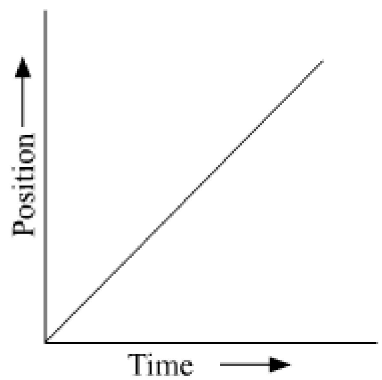
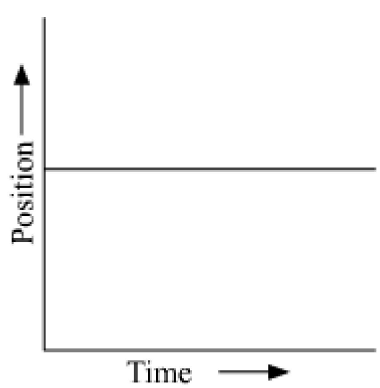
3. Projectile Motion

- An object is at rest when the position of the object does not change with time and with respect to its surroundings.
- An object is in motion when the position of the object changes with time and with respect to its surroundings.
- Rest and motion are relative.
- If the distance covered by an object is much greater than its size during its motion, then the object is considered as point mass object.
- Distance or path length — Total length of the path covered by a body (scalar quantity)
- Displacement — Shortest distance between initial and final positions measured along a particular direction

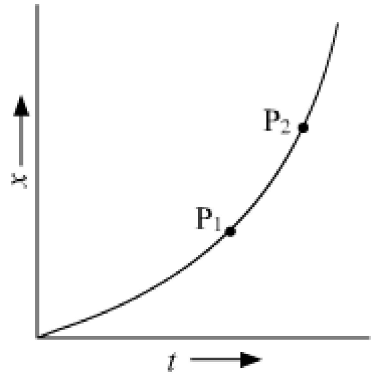
- **Uniform motion (object moving with a constant velocity):**



- **Stationary object (object at rest)**:



- **Average velocity (slope of the *x*-*t* graph)**



$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

∴ Average velocity = slope of $\overline{P_1P_2}$

- **Average speed** = Total path lengthTotal time interval
- [No direction is considered]
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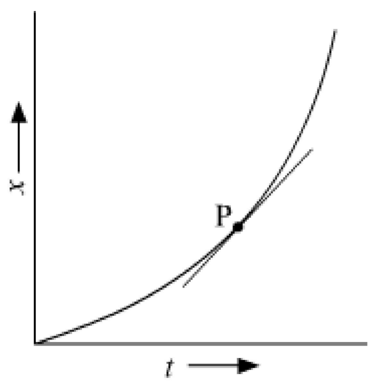
- **Instantaneous velocity:**



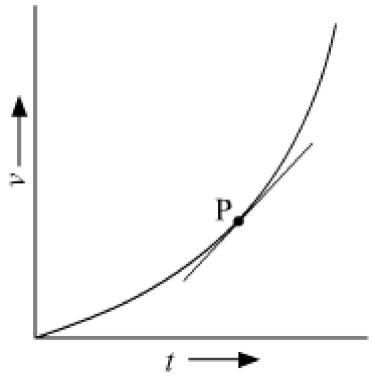
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

= slope of the tangent at point P

- **Average acceleration:**

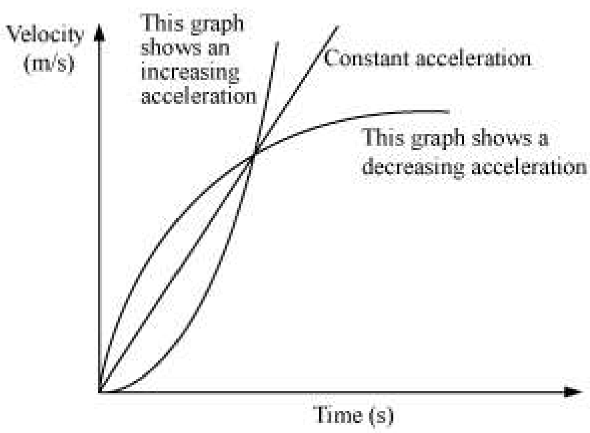
a=v2-v1t2-t1=ΔvΔt

- **Instantaneous acceleration**:

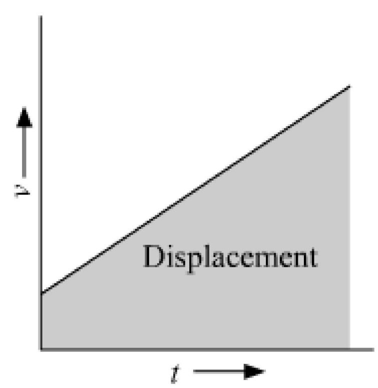


$a=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv}{dt}$ = slope of the tangent at point P

- Velocity-time graph showing constant acceleration, increasing acceleration and decreasing acceleration:



- **Area under the *v*-*t* curve is equal to the displacement of the body.**



Equations of motions (Kinematic equations) [When acceleration is uniform]

- Velocity-time relation

  $v=u+at$

- Disatnce-time relation

  s=ut + 12at2

- Velocity-displacent relation

  v2-u2=2as

- Distance travelled in n^{th}second of uniformly accelerated motion is given by the relation,

$$D_n = u + \frac{a}{2}(2n-1)$$

Galileo's law of odd number

- The ratios of the distance covered by a body falling from the rest increase by odd numbers from one second to the next.That means, distances covered by each will increase by factors of 1, 3, 5, 7, …

Relative Velocity

- The relative velocity of a body **A** with respect to another body **B** (v_{AB}) is the time rate at which **A** changes its position with respect to **B**.
  - **Case 1: Both bodies move in the same direction**: If **A** and **B** are moving in the same direction, then the resultant relative velocity is $v_{AB} = v_A - v_B$
  - **Case 2: The bodies move in opposite directions**: If **A** and **B** are moving in the opposite directions, then the resultant relative velocity is $v_{AB} = v_A + v_B$

- a→=v'→-v→Δt=Δv→ΔtDisplacement vector can be written as

Δr→=r'→-r→=x'-xi^+y'-yj^+z'-zk^=Δxi^+Δyj^+Δzk^

- **Velocity:**
  - Average velocity, v→=Δr→Δt
  - Instantaneous velocity, v→=dr→dt=vxi^+vy j^+vzk^

  Where, vx=dxdt, vy=dxdt, vz=dxdt

- **Acceleration:**
  - Average acceleration, a→=v'→-v→Δt=Δv→Δt
  - Instantaneous acceleration, a→=dv→dt=axi^+ay j^+azk^

  Where, ax=dvxdt, ay=dvydt, az=dvzdt

- Motion in a plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

- Relative Velocity in Two Dimensions**:** If two objects A and B are moving with velocities v→A and v→B.
  - velocity of object A relative to that of B is

    v→AB=v→A-v→B

  - velocity of object B relative to that of A is

    v→BA=v→B-v→A

- If the man wants to protect himself from rain, then he should hold an umbrella at an angle θ=tan-1vmvr towards his motion with the vertical.

- The motion of a projectile may be thought of as the result of horizontal and vertical components.
- Vertical component comes under accelerated motion because of acceleration due to gravity acting downwards but horizontal component is under uniform motion.
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- Both the components act independently.
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- **Equation of the path of a projectile:**

y = x tanθ -12gu2 cos2θx2

This is the equation of a parabola. Hence, the path of a projectile is a parabola.

- **Time of flight (T):** It is total time for which an object is in flight.

Total time of flight = Time of ascent + Time of descent

T = 2 usinθg

- **Maximum height:** Maximum height *h* reached by the projectile,

h = u2sin2θ2g

- **Horizontal Range:** It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground. It is denoted by *R*.

R= u2sin2θg

Horizontal range is maximum if the angle of projection is 45°.

- **Angle of projection:** It is the angle made by velocity of projection with the horizontal.

Angle of projection,
θ0 = tan-14HR
Cases :
When $\theta_o = 45^o$,
R = u2g
$R_{max} = 4H_{max}$
When $\theta_o = 90^o$,
H = u22g
R = 0