Solved Paper–5

Class 9th, Mathematics, SA–2

Time: 3hours **Max. Marks 90**

General Instructions

1. All questions are compulsory.
2. Draw neat labeled diagram wherever necessary to explain your answer.
3. Q.No. 1 to 8 are of objective type questions, carrying 1 mark each.
4. Q.No.9 to 14 are of short answer type questions, carrying 2 marks each.
5. Q. No. 15 to 24 carry 3 marks each. Q. No. 25 to 34 carry 4 marks each.

1. Abscissa of all the points on the x-axis is
   (A) 0 (B) 1
   (C) 2 (D) any number

2. In Δ ABC, BC = AB and $\angle B = 80°$. Then $\angle A$ is equal to
   (A) 80° (B) 40°
   (C) 50° (D) 100°

3. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is :
   (A) 17 cm (B) 15 cm
   (C) 4 cm (D) 8 cm

4. The equation $2x + 5y = 7$ has a unique solution, if x, y are :
   (A) Natural numbers (B) Positive real numbers
   (C) Real numbers (D) Rational numbers

5. The marks obtained by 17 students in a mathematics test (out of 100) are given below :
   91, 82, 100, 100, 96, 65, 82, 76, 79, 90, 46, 64, 72, 68, 66, 48, 49.
   The range of the data is :
   (A) 46 (B) 54
   (C) 90 (D) 100

6. In a cylinder, radius is doubled and height is halved, curved surface area will be
   (A) halved (B) doubled
   (C) same (D) four times

7. Two coins are tossed 1000 times and the outcomes are recorded as below :

Number of heads	2	1	0
Frequency	200	550	250

Based on this information, the probability for at most one head is

(A)/15 (B)1/4

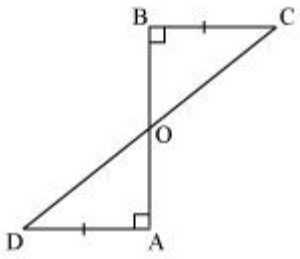
(C)4/5 (D)3/4

8. The number of planks of dimensions (4 m × 50 cm × 20 cm) that can be stored in a pit which is 16 m long, 12m wide and 4 m deep is

(A) 1900 (B) 1920

(C) 1800 (D) 1840

9. AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.



10. Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm $\left[\text{Assume } \pi = \frac{22}{7}\right]$

11. The heights of 50 students, measured to the nearest centimeters, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

(i) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

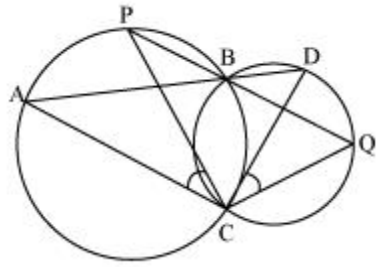
(ii) What can you conclude bout their heights from the table?

12.

Blood group	Number of students
A	9
B	6
AB	3
O	12
Total	30

The above frequency distribution table represents the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

13. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the given figure). Prove that ∠ACP = ∠QCD.



14. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0 1 2 2 1 2 3 1 3 0
1 3 1 1 2 2 0 1 2 1
3 0 0 1 1 2 3 2 2 0

Prepare a frequency distribution table for the data given above.

15. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

16. The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

17. Construct a triangle XYZ in which $\angle Y = 30°$, $\angle Z = 90°$ and XY + YZ + ZX = 11 cm.

18. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

19. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

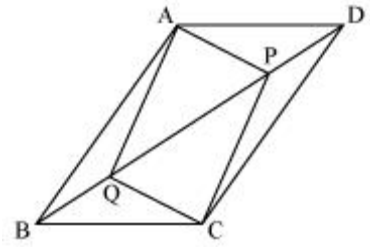
Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.
(ii) Which political party won the maximum number of seats?

20. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is
(i) 2 units (ii) 0 units

21. Find the volume of a sphere whose radius is
(i) 7 cm (ii) 0.63 m $\left[\text{Assume } \pi = \frac{22}{7}\right]$

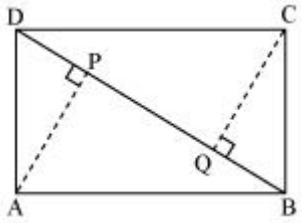
22. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:



(i) $\Delta APD \cong \Delta CQB$
(ii) AP = CQ
(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP
(v) APCQ is a parallelogram

23. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



(i) $\Delta APB \cong \Delta CQD$
(ii) AP = CQ

24. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	**Number of leaves**
118 − 126	3
127 − 135	5
136 − 144	9
145 − 153	12
154 − 162	5
163 − 171	4
172 − 180	2

(i) Draw a histogram to represent the given data.
(ii) Is there any other suitable graphical representation for the same data?
(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

25. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

26. Construct an equilateral triangle, given its side and justify the construction

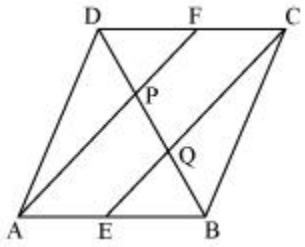
27. Give the geometric representations of $2x + 9 = 0$ as an equation
(1) in one variable

(2) in two variables

28. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

29. Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

30. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



31. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data.

32. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

33. Twenty seven solid iron spheres, each of radius *r* and surface area S are melted to form a sphere with surface area S'. Find the
(i) radius *r*' of the new sphere, (ii) ratio of S and S'.

34. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

Solutions

1. D
2. C
3. B
4. A
5. B
6. B
7. D
8. B

9. In ΔBOC and ΔAOD,
∠BOC = ∠AOD (Vertically opposite angles)
∠CBO = ∠DAO (Each 90º)
BC = AD (Given)
∴ ΔBOC ≅ ΔAOD (AAS congruence rule)
∴ BO = AO (By CPCT)
⇒ CD bisects AB.

10. (i) Radius (r) of cone = 6 cm
Height (h) of cone = 7 cm
Volume of cone $= \frac{1}{3}\pi r^2 h$
$= \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{ cm}^3$
$= (12 \times 22) \text{ cm}^3$
$= 264 \text{ cm}^3$
Therefore, the volume of the cone is 264 cm^3.
(ii) Radius (r) of cone = 3.5 cm
Height (h) of cone = 12 cm
Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{ cm}^3$$

$$= \left(\frac{1}{3} \times 22 \times \frac{1}{2} \times 3.5 \times 12\right) \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

Therefore, the volume of the cone is 154 cm^3.

11. (i) A grouped frequency distribution table has to be constructed taking class intervals 160 − 165, 165 − 170, etc. By observing the data given above, the required table can be constructed as follows.

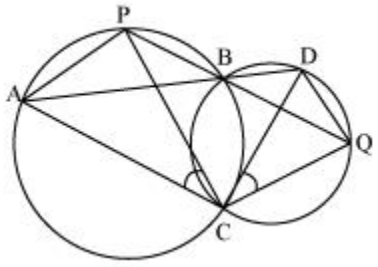
Height (in cm)	**Number of students (frequency)**
150 − 155	12
155 − 160	9
160− 165	14
165 − 170	10
170 − 175	5
Total	50

(ii) It can be concluded that more than 50% of the students are shorter than 165 cm.

12. Number of students having blood group AB = 3
Total number of students = 30

Hence, required probability, $P = \frac{3}{30} = \frac{1}{10}$

13.



Join chords AP and DQ.
For chord AP,
$\angle PBA = \angle ACP$ (Angles in the same segment) ... (1)
For chord DQ,
$\angle DBQ = \angle QCD$ (Angles in the same segment) ... (2)
ABD and PBQ are line segments intersecting at B.
$\therefore \angle PBA = \angle DBQ$ (Vertically opposite angles) ... (3)
From equations (1), (2), and (3), we obtain
$\angle ACP = \angle QCD$

14. By observing the data given above, the required frequency distribution table can be constructed as follows.

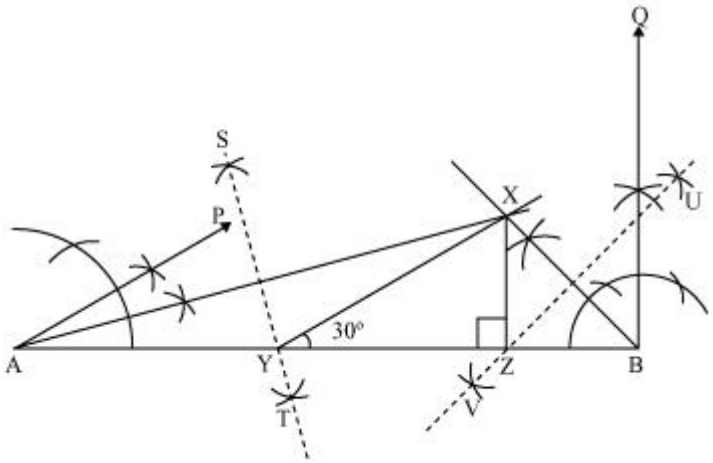
Number of heads	**Number of times (frequency)**
0	6
1	10
2	9
3	5
Total	30

15. It can be observed that point (2, 14) satisfies the equation $7x - y = 0$ and $x - y + 12 = 0$.
Therefore, $7x - y = 0$ and $x - y + 12 = 0$ are two lines passing through point (2, 14).
As it is known that through one point, infinite number of lines can pass through, therefore, there are infinite lines of such type passing through the given point.

16. Let the common ratio between the angles be x. Therefore, the angles will be $3x$, $5x$, $9x$, and $13x$ respectively.
As the sum of all interior angles of a quadrilateral is 360°,
$\therefore 3x + 5x + 9x + 13x = 360^\circ$
$30x = 360^\circ$
$x = 12^\circ$
Hence, the angles are

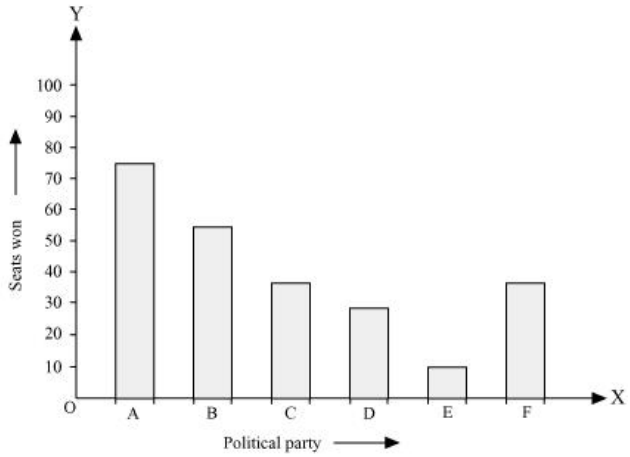
$3x = 3 \times 12 = 36^o$
$5x = 5 \times 12 = 60^o$
$9x = 9 \times 12 = 108^o$
$13x = 13 \times 12 = 156^o$

17. The below given steps will be followed to construct the required triangle.
Step I: Draw a line segment AB of 11 cm.
(As XY + YZ + ZX = 11 cm)
Step II: Construct an angle, ∠PAB, of 30° at point A and an angle, ∠QBA, of 90° at point B.
Step III: Bisect ∠PAB and ∠QBA. Let these bisectors intersect each other at point X.
Step IV: Draw perpendicular bisector ST of AX and UV of BX.
Step V: Let ST intersect AB at Y and UV intersect AB at Z.
Join XY, XZ.
ΔXYZ is the required triangle.



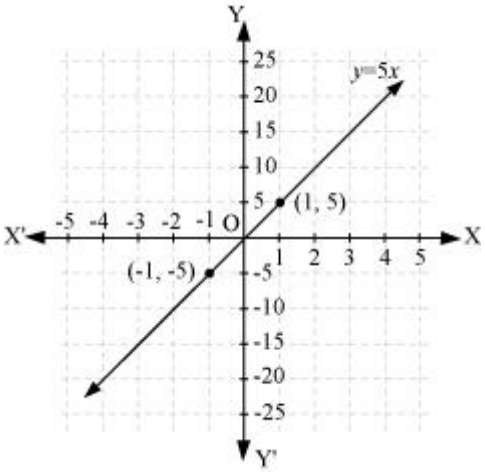
18. Radius (r) of pit $= \left(\frac{3.5}{2}\right)$ m $= 1.75$ m
Height (h) of pit = Depth of pit = 12 m
Volume of pit $= \frac{1}{3}\pi r^2 h$
$= \left[\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\right]$ cm^3
= 38.5 m^3
Thus, capacity of the pit = (38.5 × 1) kilolitres = 38.5 kilolitres

19. (i) By taking polling results on x-axis and seats won as y-axis and choosing an appropriate scale (1 unit = 10 seats for y-axis), the required graph of the above information can be constructed as follows.



Here, the rectangle bars are of the same length and have equal spacing in between them.
(ii) Political party 'A' won maximum number of seats.

20. Let the distance travelled and the work done by the body be x and y respectively.
Work done $\propto$ distance travelled
$y \propto x$
$y = kx$
Where, k is a constant
If constant force is 5 units, then work done $y = 5x$
It can be observed that point (1, 5) and (−1, −5) satisfy the above equation. Therefore, these are the solutions of this equation. The graph of this equation is constructed as follows.



(i)From the graphs, it can be observed that the value of y corresponding to $x = 2$ is 10. This implies that the work done by the body is 10 units when the distance travelled by it is 2 units.

(ii) From the graphs, it can be observed that the value of y corresponding to $x = 0$ is 0. This implies that the work done by the body is 0 units when the distance travelled by it is 0 unit.

21. (i) Radius of sphere = 7 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times (7)^3\right] \text{ cm}^3$$

$$= \left(\frac{4312}{3}\right) \text{ cm}^3$$

$$= 1437\frac{1}{3} \text{ cm}^3$$

Therefore, the volume of the sphere is $1437\frac{1}{3}$ cm^3.

(ii) Radius of sphere = 0.63 m

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times (0.63)^3\right] \text{ m}^3$$

$$= 1.0478 \text{ m}^3$$

Therefore, the volume of the sphere is 1.05 m^3 (approximately).

22. (i) In ΔAPD and ΔCQB,

∠ADP = ∠CBQ (Alternate interior angles for BC || AD)

AD = CB (Opposite sides of parallelogram ABCD)

DP = BQ (Given)

∴ ΔAPD ≅ ΔCQB (Using SAS congruence rule)

(ii) As we had observed that ΔAPD ≅ ΔCQB,

∴ AP = CQ (CPCT)

(iii) In ΔAQB and ΔCPD,

∠ABQ = ∠CDP (Alternate interior angles for AB || CD)

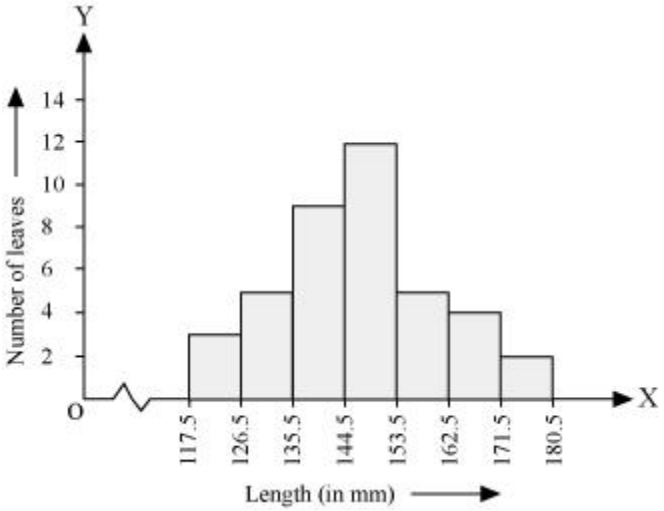
AB = CD (Opposite sides of parallelogram ABCD)

BQ = DP (Given)
∴ ΔAQB ≅ ΔCPD (Using SAS congruence rule)
(iv) As we had observed that ΔAQB ≅ ΔCPD,
∴ AQ = CP (CPCT)
(v) From the result obtained in (ii) and (iv),
AQ = CP and
AP = CQ
Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

23. (i) In ΔAPB and ΔCQD,
∠APB = ∠CQD (Each 90°)
AB = CD (Opposite sides of parallelogram ABCD)
∠ABP = ∠CDQ (Alternate interior angles for AB || CD)
∴ ΔAPB ≅ ΔCQD (By AAS congruency)
(ii) By using the above result
ΔAPB ≅ ΔCQD, we obtain
AP = CQ (By CPCT)

24. (i) It can be observed that the length of leaves is represented in a discontinuous class interval having a difference of 1 in between them. Therefore, $\frac{1}{2} = 0.5$ has to be added to each upper class limit and also have to subtract 0.5 from the lower class limits so as to make the class intervals continuous.

Length (in mm)	**Number of leaves**
117.5 − 126.5	3
126.5 − 135.5	5
135.5 − 144.5	9
144.5 − 153.5	12
153.5 − 162.5	5
162.5 − 171.5	4
171.5 − 180.5	2



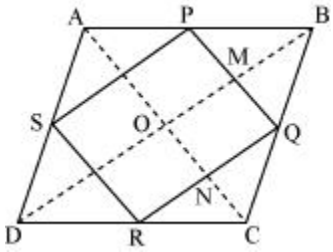
Taking the length of leaves on *x*-axis and the number of leaves on *y*-axis, the histogram of this information can be drawn as above.

Here, 1 unit on *y*-axis represents 2 leaves.

(ii) Other suitable graphical representation of this data is frequency polygon.

(iii) No, as maximum number of leaves (i.e., 12) has their length in between 144.5 mm and 153.5 mm. It is not necessary that all have their lengths as 153 mm.

25.



In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

∴ PQ || AC and PQ = $\frac{1}{2}$ AC (Using mid-point theorem) ... (1)

In ΔADC,

R and S are the mid-points of CD and AD respectively.

∴ RS || AC and RS = $\frac{1}{2}$ AC (Using mid-point theorem) ... (2)

From equations (1) and (2), we obtain

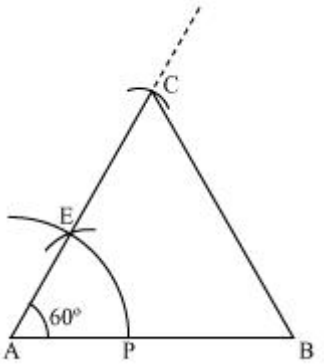
PQ || RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,
MQ || ON (∵ PQ || AC)
QN || OM (∵ QR || BD)
Therefore, OMQN is a parallelogram.
⇒ ∠MQN = ∠NOM
⇒ ∠PQR = ∠NOM
However, ∠NOM = 90° (Diagonals of a rhombus are perpendicular to each other)
∴ ∠PQR = 90°
Clearly, PQRS is a parallelogram having one of its interior angles as 90º.
Hence, PQRS is a rectangle.

26. Let us draw an equilateral triangle of side 5 cm. We know that all sides of an equilateral triangle are equal. Therefore, all sides of the equilateral triangle will be 5 cm. We also know that each angle of an equilateral triangle is 60º.
The below given steps will be followed to draw an equilateral triangle of 5 cm side.
Step I: Draw a line segment AB of 5 cm length. Draw an arc of some radius, while taking A as its centre. Let it intersect AB at P.
Step II: Taking P as centre, draw an arc to intersect the previous arc at E. Join AE.
Step III: Taking A as centre, draw an arc of 5 cm radius, which intersects extended line segment AE at C. Join AC and BC. ΔABC is the required equilateral triangle of side 5 cm.



Justification of Construction:
We can justify the construction by showing ABC as an equilateral triangle i.e., AB = BC = AC = 5 cm and ∠A = ∠B = ∠C = 60°.
In ΔABC, we have AC = AB = 5 cm and ∠A = 60°.
Since AC = AB,
∠B = ∠C (Angles opposite to equal sides of a triangle)
In ΔABC,

$\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

$\Rightarrow 60° + \angle C + \angle C = 180°$

$\Rightarrow 60° + 2\angle C = 180°$

$\Rightarrow 2\angle C = 180° - 60° = 120°$

$\Rightarrow \angle C = 60°$

$\therefore \angle B = \angle C = 60°$

We have, $\angle A = \angle B = \angle C = 60°$ ... (1)

$\Rightarrow \angle A = \angle B$ and $\angle A = \angle C$

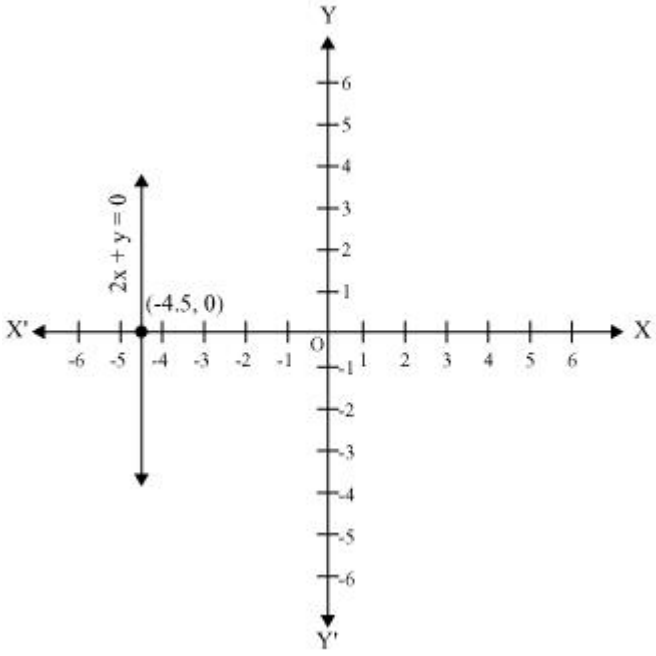
$\Rightarrow BC = AC$ and $BC = AB$ (Sides opposite to equal angles of a triangle)

$\Rightarrow AB = BC = AC = 5$ cm ... (2)

From equations (1) and (2), ΔABC is an equilateral triangle.

27. (1) In one variable, $2x + 9 = 0$ represents a point $x = \frac{-9}{2} = -4.5$ as shown in the following figure.

(2) In two variables, $2x + 9 = 0$ represents a straight line passing through point $(-4.5, 0)$ and parallel to y-axis. It is a collection of all points of the plane, having their x-coordinate as 4.5.



28. Radius (r) of metallic ball = $\left(\frac{4.2}{2}\right)$ cm = 2.1 cm

Volume of metallic ball = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (2.1)^3\right]$ cm^3

$= 38.808$ cm^3

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

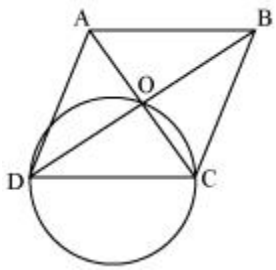
Mass = Density × Volume

= (8.9 × 38.808) g

= 345.3912 g

Hence, the mass of the ball is 345.39 g (approximately).

29.



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

∴ ∠COD = 90°

Also, in rhombus, the diagonals intersect each other at 90°.

∠AOB = ∠BOC = ∠COD = ∠DOA = 90°

Clearly, point O has to lie on the circle.

30. ABCD is a parallelogram.

∴AB || CD

And hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}$ AB = $\frac{1}{2}$ CD

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

⇒ AF || EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

⇒ DP = PQ ... (1)

Similarly, in ΔAPB, E is the mid-point of side AB and EQ || AP (as AF || EC).

Therefore, by using the converse of mid-point theorem, it can be said that

Q is the mid-point of PB.

⇒ PQ = QB ... (2)

From equations (1) and (2),

DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

31. The marks of 15 students in mathematics test are

41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

$$\text{Mean of data} = \frac{\text{Sum of all observation}}{\text{Total number of observation}}$$

$$= \frac{41+39+48+52+46+62+54+40+96+52+98+40+42+52+60}{15}$$

$$= \frac{822}{15} = 54.8$$

Arranging the scores obtained by 15 students in an ascending order,

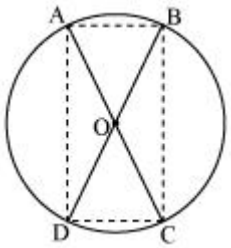
39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98

As the number of observations is 15 which is odd, therefore, the median of data will be $\frac{15+1}{2} = 8^{th}$ observation whether the data is arranged in an ascending or descending order.

Therefore, median score of data = 52

Mode of data is the observation with the maximum frequency in data. Therefore, mode of this data is 52 having the highest frequency in data as 3.

32.



Let two chords AB and CD are intersecting each other at point O.
In ΔAOB and ΔCOD,
OA = OC (Given)
OB = OD (Given)
∠AOB = ∠COD (Vertically opposite angles)
ΔAOB ≅ ΔCOD (SAS congruence rule)
AB = CD (By CPCT)
Similarly, it can be proved that ΔAOD ≅ ΔCOB
∴ AD = CB (By CPCT)
Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram.
We know that opposite angles of a parallelogram are equal.
∴ ∠A = ∠C
However, ∠A + ∠C = 180° (ABCD is a cyclic quadrilateral)
⇒ ∠A + ∠A = 180°
⇒ 2 ∠A = 180°
⇒ ∠A = 90°
As ACBD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.
∠A is the angle subtended by chord BD. And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

33. (i)Radius of 1 solid iron sphere = r

Volume of 1 solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid iron spheres $= 27 \times \frac{4}{3}\pi r^3$

27 solid iron spheres are melted to form 1 iron sphere. Therefore, the volume of this iron sphere will be equal to the volume of 27 solid iron spheres. Let the radius of this new sphere be r'.

Volume of new solid iron sphere $= \frac{4}{3}\pi r'^3$

$\frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$r'^3 = 27r^3$

$r' = 3r$

(ii) Surface area of 1 solid iron sphere of radius $r = 4\pi r^2$

Surface area of iron sphere of radius $r' = 4\pi (r')^2$

$= 4\pi (3r)^2 = 36\pi r^2$

$$\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$$

34. Arranging the data in an ascending order,

14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28

It can be observed that 14 has the highest frequency, i.e. 4, in the given data. Therefore, mode of the given data is 14.