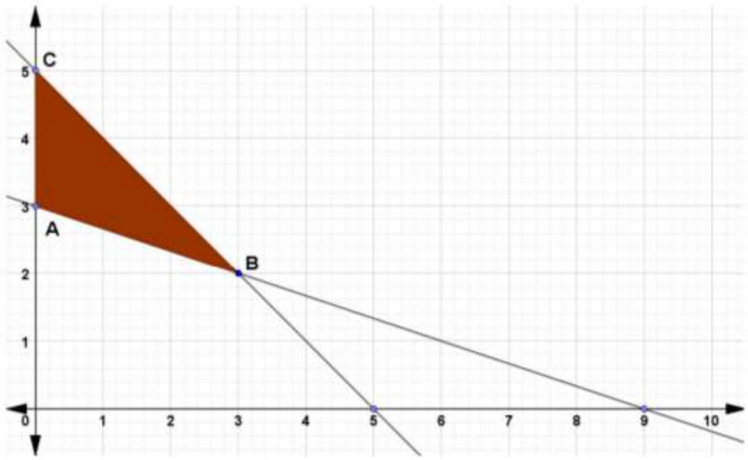
Linear Programming

Que 1: ***Marks :(6)***

1. The feasible region of a LPP is given below:



(i) Find the constraints satisfying the above feasible region.
(ii) If $Z = 11x + 7y$ is an objective function, then find the maximum and minimum value of Z.

Ans:

(i) Equation of line through B and C is $x + y = 5$

Equation of line through A and B is

$$\frac{x}{9} + \frac{y}{3} = 1$$

$$\Rightarrow x + 3y = 9$$

Hence constraints are

$$x + y \leq 5$$

$$x + 3y \geq 9$$

$$y \geq 0$$

(ii) Point A (0,3)

$$Z = 0 + 7(3) = 21$$

For point B solve equations $x + y = 5$ and $x + 3y = 9$

Point B is (3,2)

$$Z = 11(3) + 7(2) = 47$$

Point C is (0,5)

$$Z = 0 + 7(5) = 35$$

Maximum value Z = 47 at B (3,2)
Maximum value Z = 21 at B (0,3)

Que 2: ***Marks :(6)***

5. A (0,10), B(5,5), C (15,15), D (0,20) are the corner points of a feasible region of a LPP. At C (15,15) and D (0,20) the objection function has multiple optimal solution of Z = 180.
(i) Find the objective function.
(ii) Find the maximum and minimum value of Z.

Ans:

(i) Let $Z = ax + by$

$15a + 15b = 180 \Rightarrow a + b = 12$

$(0)a + 20b = 180 \Rightarrow b = 9$

$\Rightarrow a = 12 - 9 = 3$

(ii)

Corner points	$Z = 3x + 9y$
A (0,0)	90
B (5,5)	60
C (15,15)	180
D (0,20)	180

Que 3: ***Marks :(6)***

4. The following table gives the solution of a LPP.

Corner points	Values of Objective function Z
A (0,0)	0
B (3,0)	12
C (2,3)	11
D (0,5)	5

(i) Find the objective function
(ii) Draw the feasible region.
(iii) Find the constraints satisfying the above feasible region.

Ans:

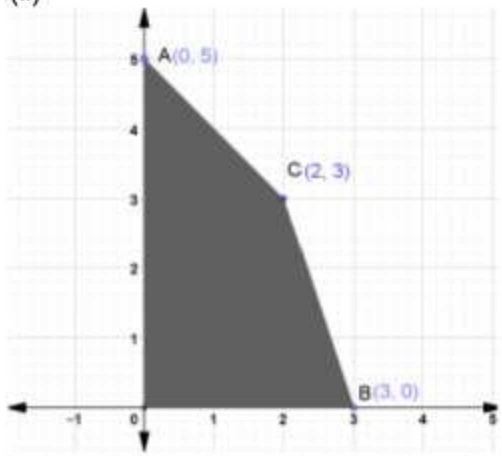
(i) Let Z = ax + by

At point B (3,0), z = 12 $\Rightarrow 3a + b(0) = 12 \Rightarrow a = 4$

At point D (0,5), z = 12 $\Rightarrow a(0) + b(5) = 5 \Rightarrow b = 1$

$\Rightarrow Z = 4x + y$

(ii)



(iii) Equation of the line Passing through (0,3) and (2,3)

$$y-0=-3(x-3)$$

$$3x+y=9$$

Equation of the line Passing through (0,3) and (2,3)

$$y-5=-1(x-0)$$

$$x+y=5$$

Constraints:

$$3x+y\le 9$$

$$x+y\le 5$$

$$x\ge 0, y\ge 0$$

Que 4: ***Marks :(6)***

3. In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs 2 and Re 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Ans:

Let x-number of x tablets
y-number of y tablets.

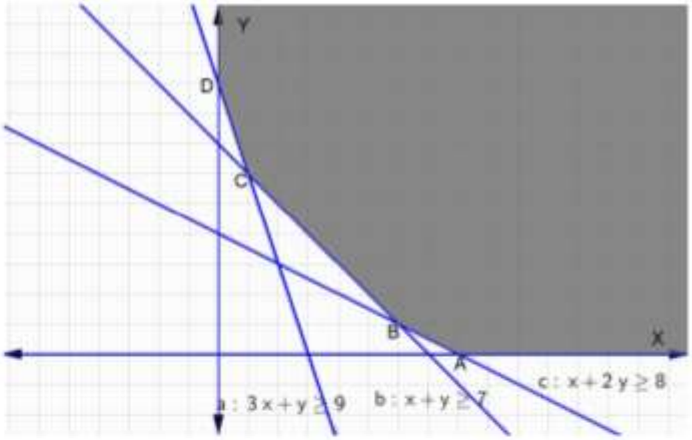
Objective function: $Z=2x+y$

Constraints:

$$6x+2y\ge 18 \Rightarrow 3x+y\ge 9$$

$$3x+3y\ge 21 \Rightarrow x+y\ge 7$$

$$2x+4y\ge 16 \Rightarrow x+2y\ge 8$$

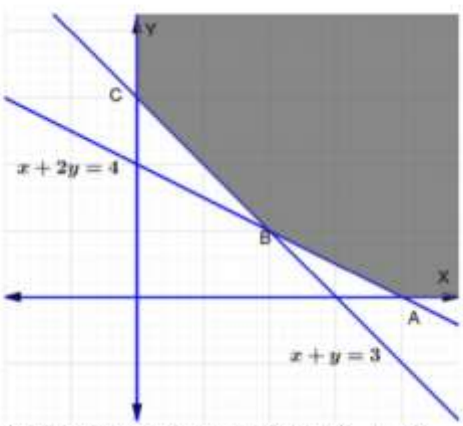


Point	Value of Objective function
A(8,0)	Z = 16
B(6,1)	Z = 13
C(1,6)	Z = 8
D(0,9)	Z = 9

The feasible region is unbounded. Draw the region $2x + y < 8$
No points are common with the feasible region. So minimum value is 8. So the person should take 1 x tablet and 6 y tablets.

Que 5: ***Marks :(6)***

2. The figure given below shows the feasible region of a LPP.



(i) Find the corner points A, B, C
(ii) If $Z = 4x + y$ is an objective function, then find value of Z at A, B, C.
(iii) Find the minimum value of objective function.

Ans:

(i) Solve the equations $x + y = 3$ and $x + 2y = 4$ to get point B (2,1)

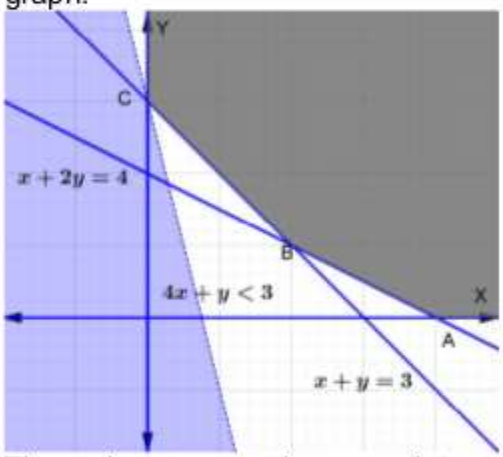
From equation $x + y = 3$ we get the point C (0,3)

From equation $x + 2y = 4$ we get the point A (4,0)

(ii)

Points	Value of objective function
A	$Z = 4(4) + 0 = 16$
B	$Z = 4(2) + 1 = 9$
C	$Z = 4(0) + 3 = 3$

(iii) Here the feasible region is unbounded. Draw the line $4x + y < 3$ in the given graph.



The region $4x + y < 3$ has no points common with the feasible region. Hence minimum value is 3.