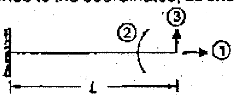
# Matrix Method of Structural Analysis

**Q.1** The flexibility method in structural analysis starts with
(a) compatible deformations.
(b) equilibrium of forces.
(c) force-deformation relation.
(d) kinematically-admissible deformations.

**Q.2** The stiffness method of structural analysis always starts with
(a) force-deformation relation.
(b) equilibrium conditions.
(c) compatible deformations.
(d) none of these.

**Q.3** The cross-stiffness coefficients in elastic structural analysis are
(a) always symmetrical.
(b) symmetrical only in prismatic members.
(c) symmetrical in symmetrical members only.
(d) have no relation at all.

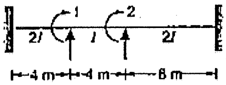
**Q.4** The stiffness matrix for the cantilever beam with reference to the coordinates, as shown below is




(a) $\begin{bmatrix} \frac{AE}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{AE}{L} & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & 0 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{AE}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^3} & \frac{12EI}{L^3} \end{bmatrix}$

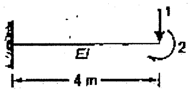
**Q.5** Displacement coordinates for a beam are shown in the given figure. The stiffness matrix is given by




(a) $\begin{bmatrix} 3EI & EI \\ EI & 2EI \end{bmatrix}$ (b) $\begin{bmatrix} 3EI & -0.5EI \\ -0.5EI & 2EI \end{bmatrix}$

(c) $\begin{bmatrix} 3EI & 0 \\ 0 & 2EI \end{bmatrix}$ (d) $\begin{bmatrix} 3EI & 0.5EI \\ 0.5EI & 2EI \end{bmatrix}$

**Q.6** The flexibility matrix of the beam shown in the given figure is

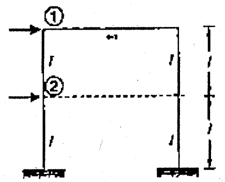



(a) $\begin{bmatrix} \frac{64}{3EI} & -\frac{8}{EI} \\ -\frac{8}{EI} & \frac{64}{3EI} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & -\frac{64}{3EI} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & \frac{4}{EI} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{4}{EI} & \frac{8}{EI} \end{bmatrix}$

**Q.7** The number of unknowns to be determined in the stiffness method is equal to
(a) the static indeterminacy
(b) the kinematic indeterminacy
(c) the sum of kinematic indeterminacy and static indeterminacy
(d) two times the number of supports

**Q.8** The beams in the two storey frame shown in the figure below have a cross section such that the flexural rigidity may be considered infinite. Which among the following is the stiffness matrix for the structure in respect of the global coordinates 1 and 2?

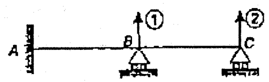



(a) $\frac{24EI}{L^3}\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}$ (b) $\frac{24EI}{L^3}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$

(c) $\frac{24EI}{L^3}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$ (d) $\frac{24EI}{L^3}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$

**Q.9** Flexibility matrix of the beam shown below is

$$\delta = \frac{1}{3EI}\begin{bmatrix}1 & 2\\2 & 8\end{bmatrix}$$

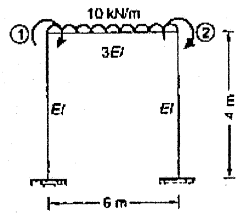



If support $B$ settles by $\frac{\Delta}{EI}$ units, what is the reaction at $B$?

(a) 0.75 Δ (b) 3.0 Δ

(c) 6.0 Δ (d) 24.0 Δ

**Q.10** Considering only flexural deformations, which is the stiffness matrix for the plane frame shown in the figure given below?




(a) $\begin{bmatrix}4 & 3\\3 & 4\end{bmatrix}EI$ (b) $\begin{bmatrix}3 & 1\\1 & 3\end{bmatrix}EI$

(c) $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}EI$ (d) $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}EI$

**Q.11** The stiffness matrix of a beam element is

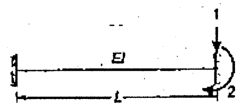
$$\left(\frac{2EI}{L}\right)\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

Which one of the following is its flexibility matrix?

(a) $\left(\frac{L}{2EI}\right)\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ (b) $\left(\frac{L}{6EI}\right)\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$

(c) $\left(\frac{L}{5EI}\right)\begin{bmatrix}1 & -2\\-2 & 1\end{bmatrix}$ (d) $\left(\frac{L}{6EI}\right)\begin{bmatrix}1 & -2\\-2 & 1\end{bmatrix}$

**Q.12** A cantilever beam is shown in the figure below with its degrees of freedom. The coefficient of stiffness $K_{11}$ is given by




(a) $\frac{L^3}{3EI}$ (b) $\frac{-L^3}{3EI}$

(c) $\frac{-12EI}{L^3}$ (d) $\frac{12EI}{L^3}$

**Q.13** In a linear elastic structural element

(a) stiffness is directly proportional to flexibility.

(b) stiffness is inversely proportional to flexibility.

(c) stiffness is equal to flexibility.

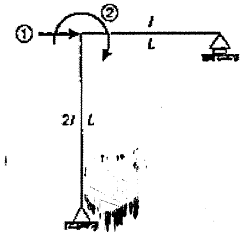
(d) stiffness and flexibility are not related.

**Q.14** In a non-prismatic beam element $AB$, the cofactors $C_{AB}$ and $C_{BA}$ are 0.6 and 0.5 respectively. If the stiffness $k_A = 0.4\ EI$ then the stiffness $k_B$ will be

(a) 0.24 $EI$ (b) 0.32 $EI$

(c) 0.38 $EI$ (d) 0.48 $EI$

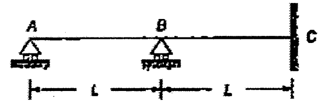
**Q.15** What is the stiffness matrix for the structure shown with loadings symbolised as (1) and (2)?




(a) $\frac{EI}{L^2}\begin{bmatrix}\frac{6}{L} & -6\\-6 & 9L\end{bmatrix}$ (b) $\frac{EI}{L}\begin{bmatrix}\frac{24}{L} & 6\\6 & 9L\end{bmatrix}$

(c) $\frac{EI}{L}\begin{bmatrix}24 & \frac{-12}{L}\\\frac{-12}{L} & 9L\end{bmatrix}$ (d) $\frac{EI}{L^2}\begin{bmatrix}24 & 12\\12 & \frac{11}{L}\end{bmatrix}$

**Q.16** Find the flexibility matrix of the continuous beam shown below by considering that support $B$ sinks downward and support $C$ rotates in anticlockwise direction.




(a) $\frac{L}{12EI}\begin{bmatrix}2L^2 & L\\L & 8\end{bmatrix}$

(b) $\frac{L}{6EI}\begin{bmatrix}2L^2 & 3L\\3L & 8\end{bmatrix}$

(c) $\frac{L}{6EI}\begin{bmatrix}2L^2 & L\\L & 8\end{bmatrix}$

(d) $\frac{L}{12EI}\begin{bmatrix}2L^2 & 3L\\3L & 8\end{bmatrix}$

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## Answers Matrix Method of Structural Analysis

1. (b) 2. (c) 3. (a) 4. (b) 5. (d) 6. (c) 7. (b) 8. (a) 9. (c) 10. (b)

11. (b) 12. (d) 13. (b) 14. (d) 15. (a) 16. (d)

## Explanations Matrix Method of Structural Analysis

**1.** (b)

In flexibility method, unknowns are forces, so it always starts with equilibrium of forces. Hence flexibility method is also called force method. While in stiffness method displacements are taken as unknown quantities, hence called displacement method.

**3.** (a)

According to Maxwell's law or reciprocal theorem, coefficient of stiffness matrix are always symmetrical.

**4.** (b)

To generate Ist column of stiffness matrix, apply unit displacement in direction (1) only without any displacement in direction (2) and (3).

$k_{11}$ = force required in direction 1 to cause unit displacement in direction 1.

$\therefore \quad k_{11} = \frac{AE}{L}$

Similarly $\quad k_{21} = 0$

$k_{31} = 0$

**To generate IInd column, apply unit displacement in direction (2) only without any displacement in direction (1) and (3)**

$\therefore \quad k_{12} = 0$

$k_{22} = \frac{4EI}{L}$

$$k_{32} = \frac{6EI}{L^2}$$

**To generate III$^{rd}$ column, apply unit displacement in direction (3) without any displacement in direction (1) and (2)**

$\therefore$ $k_{13} = 0$

$$k_{23} = \frac{6EI}{L^2}$$

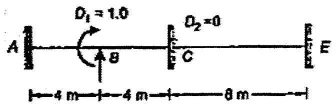
$$k_{33} = \frac{12EI}{L^3}$$

So stiffness matrix

$$k = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$$

5. (d)

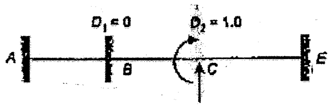
Stiffness matrix can be obtained by making second co-ordinate i.e. rotation in the direction of 2 as zero and considering unit rotation in the direction of co-ordinate 1.




The beam $AB$ has rotational stiffness $\frac{4E(2I)}{4} = 2EI$ and beam $BC$ has rotational stiffness $\frac{4E(I)}{4} = EI$. So moment required in the direction of 1 to produce unit rotation will be $2EI + EI = 3EI$. Thus $K_{11} = 3EI$.

The moment generated at point $C$ in the direction of co-ordinate 2 is $\frac{1}{2} \times EI = 0.5EI$ as the carry over factor is half. So $K_{21} = 0.5EI$.

Similarly make $D_1 = 0$ and $D_2 = 1.0$




$$K_{22} = \frac{4EI}{4} + \frac{4E(2I)}{8} = 2EI$$

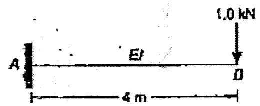
$$K_{12} = 0.5EI$$

$$\text{Stiffness matrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 3EI & 0.5EI \\ 0.5EI & 2EI \end{bmatrix}$$

6. (c)

Elements of flexibility matrix can be obtained by applying unit force in the direction of any one co-ordinate and calculating displacement in the direction of both co-ordinates.




Applying unit load in the direction of 1.

Deflection at $B$,

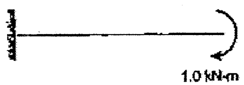
$$f_{11} = \frac{1 \times 4^3}{3EI} = \frac{64}{3EI}$$

Rotation at $B$, $f_{21} = \frac{1 \times 4^2}{2EI} = \frac{8}{EI}$

The positive sign is used when the displacement is in the direction of co-ordinate, otherwise negative sign is used.

Applying unit moment in the direction of 2, deflection at $B$

$$f_{12} = \frac{1 \times 4^2}{2EI} = \frac{8}{EI}$$




Rotation at $B$, $f_{22} = \frac{1 \times 4}{EI} = \frac{4}{EI}$

So flexibility matrix

$$= \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & \frac{4}{EI} \end{bmatrix}$$

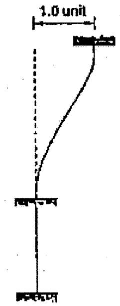
7. (b)

The stiffness or displacement method of analysis is based on kinematic degree of indeterminacy or independent degrees of freedom at joint.

8. (a)

For first column of matrix $D_1 = 1.0$; $D_2 = 0$

The shape of deformed columns is shown below.

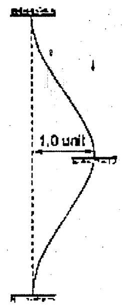



$$\therefore \quad s_{11} = \frac{12EI}{L^3} + \frac{12EI}{L^3} = \frac{24EI}{L^3}$$

$$s_{21} = -\frac{24EI}{L^3}$$

For second column of matrix

$D_1 = 0$; $D_2 = 1.0$




The shape of the deformed columns is shown.

$$\therefore \quad s_{12} = -\frac{24EI}{L^3}$$

$$s_{22} = \frac{24EI}{L^3} + \frac{24EI}{L^3} = \frac{48EI}{L^3}$$

9. (c)

We know that

$$D_Q = D_{QS} + FQ$$

where $D_Q$ is the final displacement matrix corresponding to the redundants in the actual structure.

$$D_{QS} = D_{QL} + D_{QT} + D_{QP} + D_{QR}$$

$D_{QS}$ includes the effects of external loads ($D_{QL}$), temperature ($D_{QT}$), pre-strain effects ($D_{QP}$), support settlements, etc.

$F$ is the Flexibility matrix and $Q$ is the unknown reactions matrix.

Assuming upward displacements as positive.

$$D_{QL} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \; D_{QP} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D_{QT} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \; D_{QR} = \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix}$$

$$\therefore \quad D_{QS} = \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix}$$

and $$Q = \begin{bmatrix} V_B \\ V_C \end{bmatrix}$$

Final displacement, $D_Q = 0$

$\therefore$ $D_{QS} + FQ = 0$

$\Rightarrow$ $FQ = -D_{QS}$

$\Rightarrow$ $Q = -F^{-1} D_{QS}$

$$\Rightarrow \quad \begin{bmatrix} V_B \\ V_C \end{bmatrix} = -\frac{3EI}{4} \begin{bmatrix} 8 & -2 \\ -2 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} -\frac{\Delta}{EI} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_B \\ V_C \end{bmatrix} = -\frac{3EI}{4} \times \left(-\frac{\Delta}{EI}\right) \begin{bmatrix} 8 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \frac{3\Delta}{4} \begin{bmatrix} 8 + (-2) \times 0 \\ (-2) \times 1 + 1 \times 0 \end{bmatrix}$$

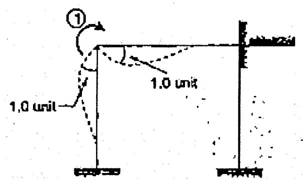
$$\Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \frac{3\Delta}{4}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$\therefore \quad V_B = \frac{3\Delta}{4}\times 8 = 6\,\Delta$$

and $$V_C = \frac{3\Delta}{4}\times(-2) = -1.5\,\Delta$$

10. (b)

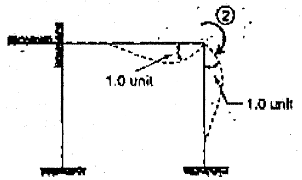
Keeping $D_1 = 1.0$ and $D_2 = 0$, the frame will be




$$s_{11} = \frac{4\times(3EI)}{6}+\frac{4EI}{4} = 3EI$$

$$s_{21} = \frac{2\times(3EI)}{6} = EI$$

Keeping $D_1 = 0$ and $D_2 = 1.0$, the frame will be




$$s_{12} = \frac{2\times 3EI}{6} = EI$$

and $$s_{22} = 3EI$$

11. (b)

The flexibility and stiffness matrices are inverse of each other and both are symmetrical.

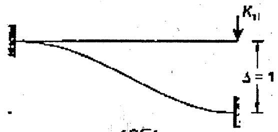
The value of determinant of stiffness matrix is

$$\frac{2EI}{L}\times 3 = \frac{6EI}{L}$$

The flexibility matrix will be $= \dfrac{L}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

12. (d)

$K_{11}$ = Force required along degree of freedom '1' to produce unit displacement in the direction of degree of freedom '1' and degree of freedom '2' must be locked.




$$\therefore \quad K_{11} = \frac{12EI}{L^3}$$

14. (d)

Relationship between cofactor and stiffness factor is

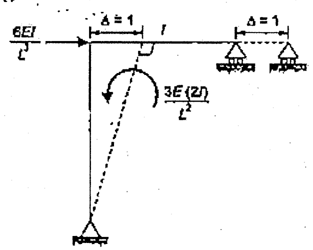
$$C_{AB}\,k_A = C_{BA}\,k_B$$

$$\Rightarrow \quad 0.6\times 0.4\,EI = 0.5\times k_B$$

$$\therefore \quad k_B = 0.48\,EI$$

15. (a)

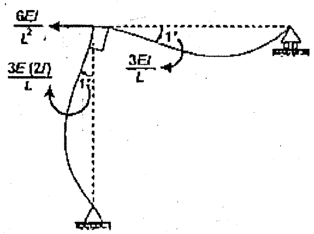
(i) Provide unit deflection in the direction of (1).




$$k_{11} = \frac{6EI}{L^3}$$

$$k_{12} = -\frac{6EI}{L^2}$$

(ii) Provide unit rotation in the direction of (2).




$$k_{12} = -\frac{6EI}{L^2}$$

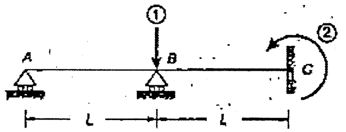
$$k_{22} = \frac{3E(2I)}{L}+\frac{3EI}{L} = \frac{9EI}{L}$$

∴ Stiffness matrix is,

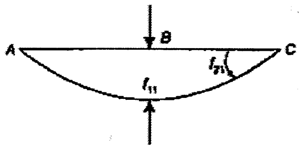
$$k = \begin{bmatrix} \frac{6EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{9EI}{L} \end{bmatrix}$$

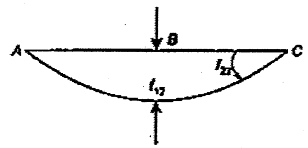
$$= \frac{EI}{L^2}\begin{bmatrix} \frac{6}{L} & -6 \\ -6 & 9L \end{bmatrix}$$

16. (d)




The elements of the flexibility matrix are obtained by applying unit values of redundants at the coordinates one after the other as shown below




$$f_{11} = \frac{(2L)^3}{48EI} = \frac{L^3}{6EI}$$

$$f_{21} = f_{12} = \frac{L^2}{4EI}$$

$$f_{22} = \frac{2L}{3EI}$$

$$[F] = \frac{L}{12EI}\begin{bmatrix} 2L^2 & 3L \\ 3L & 8 \end{bmatrix}$$

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