2 Inverse Trigonometric Functions

Fastrack Revision

- Trigonometric functions are not one-one and onto on their natural domains and ranges, so their inverse do not exists in all values but their inverse may exists in some interval of their domains and ranges.
  Thus, we can say that inverse of trigonometric functions are defined within restricted domains of corresponding trigonometric functions.
  Suppose $y = f(x) = \sin x$ is a function. Then, its inverse is $x = \sin^{-1} y$.
- The value of inverse trigonometric function which lies in the range of its principal branch, is called the principal value.
- **Principal Value Branch Table**

Function	Domain	Range (Principal value)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$
$\sec^{-1} x$	$R-(-1, 1)$	$[0, \pi]-\{\pi/2\}$
$\operatorname{cosec}^{-1} x$	$R-(-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

- The graph of the inverse of the given function can be obtained by interchanging the coordinate axes in the graph of that function.
- $\sin^{-1} x = \theta \Leftrightarrow \sin\theta = x$
- $\cos^{-1} x = \theta \Leftrightarrow \cos\theta = x$
- $\tan^{-1} x = \theta \Leftrightarrow \tan\theta = x$
- $\sin^{-1}(\sin\theta) = \theta$ only if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
- $\cos^{-1}(\cos\theta) = \theta$ only if $0 \le \theta \le \pi$
- $\tan^{-1}(\tan\theta) = \theta$ only if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
- $\cot^{-1}(\cot\theta) = \theta$ only if $0 < \theta < \pi$
- $\sec^{-1}(\sec\theta) = \theta$ only if $0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$
- $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$ only if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \theta \ne 0$
- $\sin(\sin^{-1} x) = x$, if $-1 \le x \le 1$
- $\cos(\cos^{-1} x) = x$, if $-1 \le x \le 1$
- $\tan(\tan^{-1} x) = x$, if $-\infty < x < \infty$
- $\cot(\cot^{-1} x) = x$, if $-\infty < x < \infty$
- $\sec(\sec^{-1} x) = x$, if $x \le -1$ or $x \ge 1$
- $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$, if $x \le -1$ or $x \ge 1$

Practice Exercise

Multiple Choice Questions

Q 1. If $\tan^{-1} x = y$, then: *(CBSE SQP 2021 Term-1)*

a. $-1 < y < 1$
b. $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
c. $\frac{-\pi}{2} < y < \frac{\pi}{2}$
d. $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

Q 2. If $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, then x belongs to:

a. $[-1, 1]$
b. $\left[-\frac{1}{\sqrt{2}}, 1\right]$
c. $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$
d. None of these

Q 3. $2\cos^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$ is true for:

a. all x
b. $x > 0$
c. $x \in [-1, 1]$
d. $\frac{1}{\sqrt{2}} \le x \le 1$

Q 4. What is the domain of the function $\cos^{-1}(2x-3)$? *(CBSE 2021 Term-1)*

a. $[-1, 1]$
b. $(1, 2)$
c. $(-1, 1)$
d. $[1, 2]$

Q 5. The domain of $y = \cos^{-1}(x^2-4)$ is: *(NCERT EXEMPLAR)*

a. $[3, 5]$
b. $[0, \pi]$
c. $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$
d. $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Q 6. The domain of the function defined by $f(x) = \sin^{-1} x + \cos x$ is: *(NCERT EXEMPLAR)*

a. $[-1, 1]$
b. $[-1, \pi+1]$
c. $(-\infty, \infty)$
d. ϕ

Q 7. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is: *(NCERT EXEMPLAR)*

a. $[1, 2]$
b. $[-1, 1]$
c. $[0, 1]$
d. None of these

Q 8. The domain of the function $y = \sin^{-1}(-x^2)$ is: *(NCERT EXEMPLAR)*

a. $[0, 1]$
b. $(0, 1)$
c. $[-1, 1]$
d. ϕ

Q 9. Range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is:

a. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ b. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

c. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ d. None of these

Q 10. The principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is:

(NCERT EXEMPLAR; CBSE 2021 Term-1)

a. $\frac{\pi}{12}$ b. π c. $\frac{\pi}{3}$ d. $\frac{\pi}{6}$

Q 11. The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is:

(CBSE 2021 Term-1, CBSE 2018)

a. π b. $-\frac{\pi}{2}$ c. 0 d. $2\sqrt{3}$

Q 12. The principal value of $\cot^{-1}(-1)$ is:

a. $\frac{\pi}{4}$ b. $\frac{\pi}{3}$

c. $\frac{3\pi}{4}$ d. None of these

Q 13. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to: *(NCERT EXEMPLAR)*

a. π b. $-\frac{\pi}{3}$ c. $\frac{\pi}{3}$ d. $\frac{2\pi}{3}$

Q 14. $\cot^{-1}(-\sqrt{3}) =$

a. $\frac{5\pi}{6}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{2}$ d. $\frac{\pi}{4}$

Q 15. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$

a. $\frac{5\pi}{6}$ b. $\frac{\pi}{6}$ c. $\frac{4\pi}{9}$ d. $\frac{2\pi}{3}$

Q 16. $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) =$

a. $\frac{\pi}{6}$ b. $\frac{\pi}{3}$ c. $\frac{5\pi}{6}$ d. $\frac{2\pi}{3}$

Q 17. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to: *(CBSE 2023)*

a. 1 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$

Q 18. The value of $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ is:

a. 1 b. –1

c. 0 d. None of these

Q 19. The value of $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is:

a. 1 b. –1

c. 0 d. None of these

Q 20. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to: *(NCERT EXERCISE)*

a. $\frac{7\pi}{6}$ b. $\frac{5\pi}{6}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{6}$

Q 21. The value of $\tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$ is:

a. $\frac{\pi}{3}$ b. $-\frac{\pi}{3}$ c. $\frac{\pi}{4}$ d. $-\frac{\pi}{4}$

Q 22. The value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$ is: *(CBSE 2020)*

a. $-\frac{\pi}{6}$ b. $\frac{\pi}{8}$ c. $-\frac{\pi}{8}$ d. $\frac{\pi}{12}$

Q 23. The principal value of $\cos^{-1}\left(-\sin\frac{7\pi}{6}\right)$ is:

a. $\frac{5\pi}{3}$ b. $\frac{7\pi}{6}$

c. $\frac{\pi}{3}$ d. None of these

Q 24. The value of the expression $\sin[\cot^{-1}\{\cos(\tan^{-1} 1)\}]$ is: *(NCERT EXEMPLAR)*

a. 0 b. 1 c. $\frac{1}{\sqrt{3}}$ d. $\sqrt{\frac{2}{3}}$

Q 25. The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is:

a. $\frac{\pi}{2}$ b. $\frac{5\pi}{3}$ c. $\frac{10\pi}{3}$ d. 0

Q 26. $\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$ is equal to:

a. $\frac{6}{26}$ b. $\frac{24}{25}$ c. $\frac{4}{5}$ d. $-\frac{24}{25}$

Q 27. $\sin(\tan^{-1} x)$, where $|x| < 1$, is equal to:

(CBSE SQP 2021 Term-1)

a. $\frac{x}{\sqrt{1-x^2}}$ b. $\frac{1}{\sqrt{1-x^2}}$ c. $\frac{1}{\sqrt{1+x^2}}$ d. $\frac{x}{\sqrt{1+x^2}}$

Q 28. $2\sin^{-1}\sqrt{\frac{1-x}{2}} =$

a. $\cos^{-1} x$ b. $2\cos^{-1}\sqrt{\frac{1+x}{2}}$

c. Both a and b. d. None of these

Q 29. If $0 < x < 1$, then $\sqrt{1+x^2}\,[(x\cos[\cot^{-1} x] + \sin[\cot^{-1} x])^2 - 1]^{1/2} =$

a. $\frac{x}{\sqrt{1+x^2}}$ b. x c. $x\sqrt{1+x^2}$ d. $\sqrt{1+x^2}$

Q 30. $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is equal to:

a. $\sqrt{\frac{x^2+2}{x^2+3}}$ b. $\sqrt{\frac{x^2+2}{x^2+1}}$

c. $\sqrt{\frac{x^2+1}{x^2+2}}$ d. None of these

Q 31. If $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1} x)$, then $x =$

a. $\frac{1}{2}$ b. 1 c. 0 d. $-\frac{1}{2}$

Q 32. If $\cot(\cos^{-1} x) = \sec\left(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right)$, then x is equal to:

a. $\frac{b}{\sqrt{2b^2-a^2}}$ b. $\frac{a}{\sqrt{2b^2-a^2}}$

c. $\frac{\sqrt{2b^2-a^2}}{a}$ d. $\frac{\sqrt{2b^2-a^2}}{b}$

Q 33. The simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is:

(NCERT EXERCISE; CBSE 2021 Term-1)

a. $\frac{\pi}{4} - \frac{x}{2}$ b. $\frac{\pi}{4} + \frac{x}{2}$

c. $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$ d. $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x$

Q 34. The simplest form of

$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right), \pi < x < \frac{3\pi}{2}$ is:

(CBSE SQP 2021 Term-1)

a. $\frac{\pi}{4}-\frac{x}{2}$ b. $\frac{3\pi}{2}-\frac{x}{2}$ c. $-\frac{x}{2}$ d. $\pi-\frac{x}{2}$

Q 35. If $\tan(\sec^{-1}x)=\sin\left(\cos^{-1}\frac{1}{\sqrt{5}}\right)$, then x is equal to:

a. $\pm\frac{3}{\sqrt{5}}$ b. $\pm\frac{\sqrt{5}}{3}$

c. $\pm\frac{\sqrt{3}}{5}$ d. None of these

Q 36. The number of triplets (x, y, z) satisfies the equation $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ is:

a. 1 b. 2

c. 0 d. infinite

Q 37. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, then $xy+yz+zx$ is equal to:

a. −3 b. 0 c. 3 d. −1

Q 38. If $\cos^{-1}\alpha+\cos^{-1}\beta+\cos^{-1}\gamma=3\pi$, then $\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$ equals:

(NCERT EXEMPLAR)

a. 0 b. 1 c. 6 d. 12

Assertion & Reason Type Questions

Directions (Q. Nos. 39-47): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 39. Assertion (A): All trigonometric functions have their inverses over their respective domains.

Reason (R): The inverse of $\tan^{-1}x$ exists for some $x\in R$. *(CBSE 2023)*

Q 40. Assertion (A): The value of $\sin\left[\tan^{-1}(-\sqrt{3})+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.

Reason (R): $\tan^{-1}(-x)=-\tan^{-1}x, x\in R$

and $\cos^{-1}(-x)=\cos^{-1}x, x\in[-1, 1]$

Q 41. Assertion (A): If $2(\sin^{-1}x)^2-5(\sin^{-1}x)+2=0$, then x has 2 solutions.

Reason (R): $\sin^{-1}(\sin x)=x$, if $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$

Q 42. Assertion (A): The domain of the function $\sec^{-1}2x$ is $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$.

Reason (R): $\sec^{-1}(-2)=\frac{\pi}{4}$ *(CBSE SQP 2022-23)*

Q 43. Assertion (A): If $\alpha\in\left(-\frac{\pi}{2},0\right)$, then

$$\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)=0.$$

Reason (R): If $\alpha\in\left(-\frac{\pi}{2},0\right)$, then $\sin^{-1}(\sin\alpha)=\alpha$ and $\cos^{-1}(\cos\alpha)=-\alpha$.

Q 44. Assertion (A): Range of $f(x)=\cot^{-1}(2x-x^2)$ is $(0,\pi)$.

Reason (R): $\cot^{-1}x$ is defined for all $x\in R$.

Q 45. Assertion (A): The domain for

$$f(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)\text{ is }\{0, 1\}.$$

Reason (R): $\sin^{-1}x$ is defined only if $x\in[-1, 1]$.

Q 46. Assertion (A): Principal value of $\sin^{-1}\left\{\sin\left(\frac{2\pi}{3}\right)\right\}$ is $\frac{\pi}{3}$.

Reason (R): Interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ lies in principal value branch of arc sine function.

Q 47. Assertion (A): Range of $f(x)=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$ is $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$.

Reason (R): $f(x)=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$ is defined for all $x\in[-1, 1]$.

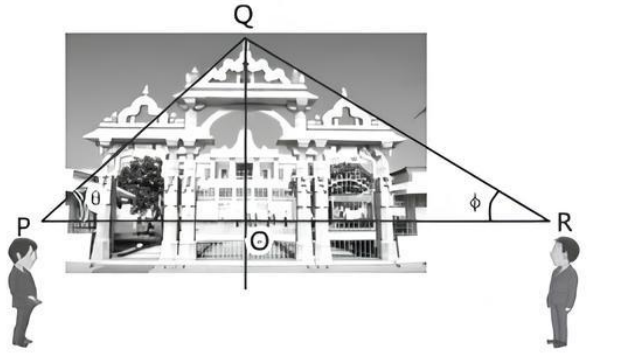
Answers

1. (c)	2. (c)	3. (d)	4. (d)	5. (d)	6. (a)	7. (a)	8. (c)	9. (c)	10. (a)
11. (b)	12. (c)	13. (b)	14. (a)	15. (a)	16. (c)	17. (a)	18. (b)	19. (a)	20. (b)
21. (d)	22. (c)	23. (c)	24. (d)	25. (d)	26. (d)	27. (d)	28. (c)	29. (c)	30. (c)
31. (d)	32. (a)	33. (c)	34. (a)	35. (a)	36. (a)	37. (c)	38. (c)	39. (d)	40. (c)
41. (d)	42. (c)	43. (a)	44. (d)	45. (d)	46. (a)	47. (c)			

Case Study Based Questions

Case Study 1

Two persons on either side of a temple of $60\sqrt{3}$ m height observes its top at the angles of elevations θ and ϕ respectively (as shown in the given figure). The distance between the two persons is 240 m and the distance between the first person *P* and the temple is 60 m.



Based on the above information, solve the following questions:

Q 1. $\angle RPQ = \theta =$

a. $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ b. $\sin^{-1}\left(\frac{1}{2}\right)$

c. $\sin^{-1}(2)$ d. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Q 2. $\angle RPQ = \theta =$

a. $\cos^{-1}\left(\frac{1}{2}\right)$ b. $\cos^{-1}\left(\frac{2}{5}\right)$

c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d. $\cos^{-1}\left(\frac{4}{5}\right)$

Q 3. $\angle QRP = \phi =$

a. $\tan^{-1}\left(\frac{1}{2}\right)$ b. $\tan^{-1}(2)$

c. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ d. $\tan^{-1}(\sqrt{3})$

Q 4. $\angle PQR =$

a. $\frac{\pi}{4}$ b. $\frac{\pi}{6}$

c. $\frac{\pi}{2}$ d. $\frac{\pi}{3}$

Q 5. Domain and range of $\tan^{-1} x =$

a. $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

b. $R, (0, \pi)$

c. $R - (-1, 1), [0, \pi] - \left\{\frac{\pi}{2}\right\}$

d. $R - (-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - (0)$

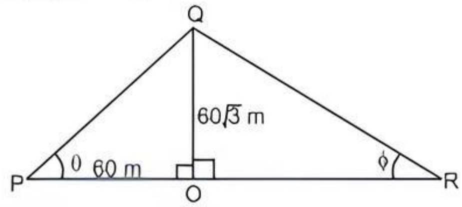
Solutions

1. Given that,

Height of the temple, $OQ = 60\sqrt{3}$ m

Distance between two persons, $PR = 240$ m

and the distance between the first person *P* and the temple, $OP = 60$ m



Now, in right-angled ΔQOP,

$$\tan\theta = \frac{OQ}{OP} = \frac{60\sqrt{3}}{60}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3} = \tan 60°$$

$$\therefore \quad \theta = 60°$$

Now, $\sin\theta = \sin 60° \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

So, option (d) is correct.

2. $\cos\theta = \cos 60° \Rightarrow \cos\theta = \frac{1}{2}$ $[\because \theta = 60°]$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

So, option (a) is correct.

3. $\because$ $PR = 240$ m and $OP = 60$ m

$\therefore$ $OR = PR - OP = 240 - 60 = 180$ m

Now, in right-angled ΔQOR,

$$\tan\phi = \frac{OQ}{OR} = \frac{60\sqrt{3}}{180}$$

$$\Rightarrow \quad \tan\phi = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

So, option (c) is correct.

4.

TiP

Sum of all internal angles of a triangle is equal to 180°.

In ΔPQR, $\angle RPQ + \angle QRP + \angle PQR = 180°$

$$\therefore \quad \theta + \phi + \angle PQR = 180°$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \angle PQR = 180°$$

$$\Rightarrow \sin^{-1}(\sin 60°) + \tan^{-1}(\tan 30°) + \angle PQR = 180°$$

$$\Rightarrow \quad 60° + 30° + \angle PQR = 180°$$

TR!CKS

- $\sin^{-1}(\sin\theta) = \theta$, *if* $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,
- $\tan^{-1}(\tan\theta) = \theta$, *if* $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \quad \angle PQR = 180° - 90° = 90°$

$\therefore \quad \angle PQR = 90° \times \frac{\pi}{180°} = \frac{\pi}{2}$

So, option (c) is correct.

5. Domain of $\tan^{-1} x$ is R.

and range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

So, option (a) is correct.

Case Study 2

The value of an inverse trigonometric function which lies in its principal value branch is called the principal value of that inverse trigonometric function.

For example: $\sin^{-1}\left\{\sin\left(\frac{-\pi}{3}\right)\right\} = \frac{-\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Based on the above information, solve the following questions:

Q 1. The value of $\sin^{-1}\left\{\sin\left(\frac{2\pi}{3}\right)\right\} + \cos^{-1}\left\{\cos\left(\frac{4\pi}{3}\right)\right\}$ is:

a. π b. $\frac{\pi}{2}$ c. $\frac{\pi}{3}$ d. $\frac{2\pi}{3}$

Q 2. The value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is:

a. $\frac{-\pi}{3}$ b. $\frac{\pi}{3}$ c. $\frac{\pi}{6}$ d. $\frac{-\pi}{6}$

Q 3. $\sin^{-1}\left\{\sin\left(\frac{4\pi}{5}\right)\right\}$ is equal to:

a. $\frac{4\pi}{5}$ b. $\frac{3\pi}{5}$ c. $\frac{\pi}{5}$ d. $\frac{-\pi}{5}$

Q 4. $\tan^{-1}\left\{\tan\left(\frac{5\pi}{6}\right)\right\}$ is equal to:

a. $\frac{-\pi}{6}$ b. $\frac{5\pi}{6}$ c. $\frac{\pi}{6}$ d. $\frac{7\pi}{6}$

Q 5. The value of $\sin^{-1}\left(\frac{-1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is:

a. $\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c. $\frac{-\pi}{3}$ d. $\frac{4\pi}{3}$

Solutions

1. $\sin^{-1}\left\{\sin\left(\frac{2\pi}{3}\right)\right\} + \cos^{-1}\left\{\cos\left(\frac{4\pi}{3}\right)\right\}$

$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(2\pi - \frac{2\pi}{3}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$

$= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi \qquad \left[\because \frac{2\pi}{3} \in [0, \pi]\right]$

So, option (a) is correct.

2. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\sin\left(-\frac{\pi}{3}\right) = -\frac{\pi}{3}$

So, option (a) is correct.

3. $\sin^{-1}\left[\sin\left(\frac{4\pi}{5}\right)\right] = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{5}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{\pi}{5}\right)\right] = \frac{\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So, option (c) is correct.

4. $\tan^{-1}\left\{\tan\left(\frac{5\pi}{6}\right)\right\} = \tan^{-1}\left\{\tan\left(\pi - \frac{\pi}{6}\right)\right\} = \tan^{-1}\left(-\tan\frac{\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, option (a) is correct.

5. $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right) - \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

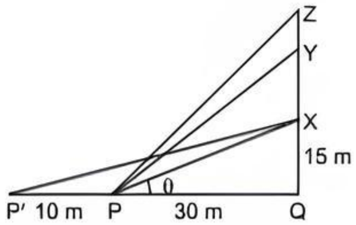
$= \sin^{-1}\sin\left(-\frac{\pi}{6}\right) - \cos^{-1}\cos\left(\frac{\pi}{6}\right)$

$= -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{2\pi}{6} = -\frac{\pi}{3}$

So, option (c) is correct.

Case Study 3

The Government of India is planning to fix a hoarding at the face of a tower side by the road of a busy place for awareness on COVID-19 protocol. Sanjeev, Rohit and Alok are three engineers who are working on this project. 'P' is considered to be a person viewing the hoarding 30 metres away from the tower, standing at the edge of a pathway nearby. Sanjeev, Rohit and Alok suggested to the firm to place the hoarding at three different locations namely X, Y and Z. 'X' is at the height of 15 metres from the ground level. For the viewer P, the angle of elevation of 'Y' is double the angle of elevation of 'X'. The angle of elevation of 'Z' is triple the angle of elevation of 'X' for the same viewer.



Look at the given figure and based on the given information, solve the following questions:

Q 1. Find the measure of ∠XPQ.

Or

Find the measure of ∠XP′Q

Q 2. Find the measure of ∠YPQ.

Q 3. Find the measure of ∠ZPQ.

Solutions

1. Given that, XQ = 15 m and PQ = 30 m

Now in right-angled ΔPQX,

$\tan\theta = \frac{XQ}{PQ} \qquad [\because \text{say } \angle XPQ = \theta]$

$$\Rightarrow \quad \tan\theta = \frac{15}{30} = \frac{1}{2}$$

$$\angle XPQ = \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Or

Given, XQ = 15 m, PQ = 30 m and PP′ = 10 m

$\therefore \quad P'Q = PP' + PQ = 10 + 30 = 40$ m

Now in right-angled $\Delta XQP'$,

$$\tan \angle XP'Q = \frac{XQ}{P'Q} = \frac{15}{40} = \frac{3}{8}$$

$$\therefore \quad \angle XP'Q = \tan^{-1}\left(\frac{3}{8}\right)$$

2. According to the question, $\angle YPQ = 2 \times \angle XPQ = 2\theta$

Now in right-angled ΔYQP, $\tan \angle YPQ = \tan 2\theta$

$$= \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2 \times \frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}$$

$$\left[\because \tan\theta = \frac{1}{2} \text{ and } \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\therefore \quad \angle YPQ = \tan^{-1}\left(\frac{4}{3}\right)$$

3. According to the question,

$$\angle ZPQ = 3 \times \angle XPQ = 3\theta$$

Now in right-angled ΔZQP,

$$\tan \angle ZPQ = \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$$\left[\because \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right]$$

$$= \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1-3\left(\frac{1}{2}\right)^2} \qquad \left[\because \tan\theta = \frac{1}{2}\right]$$

$$= \frac{\frac{3}{2} - \frac{1}{8}}{1 - 3 \times \frac{1}{4}} = \frac{12-1}{8} \times \frac{4}{1} = \frac{11}{2}$$

$$\therefore \quad \angle ZPQ = \tan^{-1}\left(\frac{11}{2}\right)$$

Case Study 4

The value of an inverse trigonometric function which lies in its principal value branch is called the principal value of that inverse trigonometric function.

When we refer to the function $\sin^{-1}$, we take it as the function whose domain is $[-1, 1]$ and range is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. The branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch.

Based on the given information, solve the following questions:

Q 1. Find the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$.

Q 2. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.

Q 3. Find the principal value of $\tan^{-1}(-\sqrt{3})$.

Or

Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

Solutions

1. $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$$= \sin^{-1}\sin\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

2. $\cos^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\left[-\cos\left(\frac{\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right]$

$$= \cos^{-1}\cos\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} \in [0, \pi]$$

3. $\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]$

TR!CKS

- *Range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.*
- *Range of $\cot^{-1}x$ is $(0, \pi)$.*

$$= -\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Or

$$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \cot^{-1}\left[-\cot\left(\frac{\pi}{3}\right)\right] = \cot^{-1}\left[\cot\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \cot^{-1}\cot\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} \in [0, \pi]$$

COMMON ERR!R

Without knowing the range of $\cot^{-1}x$, students solve this question and get the wrong answer.

e.g., $\cot y = \cot\left(-\frac{\pi}{3}\right) \Rightarrow y = -\frac{\pi}{3}$

while, $y = -\frac{\pi}{3} \notin (0, \pi)$

Very Short Answer Type Questions

Q 1. Find the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Q 2. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

(NCERT EXERCISE)

Q 3. Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$.

(NCERT EXERCISE)

Q 4. Find the principal value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$.

Q 5. If $\sin^{-1}\left(\frac{1}{2}\right) = \tan^{-1} x$, find the value of x.

Q 6. Find the principal value of $\sin^{-1}\left(\sin\frac{7\pi}{4}\right)$.

Q 7. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ *(NCERT EXEMPLAR)*

Q 8. Prove that $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x)$, $x \ge 1$ or $x \le -1$.

Q 9. Prove that $\sin^{-1}(-x) = -\sin^{-1} x$, $x \in [-1, 1]$.

Q 10. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $x \in [-1, 1]$.

Q 11. If $\tan^{-1}\left(\frac{3}{4}\right) = A$, then find the value of $\sin 2A$.

Q 12. If $\cot^{-1}\left(\frac{4}{3}\right) = x$, then find $\cos 2x$.

Q 13. Prove that $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$.

(NCERT EXERCISE)

Q 14. Prove that $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$.

Short Answer Type Questions

Q 1. Write the domain and range (principal value branch) of the following functions:

$f(x) = \tan^{-1} x$ *(CBSE 2023)*

Q 2. Find the value of $\sin^{-1}\left\{\cos\left(\frac{33\pi}{5}\right)\right\}$.

(CBSE SQP 2023-24)

Q 3. Prove that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

(NCERT EXERCISE; CBSE 2018)

Q 4. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$. *(CBSE SQP 2022-23)*

Q 5. Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$ *(CBSE 2023)*

Q 6. Prove that $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

Q 7. Prove that $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \frac{1}{2}\tan^{-1} x$.

(NCERT EXERCISE)

Q 8. Find the domain of $y = \sin^{-1}(x^2 - 4)$.

(CBSE 2023, CBSE SQP 2023-24)

Q 9. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1$.

(CBSE 2023)

Q 10. Find the value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right).$$

(NCERT EXERCISE)

Q 11. Prove that

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \left(\frac{\pi}{4} + \frac{x}{2}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}.$$

(NCERT EXERCISE)

Q 12. Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$. *(CBSE 2023)*

Q 13. In a principal branch, draw the graph of the function $f(x) = \tan^{-1} x$.

Long Answer Type Questions

Q 1. Prove that:

$$\tan^{-1} 2x + \tan^{-1}\frac{4x}{1-4x^2} = \tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right);$$

$$|x| < \frac{1}{2\sqrt{3}}.$$

(CBSE 2017)

Q 2. If $\sin(\pi\cos x) = \cos(\pi\sin x)$, then prove that $x = \frac{1}{2}\sin^{-1}\frac{3}{4}$.

Q 3. Prove that:

$$\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}.$$

(CBSE 2017)

Q 4. Prove that:

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2;$$

$-1 < x < 1$. *(CBSE 2017)*

Q 5. Solve the following for x:

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

(CBSE 2017)

Q 6. Evaluate:

$$\cot^{-1}\left\{\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}.$$

Q 7. Solve for x: $\sin^{-1}(1-x) - 2\sin^{-1}(x) = \frac{\pi}{2}$.

(NCERT EXERCISE; CBSE 2020)

Solutions

Very Short Answer Type Questions

1. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\sin^{-1}\left(-\sin\frac{\pi}{3}\right)$

$$=\sin^{-1}\left\{\sin\left(-\frac{\pi}{3}\right)\right\}=-\frac{\pi}{3}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$$

TR!CK

$\sin^{-1}(\sin x)=x$

2. Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$

Then, $-\frac{1}{2}=\cos y$

or $\cos y=-\frac{1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)$

$\Rightarrow\quad \cos y=\cos\frac{2\pi}{3}\Rightarrow y=\frac{2\pi}{3}$

We know that the range of principal value of $\cos^{-1}$ is $[0,\pi]$.

$\therefore$ Principal value of $\cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}\in[0,\pi]$

3. Let $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=y\Rightarrow \cot y=\frac{-1}{\sqrt{3}}=-\cot\left(\frac{\pi}{3}\right)$

$$=\cot\left(\pi-\frac{\pi}{3}\right)$$

$$=\cos\left(\frac{2\pi}{3}\right)$$

$\Rightarrow\quad y=\frac{2\pi}{3}$

We know that the range of principal value of $\cot^{-1}$ is $(0,\pi)$.

Hence, the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{2\pi}{3}\in[0,\pi]$.

4. Let $y=\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\Rightarrow\quad \operatorname{cosec} y=\frac{-2}{\sqrt{3}}\Rightarrow \operatorname{cosec} y=-\operatorname{cosec}\frac{\pi}{3}$

$\Rightarrow\quad \operatorname{cosec} y=\operatorname{cosec}\left(-\frac{\pi}{3}\right)$

$\Rightarrow\quad y=\frac{-\pi}{3}$

Range of the principal branch of the function $\operatorname{cosec}^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$.

Hence, the principal value of $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is $\left(-\frac{\pi}{3}\right)$.

5. Given, $\sin^{-1}\left(\frac{1}{2}\right)=\tan^{-1}x$

$\Rightarrow\quad \sin^{-1}\left(\sin\frac{\pi}{6}\right)=\tan^{-1}x$

$\Rightarrow\quad \frac{\pi}{6}=\tan^{-1}x\qquad [\because \sin^{-1}\sin x=x]$

$\Rightarrow\quad x=\tan\frac{\pi}{6}=\tan 30^\circ$

$\therefore\quad x=\frac{1}{\sqrt{3}}$

6. Let $y=\sin^{-1}\left(\sin\frac{7\pi}{4}\right)$

$\therefore$ Range of the principal value of the function $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$\therefore\quad y=\sin^{-1}\left(\sin\frac{7\pi}{4}\right)=\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{4}\right)\right]$

$=\sin^{-1}\left[-\sin\left(\frac{\pi}{4}\right)\right]\qquad [\because \sin(2\pi-\theta)=-\sin\theta]$

$=\sin^{-1}\sin\left(-\frac{\pi}{4}\right)$

$=-\frac{\pi}{4}\qquad [\because \sin^{-1}(\sin x)=x,\ -\sin\theta=\sin(-\theta)]$

7. $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)=\tan^{-1}\tan\left(\pi+\frac{\pi}{8}\right)$

$$=\tan^{-1}\left(\tan\frac{\pi}{8}\right)=\frac{\pi}{8}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$$

$[\because \tan^{-1}(\tan x)=x]$

8. $\because\quad \sec(\sec^{-1}(x))=x$

$\therefore\quad \frac{1}{\cos(\sec^{-1}(x))}=x\qquad \left[\because \sec\theta=\frac{1}{\cos\theta}\right]$

$\Rightarrow\quad \cos(\sec^{-1}(x))=\frac{1}{x}$

$\Rightarrow\quad \sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)$

Hence, $\cos^{-1}\left(\frac{1}{x}\right)=\sec^{-1}(x)$ **Hence proved.**

9. Let $\sin^{-1}(-x)=\theta\Rightarrow -x=\sin\theta\Rightarrow x=-\sin\theta$

$\Rightarrow\quad x=\sin(-\theta)$

$\Rightarrow\quad \sin^{-1}x=-\theta$

$\Rightarrow\quad \theta=-\sin^{-1}x$

$\Rightarrow\quad \sin^{-1}(-x)=-\sin^{-1}x$ **Hence proved.**

10. Let $\cos^{-1}(-x)=\theta\Rightarrow -x=\cos\theta$

$\Rightarrow\quad x=-\cos\theta=\cos(\pi-\theta)$

$\Rightarrow\quad \cos^{-1}x=\pi-\theta\Rightarrow \theta=\pi-\cos^{-1}x$

$\Rightarrow\quad \cos^{-1}(-x)=\pi-\cos^{-1}x$ **Hence proved.**

11. Given that, $\tan^{-1}\left(\frac{3}{4}\right)=A\Rightarrow \tan A=\frac{3}{4}$

$\therefore\quad \sin 2A=\frac{2\tan A}{1+\tan^2 A}=\frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2}$

$$=\frac{3/2}{1+\frac{9}{16}}=\frac{3/2}{\frac{25}{16}}=\frac{16}{25}\times\frac{3}{2}=\frac{24}{25}$$

12. Here, $\cot^{-1}\left(\frac{4}{3}\right)=x$

$\Rightarrow\quad \cot x=\frac{4}{3}\Rightarrow \tan x=\frac{3}{4}$

Now, $\cos 2x=\frac{1-\tan^2 x}{1+\tan^2 x}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}=\frac{7}{25}$

13. Let $x = \sin\theta$, then $\sin^{-1} x = \theta$

$\therefore \sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$

$= 2\theta = 2\sin^{-1} x$ **Hence proved.**

14. Let $\cos^{-1} x = A \Rightarrow x = \cos A$

$\therefore \cos 2A = 2\cos^2 A - 1 = 2x^2 - 1$

$\Rightarrow 2A = \cos^{-1}(2x^2-1)$

$\Rightarrow 2\cos^{-1} x = \cos^{-1}(2x^2-1)$ **Hence proved.**

Short Answer Type Questions

1. The domain and range of the function $f(x) = \tan^{-1} x$ are $(-\infty, \infty)$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ respectively.

2. $\sin^{-1}\left\{\cos\left(\frac{33\pi}{5}\right)\right\} = \sin^{-1}\left\{\cos\left(\frac{30\pi+3\pi}{5}\right)\right\}$

$= \sin^{-1}\left\{\cos\left(6\pi + \frac{3\pi}{5}\right)\right\}$

$= \sin^{-1}\left\{\cos\left(\frac{3\pi}{5}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(-\frac{\pi}{10}\right)\right\} = -\frac{\pi}{10} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. Let $\sin^{-1} x = A \Rightarrow \sin A = x$

$\because \sin 3A = 3\sin A - 4\sin^3 A, -\frac{\pi}{2} \le 3A \le \frac{\pi}{2}$

$\therefore \sin 3A = 3x - 4x^3, -\frac{\pi}{6} \le A \le \frac{\pi}{6}$

$\Rightarrow 3A = \sin^{-1}(3x-4x^3), -\frac{\pi}{6} \le \sin^{-1} x \le \frac{\pi}{6}$

$\Rightarrow 3\sin^{-1} x = \sin^{-1}(3x-4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Hence proved.

COMMON ERROR

Some students commit error while writing the formula,
$\sin 3x = 3\sin x - 4\sin^3 x$

4. $\sin^{-1}\left\{\sin\left(\frac{13\pi}{7}\right)\right\} = \sin^{-1}\left\{\sin\left(2\pi - \frac{\pi}{7}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(-\frac{\pi}{7}\right)\right\} = -\frac{\pi}{7} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

5. $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\frac{7\pi}{3}\right)\right]$

$[\because \cos(-\theta) = \cos\theta]$

$= \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{3}\right)\right]$

$= \cos^{-1}\cos\left(\frac{\pi}{3}\right)$

$[\because \cos(2\pi+\theta) = \cos\theta]$

$= \frac{\pi}{3} \in [0, \pi]$

6. Let $\tan^{-1} x = \theta$, then $x = \tan\theta$

We know that, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \cos 2\theta = \frac{1-x^2}{1+x^2} \Rightarrow 2\theta = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow 2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ **Hence proved.**

7. Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

LHS $= \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} = \tan^{-1}\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}$

$[\because 1+\tan^2\theta = \sec^2\theta]$

$= \tan^{-1}\frac{\left(\frac{1}{\cos\theta}\right)-1}{\left(\frac{\sin\theta}{\cos\theta}\right)} = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

TR!CKS

- *$1 - \cos\theta = 2\sin^2\frac{\theta}{2}$*
- *$\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$*

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\theta$ $[\because \tan^{-1}(\tan\theta) = \theta]$

$= \frac{1}{2}\tan^{-1} x = \text{RHS}$ **Hence proved.**

8. Given, $y = \sin^{-1}(x^2-4)$

TR!CK

Domain of $\sin^{-1}x$ is $x \in [-1, 1]$.

$\therefore -1 \le x^2 - 4 \le 1$

$\Rightarrow -1+4 \le x^2-4+4 \le 1+4$

$\Rightarrow 3 \le x^2 \le 5$

Consider $x^2 \ge 3$ and $x^2 \le 5$

$\Rightarrow x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$ and $x \in [-\sqrt{5}, \sqrt{5}]$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Hence, domain of y is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.

9. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1}(1)$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] + \frac{\pi}{4}$

$= \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right] + \frac{\pi}{4}$

$= \tan^{-1}\left[2\times\frac{1}{2}\right] + \frac{\pi}{4}$

$= \tan^{-1}(1) + \frac{\pi}{4} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

10. Let $x = \tan^{-1}(1) \Rightarrow \tan x = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \quad x = \frac{\pi}{4}$

where principal value of $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let $\quad y = \cos^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$[\because \cos(\pi - \theta) = -\cos\theta]$

$\Rightarrow \quad y = \frac{2\pi}{3}$

where principal value of $y \in [0, \pi]$.

$\therefore \quad \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\quad z = \sin^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \quad \sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \quad z = -\frac{\pi}{6}$

where principal value of $z \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \quad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= x + y + z = \frac{\pi}{4} + \frac{2\pi}{3} + \left(-\frac{\pi}{6}\right)$

$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{11\pi - 2\pi}{12} = \frac{9}{12}\pi = \frac{3}{4}\pi$

11. LHS $= \tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$

$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right]$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}\right]$

$[\because x^2 - y^2 = (x - y)(x + y)$ and $x^2 + y^2 - 2xy = (x - y)^2]$

$= \tan^{-1}\left\{\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right\}$

$= \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right] = \tan^{-1}\left[\frac{\tan\left(\frac{\pi}{4}\right) + \tan\frac{x}{2}}{1 - \tan\left(\frac{\pi}{4}\right)\tan\frac{x}{2}}\right]$

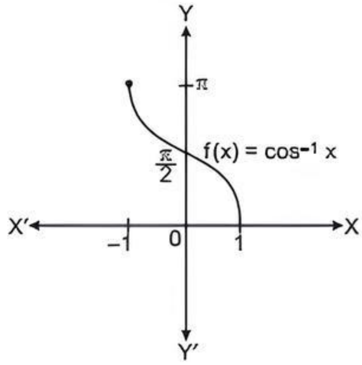
$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right\} \quad \left[\because \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B} = \tan(A + B)\right]$

$= \frac{\pi}{4} + \frac{x}{2} =$ RHS

Hence proved.

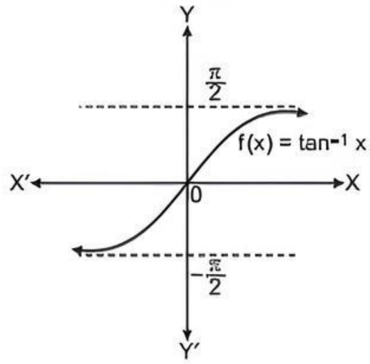
12. In a principal branch, $f(x) = \cos^{-1}$ is a function whose domain is $[-1, 1]$ and range is $[0, \pi]$.

The graph of $f(x) = \cos^{-1} x$ is shown below:



13. In a principal branch, $f(x) = \tan^{-1}$ is a function whose domain is R and range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The graph of $f(x) = \tan^{-1} x$ is shown below:



Long Answer Type Questions

1. LHS $= \tan^{-1}(2x) + \tan^{-1}\frac{4x}{1 - 4x^2}$

$= \tan^{-1}(2x) + \tan^{-1}\frac{2(2x)}{1 - (2x)^2}$

$= \theta + \tan^{-1}\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)$

[Put $2x = \tan\theta \Rightarrow \theta = \tan^{-1}(2x)$]

$= \theta + \tan^{-1}(\tan 2\theta) \quad \left[\because \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}\right]$

$= \theta + 2\theta = 3\theta = 3\tan^{-1}(2x)$

RHS $= \tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right)$

$= \tan^{-1}\left(\frac{3(2x) - (2x)^3}{1 - 3\cdot(2x)^2}\right)$

$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$= \tan^{-1}(\tan 3\theta) \quad \left[\because \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right]$

$= 3\theta = 3\tan^{-1}(2x)$

$\therefore$ LHS = RHS

Hence proved.

2. Given that $\sin(\pi \cos x) = \cos(\pi \sin x)$

$$\Rightarrow \quad \sin(\pi\cos x) = \sin\left[\frac{\pi}{2} + \pi \sin x\right] \quad \left[\because \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta\right]$$

$$\Rightarrow \quad \pi\cos x = \frac{\pi}{2} + \pi\sin x$$

$$\Rightarrow \quad \cos x = \frac{1}{2} + \sin x$$

$$\Rightarrow \quad \cos x - \sin x = \frac{1}{2}$$

$$\Rightarrow \quad (\cos x - \sin x)^2 = \frac{1}{4}$$

$$\Rightarrow \cos^2 x + \sin^2 x - 2\sin x \cdot \cos x = \frac{1}{4}$$

$$\Rightarrow \quad 1 - \sin 2x = \frac{1}{4} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad \sin 2x = 1 - \frac{1}{4} = \frac{3}{4} \quad [\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$$

$$\Rightarrow \quad 2x = \sin^{-1}\frac{3}{4}$$

$$\Rightarrow \quad x = \frac{1}{2}\sin^{-1}\frac{3}{4}$$

Hence proved.

3. Let $\cos^{-1}\left(\frac{a}{b}\right) = \theta \Rightarrow \cos\theta = \frac{a}{b}$ ...(1)

$$\text{LHS} = \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$$

$$= \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{\theta}{2}} + \frac{\tan\frac{\pi}{4} - \tan\frac{\theta}{2}}{1 + \tan\frac{\pi}{4}\cdot\tan\frac{\theta}{2}}$$

$$\left[\because \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \cdot \tan B}\right]$$

$$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \frac{\left(1+\tan\frac{\theta}{2}\right)^2 + \left(1-\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)\left(1+\tan\frac{\theta}{2}\right)}$$

$$= \frac{1+\tan^2\frac{\theta}{2} + 2\tan\frac{\theta}{2} + 1 + \tan^2\frac{\theta}{2} - 2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$= \frac{2\left(1+\tan^2\frac{\theta}{2}\right)}{\left(1-\tan^2\frac{\theta}{2}\right)} = \frac{2}{\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}$$

$$= \frac{2}{\cos\theta} \quad \left[\because \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right]$$

$$= \frac{2}{\left(\frac{a}{b}\right)} \quad \text{[from eq. (1)]}$$

$$= \frac{2b}{a} = \text{RHS}$$

Hence proved.

4. $$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

Put $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x^2$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+2\cos^2\theta-1}+\sqrt{1-1+2\sin^2\theta}}{\sqrt{1+2\cos^2\theta-1}-\sqrt{1-1+2\sin^2\theta}}\right)$$

$$[\because \cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1]$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}\right)$$

TR!CK

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\} = \frac{\pi}{4} + \theta \quad [\because \tan^{-1}\tan x = x]$$

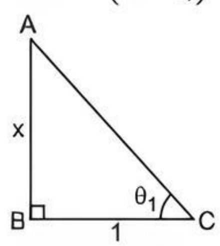
$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2 \quad \left[\because \theta = \frac{1}{2}\cos^{-1}x^2\right]$$

$= \text{RHS}$

Hence proved.

5. Given equation is

$$\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right) \quad ...(1)$$



Let $\theta_1 = \tan^{-1}x$

$\Rightarrow \tan\theta_1 = x$

Let ABC be a right angled triangle with ∠B = 90°.

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

In right-angled ΔABC,

$$AC^2 = AB^2 + BC^2$$

[by Pythagoras theorem]

$$\Rightarrow \quad AC^2 = x^2 + 1$$

$$\Rightarrow \quad AC = \sqrt{1+x^2}$$

$$\therefore \quad \cos\theta_1 = \frac{BC}{AC} = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \theta_1 = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$

$\Rightarrow \qquad \tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1+x^2}} \qquad ...(2)$

Let $\qquad \theta_2 = \cot^{-1}\left(\dfrac{3}{4}\right)$

$\Rightarrow \qquad \cot \theta_2 = \dfrac{3}{4}$

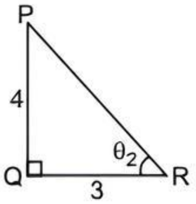
Let PQR be a right angled triangle with $\angle Q = 90°$.
In right-angled ΔPQR,

$PR^2 = PQ^2 + QR^2$ [by Pythagoras theorem]

$\Rightarrow \qquad PR^2 = (4)^2 + (3)^2$

$= 16 + 9 = 25$

$\therefore \qquad PR = 5$



Now, $\quad \sin \theta_2 = \dfrac{PQ}{PR} = \dfrac{4}{5}$

$\Rightarrow \qquad \theta_2 = \sin^{-1}\left(\dfrac{4}{5}\right)$

$\Rightarrow \quad \cot^{-1}\left(\dfrac{3}{4}\right) = \sin^{-1}\left(\dfrac{4}{5}\right) \qquad ...(3)$

From eqs. (1), (2) and (3), we get

$\cos\left(\cos^{-1} \dfrac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1} \dfrac{4}{5}\right)$

$\Rightarrow \qquad \dfrac{1}{\sqrt{1+x^2}} = \dfrac{4}{5}$

$[\because \cos(\cos^{-1} x) = x, \sin(\sin^{-1} x) = x]$

$\Rightarrow \qquad \dfrac{1}{1+x^2} = \dfrac{16}{25}$ (squaring on both sides)

$\Rightarrow \qquad 1 + x^2 = \dfrac{25}{16}$

$\Rightarrow \qquad x^2 = \dfrac{25}{16} - 1 = \dfrac{9}{16}$

$\therefore \qquad x = \pm \dfrac{3}{4}$

6. $\sqrt{1-\sin x} = \sqrt{\left(\sin^2 \dfrac{x}{2} + \cos^2 \dfrac{x}{2}\right) - \left(2\sin \dfrac{x}{2} \cdot \cos \dfrac{x}{2}\right)}$

$\left[\because \sin^2 \theta + \cos^2 \theta = 1 \text{ and } \sin \theta = 2\sin \dfrac{\theta}{2} \cdot \cos \dfrac{\theta}{2}\right]$

$= \sqrt{\left(\cos \dfrac{x}{2} - \sin \dfrac{x}{2}\right)^2} = \cos \dfrac{x}{2} - \sin \dfrac{x}{2}$

and $\sqrt{1+\sin x} = \sqrt{\left(\sin^2 \dfrac{x}{2} + \cos^2 \dfrac{x}{2}\right) + \left(2\sin \dfrac{x}{2} \cdot \cos \dfrac{x}{2}\right)}$

$= \sqrt{\left(\sin \dfrac{x}{2} + \cos \dfrac{x}{2}\right)^2}$

$= \sin \dfrac{x}{2} + \cos \dfrac{x}{2}$

$\therefore \cot^{-1}\left\{\dfrac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right\}$

$= \cot^{-1}\left\{\dfrac{\left(\cos \dfrac{x}{2} - \sin \dfrac{x}{2}\right) + \left(\sin \dfrac{x}{2} + \cos \dfrac{x}{2}\right)}{\left(\cos \dfrac{x}{2} - \sin \dfrac{x}{2}\right) - \left(\sin \dfrac{x}{2} + \cos \dfrac{x}{2}\right)}\right\}$

$= \cot^{-1}\left\{\dfrac{\cos \dfrac{x}{2} - \sin \dfrac{x}{2} + \sin \dfrac{x}{2} + \cos \dfrac{x}{2}}{\cos \dfrac{x}{2} - \sin \dfrac{x}{2} - \sin \dfrac{x}{2} - \cos \dfrac{x}{2}}\right\}$

$= \cot^{-1}\left\{\dfrac{2\cos \dfrac{x}{2}}{-2\sin \dfrac{x}{2}}\right\} = \cot^{-1}\left(-\cot \dfrac{x}{2}\right)$

$= \cot^{-1}\left\{\cot\left(\pi - \dfrac{x}{2}\right)\right\} = \pi - \dfrac{x}{2}$

$[\because -\cot \theta = \cot(\pi - \theta) \text{ and } \cot^{-1}(\cot x) = x]$

7. Given, $\sin^{-1}(1-x) - 2\sin^{-1}(x) = \dfrac{\pi}{2}$

$\Rightarrow \qquad \sin^{-1}(1-x) = \dfrac{\pi}{2} + 2\sin^{-1} x$

$\Rightarrow \qquad 1 - x = \sin\left\{\dfrac{\pi}{2} + 2\sin^{-1} x\right\}$

$\Rightarrow \qquad 1 - x = \cos(2\sin^{-1} x) \qquad ...(1)$

$[\because \sin(90° + \theta) = \cos \theta]$

Put $\qquad 2\sin^{-1} x = \theta$

$\Rightarrow \qquad x = \sin \dfrac{\theta}{2}$

$\Rightarrow \qquad x^2 = \sin^2 \dfrac{\theta}{2} = \dfrac{1 - \cos \theta}{2}$

$\left[\because \cos \theta = 1 - 2\sin^2 \dfrac{\theta}{2}\right]$

$\Rightarrow \qquad 2x^2 = 1 - \cos(2\sin^{-1} x) \quad [\because \theta = 2\sin^{-1} x]$

$\Rightarrow \quad \cos(2\sin^{-1} x) = 1 - 2x^2 \qquad ...(2)$

From eqs. (1) and (2), we get

$1 - x = 1 - 2x^2$

$\Rightarrow \qquad 2x^2 - x = 0$

$\Rightarrow \qquad x(2x - 1) = 0$

$\Rightarrow \qquad x = 0 \text{ or } 2x - 1 = 0$

$\therefore \qquad x = 0 \text{ or } x = \dfrac{1}{2}$

For $x = \dfrac{1}{2}$, $\quad$ LHS $= \sin^{-1}\left(1 - \dfrac{1}{2}\right) - 2\sin^{-1}\left(\dfrac{1}{2}\right)$

$= \sin^{-1}\left(\dfrac{1}{2}\right) - 2\sin^{-1}\left(\dfrac{1}{2}\right)$

$= \dfrac{\pi}{6} - 2 \times \dfrac{\pi}{6} = \dfrac{\pi}{6} - \dfrac{\pi}{3} = -\dfrac{\pi}{6} \neq$ RHS

$\therefore \ x = \dfrac{1}{2}$ is not a solution of given equation.

Hence, $x = 0$ is the only solution.

Chapter Test

Multiple Choice Questions

Q 1. The value of $\sin^{-1}\left\{\cos\left(\frac{43\pi}{5}\right)\right\}$ is:

a. $\frac{3\pi}{5}$
b. $\frac{-7\pi}{5}$
c. $\frac{\pi}{10}$
d. $-\frac{\pi}{10}$

Q 2. The domain of $\sin^{-1} 2x$ is:

a. [0, 1]
b. [−1, 1]
c. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
d. [−2, 2]

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 3. **Assertion (A):** The principal value of $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{1}{2}\right)\right]$ is $\frac{\pi}{3}$.

Reason (R): The least numerical value, either positive or negative of angle θ is called principal value of the inverse trigonometric function.

Q 4. **Assertion (A):** The minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}, n \in N$ is valid, is 5.

Reason (R): The graph of inverse trigonometric function can be obtained from the graph of their corresponding function by interchanging X and Y-axes.

Case Study Based Question

Q 5. As the trigonometric functions are periodic functions, so these functions are many-one. Trigonometric functions are not one-one and onto over their natural domain and range, so their inverse do not exist. But if we restrict their domain and range, then their inverse may exists.

Based on the given information, solve the following questions.

(i) Find the value of $\tan^{-1}(-1)$ in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

(ii) Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.

(iii) Find the value of

$$\tan^{-1}(\sqrt{3}) + \cot^{-1}(\sqrt{3}) + \tan^{-1}(\cos(0)).$$

Or

Find the value of $\tan^{-1}\left[2\sin\left\{2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$.

Very Short Answer Type Questions

Q 6. Find the set of values of $\sec^{-1}\left(\frac{1}{2}\right)$.

Q 7. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$.

Short Answer Type Questions

Q 8. Draw the graph of the principal branch of the function $f(x) = \sin^{-1} x$.

Q 9. Find the value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$.

Q 10. Find the value of $2\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right)$.

Q 11. Find the simplified form of the following in terms of $\tan^{-1}$.

$$\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right), \text{where } x \in \left[\frac{-3\pi}{4}, \frac{\pi}{4}\right].$$

Q 12. Find the number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $\left[\frac{\pi}{2}, \pi\right]$.

Long Answer Type Questions

Q 13. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?

Q 14. Find the value of

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right).$$