SECTION - I

Straight Objective Type

1. x, y, z are in GP

$\Rightarrow \quad y^2 = xz$

$\Rightarrow \quad 2\log y = \log x + \log z$

$\Rightarrow \quad \text{logy} - \text{logz} = \text{logx} - \text{logy}$

$\Rightarrow \quad (1+\text{logy}) - (1+\text{logz})$

$= (1+\text{logx}) - (1+\text{logy})$

$\Rightarrow \quad 1+\log x, 1+\log y, 1+\log z$ are in AP

$\Rightarrow \quad \dfrac{1}{1+\log x}, \dfrac{1}{1+\log \text{y}}, \dfrac{1}{1+\log z}$

are in HP

Hence (b) is the correct option.

2. $a_1, a_2, \ldots\ldots\ldots a_{10}$ be in AP

$h_1, \text{h}_2, \ldots\ldots\ldots \text{h}_{10}$ be in AP

If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then $a_4 h_7 = 5$

Hence (c) is the correct option

3. If $(1.05)^{50} = 11.658$ then

$$\sum_{n=1}^{49}(1.05)^n \quad = 10.5 + (1005)^2 + (1.05)^3 + \ldots\ldots\ldots\ldots + (1.05)^{49}$$

$$= \frac{(1.05)\left[(1.05)^{49} - 1\right]}{1.05 - 1}$$

$$= \frac{(1.05)^{50} - 1.05}{0.05} = \frac{11.658 - 1.05}{0.05}$$

$= 212.16$

Hence (c) is the correct option.

4. $\alpha.\beta$ are the roots of $x^2 - x + p = 0$

$\Rightarrow$ $\alpha + \beta = 1$

$\alpha\beta = p$

γ, δ are the roots of $x^2 - 4x + q = 0$ then

$\gamma, \delta = 4$

$\gamma\delta = q$

$\alpha, \beta, \gamma, \delta$ are –2, –32.

Hence (a) is the correct option.

5. The sum of first 2n terms of 2, 5, 8.............

$= \frac{2n}{2}[4 + (2n-1)3]$

$= n[6n+1] = 6n^2 + n$

The Sun of first n terms of 57, 59, 61...........

$= \frac{n}{2}[104 + (n+1)2]$

$= \frac{n}{2}[2\text{n}+102]$

$= n(n+51) = n^2 + 51n$

$6n^2 + n = n^2 + 51n$

$\Rightarrow 5n^2 = 50n$

$\Rightarrow n = 10$

Hence (a) is the correct option.

6. $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{2^2 - 1}$

$a_2 \quad = \frac{3}{2} < 2$

$a_3 \quad = 1 + \frac{1}{2} + \frac{1}{3} < 2$

$a_4 \quad = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} < 3$

$a_{200} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{2^{100} - 1} \le 100$

Hence (b) is the correct option.

7. $a = x$

$\frac{a}{1-r} = 5 \quad \Rightarrow \frac{x}{1-r} = 5$

$\Rightarrow x = 5(1-r)$

for an infinite G.P.

$r < 1$

$1 - r < 0$

$5(1-\mathrm{r}) < 0$

$\Rightarrow x < 0$

Hence (b) is the correct option.

8. If a, b, c are in A.P.

$\Rightarrow 2B = a + c$

If $1, \frac{1}{2}\log_2 3^{1-x} + 2, \log_3\left(4.3^x - 1\right)$

are *n* AP

$\log_3 3^{1-x} + \log_3 9 = \log_3^3 + \log_3\left(4.3^x - 1\right)$

$\Rightarrow \log_3 3^{1-x} 3^2 = \log_3 2\left(4.3^x - 1\right)$

$\log_3 3^{3-x} \qquad = \log_3 + 3^{x+1} - 3$

$\Rightarrow 3^{3-x} \qquad = 4.3^{x+1} - 3$

$\Rightarrow \qquad x = \log_3^4$

Hence (a) is the correct option.

9. $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right) \quad i=\sqrt{-1}$

$\left(1+i^2\right)+\left(i^2+i^3\right)+.........+\left(i^{13}+i^{14}\right)$

$=i+2\left(i^2+i^3+.......+i^{13}\right)+i^{14}$

$=-i$

Hence (c) is the correct option.

10. The roots of the equation

$\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+\left(8+2\sqrt{5}\right)=0$

are α,β

Then $\alpha+\beta=\dfrac{4+\sqrt{5}}{5+\sqrt{2}}$

$\alpha\beta=\dfrac{8+2\sqrt{5}}{5+\sqrt{2}}$

Harmonic mean of $\alpha,\beta=\dfrac{2\alpha\beta}{\alpha+\beta}$

$$2\frac{\dfrac{\left(8+2\sqrt{5}\right)}{\left(5+\sqrt{2}\right)}}{\dfrac{4+\sqrt{5}}{\left(5+\sqrt{2}\right)}}$$

$$=4\left(\frac{4+\sqrt{5}}{\left(4+\sqrt{5}\right)}\right)$$

$=4$

Hence (b) is the correct option.

11. If $s_n = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots\ldots\ldots$

$$+ \frac{1}{1+2+3+2007}$$

then t_n

$$= \frac{1}{\frac{n(n+1)}{2}} = \frac{2}{n(n+1)} = \left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$\therefore s_n = 2\sum\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$= 2\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{2007} - \frac{1}{2006}\right]$$

$$= 2\left[1 - \frac{1}{2008}\right]$$

$$= 2\left[\frac{2007}{2008}\right] = \frac{4014}{2008}$$

Statement 1 is true
Statement 2 is true and Statement 2 is correct explanation for Statement 1.
Hence (a) is the correct option.

12. $a = 2$

$5a + 10d \quad = \frac{1}{4}(5a + 35d)$

$\Rightarrow 20a + 40d \quad = 5a + 35d$

$\Rightarrow 15a + 5d \quad = 0$

$\Rightarrow 30a + 5d \quad = 0$

$\Rightarrow d = -6$

$\therefore t_{20} \qquad = 2+(20-1)(-6)$

$= 2-114 = -112$

$\therefore$ Statement I is true

Statement 2 is true

Statement 2 is a correct explanation for Statement 1.

Hence (a) is the correct option.

13. $s_1 = \sum n \qquad s_2 = \sum n^2 \qquad s_3 = \sum n^3$

Then $\quad s_3\left(1+8s_1\right)$

$$= \frac{n^2\left(n+1\right)^2}{4}\left(1+8.\frac{n(n+1)}{2}\right)$$

$$= \frac{n^2(n+1)^2}{4}(1+4n(n+1))$$

$$= \frac{n^2(n+1)^2}{4}(2n+1)^2$$

$$= \frac{9n^2(n+1)^2(2n+1)^2}{36}$$

$$= 9\left(\frac{n(n+1)(2n+1)}{6}\right)^2$$

$$= 9\left(\sum n^2\right) = 9s_2^2$$

Statement 1 is true.

Statement 2 is true.

Statement 1 is a correct explanation for the Statement 1,

Hence (a) is the correct option.

14. $a = 50 \quad t_n = r^{n-1} \quad t_4 = 50(r)4^{-1}$

$1350 \quad = 50\sqrt{3}$

$$\frac{1350}{50} = \sqrt{3}$$

$\Rightarrow r \quad = 3$

5th term = $t_5 \quad ar^4$

$=50\times(3)^4$

$=50\times81=4050$

Statement 1 is true.

Statement 2 is true.

Statement 2 is a correct explanation for the Statement 1.

Hence (a) is the correct option.

SECTION - III

Linked Comprehension Type

15. The relation between A.M., G.M. and HM is

$G^2 = AH$

Since if a and 6 are numbers

Then G = $\sqrt{ab}$

A = $\frac{a+b}{2}$

H = $\frac{2ab}{a+b}$

G2 = ab

AH = $\frac{a+b}{2}\times\frac{2ab}{a+b}$

$\Rightarrow G^2 = AH$

Hence (b) is the correct option.

16. $a, A_1, A_2, A_3..............A_n, b$ are in AP

$b=a+(\text{n}+2-1)\text{d}$

$b=a+(\text{n}+1)\text{d}$

$b-a=(\text{n}+1)\text{d}$

$d=\frac{b-a}{n+1}$

$\therefore$ common difference $d = \dfrac{b-a}{n+1}$

Hence (c) is the correct option,

17. $\dfrac{a_1 + a_2 + ... + a_n}{n} \geq n\sqrt{a_1 a_2 .... a_n}$

$\geq \dfrac{n}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + ... + \dfrac{1}{an}}$

i.e. $A \geq G \geq +1$

Hence (a) is the correct option.

18. $S1 = S_1 = \sum n, S_2 = \sum n^2, S_3 = \sum n^3$

$9S_2^2 = S_3(1+8S_1)$

Hence (b) is the correct option.

19. $1^2 - 2^2 + 3^2 - 4^2 + ... - 1999^2$

$= 1^2 + (3^2 - 2^2) + (5^2 - 4^2) + ... + (1999^2 - 1998^2)$

$= 1 + 2 + 3 + 4 + 5 + ... + 1998 + 1999$

$= 1999\dfrac{(1999+1)}{2} = 19,99,000$

Hence (b) is the correct option.

20. $\left(1 - \dfrac{1}{2^2}\right)\left(1 - \dfrac{1}{3^2}\right)\left(1 - \dfrac{1}{4^2}\right)\left(1 - \dfrac{1}{5^2}\right)$

$...........\left(1 - \dfrac{1}{n^2}\right) = \dfrac{n+1}{2n}$

$\Rightarrow \left(1 - \dfrac{1}{2^2}\right)\left(1 - \dfrac{1}{3^2}\right)\left(1 - \dfrac{1}{4^2}\right)\left(1 - \dfrac{1}{5^2}\right)......$

$\left(1 - \dfrac{1}{2n^2}\right) = \dfrac{2007}{4012}$

Hence (a) is the correct option.

21. $\sum 100 = \frac{100(100+1)}{2}$

$= 50(101) = 5050$

Hence (a) is the correct option.

22. 1, 4, 7,..................

$a = 1,\ d = 3,\ S_n = 715$

$S_n \quad = \frac{n}{2}[2a + (n-1)d]$

$715 \quad = \frac{n}{2}[2 + (n+1)3]$

$1430 \quad = n[3n-1]$

$\Rightarrow 3n^2 - n - 1430 = 0$

$\Rightarrow n = 22$

Hence (b) is the correct option.

23. Multiple of 3 between 1 and 100 are

3, 6, 9, ... 99

$\therefore$ 3 + 6 + 9 + ... + 99

$= 33\left(\frac{3+99}{2}\right)$

$= 33 \times \frac{102}{2}$

$= 1683$

Hence (a) is the correct option.

24. $2, 2\sqrt{2}, 4, ...$

First term a = 2, common ratio = $\sqrt{2}$

n^{th} term $t_n \ = a.r^{n-1}$

$64 \quad = 2.(r\,2)^{n-1}$

$32 = \left(\sqrt{2}\right)^{n-1}$

$2^5 = 2\frac{n-1}{2}$

$\Rightarrow n-1=10 \quad n=11$

Hence (c) is the correct option.

25. First term = a = 50

$t_4 = ar^3 = 1350$

$r^3 = \frac{1350}{50} = 27$

$r = 3$

5^{th} term $t_5 = ar^4 = 50\times(3)^4 = 4050$

Hence (a) is the correct option.

26. 6th term $t_6 = ar^5 = 24$

13^{th} term $t^{13} = ar^{12} = \frac{3}{16}$

$$\Rightarrow \frac{ar^{12}}{ar^5} = \frac{\frac{1}{24}}{\frac{3}{16}} = \frac{\frac{\frac{1}{8}}{24\times16}}{\frac{1}{128}}$$

$r^7 = \frac{1}{7}$

$r = \frac{1}{2}$

$a\left(\frac{1}{32}\right) = 14$

$a = 24\times32$

25^{th} term $= ar^{24}$

$$= 24 \times 32 \times \frac{1}{2^{24}}$$

$$= \frac{3 \times 2^3 + 2^5}{2^{24}}$$

$$= 3 \times 2$$

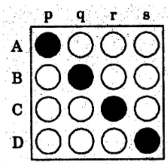
$$= 3 \times 2^{-16}$$

Hence (b) is the correct option.

SECTION - IV

Matrix - Match Type

27.



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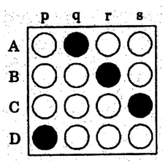

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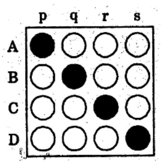

30.