - OE = h\cot\theta - k\cos\phi \quad \text{(using (ii))}$$

In ΔBCD, we have

$$\cot\alpha = \frac{CD}{BD} \Rightarrow \cot\alpha = \frac{AE}{AB - AD} \quad [\because CD = AE]$$

$$\Rightarrow \cot\alpha = \frac{h\cot\theta - k\cos\phi}{h - k\sin\phi} \quad \text{(using (i))}$$

$$\Rightarrow h\cot\alpha - k\sin\phi\cot\alpha = h\cot\theta - k\cos\phi$$

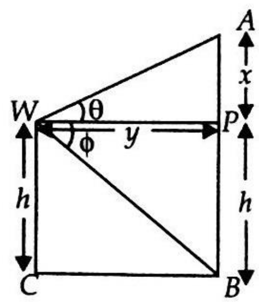
$$\Rightarrow h\cot\theta - h\cot\alpha = k\cos\phi - k\sin\phi\cot\alpha$$

$$\Rightarrow h(\cot\theta - \cot\alpha) = k(\cos\phi - \sin\phi\cot\alpha)$$

$$\Rightarrow h = \frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$$

OR

Let W be the window and AB be the house on the opposite side. Then WP is the width of the street, height of the window = h metres = BP. Let $AP = x$ metres and $WP = y$ metres



In ΔBPW, right angled at P,

$$\tan\phi = \frac{BP}{WP}$$

$$\Rightarrow \tan\phi = \frac{h}{y}$$

Now, in ΔAPW, we have

$$\Rightarrow y = \frac{h}{\tan\phi} = h\cot\phi$$

$$\tan\theta = \frac{AP}{WP} \Rightarrow \tan\theta = \frac{x}{y} \Rightarrow x = y\tan\theta$$

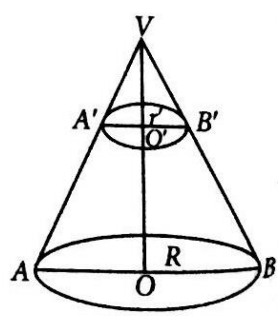
$\Rightarrow x = h\cot\phi\tan\theta$ $\quad [\because y = h\cot\phi]$

Height of the opposite house = $AP + BP$

$= x + h = h\cot\phi\tan\theta + h = h(\cot\phi\tan\theta + 1)$

$= h(1 + \tan\theta\cot\phi)$.

26. Let VAB be a hollow cone of height H, slant height L and the base radius R. Suppose this cone is cut by a plane parallel to the base such that O' is the centre of the circular section of the cone. Let h be the height, l be the slant height and r be the base radius of the smaller cone $VA'B'$.



In $\Delta VO'A'$ and VOA

$\angle A = \angle A'$

$\angle O = \angle O'$

Therefore by AA postulate

$\angle VO'A' \sim \Delta VOA$

$$\Rightarrow \frac{h}{H} = \frac{r}{R} = \frac{l}{L} \quad \text{...(i)}$$

Curved surface area of the frustum $ABB'A = \frac{8}{9} \times$ curved surface area of the cone

$$\Rightarrow \frac{8}{9}\pi RL = \pi(R+r)(L-l)$$

$$\Rightarrow \left(\frac{R+r}{R}\right)\times\left(\frac{L-l}{L}\right)=\frac{8}{9} \Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right)=\frac{8}{9}$$

$$\Rightarrow \left(1+\frac{h}{H}\right)\left(1-\frac{h}{H}\right)=\frac{8}{9}\text{(from (1))}$$

$$\Rightarrow \left(1-\left(\frac{h}{H}\right)^2\right)=\frac{8}{9} \Rightarrow \frac{h^2}{H^2}=1-\frac{8}{9}$$

$$\Rightarrow \frac{h^2}{H^2}=\frac{1}{9} \Rightarrow \frac{h}{H}=\frac{1}{3}$$

Hence, required ratio $= \frac{h}{H-h}$

$$= \frac{\frac{H}{3}}{\frac{2H}{3}} = \frac{1}{2} = 1:2.$$

OR

TSA of the top = CSA of hemisphere + CSA of cone

Now, the curved surface of the hemisphere

$$= \frac{1}{2}(4\pi r^2) = 2\pi r^2 = \left(2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\right)\text{cm}^2$$

Also, the height of the cone = height of the top - height (radius) of the hemispherical part

$$= \left(5-\frac{3.5}{2}\right)\text{cm} = 3.25 \text{ cm}$$

So, the slant height of the cone (l)

$$\sqrt{r^2+h^2} = \sqrt{\left(\frac{3.5}{2}\right)^2+(3.25)^2}$$

= 3.7 cm (approx,)

Therefore, CSA of cone $= \pi rl$

$$\left(\frac{22}{7}\times\frac{3.5}{2}\times 3.7\right)\text{cm}^2$$

This gives the surface area of the top as

$$\left(2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\right)\text{cm}^2 + \left(\frac{22}{7}\times\frac{3.5}{2}\times 3.7\right)\text{cm}^2$$

$$= \frac{22}{7}\times\frac{3.5}{2}(3.5+3.7)\text{ cm}^2 = 39.6 \text{ cm}^2 \text{ (approx)}$$

27. Total number of cards $= 52$

Number of cards removed $= 8$

Remaining cards $= 44$

1. Total numbers of queens $= 4$

Black queens $= 2$

$\therefore$ Probability of a black queen $= \frac{2}{44} = \frac{1}{22}$

2. Total numbers of red cards $= 26$

Number of red cards removed $= 8$

Remaining red cards $= 18$

Probability of a red card $= \frac{18}{44} = \frac{9}{22}$

3. Total numbers of jacks $= 4$

Black colour jacks $= 2$

Probability of a black jack $= \frac{2}{44} = \frac{1}{22}$

4. Total number of picture cards $= 12$

Number of picture cards removed $= 6$

Remaining picture cards $= 6$

Probability of a picture card $= \frac{6}{44} = \frac{3}{22}$

28. Given, AC and AD are tangents at C and D and $\angle BCD = 44°$

Clearly, $\angle OCA = 90°$

[Angle between tangent and radius]

Now, $\angle OCA = \angle OCD + \angle ACD$

$\Rightarrow \angle ACD = \angle OCA - \angle OCD$

$\Rightarrow \angle ACD = 90° - 44° \Rightarrow \angle ACD = 46°$

As, $AC = AD$ [Tangents drawn from an external point are equal in length]

So, $\angle ADC = \angle ACD = 46°$

[$\because$ Angles apposite to the equal sides are equal]

$\Rightarrow \angle CAD + 46° + 46° = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle CAD = 180° - 92° = 88°$

Again, $\angle COD = 180° - \angle CAD$

$\Rightarrow \angle CAD = 180° - 88° = 92°$

[$\because$ In quadrilateral $ODAC$, $\angle C + \angle D = 90°$ and also sum of all interior angles of a quadrilateral is $360°$]

Further, $\angle OBD = \angle ODB$

[$\because$ $OB = OD$ radii of circle]

$\Rightarrow$ $2\angle OBD = \angle COD$ [Exterior angle theorem]

$\Rightarrow$ $\angle CAD = 92° \Rightarrow \angle CBD = \frac{1}{2} \times 92° = 46°$

Thus, $\angle CBD = 46°$, $\angle ADC = 46°$, $\angle CAD = 88°$ and $\angle ACD = 46°$

29. The given series is an Inclusive series. Making it an exclusive series, we get

(Age in years)	Frequency (f_i)	Mid-value (x_i)	$u_i = \frac{x_i - a}{h} = \frac{(x_i - 32)}{5}$	$f_i \times u_i$
14.5-19.5	3	17	–3	–9
19.5-24.5	13	22	–2	–26
24.5-29.5	21	27	–1	–21
29.5-34.5	15	32 = a	0	0
34.5-39.5	5	37	1	5
39.5-44.5	4	42	2	8
44.5-49.5	2	47	3	6
	$n = \Sigma f_i$ = 63			$\Sigma(f_i \times u_i)$ = (19 – 56) = – 37

Thus, $a = 32$, $h = 5$, $n = 63$ and $\Sigma(f_i \times u_i) = -37$

$$\therefore \quad \text{Mean, } \bar{x} = a + h \times \left\{\frac{1}{n}\Sigma f_i u_i\right\}$$

$$= 32 + \left[5 \times \left(\frac{-37}{63}\right)\right] = (32 - 2.936) = 29.06$$

30. Given, $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = 1 - \sin^2\theta \quad \Rightarrow \quad \sin\theta = \cos^2\theta$

$= (\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta)$

$+ 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\cos^4\theta + \cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$

$[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3]$

$= (\sin^2\theta + \cos^2\theta)^3 + 2(\sin^2\theta + \cos^2\theta - 1)$

$[\because \cos^2\theta = \sin\theta \Rightarrow \cos^4\theta = \sin^2\theta]$

$= 1 + 2(1 - 1) = 1$

OR

We have, L.H.S. $= \left(\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2$

$$= \frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\cos\theta-2\sin\theta\cos\theta}{1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\cos\theta+2\sin\theta\cos\theta}$$

$$= \frac{1+1+2\sin\theta-2\cos\theta\,(1+\sin\theta)}{1+1+2\sin\theta+2\cos\theta\,(1+\sin\theta)}$$

$$= \frac{2+2\sin\theta-2\cos\theta\,(1+\sin\theta)}{2+2\sin\theta+2\cos\theta\,(1+\sin\theta)}$$

$$= \frac{2\{(1+\sin\theta)-\cos\theta\,(1+\sin\theta)\}}{2\{(1+\sin\theta)+\cos\theta\,(1+\sin\theta)\}}$$

$$= \frac{(1+\sin\theta)(1-\cos\theta)}{(1+\sin\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$$