Trigonometric Function

OLYMPIAD EXCELLENCE BOOK MATHEMATICS

QUESTIONS

1. $\frac{\sec^2\theta-1}{\tan^2\theta}=?$

(a) 1 (b) 2 (c) 3 (d) 5

2. $\frac{3-4\sin^2\theta}{\cos^2\theta}+\tan^2\theta$ is

(a) 1 (b) 2 (c) 3 (d) None of these

3. Find the value of, $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

(a) $2\sec\theta$ (b) $\sec\theta$ (c) $2\operatorname{cosec}\theta$ (d) None of these

4. If $\tan\theta=\frac{a}{b}$, find the value of $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}$

(a) $\frac{a^2-b^2}{a^2+b^2}$ (b) $\frac{b^2-a^2}{b^2+a^2}$ (c) $\frac{a^2+b^2}{a^2-b^2}$ (d) None of these

5. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, then $\cos\theta-\sin\theta=$

(a) $\sqrt{2}\cos\theta$ (b) $\sqrt{2}\sin\theta$ (c) $\cos\theta$ (d) None of these

6. Which of the following is possible?

(a) $\cos\theta=x+\frac{1}{x}$ (b) $\sec\theta=\frac{x^2}{1+x^2}$ (c) $\operatorname{cosec}\theta=\frac{x^2}{1+x^2}$ (d) $\tan\theta=\frac{x^2+x+1}{x^2-3x+5}$

7. If $\tan\theta=\frac{x}{y}$, where x and y are whole numbers, find $\sin\theta$.

(a) $\frac{y}{\sqrt{y^2-x^2}}$ (b) $\frac{x}{\sqrt{x^2-y^2}}$ (c) $\frac{y}{\sqrt{x^2+y^2}}$ (d) $\frac{x}{\sqrt{y^2-x^2}}$

8. If $\sin A+\sin^2 A=1$, find the value of the expression $(\cos^2 A+\cos^4 A)$.

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

9. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{\sqrt{3}+1}{\sqrt{3}-1}$, find the acute angle θ.

(a) 90° (b) 45° (c) 30° (d) 60°

10. $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ is equal to

(a) $2\sec^2\theta$ (b) $2\cos^2\theta$ (c) 0 (d) 1

11. $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=$

(a) $\frac{2}{1-2\cos^2\theta}$ (b) $\frac{2}{2\sin^2\theta-1}$ (c) both (A) & (B) (d) None of these

12. If $\frac{1+\sin\alpha}{1-\sin\alpha}=\frac{m^2}{n^2}$, then sin α is:

(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{m^2-n^2}{m^2+n^2}$ (c) $\frac{m^2+n^2}{n^2+m^2}$ (d) $\frac{n^2-m^2}{m^2+n^2}$

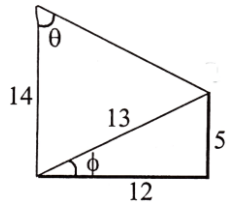
13. If $sec\theta+tan\theta=2$, then find the value of sin θ.

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $-\frac{3}{5}$ (d) $-\frac{2}{5}$

14. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=$ _____

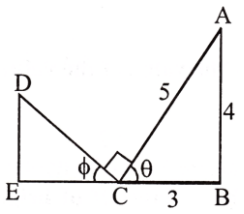
(a) $sec\theta+tan\theta$ (b) $sec\theta-cot\theta$ (c) $cosec\theta+tan\theta$ (d) $cosec\theta-tan\theta$

15. What is the value of $\tan\theta$ in the adjoining figure?



(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{5}{12}$

16. In the given fig. What is the value of $\cos\phi$?



(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

17. If $x = m\ sin\theta$ and $y = n\ cos\theta$, What is the value of $n^2x^2 + m^2y^2$?

(a) mn (b) mn^2 (c) 1 (d) m^2n^2

18. If $\frac{\cos 20^\circ}{\sin 70^\circ} + \frac{2\cos\theta}{\sin(90^\circ - \theta)} = \frac{k}{2}$, what is the value of k?

(a) 4 (b) 5 (c) 6 (d) 1

19. Which of the following is equal to $2\ tan^2A - 2\ sec^2A - 5\ cot^2A + 5\ \cos ec^2A$?

(a) 2 (b) 3 (c) 4 (d) 5

20. Which of the following is equal to a $\left(1 + tan^2\theta\right)\ (1 - sin\theta)(1 + sin\theta)(1 + cos\theta)(1 - cos\theta)\ \left(1 + cot^2\theta\right)$?

(a) 1 (b) 2 (c) 3 (d) 4

21. If $1 + 2sin^2\theta\ cos^2\theta = sin^2\theta + cos^2\theta + 4k\ sin^2\theta\ \ cos^2\theta$ what is the value of k?

(a) 1 (b) – 1 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

22. Which of the following is equal to $\sin\theta\cos\theta - \frac{\sin\theta\cos(90^\circ - \theta)\cos\theta}{\sec(90^\circ - \theta)} - \frac{\cos\theta\sin(90^\circ - \theta)\sin\theta}{\cos ec(90^\circ - \theta)}$?

(a) 1 (b) 0 (c) – 1 (d) – 2

23. If $cos\theta + sec\theta = 2$. Which of the following is equal to $cos^3\theta + sec^3\theta$?

(a) 0 (b) 1 (c) 2 (d) 3

24. If $sin\ A + cosec\ A = 2$, then what is the value of the value of $\frac{\sin^4 A + 1}{\sin^2 A}$?

(a) 0 (b) 1 (c) 2 (d) $6/\sqrt{2}$

25. A vertical tower is surmounted by a flagstaff of height h metres. At a point on the ground, the angels of elevation of the bottom and top of the flagstaff are α and β respectively. What is the height of the tower in meters?

(a) $\frac{h\tan\alpha}{\tan\beta + \tan\alpha}$ (b) $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$ (c) $\frac{h\tan\alpha}{\tan\beta}$ (d) $\frac{h\tan\beta}{\tan\beta - \tan\alpha}$

26. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its center is β. What is the height of the center of the balloon?

(a) $r\sin\beta\sec\frac{\alpha}{2}$ (b) $r\sin\beta\cos\frac{\alpha}{2}$ (c) $r\sin\beta\ cosec\frac{\alpha}{2}$ (d) $r\cos\beta\ cosec\frac{\alpha}{2}$

27. If the angle of elevation of a cloud from a point *h* metres above a lake is α and the angle of depression of its reflection in the lake is β, What is the distance of the cloud from the point of observation? (in 'm')

(a) 1 (b) $\frac{h(\tan\beta - \tan\alpha)}{\tan\beta + \tan\alpha}$ (c) $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$ (d) None of these

28. A 1.5 m tall boy is standing at some distance from a 63/2 m tall building. The angle of elevation from his eyes to the top of the building increases from $30°$ to $60°$ as he walks towards the building. What is the distance he walked towards the building?

(a) $\frac{20}{\sqrt{3}}m$ (b) $30\sqrt{3}m$ (c) $20\sqrt{3}m$ (d) $\frac{30}{\sqrt{3}}m$

29. If $sec\ \theta + tan\ \theta = m$ and $sec\ \theta - tan\ \theta = n$ then what is the value of mn?

(a) 0 (b) 1 (c) 2 (d) 3

30. If $7\ sin^2\theta + 3\ cos^2\theta = 4$, then which of the following is equal to tan θ?

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

ANSWER - KEY				
1. A	**2.** C	**3.** C	**4.** A	**5.** B
6. D	**7.** B	**8.** A	**9.** D	**10.** A
11. C	**12.** B	**13.** A	**14.** A	**15.** B
16. A	**17.** D	**18.** C	**19.** B	**20.** A
21. C	**22.** B	**23.** C	**24.** C	**25.** B
26. C	**27.** C	**28.** C	**29.** B	**30.** C

SOLUTIONS

1. (A): $\dfrac{\sec\theta-1}{\tan^2\theta}=\dfrac{\tan^2\theta}{\tan^2\theta}=1$

2. (C): $\dfrac{3}{\cos^2\theta}-\dfrac{4\sin^2\theta}{\cos^2\theta}+\tan^2\theta$

3. (C): $\sqrt{\dfrac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}+\sqrt{\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$=\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{1-\cos\theta}{\sin\theta}=\dfrac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$=\dfrac{2}{\sin\theta}=2\,\mathrm{cosec}\theta$

4. (A): $\dfrac{a\dfrac{\sin\theta}{\cos\theta}-b\dfrac{\cos\theta}{\cos\theta}}{a\dfrac{\sin\theta}{\cos\theta}+b\dfrac{\cos\theta}{\cos\theta}}=\dfrac{a\times\dfrac{a}{b}-b}{a\times\dfrac{a}{b}+b}$

$=\dfrac{a^2-b^2}{a^2+b^2}$

5. (B):

$(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2=\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta$

$\Rightarrow \left(\sqrt{2}\cos\theta\right)^2+\left(\cos\theta-\sin\theta\right)^2=1+1$

$\Rightarrow 2\cos^2\theta+(\cos\theta-\sin\theta)^2=2$

$\Rightarrow (\cos\theta-\sin\theta)^2=2-2\cos^2\theta$

$\Rightarrow (\cos\theta-\sin\theta)^2=2\sin^2\theta$

$\Rightarrow \cos\theta-\sin\theta=\sqrt{2}\sin\theta$

6. (D): Put $x=1$ and verify in all the options. Since $\cos\theta\le 1$, but $x+\dfrac{1}{x}\not\le 1$ always.

$\therefore$ Option (A) is not true.

Further, $\sec\theta\ge 1$ but $\dfrac{x^2}{1+x^2}<1$ always.

$\therefore$ Option (B) is not possible.

Similarly, since $cosec\theta \geq 1$ always.

$\therefore$ Options (C) is not possible.

Hence, by elimination method, option (D) is true.

Aliter: You can also verify for option (D) separately as $-\infty < \tan\theta < +\infty$

7. (B): $\tan\theta = \frac{x}{y} \Rightarrow \cot\theta = \frac{y}{x} \Rightarrow \cos ec\theta$

$$= \sqrt{1+\left(\frac{y}{x}\right)^2} = \frac{\sqrt{x^2+y^2}}{x}$$

$$\therefore \quad \sin\theta = \frac{x}{\sqrt{x^2+y^2}}$$

8. (A): $\sin A = 1 - \sin^2 A = \cos^2 A$

$$\Rightarrow \quad \cos^2 A + \cos^4 A = \cos^2 A + \sin^2 A = 1$$

9. (D): $\frac{1+\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$

$$\Rightarrow \quad \frac{1+\cot\theta}{1-\cot\theta} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \quad \cot\theta = \frac{1}{\sqrt{3}} = \cot 60° \Rightarrow \theta = 60°$$

10. (A) $\frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

11. (C): $\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{\sin^2\theta-\cos^2\theta}$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta-\cos^2\theta} = \frac{2}{1-2\cos^2\theta} = \frac{2}{2\sin^2\theta-1}$$

12. (B): $n^2(1+\sin\alpha) = m^2(1-\sin\alpha)$

$\Rightarrow \quad n^2 + n^2 \sin\alpha = m^2 - m^2 \sin\alpha$

$\Rightarrow \quad \left(m^2 + n^2\right)\sin\alpha = m^2 - n$

$\Rightarrow \quad \sin\alpha = \dfrac{m^2 - n^2}{m^2 + n^2}$

13. (A): $\sec\theta + \tan\theta = 2$

$\therefore \quad \sec\theta - \tan\theta = \dfrac{1}{2}$

$\{\text{since}(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1\}$

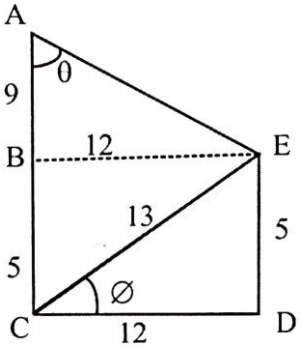
Solving, we get, $\sec\theta = \dfrac{5}{4}$ and $\tan\theta = \dfrac{3}{4}$

$\therefore \quad \sin\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{3}{5}$

14. (A): $\sqrt{\dfrac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} = \sqrt{\dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

15. (B):



$\therefore \quad \tan\theta = \dfrac{12}{9} = \dfrac{4}{3}$

16. (A): $\phi + \theta + 90° = 180°$

$\Rightarrow \quad \phi + \theta = 90°$

$\Rightarrow \quad \phi = 90° - \theta$

$\Rightarrow \quad \cos\phi = \cos(90° - \theta)$

$\Rightarrow \quad \cos\phi = \sin\theta$

$\Rightarrow \quad \cos\phi = \dfrac{4}{5}$

17. (D): $x = m\sin\theta$

$\Rightarrow \quad x^2 = m^2 \sin^2\theta$

$y = n\cos\theta$

$\Rightarrow \quad y^2 = n^2 \cos^2\theta$

$n^2x^2 = m^2n^2 \sin^2\theta$ _____(1)

$m^2y^2 = m^2y^2 \cos^2\theta$ _____(2)

$\therefore \quad n^2x^2 + m^2y^2 = m^2n^2$

18. (C): $\dfrac{\sin 70°}{\sin 70°} + \dfrac{2\cos\theta}{\cos\theta} = \dfrac{k}{2}$

$\Rightarrow \quad 1 + 2 = \dfrac{k}{2}$

$\Rightarrow \quad 6 = k$

19. (B): $2\left(\tan^2 A - \sec^2 A\right) - 5\left(\cot^2 A - \operatorname{cosec}^2 A\right)$

$\Rightarrow \quad -2 - 5(-1) = -2 + 5 = 3$

20. (A): $\left(\sec^2\theta\right) \times \left(1 - \sin^2\theta\right)\left(1 - \cos^2\theta\right)\left(\operatorname{cosec}^2\theta\right)$

$= \sec^2\theta \times \cos^2\theta \times \sin^2\theta \times \operatorname{cosec}^2\theta = 1$

21. (C): $1 + 2\sin^2\theta\cos^2\theta = 1 + 4k\sin^2\theta\cos^2\theta$

$\Rightarrow \quad 0 = \sin^2\theta\cos^2\theta(4\text{k} - 2)$

$\Rightarrow \quad 0 = 4K - 2$

$\Rightarrow \quad K = \dfrac{1}{2}$

22. (B): $\sin\theta\cos\theta - \dfrac{\sin\theta.\sin\theta\cos\theta}{\operatorname{cosec}\theta} - \dfrac{\cos\theta.\cos\theta\sin\theta}{\sec\theta}$

$= \sin\theta\cos\theta - \sin\theta\cos\theta\left[\sin^2\theta + \cos^2\theta\right]$

$= \sin\theta\cos\theta - \sin\theta\cos\theta = 0$

23. (C): $\cos\theta + \dfrac{1}{\cos\theta} = 2$

$\Rightarrow \quad \cos^2\theta + 1 = 2\cos\theta$

$\Rightarrow \quad \cos^2\theta - 2\cos\theta + 1 = 0$

$\Rightarrow \quad (\cos\theta - 1)^2 = 0$

$\Rightarrow \quad \cos\theta = 1$

$\therefore \quad \sec\theta = 1$

$\therefore \quad \cos^3\theta + \sec^3\theta = 1 + 1 = 2$

Mind of a mathematician

A thoughtful student will straightaway see $\left(\cos\theta + \frac{1}{\cos\theta}\right)$ as equivalent to $x + \frac{1}{x}$ whose minimum value is attained when $x = 1$ and hence, $x + \frac{1}{x} = 2$ (min value)

24. (C): $\sin A + \frac{1}{\text{sinA}} = 2$

$\Rightarrow \quad \sin^2 A + 1 = 2\sin A$

$\Rightarrow \quad \sin^2 A - 2\sin A + 1 = 0$

$\Rightarrow \quad (\text{sinA} - 1)^2 = 0$

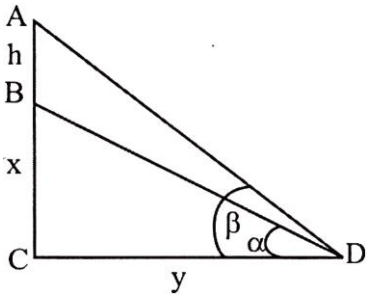
$\Rightarrow \quad \sin A = 1$

$\therefore \quad \frac{\sin^4 A + 1}{\sin^2 A} = \frac{1+1}{1} = 2$

Again here, you can straightaway conclude that $\sin A = 1$

$\therefore \quad \frac{\sin^4 A + 1}{\sin^2 A} = \frac{1+1}{1} = 2$

25. (B):



In ΔACD, $\tan\beta = \frac{x+h}{y}$

$y = \frac{x+h}{\tan\beta}$ .....(1)

In ΔBCD, $\tan\alpha = \frac{x}{y}$

$\therefore y = \frac{x}{\tan\alpha}$ .....(2)

From (1) and (2), $\frac{x+h}{\tan\beta} = \frac{x}{\tan\alpha}$

$\Rightarrow \quad x\tan\alpha + h\tan\alpha = x\tan\beta$

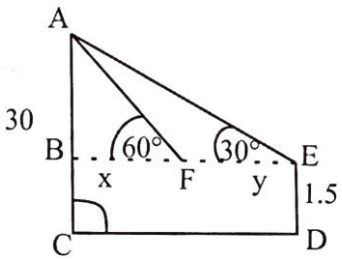
$\Rightarrow \quad h\tan\alpha = x(\tan\beta - \tan\alpha)$

$\Rightarrow \quad x = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

26. (C) Not Available

27. (C) Not Available

28. (C)



In ΔABF, $\tan 60° = \frac{30}{x}$

$\therefore\ x = \frac{30}{\tan 60°} = \frac{30}{\sqrt{3}}$

In ΔABE, $\tan 30° = \frac{30}{x+y}$

$\therefore \quad x + y = \frac{30}{\tan 30°}$

$\Rightarrow \quad x + y = 30\sqrt{3}$

$\Rightarrow \quad \frac{30}{\sqrt{3}} + y = 30\sqrt{3}$

$\Rightarrow \quad y = 30\sqrt{3} - \frac{30}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

29. (B): $mn = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta = 1$

30. (C): $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow \quad 4\sin^2\theta + 3 = 4$

$\Rightarrow \quad 4\sin^2\theta = 1$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$