

CBSE Board
Class XII Mathematics
Sample Paper - 1
Term 2 – 2021-22

Time: 2 hours

Total Marks: 40

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
 2. Section - A has 6 short answer type (SA1) questions of 2 marks each
 3. Section - B has 4 short answer type (SA2) questions of 3 marks each.
 4. Section - C has 4 long answer type questions (LA) of 4 marks each.
 5. There is an internal choice in some of the questions.
 6. Q14 is a case-based problem having 2 sub parts of 2 marks each.
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Section A

Q1 – Q6 are of 2 marks each.

1. Integrate $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

OR

Integrate $\int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$

2. Check whether the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$ is homogeneous.
3. Write the direction cosines of the vectors $-2\hat{i} + \hat{j} - 5\hat{k}$.
4. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$
5. Bag A contains 3 white and 4 red balls, and bag B contains 6 white and 3 red balls. An unbiased coin, twice as likely to come up heads as tails, is tossed once. If it shows head, a ball drawn from bag A, otherwise, from bag B. Given that a white ball was drawn, what is the probability that the coin came up tail?

6. When 150 identical coins are tossed the probability that the coins will show head is p . Suppose the probability of showing heads on 75 coins is equal to the probability of showing heads on 76 coins with the condition, $0 < p < 1$, then find the value of p .

Section B

Q7 – Q10 are of 3 marks each

7. Integrate: $\int \frac{dx}{3 - 10x - 25x^2}$

8. Solve the given differential equation $(x + 1) \frac{dy}{dx} = 2e^{-y} - 1$ if $y(0) = 0$.

OR

Find the particular solution of this differential equation $2ye^{x/y} dx + (y - 2x e^{x/y}) dy = 0$ if it is homogeneous, given that $x = 0$ when $y = 1$.

9. Find λ if the vectors $\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other

10. The vector equations of two lines are:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}) \text{ and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} - 3\hat{j} + \hat{k})$$

Find the shortest distance between the above lines.

OR

Find the vector and Cartesian equation of the plane through $3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the lines

$$\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$$

$$\vec{r} = \hat{i} - 3\hat{j} + \hat{k} + \mu(-5\hat{i} + 4\hat{j})$$

Section C

Q11 – Q14 are of 4 marks each

11. Integrate $\int (3x-2)\sqrt{x^2+x+1}dx$

12. Using integration, find the area between the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

OR

Find the area of the smaller region bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and the line } \frac{x}{a} + \frac{y}{b} = 1$$

13. Find the vector equation of the plane passing through three points with position vector $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda (2\hat{i} - 2\hat{j} + \hat{k})$.

14. Case Study

Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

- All the four cards are spades?
- Only 3 cards are spades?

Solution

Section A

$$\begin{aligned} 1. \quad I &= \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx \\ I &= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx \\ I &= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx \end{aligned}$$

$$I = \int (\operatorname{cosec}^2 x - \sec^2 x) dx$$

$$I = -\cot x - \tan x + c$$

OR

$$I = \int_2^8 \frac{\sqrt[3]{x+1}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx \dots (i)$$

$$I = \int_2^8 \frac{\sqrt[3]{10-x+1}}{\sqrt[3]{10-x+1} + \sqrt[3]{11-(10-x)}} dx$$

$$I = \int_2^8 \frac{\sqrt[3]{11-x}}{\sqrt[3]{11-x} + \sqrt[3]{1+x}} dx \dots (ii)$$

Adding (i) and (ii)

$$2I = \int_2^8 \frac{\sqrt[3]{x+1} + \sqrt[3]{11-x}}{\sqrt[3]{x+1} + \sqrt[3]{11-x}} dx$$

$$I = \frac{1}{2} \int_2^8 dx$$

$$I = \frac{1}{2} [x]_2^8$$

$$I = 3$$

2. Given DE is $2ye^{x/y}dx + y - 2xe^{x/y} dy = 0$

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}} \quad \dots 1$$

Let

$$F_{x,y} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$$

Then,

$$F_{\lambda x, \lambda y} = \frac{\lambda \left(2xe^{\frac{x}{y}} - y \right)}{\lambda \left(2ye^{\frac{x}{y}} \right)} = \lambda^0 [F_{x,y}]$$

Thus, $F(x, y)$ is a homogeneous function of degree zero.

3. Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$, then

$$|\vec{a}| = \sqrt{(-2)^2 + (1)^2 + (-5)^2}$$

$$\Rightarrow |\vec{a}| = \sqrt{4 + 1 + 25}$$

$$\Rightarrow |\vec{a}| = \sqrt{30}$$

$$\text{Now, } \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{-2\hat{i} + \hat{j} - 5\hat{k}}{\sqrt{30}} = -\frac{2}{\sqrt{30}}\hat{i} + \frac{1}{\sqrt{30}}\hat{j} - \frac{5}{\sqrt{30}}\hat{k}$$

Thus, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ are $-\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}$ and $-\frac{5}{\sqrt{30}}$.

4. Let ϕ be the angle between the line and plane and θ be the angle between the line and normal to plane

$$\phi = (90 - \theta)$$

Let \vec{b} be vector parallel to line

$$\Rightarrow \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Let \hat{n} be normal to plane

$$\Rightarrow \hat{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$$

$$\cos\theta = \frac{\left| (2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (10\hat{i} + 2\hat{j} - 11\hat{k}) \right|}{\sqrt{2^2 + 3^2 + 6^2} \sqrt{10^2 + 2^2 + 11^2}}$$

$$\Rightarrow \cos(90 - \phi) = \left| \frac{-40}{7 \times 15} \right| = \left| \frac{-8}{21} \right|$$

$$\Rightarrow \sin\phi = \frac{8}{21} \Rightarrow \phi = \sin^{-1}\left(\frac{8}{21}\right)$$

5.

Let W be the white ball was drawn and T be the tail come up.

$$P\left(\frac{T}{W}\right) = \frac{P(T \cap W)}{P(W)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{9}}{\frac{1}{2} \times \frac{6}{9} + \frac{1}{2} \times \frac{3}{7}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{3}{14}}$$

$$= \frac{14}{17}$$

6. Let X be the number of coins showing heads.

Let X be a binomial variate with parameter $n = 150$ and p .

$$P(X = 75) = P(X = 76)$$

$$\Rightarrow {}^{150}C_{75} p^{75} (1-p)^{75} = {}^{150}C_{76} p^{76} (1-p)^{74}$$

$$\Rightarrow \frac{150!}{75! 75!} (1-p) = \frac{150!}{76! 74!} p$$

$$\Rightarrow \frac{1}{75} (1-p) = \frac{1}{76} p$$

$$\Rightarrow \frac{p}{1-p} = \frac{76}{75}$$

$$\Rightarrow 75p = 76 - 76p$$

$$\Rightarrow 151p = 76$$

$$\Rightarrow p = \frac{76}{151}$$

Section B

$$\begin{aligned}
 \mathbf{7.} \quad I &= \int \frac{dx}{3 - 10x - 25x^2} \\
 I &= -\frac{1}{25} \int \frac{dx}{x^2 + \frac{10}{25}x - \frac{3}{25}} \\
 I &= -\frac{1}{25} \int \frac{dx}{x^2 + \frac{10}{25}x - \frac{3}{25}} \\
 I &= -\frac{1}{25} \int \frac{dx}{x^2 + \frac{10}{25}x + \frac{1}{25} - \frac{3}{25} - \frac{1}{25}} \\
 I &= -\frac{1}{25} \int \frac{dx}{\left(x + \frac{1}{5}\right)^2 - \left(\frac{2}{5}\right)^2} \\
 I &= -\frac{1}{25} \times \frac{1}{2 \times \frac{2}{5}} \log \left| \frac{5x + 1 - 2}{5x + 1 + 2} \right| + c \\
 I &= -\frac{1}{20} \log \left| \frac{5x - 1}{5x + 3} \right| + c
 \end{aligned}$$

$$\mathbf{8.} \quad \text{Given DE is } (x+1) \frac{dy}{dx} = 2e^{-y} - 1 \quad \dots (1)$$

$$\begin{aligned}
 \Rightarrow \frac{dy}{2e^{-y} - 1} &= \frac{dx}{(x+1)} \\
 \Rightarrow \int \frac{dy}{2e^{-y} - 1} &= \int \frac{dx}{(x+1)} \\
 \Rightarrow -\int \frac{e^y dy}{2 - e^y} &= \int \frac{dx}{(x+1)}
 \end{aligned}$$

$$\begin{aligned}
-\log|2 - e^y| &= \log|x + 1| + c \\
\Rightarrow \log|(x + 1)(2 - e^y)| &= -c \\
\Rightarrow |(x + 1)(2 - e^y)| &= e^{-c} \\
\Rightarrow (x + 1)(2 - e^y) &= \pm e^{-c} = A \dots (\text{say}) \\
\Rightarrow (x + 1)(2 - e^y) &= A \dots (\text{ii}) \\
x = 0, y = 0 \\
(0 + 1)(2 - e^0) &= A \\
\Rightarrow 1(2 - 1) &= A \\
\Rightarrow A &= 1 \\
\text{Substituting in (ii), we get} \\
(x + 1)(2 - e^y) &= 1
\end{aligned}$$

OR

Let $x = vy$

Differentiating w.r.t. y , we get

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

Substituting the value of x and $\frac{dx}{dy}$ in equation (1), we get

$$v + y \frac{dv}{dy} = \frac{2vye^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$$

$$y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$

$$y \frac{dv}{dy} = -\frac{1}{2e^v}$$

$$2e^v dv = \frac{-dy}{y}$$

$$\int 2e^v . dv = - \int \frac{dy}{y}$$

$$2e^v = -\log |y| + C$$

Substituting the value of v , we get

$$2e^{\frac{x}{y}} + \log |y| = C \dots 2$$

Substituting $x = 0$ and $y = 1$ in equation (2), we get

$$2e^0 + \log |1| = C \Rightarrow C = 2$$

Substituting the value of C in equation (2), we get

$2e^{\frac{x}{y}} + \log|y| = 2$, Which is the particular solution of the given differential equation.

9.

Let \hat{n} be the unit vector along the sum of vectors $\vec{b} + \vec{c}$:

$$\hat{n} = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 6^2 + 2^2}}$$

The scalar product of \vec{a} and \hat{n} is 1. Thus,

$$\vec{a} \cdot \hat{n} = (\hat{i} + \hat{j} + \hat{k}) \cdot \left(\frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 6^2 + 2^2}} \right)$$

$$\Rightarrow 1 = \frac{1(2 + \lambda) + 1 \cdot 6 - 1 \cdot 2}{\sqrt{(2 + \lambda)^2 + 6^2 + 2^2}}$$

$$\Rightarrow \sqrt{(2 + \lambda)^2 + 6^2 + 2^2} = 2 + \lambda + 6 - 2$$

$$\Rightarrow \sqrt{(2 + \lambda)^2 + 6^2 + 2^2} = \lambda + 6$$

$$\Rightarrow (2 + \lambda)^2 + 40 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 + 40 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 4\lambda + 44 = 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

10. The given equations of the lines are

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}) \text{ and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} - 3\hat{j} + \hat{k})$$

On comparing with standard equation of line

We have $a_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $b_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$$a_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}, b_2 = 2\hat{i} - 3\hat{j} + \hat{k}$$

Therefore, $a_2 - a_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\begin{aligned}
\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} \\
&= -9\hat{i} + 3\hat{j} + 9\hat{k} \\
\therefore (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) &= (3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k}) \\
&= (3)(-9) + 3(3) + 3(9) \\
&= -27 + 9 + 27 = 9
\end{aligned}$$

$$\begin{aligned}
\text{And, } |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(-9)^2 + 3^2 + 9^2} \\
&= \sqrt{81 + 9 + 81} = \sqrt{171} = 3\sqrt{19} \\
\therefore \text{Shortest Distance} &= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|
\end{aligned}$$

OR

Required plane passes through the point with position vector $\vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$ and parallel to vectors $\vec{b} = 2\hat{i} - 5\hat{j} - \hat{k}$ and $\vec{d} = -5\hat{i} + 4\hat{j}$

The equation of the plane is,

$$(\vec{r} - \vec{r}_1) \cdot (\vec{b} \times \vec{d}) = 0$$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{vmatrix} = 4\hat{i} + 5\hat{j} - 17\hat{k}$$

So the equation is,

$$(\vec{r} - (3\hat{i} - \hat{j} + 2\hat{k})) \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) = 0$$

$$\Rightarrow \vec{r} \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) - (3 \cdot 4 + (-1) \cdot 5 + 2(-17)) = 0$$

$$\Rightarrow \vec{r} \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) + 27 = 0 \text{ [Vector equation]}$$

$$\Rightarrow 4x + 5y - 17z + 27 = 0 \text{ [Cartesian equation]}$$

Section C

11. $I = \int (3x-2)\sqrt{x^2+x+1}dx$

We express

$$3x-2 = A \frac{d}{dx}(x^2+x+1) + B$$

$$\Rightarrow 3x-2 = A(2x+1) + B$$

$$\Rightarrow 3x-2 = 2Ax + A + B$$

$$\Rightarrow 2A = 3 \text{ i.e. } A = 3/2$$

$$\Rightarrow A + B = -2$$

$$\Rightarrow B = -7/2$$

$$I = \int \left(\frac{3}{2}(2x+1) - \frac{7}{2} \right) \sqrt{x^2+x+1} dx$$

$$I = \int \left(\frac{3}{2}(2x+1) \sqrt{x^2+x+1} \right) dx - \int \frac{7}{2} \sqrt{x^2+x+1} dx$$

$$I = I_1 - I_2$$

$$I_1 = \int \left(\frac{3}{2}(2x+1) \sqrt{x^2+x+1} \right) dx$$

Put $t = x^2 + x + 1 \Rightarrow (2x + 1)dx = dt$

$$I_1 = \frac{3}{2} \int \sqrt{t} dt = \frac{3}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = t^{\frac{3}{2}} + c = \sqrt[3]{x^2+x+1} + c$$

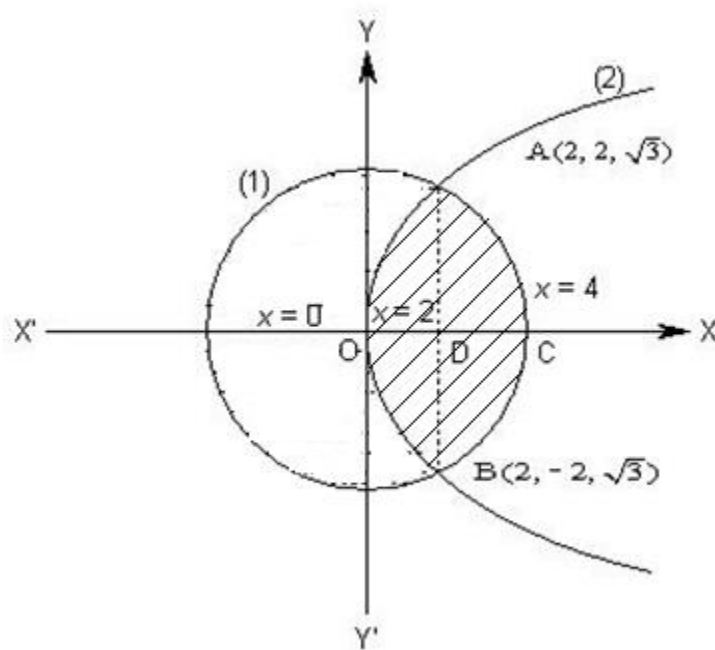
$$I_2 = \int \frac{7}{2} \sqrt{x^2+x+1} dx$$

$$I_2 = \frac{7}{2} \int \sqrt{x^2+x+\frac{1}{4}+1-\frac{1}{4}} dx$$

$$I_2 = \frac{7}{2} \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx = \frac{7}{4} \left(x+\frac{1}{2}\right) \sqrt{x^2+x+1} + \frac{3}{8} \log \left| x + \sqrt{x^2+x+1} \right| + c$$

$$I = \sqrt[3]{x^2+x+1} + \frac{7}{4} \left(x+\frac{1}{2}\right) \sqrt{x^2+x+1} + \frac{3}{8} \log \left| x + \sqrt{x^2+x+1} \right| + c$$

12. $x^2 + y^2 = 16 \dots (i)$ and $y^2 = 6x \dots (ii)$



Points of intersection of curve (i) and (ii) is

$\therefore A(2, 2\sqrt{3})$ and $B(2, -2\sqrt{3})$

Also $C(4, 0)$.

Area $OBCAO = 2 (\text{Area ODA} + \text{Area DCA})$

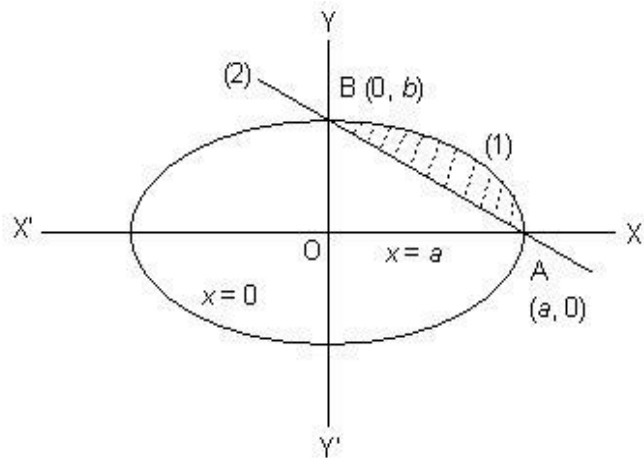
$$\begin{aligned}
 &= 2 \left[\int_0^2 y_2 dx + \int_2^4 y_1 dx \right] \\
 &= 2 \left[\int_0^2 \sqrt{6x} dx + \int_2^4 \sqrt{16-x^2} dx \right] \\
 &= 2 \left[\sqrt{6} \cdot \left\{ \frac{2}{3} x^{3/2} \right\}_0^2 + \left\{ \frac{x\sqrt{16-x^2}}{2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right\}_2^4 \right]
 \end{aligned}$$

$$\begin{aligned}
 &= 2 \left[\frac{2\sqrt{6}}{3} \cdot 2\sqrt{2} + \left\{ 0 + 8 \sin^{-1} 1 \right\} - \left\{ \frac{2.2\sqrt{3}}{2} + 8 \sin^{-1} \frac{1}{2} \right\} \right] \\
 &= \frac{16\sqrt{3}}{3} + 16 \cdot \frac{\pi}{2} - \left(4\sqrt{3} + 16 \cdot \frac{\pi}{6} \right) \\
 &= \left(\frac{4\sqrt{3}}{3} + \frac{16}{3} \pi \right) \text{sq. units.}
 \end{aligned}$$

OR

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(1)$$

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(2)$$



Equation (i) gives $y = b \sqrt{1 - \frac{x^2}{a^2}}$

Equation (ii) gives $y = b \left(1 - \frac{x}{a} \right)$

(Area of the smaller region)

$$= \int_0^a (y_1 - y_2) dx$$

$$\begin{aligned}
&= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
&= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - b \int_0^a \left(1 - \frac{x}{a}\right) dx \\
&= \frac{b}{a} \left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a - b \left[x - \frac{x^2}{2a} \right]_0^a \\
&= \frac{b}{a} \left[\frac{a^2}{2} \sin^{-1} 1 \right] - b \left[a - \frac{a^2}{2a} \right] \\
&= \frac{ab}{2} \cdot \frac{\pi}{2} - \frac{ab}{2} = \frac{1}{4} ab(\pi - 2) \text{ sq. units.}
\end{aligned}$$

13. Let the position vectors of the three points be,

$$\vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} + 2\hat{j} + \hat{k}.$$

So, the equation of the plane passing through the points \vec{a}, \vec{b} and \vec{c} is

$$\begin{aligned}
&\vec{r} - \vec{a} \cdot [\vec{b} - \vec{c} \times \vec{c} - \vec{a}] = 0 \\
&\Rightarrow [\vec{r} - \hat{i} + \hat{j} - 2\hat{k}] \cdot [\hat{i} - 3\hat{j} \times \hat{j} + 3\hat{k}] = 0 \\
&\Rightarrow [\vec{r} - \hat{i} + \hat{j} - 2\hat{k}] \cdot \hat{k} - 3\hat{j} - 9\hat{i} = 0 \\
&\Rightarrow \vec{r} \cdot -9\hat{i} - 3\hat{j} + \hat{k} + 14 = 0 \\
&\Rightarrow \vec{r} \cdot 9\hat{i} + 3\hat{j} - \hat{k} = 14 \quad \dots 1
\end{aligned}$$

So, the vector equation of the required plane is $\vec{r} \cdot 9\hat{i} + 3\hat{j} - \hat{k} = 14$.

The equation of the given line is $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda (2\hat{i} - 2\hat{j} + \hat{k})$.

Position vector of any point on the give line is

$$\vec{r} = 3 + 2\lambda \hat{i} + -1 - 2\lambda \hat{j} + -1 + \lambda \hat{k} \quad \dots 2$$

The point (2) lies on plane (1) if,

$$[3 + 2\lambda \hat{i} + -1 - 2\lambda \hat{j} + -1 + \lambda \hat{k}] \cdot 9\hat{i} + 3\hat{j} - \hat{k} = 14$$

$$\Rightarrow 9(3 + 2\lambda) + 3(-1 - 2\lambda) - (-1 + \lambda) = 14$$

$$\Rightarrow 11\lambda + 25 = 14$$

$$\Rightarrow \lambda = -1$$

Putting $\lambda = -1$ in (2), we have

$$\begin{aligned}
 \vec{r} &= 3 + 2\lambda \hat{i} + -1 - 2\lambda \hat{j} + -1 + \lambda \hat{k} \\
 &= 3 + 2 - 1 \hat{i} + -1 - 2 - 1 \hat{j} + -1 + -1 \hat{k} \\
 &= \hat{i} + \hat{j} - 2\hat{k}
 \end{aligned}$$

Thus, the position vector of the point of intersection of the given line and plane (1) is $\hat{i} + \hat{j} - 2\hat{k}$ and its co-ordinates are 1, 1, -2 .

14.

This is a case of bernoulli trials.

$$p = P(\text{Success}) = P(\text{getting a spade in a single draw}) = \frac{13}{52} = \frac{1}{4}$$

$$q = P(\text{Failure}) = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\text{i. All the four cards are spades} = P(X = 4) = {}^4C_4 p^4 q^0 = \left(\frac{1}{4}\right)^4 = \frac{1}{256}$$

$$\text{ii. Only 3 cards are spades} = P(X = 3) = {}^4C_3 p^3 q^1 = \frac{12}{256} = \frac{3}{64}$$