Ratio and Proportion

- In many situations, comparison between quantities is made by using division i.e., by observing how many times one quantity is in relation to the other quantity. This comparison is known as **ratio.** We denote it by using the symbol ':'.

- A ratio may be treated as a fraction. For example, 3:11 can be treated as $\frac{3}{11}$.

- We can compare two quantities in terms of ratio, if these quantities are in the same unit. If they are not, then they should be expressed in the same unit before the ratio is taken.

  For example, if we want to compare 70 paise and Rs 3 in terms of ratio then we have to convert Rs 3 into paise.

  Rs 3 = 300 paise

  Hence, required ratio $\frac{70}{300} = 7:30$

- The same ratio may occur in different situations.

  To understand this concept, let us consider the following situations.

  - Distances of Lata's home and Ravi's home from their school are 12 km and 21 km respectively. Therefore, the ratio of the distance of Lata's home to the distance of Ravi's home from their school is $\frac{12}{21} = \frac{12 \div 3}{21 \div 3} = \frac{4}{7} = 4:7$
  - Neha has Rs 20 and Saroj has Rs 35. Therefore, the ratio of the amount of money that Neha has to that of Saroj is $\frac{20}{35} = \frac{20 \div 5}{35 \div 5} = \frac{4}{7} = 4:7$

  In this way, we can come across many situations where the ratio would be 4:7.

- The order of ratio is important.

  For example, let us consider that the length and breadth of a rectangle are 80 m and 30 m respectively. The ratio of length to the breadth of rectangle is $\frac{80}{30}$. This ratio can be written as 8: 3. However, it cannot be written as 3:8.

  Therefore, the order in which quantities are taken to express their ratio is important.

- We can find equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

  For example, to find the equivalent ratios of 12:20, we proceed as follows:

  $$12:20 = \frac{12}{20} = \frac{12 \div 2}{20 \div 2} = \frac{6}{10} = 6:10$$
  $$12:20 = \frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5} = 3:5$$
  $$12:20 = \frac{12 \times 2}{20 \times 2} = \frac{24}{40} = 24:40$$
  $$12:20 = \frac{12 \times 3}{20 \times 3} = \frac{36}{60} = 36:60$$

  Therefore, 6:10, 3:5, 24:40, 36:60 are the equivalent ratios of 12:20. In this way, we can find many equivalent ratios of 12:20.

- Two ratios are equivalent, if the product of the numerator of the first ratio and the denominator of the other ratio is equal to the product of the denominator of first ratio and the numerator of the other ratio.
- For example, 14:49 and 6:21 are equivalent as:
- $14 \times 21 = 294 = 6 \times 49$
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- A ratio is always expressed in its lowest terms.
- For example, the lowest form of 45:72 is given by,

  $$45:72 = \frac{45}{72}$$
  $$= \frac{45 \div 9}{72 \div 9} \quad \text{(HCF of 45 and 72 is 9)}$$
  $$= \frac{5}{8}$$
  $$= 5:8$$

- We can also compare and arrange the ratios using the concept of equivalent ratios. For this, we make the denominators of the all the ratios equal and then compare the ratios by comparing their numerators.

- The ratio obtained on multiplying two or more ratios is known as **compound ratio**.
- When a ratio is compounded with itself, it is called **duplicate ratio** of the given ratio.
- The **sub-duplicate ratio** of $a:b$ is $\sqrt{a}:\sqrt{b}$.
- A ratio multiplied with itself three times is called **triplicate ratio**.
- The **sub-triplicate ratio** of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$.
- The **reciprocal ratio** of $a:b$ is $b:a$.

- Four quantities are said to be in proportion, if the ratio of first and second quantities is equal to the ratio of third and fourth quantities.

  For example, to check whether 8, 22, 12, and 33 are in proportion or not, we have to find the ratio of 8 to 22 and the ratio of 12 to 33.

  $$8:22=\frac{8}{22}=\frac{4}{11}=4:11$$
  $$12:33=\frac{12}{33}=\frac{4}{11}=4:11$$

  Therefore, 8, 22, 12, and 33 are in proportion as 8:22 and 12:33 are equal.

- When four terms are in proportion, the first and fourth terms are known as extreme terms and the second and third terms are known as middle terms.

  In the above example, 8, 22, 12, and 33 were in proportion. Therefore, 8 and 33 are known as extreme terms while 22 and 12 are known as middle terms.

- If two ratios are equal then we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

  For example, 8:36 and 14:63 are equal as $\frac{8}{36} = \frac{2}{9}$ 836 = 29 and 14:63 = $\frac{14}{63}=\frac{2}{9}$

  Since 8:36 and 14:63 are in proportion, we write it as 8:36 :: 14:63 or 8:36 = 14:63.

- If *a*, *b*, *c*, *d*… are some (non-zero) quantities of the same kind then *a*, *b*, *c*, *d* … are said to be in **continued proportion,** if

  $$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\ldots$$

  For example, 2, 6 and 18 are in continued proportion as $\frac{2}{6}=\frac{6}{18}$.

- Properties related to ratio and proportion.

  (1) If $a:b::c:d$, then $b:a::d:c$

  This property is called **invertendo.**

  (2) If $a:b::c:d$, then $a:c::b:d$

  This property is called **alternendo.**

(3) If $a:b::c:d$, then $(a + b): b:: (c + d):d$

This property is called **componendo.**

(4) If $a:b::c:d$, then $(a - b): b:: (c - d):d$

This property is called **dividendo.**

(5) If $a:b::c:d$, then $(a + b): (a - b) :: (c + d) : (c - d)$

This property is called **componendo and dividendo.**

(6) If $a:b::c:d$, then $a: (a - b)::c: (c - d)$

This property is called **convertendo.**

- **Theorem on equal ratios:**

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ and $p, q, r\ldots$ are non zero numbers such that $pb + qd + rf\ldots \neq 0$ then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = \frac{pa + qc + re + \ldots}{pb + qd + rf + \ldots}$

In particular, we have following formula which is commonly used.

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a + c + e}{b + d + f}$$