Mathematics

for

GUJCET-ME - 2017

Important Instructions :

1. The Mathematics test consists of 40 questions. Each question carries **1** mark. For each correct response, the candidate will get **1** mark. For each incorrect response, ¼ mark will be deducted. The maximum marks are **40**.
2. This Test is of **1 hour** duration.
3. Use **Black Ball Point Pen only** for writing particulars on OMR Answer Sheet and marking answers by darkening the circle '●'.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room / Hall. The candidates are allowed to take away this Test Booklet with them.**
6. Use of White fluid for correction is not permissible on the Answer Sheet.
7. Each candidate must show on demand his / her Admission Card to the Invigilator.
8. No candidate, without special permission of the Superintendent or Invigilator, should leave his / her seat.
9. Use of Manual Calculator is permissible.
10. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
11. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
12. The candidates will write the Correct Test Booklet Set No. as given in the Test Booklet / Answer Sheet in the Attendance Sheet. (Patrak - 01)

MATHEMATICS

1. If the length of the subnormal of a curve is constant and if it passes through the origin, then the equation of curve is ______.

(A) $x^2 + y^2 = k^2;\ k \in R$ (B) $y^2 = kx;\ k \in R$

(C) $x^2 = ky^2;\ k \in R$ (D) $x^2 - y^2 = k^2;\ k \in R$

Answer (B)

Sol. $y\frac{dy}{dx} = k_1$

$\Rightarrow \int y\,dy = \int k_1 dx$

$\Rightarrow \frac{y^2}{2} = k_1 x + c$

at x = 0, y = 0

$\therefore y^2 = 2k_1 x$

$y^2 = kx,\ k \in R$

2. The integrating factor of the differential equation $\frac{dy}{dx} = \frac{1}{x+y+2}$ is

(A) e^{x+y+2} (B) e^y

(C) e^{-y} (D) $\log|x+y+2|$

Answer (C)

Sol. $\frac{dx}{dy} = x + y + 2$

$\frac{dx}{dy} - x = y + 2$

$I.F = e^{\int -dy} = e^{-y}$

3. If $\bar{a} + \bar{b} + \bar{c} = \bar{0}$ and $|\bar{a}| = 3, |\bar{b}| = 5, |\bar{c}| = 7$ and $(\hat{\bar{a}, \bar{b}}) = \alpha$, then α = ______.

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

Answer (C)

Sol. $|a+b|^2 = |c|^2$

$|\vec{a}| + |\vec{b}| + 2\vec{a}\cdot\vec{b} = |c|^2$

$\cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

4. For $A(1, -2, 4)$, $B(5, -1, 7)$, $C(3, 6, -2)$, $D(4, 5, -1)$, the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is ______.

(A) $(2\sqrt{3}, -2\sqrt{3}, 2\sqrt{3})$ (B) $\frac{3}{13}(4, 1, 3)$

(C) $(1, -1, 1)$ (D) $(2, -2, 2)$

Answer (D)

Sol. $\overrightarrow{AB} = (4, 1, 3)$

$\overrightarrow{CD} = (1, -1, 1)$

Now projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ =

$\left(\frac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{CD}|}\right)\cdot\widehat{CD}$

$= \left(\frac{4-1+3}{\sqrt{3}}\right)\cdot\frac{(1,-1,1)}{\sqrt{3}}$

= (2, –2, 2)

5. The position vector of point A is (4, 2, –3). If p_1 is perpendicular distance of A from XY-plane and p_2 is perpendicular distance from Y-axis, then $p_1 + p_2$ = ______.

(A) 8

(B) 3

(C) 2

(D) 7

Answer (A)

Sol. $p_1 = |z|$

$p_2 = \sqrt{x_1^2 + y_1^2}$

$p_1 + p_2 = 5 + 3 = 8$

6. Plane $ax + by + cz = 1$ intersect axes in A, B, C respectively. If $G\left(\frac{1}{6}, -\frac{1}{3}, 1\right)$ is a centroid of ΔABC, then $a + b + 3c$ = ________.

(A) $\frac{4}{3}$ (B) 4

(C) 2 (D) $\frac{5}{6}$

Answer (C)

Sol. $A\left(\frac{1}{a},0,0\right)$ $B\left(0,\frac{1}{b},0\right)$ $C\left(0,0,\frac{1}{c}\right)$

centroid $\Rightarrow \left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right) = \left(\frac{1}{6},\frac{-1}{3},1\right)$

$\therefore$ a = 2, b = –1, c = 1/3

$\therefore$ a + b + 3c = 2

Only (C) is correct

7. The direction angles of the line $x = 4z + 3$, $y = 2 - 3z$ are α, β and γ, then $\cos\alpha + \cos\beta + \cos\gamma$ = _____.

(A) $\frac{2}{\sqrt{26}}$ (B) $\frac{8}{\sqrt{26}}$

(C) 1 (D) 2

Answer (A)

Sol. $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z}{1}$

$\cos\alpha = \frac{4}{\sqrt{26}}$ $\cos\beta = \frac{-3}{\sqrt{26}}$ $\cos\gamma = \frac{1}{\sqrt{26}}$

$\therefore \cos\alpha + \cos\beta + \cos\gamma = \frac{2}{\sqrt{26}}$

8. If the normal of the plane makes an angles $\frac{\pi}{4}, \frac{\pi}{4}$ and $\frac{\pi}{2}$ with positive X-axis, Y-axis and Z-axis respectively and the length of the perpendicular line segment from origin to the plane is $\sqrt{2}$, then the equation of the plane is _____.

(A) $x + y + z = \sqrt{2}$ (B) $x + y + z = 1$

(C) $x + y = 2$ (D) $x = \sqrt{2}$

Answer (C)

Sol. Dirction ratio of normal to the plane is

$\left(\cos\frac{\pi}{4},\cos\frac{\pi}{4},\cos\frac{\pi}{2}\right)$

$\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$

$\therefore \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$

$x + y = 2$

9. If $f : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{3}{5}\right\}; f(x) = \frac{3x+1}{5x-3}$, then _____.

(A) $f^{-1}(x) = 2f(x)$ (B) $f^{-1}(x) = f(x)$

(C) $f^{-1}(x) = -f(x)$ (D) $f^{-1}(x)$ doesnot exists

Answer (B)

Sol. $f(x) = \frac{3x+1}{5x-3}$

we know that if

$f(x) = \frac{ax+b}{cx+d}$

if a + d = 0

$f(x) = f^{-1}(x)$

10. $f : R \to R, f(x) = 3x + 2$

$g : R \to R, g(x) = 6x + 5$

for the given functions $(gof^{-1})(10)$ = _____.

(A) 21 (B) 29

(C) 7 (D) $\frac{8}{3}$

Answer (A)

Sol. $g\left(f^{-1}(x)\right)$

$= g\left(\frac{x-2}{3}\right)$

$= 6\left(\frac{x-2}{3}\right) + 5$

$= 2x + 1$, at x = 10

= 21

11. Relation $S = \{(1, 2), (2, 1), (2, 3)\}$ is defined on the set {1, 2, 3} is ________.

(A) not transitive (B) symmetric

(C) reflexive (D) equivalence

Answer (A)

Sol. As (1,1) is not there,

$\therefore$ it is not transitive

12. $\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$ = ____. $\left(\text{where, } 0 < x < \frac{\pi}{2}\right)$

(A) $\frac{\pi}{2}$ (B) $2x$

(C) $\pi - 2x$ (D) $\pi - x$

Answer (C)

Sol. $\frac{\pi}{2}-\cot^{-1}(\cot x)+\frac{\pi}{2}-\tan^{-1}(\tan x)$

$= \pi - 2x$

13. $\cos\left(2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}5\right)\right) =$ ______.

(A) $\frac{1}{\sqrt{2}}$ (B) 0

(C) 1 (D) –1

Answer (D)

Sol. $\tan^{-1}\frac{1}{5}+\tan^{-1}5=\frac{\pi}{2}$

$\therefore \cos 2\times\frac{\pi}{2}=\cos\pi=-1$

14. For ΔABC if $A = \tan^{-1} 2$, $B = \tan^{-1} 3$, then $C =$ _____.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

Answer (B)

Sol. $A+B+C=\pi$

$A+B=\frac{3\pi}{4}$

$\therefore C=\frac{\pi}{4}$

15. If the area of the triangle with vertices (2, 5), (7, *k*) and (3, 1) is 10, then find the value of *k*.

(A) –5 or 35

(B) 5 or –35

(C) 15 or –5

(D) –5 or –25

Answer (B)

Sol. $10=\frac{1}{2}\begin{vmatrix}2 & 5\\7 & k\\3 & 1\\2 & 5\end{vmatrix}$

k = 5 or – 35

16. If $k = p + q + r$, then the value of $\begin{vmatrix}k+r & p & q\\ r & k+p & q\\ r & p & k+q\end{vmatrix}$ is ______.

(A) $2k^2$ (B) $2k^3$

(C) k^3 (D) $3k^2$

Answer (B)

Sol. $C_1 \to C_1 + C_2 + C_3$

and taking common

k + p + q + r

$=2k\begin{vmatrix}1 & p & q\\1 & k+p & q\\1 & p & k+q\end{vmatrix}$

$= R_3 \to R_3 - R_1$

$= R_2 \to R_2 - R_1$

$=2k\begin{vmatrix}1 & p & q\\0 & k & 0\\0 & 0 & k\end{vmatrix}$

$= 2\ k^3$

17. If maximum and minimum values of $D=\begin{vmatrix}1 & -\cos\theta & -1\\ \cos\theta & 1 & -\cos\theta\\ 1 & \cos\theta & 1\end{vmatrix}$ are *p* and *q* respectively, then the value of $2p + 3q$ is _____.

(A) 16 (B) 6

(C) 14 (D) 8

Answer (C)

Sol. $D=1\left(1+\cos^2\theta\right)+\cos\theta\left(\cos\theta+\cos\theta\right)-1\left(\cos^2\theta-1\right)$

$D=1+\cos^2\theta+2\cos^2\theta-\cos^2\theta+1$

$= 2\left(1+\cos^2\theta\right)$

$\therefore$ p = 4, q = 2

$\therefore$ 2p + 3q = 14

18. If $A=\begin{bmatrix}1 & 4 & 4\\4 & 1 & 4\\4 & 4 & 1\end{bmatrix}$, then $A^2 - 6A =$ _____.

(A) $27\ I_3$ (B) $5\ I_3$

(C) $20\ I_3$ (D) $30\ I_3$

Answer (A)

Sol. $\begin{bmatrix}1 & 4 & 4\\4 & 1 & 4\\4 & 4 & 1\end{bmatrix}\Rightarrow A^2-6A=27\ I_3$

19. If $[2\ 3\ 4]\begin{bmatrix}1 & x & 3\\ 2 & 4 & 5\\ 3 & 2 & x\end{bmatrix}\begin{bmatrix}x\\ 2\\ 0\end{bmatrix}=0$, then x ______.

(A) $\frac{7}{3}$ (B) $\frac{5}{3}$

(C) $-\frac{5}{3}$ (D) $-\frac{7}{3}$

Answer (C)

Sol. $[20 \quad 20+2x \quad 21+4x]\begin{bmatrix}x\\ 2\\ 0\end{bmatrix}=0$

$\Rightarrow$ $24x + 40 = 0$

$\Rightarrow$ $x=-\frac{5}{3}$

20. $\frac{d}{dx}\left(\sqrt{3}\sin\left(2x+\frac{\pi}{3}\right)+\cos\left(2x+\frac{\pi}{3}\right)\right)=$ ______.

(A) 4 cos2x (B) –4 sin2x

(C) 4 sin2x (D) –4 cos2x

Answer (B)

Sol. $\frac{d}{dx}\left(2\sin\left(2x+\frac{\pi}{3}+\frac{\pi}{6}\right)\right)$

$\frac{d}{dx}\left(2\sin\left(2x+\frac{\pi}{2}\right)\right)$

$\frac{d}{dx}(2\cos 2x)$

$\Rightarrow$ –4sin2x

21. For the curve $f(x) = (x - 5)^2$, applying mean value theorem on [4, 6] the tangent at _____ is parallel to the chord joining A(4, 1), B(6, 1).

(A) (4, 6) (B) $\left(\frac{9}{2}, \frac{1}{4}\right)$

(C) (0, 5) (D) (5, 0)

Answer (D)

Sol. $f'(c)=2(c-5)=\frac{f(6)-f(4)}{6-4}=0$

(using LMVT)

$\therefore$ $c = 5$

$\therefore$ (5, 0)

22. Function $f(x)=\begin{cases}(\log_2 2x)^{\log_x 8}; & x\neq 1\\ (k-1)^3; & x=1\end{cases}$ is continuous at x = 1, then k = _____

(A) $e + 1$ (B) $e^{1/3}$

(C) e^3 (D) $e - 1$

Answer (A)

Sol. $f(1^-)=f(1^+)=f(1)$

$\lim_{x\to 1} e^{(\log_2 2x-1)\log_x 8}$

$\lim_{x\to 1} e^{(\log_2 2x-1)\frac{\log_2 8}{\log_2 x}}$

$e^3=(k-1)^3$

$k=e+1$

23. $\int\frac{dx}{\cos x\sqrt{1+\cos 2x+\sin 2x}}=$ ______ $+C;\left(0<x<\frac{\pi}{4}\right)$

(A) $2+\sqrt{\cot x}$ (B) $\sqrt{\tan x+1}$

(C) $\sqrt{2+2\tan x}$ (D) $\sqrt{2+2\cot x}$

Answer (C)

Sol. $\int\frac{\sec^2 x dx}{\sqrt{2+2\tan x}}$

tan$x$ = t

$\sec^2 x dx = dt$

$\therefore$ $\sqrt{2+2\tan x}$ + C

24. If $\int\frac{\sin 2x}{\sin 5x\sin 3x}dx=\frac{1}{3}\log|\sin 3x|-\frac{1}{5}$ log|$f(x)$| + C, then $f(x)$ = ____.

(A) sin 5x (B) sin 4x

(C) sin 2x (D) sin 6x

Answer (A)

Sol. $\int\frac{\sin(5x-3x)}{\sin 5x\sin 3x}dx$

$\int\frac{\sin 5x\cos 3x-\cos 5x\sin 3x}{\sin 5x\sin 3x}dx$

$\int(\cot 3x-\cot 5x)dx$

$\therefore\frac{1}{3}\log|\sin 3x|-\frac{1}{5}\log|\sin 5x|+C$

$\Sigma P(x_i)=1$

25. The probability distribution of a discrete random variable X is given in the following table :

$X = x$	0	1	2
$P(x)$	$4C^3$	$4C - 13C^2$	$7C - 1$

; C > 0 then C = ___.

(A) 2

(B) 1

(C) $\frac{1}{4}$

(D) 1 and $-\frac{1}{4}$

Answer (C)

Sol. $4C^3 + 4C - 13C^2 + 7C - 1 = 1$

$\therefore C = \frac{1}{4}$

26. The probability function of a binomial distribution is $P(x) = \binom{6}{x} p^x q^{6-x}$, x = 0, 1, 2, ..., 6. If 2P(2) = 3P(3), then p = ______.

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{5}$

Answer (A)

Sol. $2P(2) = 3P(3)$

$2 \times {}^6C_2 p^2 q^4 = 3 \times {}^6C_3 p^3 q^3$

On solving $p = \frac{1}{3}$

(as $p + q = 1$)

27. The mean and standard deviation of random variable X are 10 and 5 respectively, then $E\left(\frac{X - 15}{5}\right)^2$ = ______.

(A) 4 (B) 3

(C) 2 (D) 5

Answer (C)

Sol. $v = 5^2 = 25$

$\therefore E(x)^2 = 25 + 10^2 = 125$

$\therefore E\left(\frac{x^2 + 225 - 30x}{25}\right) = \frac{1}{25}\left(E(x)^2 + 225 - 30E(x)\right)$

= 2

28. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (25, 20) and (0, 30). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (25, 20) and (0, 30) is _______.

(A) $5p = 2q$ (B) $2p = 5q$

(C) $p = 2q$ (D) $q = 3p$

Answer (A)

Sol. $25p + 20q = 30q$

$5p = 2q$

29. Two curves $a_i x^2 + b_i y^2 = 1$; i = 1, 2 where, $a_1 \neq a_2$, $b_1 \neq b_2$, $a_1, a_2, b_1, b_2 \neq 0$ may intersect orthogonally if ______.

(A) $a_1 a_2 = b_1 b_2$

(B) $a_1^{-1} - a_2^{-1} = b_1^{-1} - b_2^{-1}$

(C) $a_1^{-1} + a_2^{-1} = b_1^{-1} + b_2^{-1}$

(D) $a_1 b_2 = a_2 b_1$

Answer (B)

Sol. If $a_1 x^2 + b_1 y^2 = 1$ and $a_2 x^2 + b_2 y^2 = 1$ are orthogonal to each other

$$\frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{b_1} - \frac{1}{b_2}$$

30. The sides of two squares are x and y respectively, such that $y = x + x^2$. The rate of change of area of second square with respect to area of first square is ______.

(A) $x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$

(C) $2x^2 + 3x + 1$ (D) $1 + 2x$

Answer (C)

Sol. $\frac{dA_2}{dA_1} = \frac{2y\frac{dy}{dt}}{2x\frac{dx}{dt}}$

$\frac{y}{x}\frac{dy}{dx}$

$\therefore \frac{y}{x} \times (1 + 2x)$

$2x^2 + 3x + 1$

31. The approximate value of $\sqrt[3]{-0.99}$ is ______.

(A) –0.9967 (B) –0.9976

(C) –1.0033 (D) –1

Answer (A)

Sol. – .9967

32. $\int \frac{dx}{\sqrt{x^{10}-x^2}}; x>1 =$ ______ + C.

(A) $\frac{1}{4}\log\left|\sqrt{x^{10}-x^2}+x^2\right|$ (B) $\frac{1}{2}\log\left|x^{10}-x^2\right|$

(C) $-\frac{1}{4}\sec^{-1}(x^4)$ (D) $\frac{1}{4}\sec^{-1}(x^4)$

Answer (D)

Sol. $\int \frac{4x^3dx}{4x^3 \times x\sqrt{x^8-1}}$

$\int \frac{4x^3dx}{4x^4\sqrt{x^8-1}}$

$\begin{pmatrix} x^4 = t \\ 4x^3dx = dt \end{pmatrix}$

$\int \frac{dt}{4t\sqrt{t^2-1}} = \frac{1}{4}\sec^{-1}(x^4)+C$

33. $\int e^{\sin x}(x\cos x - \sec x \tan x)dx =$ ___________ + C; $0 < x < \frac{\pi}{2}$.

(A) $e^{\sin x}(x - \sec x)$ (B) $e^{\sin x}(\sec x - x)$

(C) $e^{\sin x}x\cos x$ (D) $e^{\sin x}(x + \sec x)$

Answer (A)

Sol.

$\int xe^{\sin x}\cos x - \int e^{\sin x}dx - \left[\left(\sec x \cdot e^{\sin x}\right) - \int e^{\sin x}dx\right]$

$e^{\sin x}\left(x - \sec x\right) + C$

34. $\int \sin(11x)\cdot\sin^9 x\,dx =$ _____ + C.

(A) $\frac{\sin(10x)\cdot\sin^{10}x}{10}$ (B) $\frac{\sin^{11}x}{11}$

(C) $\frac{\sin(9x)\cdot\sin^9 x}{9}$ (D) $\frac{\cos(10x)\cdot\cos^{10}x}{10}$

Answer (A)

Sol. $\int \sin(10x+x)\sin^9 x\,dx$

$\int \sin 10x\cos x\sin^9 x\,dx + \int \cos 10x \sin^{10} x\,dx$

$\frac{\sin^{10}x}{10}\sin 10x - \frac{10}{10}\int \sin^{10}x\cos 10x\,dx + \int \cos(10x)\sin^{10}x\,dx$

$\frac{\sin^{10}x}{10}\sin(10x) + c$

35. $\int_{-\log 3}^{\log 3} \cot^{-1}\left(\frac{e^x-1}{e^x+1}\right)dx =$ _____

(A) $\frac{\pi}{2}\log 3$ (B) $\pi\log 3$

(C) 0 (D) $\pi\log 9$

Answer (B)

Sol. $I = \int_{-\log 3}^{\log 3} \cot^{-1}\left(\frac{e^x-1}{e^x+1}\right)dx = \int_{-\log 3}^{\log 3} \cot^{-1}\left(\frac{1-e^x}{1+e^x}\right)dx$

adding

$2I = \pi\int_{-\log 3}^{\log 3} dx$

$I = \pi\log 3$

36. $\int_0^{100\pi} |\cos x|\,dx =$ ______.

(A) 200 (B) 100

(C) 50 (D) 0

Answer (A)

Sol. $200\int_0^{\pi/2}\cos x\,dx = 200$

37. $\int_0^{\pi/2}(x - [\sin x])\,dx =$ ______.

(where $[x]$ = greatest integer not greater than x).

(A) $\frac{\pi^2}{8}-2$ (B) $\frac{\pi^2}{4}-1$

(C) $\frac{\pi^2}{8}-1$ (D) $\frac{\pi^2}{8}$

Answer (D)

Sol. $\int_0^{\pi/2} x\,dx$

$\frac{\pi^2}{8}$

38. Area bounded between two latus-rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $a > b$ is ______.

(where, e is eccentricity of the ellipse)

(A) $2b(be + a\sin^{-1} e)$ (B) $8b(be + a\sin^{-1} e)$

(C) $b(be + a\sin^{-1} e)$ (D) $4b(be + a\sin^{-1} e)$

Answer (A)

Sol. $2b\left(be + a\sin^{-1} e\right)$

39. The area of the region bounded by the curves $f(x) = \sin\pi x$ and X-axis is ______ where $x \in [-1, 2]$.

(A) 8π

(B) $\frac{8}{\pi}$

(C) $\frac{6}{\pi}$

(D) 6π

Answer (C)

Sol. $3\int_0^1 \sin(\pi x)\,dx$

$\frac{6}{\pi}$

40. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\frac{dy}{dx} = \sqrt{x}$; $x > 0$ are ______ respectively.

(A) 2 and 6

(B) 3 and 2

(C) 2 and 3

(D) 2 and degree is undefined

Answer (C)

Sol. order $\Rightarrow$ 2

degree $\Rightarrow$ 3

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