

Substitution

IMPORTANT POINTS

1. **Substitution :** The value of an expression depends on the value of its variable (s).
2. **Use of Brackets :**

The Symbols —, (), { }, [] are called brackets.

If an expression is enclosed within a bracket, it is considered a single quantity, even if it is made up of many terms.

Keep in Mind :

- While simplifying an expression containing a bracket, first of all, the terms inside the bracket are operated (combined).
- () is called a small bracket or Parenthesis.
- { } is called a middle bracket or Curly bracket.
- [] is called big or square bracket.
- If one more bracket is needed, then we use the bar bracket.
i.e. a line — is drawn over a group of terms.
Thus, in $3x + 4y - 5z$, the line over $4y - 5z$ serves as the bar bracket and is called Vinculum.

EXERCISE 20(A)

Question 1.

Fill in the following blanks, when :

$x = 3, y = 6, z = 18, a = 2, b = 8, c = 32$ and $d = 0$.

- (i) $x + y =$
- (ii) $y - x =$
- (iii) $\frac{y}{x} =$
- (iv) $c \div b =$
- (v) $z \div x =$
- (vi) $y \times d =$
- (vii) $d \div x =$
- (viii) $ab + y =$
- (ix) $a + b + x =$
- (x) $b + z - d =$
- (xi) $a - b + y =$
- (xii) $z - a - b =$
- (xiii) $d - a + x =$
- (xiv) $xy - bd =$
- (xv) $xz + cd =$

Solution:

$$(i) x + y = 3 + 6 = 9$$

$$(ii) y - x = 6 - 3 = 3$$

$$(iii) \frac{y}{x} = \frac{6}{3} = 2$$

$$(iv) c + b = \frac{c}{b} = \frac{32}{8} = 4$$

$$(v) z \div x = \frac{z}{x} = \frac{18}{3} = 6$$

$$(vi) y \times d = 6 \times 0 = 0$$

$$(vii) d \div x = \frac{d}{x} = \frac{0}{3} = 0$$

$$(viii) ab + y = 2 \times 8 + 6 = 16 + 6 = 22$$

$$(ix) a + b + x = 2 + 8 + 3 = 13$$

$$(x) b + z - d = 8 + 18 - 0 = 26$$

$$(xi) a - b + y = 2 - 8 + 6 = 8 - 8 = 0$$

$$(xii) z - a - b = 18 - 2 - 8 = 18 - 10 = 8$$

$$(xiii) d - a + x = 0 - 2 + 3 = 1$$

$$(xiv) xy - bd = 3 \times 6 - 8 \times 0 = 18 - 0 = 18$$

$$(xv) xz + cd = 3 \times 18 + 32 \times 0 = 54 + 0 = 54$$

Question 2.

Find the value of :

$$(i) p + 2q + 3r, \text{ when } p = 1, q = 5 \text{ and } r = 2$$

$$(ii) 2a + 4b + 5c, \text{ when } a = 5, b = 10 \\ \text{and } c = 20$$

$$(iii) 3a - 2b, \text{ when } a = 8 \text{ and } b = 10$$

$$(iv) 5x + 3y - 6z, \text{ when } x = 3, \\ y = 5 \text{ and } z = 4$$

$$(v) 2p - 3q + 4r - 8s, \text{ when } p = 10, \\ q = 8, r = 6, \\ \text{and } s = 2$$

$$(vi) 6m - 2n - 5p - 3q, \text{ when } m = 20, \\ n = 10, p = 2 \text{ and } q = 9$$

Solution:

$$\begin{aligned}
 (i) \quad p + 2q + 3r \\
 &= 1 + 2 \times 5 + 3 \times 2 \\
 &= 1 + 10 + 6 = \mathbf{17}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad 2a + 4b + 5c \\
 &= 2 \times 5 + 4 \times 10 + 5 \times 20 \\
 &= 10 + 40 + 100 = \mathbf{150}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad 3a - 2b &= 3 \times 8 - 2 \times 10 \\
 &= 24 - 20 = \mathbf{4}
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad 5x + 3y - 6z \\
 &= 5 \times 3 + 3 \times 5 - 6 \times 4 \\
 &= 15 + 15 - 24 = 30 - 24 = \mathbf{6}
 \end{aligned}$$

$$\begin{aligned}
 (v) \quad 2p - 3q + 4r - 8s \\
 &= 2 \times 10 - 3 \times 8 + 4 \times 6 - 8 \times 2 \\
 &= 20 - 24 + 24 - 16 \\
 &= 20 - 16 = \mathbf{4}
 \end{aligned}$$

$$\begin{aligned}
 (vi) \quad 6m - 2n - 5p - 3q \\
 &= 6 \times 20 - 2 \times 10 - 5 \times 2 - 3 \times 9 \\
 &= 120 - 20 - 10 - 27 \\
 &= 120 - 57 = \mathbf{63}
 \end{aligned}$$

Question 3.

Find the value of :

$$(i) \quad 4pq \times 2r, \text{ when } p = 5, q = 3 \text{ and } r = 1/2$$

$$(ii) \quad \frac{yx}{z}, \text{ when } x = 8, y = 4 \text{ and } z = 16$$

$$(iii) \quad \frac{a+b-c}{2a}, \text{ when } a = 5, b = 7 \text{ and } c = 2$$

Solution:

$$(i) \quad 4pq \times 2r = 4 \times 5 \times 3 \times 2 \times \frac{1}{2} = \mathbf{60}$$

$$(ii) \quad \frac{yx}{z} = \frac{4 \times 8}{16} = \frac{32}{16} = \mathbf{2}$$

$$(iii) \quad \frac{a+b-c}{2a} = \frac{5+7-2}{2 \times 5} = \frac{12-2}{10} = \frac{10}{10} = \mathbf{1}$$

Question 4.

If $a = 3$, $b = 0$, $c = 2$ and $d = 1$, find the value of :

- (i) $3a + 2b - 6c + 4d$
- (ii) $6a - 3b - 4c - 2d$
- (iii) $ab - bc + cd - da$
- (iv) $abc - bcd + cda$
- (v) $a^2 + 2b^2 - 3c^2$
- (vi) $a^2 + b^2 - c^2 + d^2$
- (vii) $2a^2 - 3b^2 + 4c^2 - 5d^2$

Solution:

- (i) $3a + 2b - 6c + 4d$
 $= 3 \times 3 + 2 \times 0 - 6 \times 2 + 4 \times 1$
 $= 9 + 0 - 12 + 4 = 13 - 12 = 1$
- (ii) $6a - 3b - 4c - 2d$
 $= 6 \times 3 - 3 \times 0 - 4 \times 2 - 2 \times 1$
 $= 18 - 0 - 8 - 2 = 18 - 10 = 8$
- (iii) $ab - bc + cd - da$
 $= 3 \times 0 - 0 \times 2 + 2 \times 1 - 1 \times 3$
 $= 0 + 0 + 2 - 3 = -1$
- (iv) $abc - bcd + cda$
 $= 3 \times 0 \times 2 - 0 \times 2 \times 1 + 2 \times 1 \times 3$
 $= 0 - 0 + 6 = 6$
- (v) $a^2 + 2b^2 - 3c^2$
 $= 3^2 + 2 \times 0^2 - 3 \times 2^2$
 $= 9 + 0 - 3 \times 4$
 $= 9 - 12 = -3$
- (vi) $a^2 + b^2 - c^2 + d^2$
 $= (3)^2 + (0)^2 - (2)^2 + (1)^2$
 $= 9 + 0 - 4 + 1 = 6$
- (vii) $2a^2 - 3b^2 + 4c^2 - 5d^2$
 $= 2(3)^2 - 3(0)^2 + 4(2)^2 - 5(1)^2$
 $= 2 \times 9 - 0 + 4 \times 4 - 5 \times 1$
 $= 18 - 0 + 16 - 5 = 34 - 5 = 29$

Question 5.

Find the value of $5x^2 - 3x + 2$, when $x = 2$.

Solution:

$$\begin{aligned}
 5x^2 - 3x + 2 &= 5(2)^2 - 3(2) + 2 \\
 &= 5(4) - 6 + 2 \\
 &= 20 + 2 - 6 = 16
 \end{aligned}$$

Question 6.

Find the value of $3x^3 - 4x^2 + 5x - 6$, when $x = -1$.

Solution:

$$\begin{aligned} & 3x^3 - 4x^2 + 5x - 6 \\ &= 3(-1)^3 - 4(-1)^2 + 5(-1) - 6 \\ &= 3 \times -1 - 4 \times 1 - 5 - 6 \\ &= -3 - 4 - 5 - 6 = -18 \end{aligned}$$

Question 7.

Show that the value of $x^3 - 8x^2 + 12x - 5$ is zero, when $x = 1$.

Solution:

$$\begin{aligned} & x^3 - 8x^2 + 12x - 5 \\ &= (1)^3 - 8(1)^2 + 12(1) - 5 \\ &= 1 - 8 + 12 - 5 \\ &= 13 - 13 = 0 \text{ Hence proved.} \end{aligned}$$

Question 8.

State true and false :

- (i) The value of $x + 5 = 6$, when $x = 1$
- (ii) The value of $2x - 3 = 1$, when $x = 0$
- (iii) $\frac{2x-4}{x+1} = -1$, when $x = 1$

Solution:

(i) True, verification $x + 5 = 6$,
When $x = 1$, $1 + 5 = 6$
 $6 = 6$ Hence proved.

(ii) $2x - 3 = 1$, when $x = 0$
 $2 \times 0 - 3 = 1$, $0 - 3 = 1$,
 $-3 = 1$ False.

(iii) $\frac{2x-4}{x+1} = -1$, when $x = 1 = \frac{2 \times 1 - 4}{1 + 1} = -1$
 $= \frac{2 - 4}{2} = -1 = \frac{-2}{2} = -1$
 $= -1 = -1$ True.

Question 9.

If $x = 2$, $y = 5$ and $z = 4$, find the value of each of the following :

$$(i) \frac{x}{2x^2}$$

$$(ii) \frac{xz}{yz}$$

$$(iii) z^x$$

$$(iv) y^x$$

$$(v) \frac{x^2 y^2 z^2}{xz}$$

$$(vi) \frac{5x^4 y^2 z^2}{2x^2}$$

$$(vii) xy + y^2 z$$

$$(viii) \frac{x^2 y^x}{x}$$

Solution:

$$(i) \frac{x}{2x^2} = \frac{2}{2(2)^2} = \frac{2}{2 \times 4} = \frac{1}{4}$$

$$(ii) \frac{xz}{yz} = \frac{2 \times 4}{5 \times 4} = \frac{2}{5}$$

$$(iii) z^x = 4^2 = 4 \times 4 = 16$$

$$(iv) y^x = 5^2 = 5 \times 5 = 25$$

$$(v) \frac{x^2 y^2 z^2}{xz} = \frac{(2)^2 \times (5)^2 \times (4)^2}{2 \times 4} \\ = (2)^{2-1} \times (5)^2 \times (4)^{2-1} \\ = 2 \times 5 \times 5 \times 4 = 200$$

$$(vi) \frac{5x^4 y^2 z^2}{2x^2} = \frac{5x^{4-2} y^2 z^2}{2} = \frac{5x^2 y^2 z^2}{2} \\ = \frac{5(2)^2 (5)^2 (4)^2}{2} = \frac{5 \times 4 \times 25 \times 16}{2} \\ = 5 \times 2 \times 25 \times 16 = 4,000$$

$$(vii) \frac{xy}{y^2 z} = \frac{x}{y^{2-1} z} = \frac{x}{yz} = \frac{2}{5 \times 4} = \frac{1}{10}$$

$$(viii) \frac{x^2 y^x}{x} = x^{2-1} y^x = xy^x = (2)(5)^2 \\ = 2 \times 25 = 50$$

Question 10.

If $a = 3$, find the values of a^2 and 2^a .

Solution:

$$a^2 = (3)^2 = 3 \times 3 = 9$$

$$2^a = (2)^3 = 2 \times 2 \times 2 = 8$$

Question 11.

If $m = 2$, find the difference between the values of $4m^3$ and $3m^4$.

Solution:

$$4m^3 = 4(2)^3 = 4 \times 2 \times 2 \times 2 = 32$$

$$3m^4 = 3(2)^4 = 3 \times 2 \times 2 \times 2 \times 2 = 48$$

$$\text{Now, a difference } 3m^4 - 4m^3 = 48 - 32 = 16$$

EXERCISE 20(B)

Question 1.

Evaluate :

- (i) $(23 - 15) + 4$
- (ii) $5x + (3x + 7x)$
- (iii) $6m - (4m - m)$
- (iv) $(9a - 3a) + 4a$
- (v) $35b - (16b + 9b)$
- (vi) $(3y + 8y) - 5y$

Solution:

- (i) $(23 - 15) + 4 = 8 + 4 = 12$
- (ii) $5x + (3x + 7x) = 5x + 10x = 15x$
- (iii) $6m - (4m - m) = 6m - 3m = 3m$
- (iv) $(9a - 3a) + 4a = 6a + 4a = 10a$
- (v) $35b - (16b + 9b) = 35b - 25b = 10b$
- (vi) $(3y + 8y) - 5y = 11y - 5y = 6y$

Question 2.

Simplify :

- (i) $12x - (5x + 2x)$
- (ii) $10m + (4n - 3n) - 5n$
- (iii) $(15b - 6b) - (8b + 4b)$
- (iv) $-(-4a - 8a)$
- (v) $x - (x - y) - (-x + y)$
- (vi) $p + (-q - r - s) - (p - q - r)$
- (vii) $(a + b) - (c + d) - (e - f)$
- (viii) $3x + (8x - 5x) - (7x - x)$
- (ix) $a - (a - b - c)$
- (x) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$
- (xi) $2m - (3m + 2n - 6n)$
- (xii) $-m - n - (-m) - m$
- (xiii) $x + y - (x + y - x)$
- (xiv) $25y - (5x - 10y + 6x - 3y)$
- (xv) $3x + (2x - x + 2)$
- (xvi) $a - (2a - 4a + 3a)$
- (xvii) $5x^2 - (3x - x^2 - 4)$
- (xviii) $-(y - x) - (x + y - 2x + y)$

Solution:

- (i) $12x - (5x + 2x) = 12x - 7x = 5x$
- (ii) $10m + (4n - 3n) - 5n$
 $= 10m + n - 5n = 10m - 4n$
- (iii) $(15b - 6b) - (8b + 4b)$
 $= 9b - 12b = -3b$
- (iv) $-(-4a - 8a) = -(-12a) = 12a$
- (v) $x - (x - y) - (-x + y)$
 $= x - x + x + y - y = x$
- (vi) $p + (-q - r - s) - (p - q - r)$
 $= p - q - r - s - p + q + r$
 $= p - p - q + q - r + r - s = -s$
- (vii) $(a + b) - (c + d) - (e - f)$
 $= a + b - c - d - e + f$
- (viii) $3x + (8x - 5x) - (7x - x)$
 $= 3x + 3x - 6x = 6x - 6x = 0$
- (ix) $a - (a - b - c) = a - a + b + c$
 $= b + c$
- (x) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$
 $= 6a^2 + a^2 - a^2 + b^2 = 6a^2 + b^2$
- (xi) $2m - (3m + 2n - 6n)$
 $= 2m - 3m - 2n + 6n$
 $= -m + 4n = 4n - m$
- (xii) $-m - n - (-m) - m$
 $= -m - n + m - m = -m - n$
- (xiii) $x + y - (x + y - x)$
 $= x + y - (x + y - x)$
 $= x + y - x - y + x$
 $= x - x + x + y - y = x$
- (xiv) $25y - (5x - 10y + 6x - 3y)$
 $= 25y - 5x + 10y - 6x + 3y$
 $= 25y + 10y + 3y - 5x - 6x$
 $= 38y - 11x$
- (xv) $3x + (2x - x + 2)$
 $= 3x + (2x - x - 2)$

$$= 3x + 2x - x - 2 = 4x - 2$$

$$(xvi) \ a - (2a - 4a + 3a)$$

$$= a - (2a - 4a - 3a)$$

$$= a - 2a + 4a + 3a = 8a - 2a = 6a.$$

$$(xvii) \ 5x^2 - (3x - x^2 - 4)$$

$$= 5x^2 - (3x - x^2 + 4) = 5x^2 - 3x + x^2 - 4$$

$$= 5x^2 + x^2 - 3x - 4 = 6x^2 - 3x - 4$$

$$(xviii) \ -(y - x) - (x + y - 2x + y)$$

$$= -(y - x) - (x + y - 2x - y)$$

$$= -y + x - x - y + 2x + y$$

$$= x - x + 2x - y - y + y = 2x - y$$

Question 3.

Simplify :

$$(i) \ x - (y - z) + x + (y - z) + y - (z + x)$$

$$(ii) \ x - [y + \{x - (y + x)\}]$$

$$(iii) \ 4x + 3(2x - 5y)$$

$$(iv) \ 2(3a - b) - 5(a - 3b)$$

$$(v) \ p + 2(q - r + p)$$

$$(vi) \ a - [-\{-(a - b - c)\}]$$

$$(vii) \ 3x - [5y - \{6y + 2(10y - x)\}]$$

$$(viii) \ 5\{a^2 - a(a - a - 2)\}$$

Solution:

$$\begin{aligned}
 (i) \quad & x - (y - z) + x + (y - z) + y - (z + x) \\
 &= x - y + z + x + y - z + y - z - x \\
 &= x + x - x - y + y + y + z - z - z \\
 &= x + y - z
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & x - [y + \{x - (y + x)\}] \\
 &= x - [y + \{x - y - x\}] \\
 &= x - [y + x - y - x] \\
 &= x - y - x + y + x \\
 &= x - x + x - y + y = x
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad & 4x + 3(2x - 5y) \\
 &= 4x + 6x - 15y \\
 &= 10x - 15y
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad & 2(3a - b) - 5(a - 3b) \\
 &= 6a - 2b - 5a + 15b \\
 &= 6a - 5a + 15b - 2b = a + 13b
 \end{aligned}$$

$$\begin{aligned}
 (v) \quad & p + 2(q - r + p) \\
 &= p + 2(q - r - p) \\
 &= p + 2q - 2r - 2p = 2q - 2r - p
 \end{aligned}$$

$$\begin{aligned}
 (vi) \quad & a - [-\{-(a - b - c)\}] \\
 &= a - [-\{-(a - b + c)\}] \\
 &= a - [-\{-a + b - c\}] \\
 &= a - [+a - b + c] \\
 &= a - a + b - c = b - c
 \end{aligned}$$

$$\begin{aligned}
 (vii) \quad & 3x - [5y - \{6y + 2(10y - x)\}] \\
 &= 3x - [5y - \{6y + 20y - 2x\}] \\
 &= 3x - [5y - 6y - 20y + 2x] \\
 &= 3x - 5y + 6y + 20y - 2x \\
 &= 3x - 2x + 6y + 20y - 5y \\
 &= x + 21y
 \end{aligned}$$

$$\begin{aligned}
 (viii) \quad & 5\{a^2 - a(a - a - 2)\} \\
 &= 5\{a^2 - a(a - a + 2)\} \\
 &= 5\{a^2 - a^2 + a^2 - 2a\} \\
 &= 5a^2 - 5a^2 + 5a^2 - 10a \\
 &= 5a^2 - 10a
 \end{aligned}$$

EXERCISE 20(C)

Question 1.

Fill in the blanks :

- (i) $2a + b - c = 2a + (\dots\dots\dots)$
- (ii) $3x - z + y = 3x - (\dots\dots\dots)$
- (iii) $6p - 5x + q = 6p - (\dots\dots\dots)$
- (iv) $a + b - c + d = a + (\dots\dots\dots)$
- (v) $5a + 4b + 4x - 2c = 4x - (\dots\dots\dots)$
- (vi) $7x + 2z + 4y - 3 = -3 + 4y + (\dots\dots\dots)$
- (vii) $3m - 2n + 6 = 6 - (\dots\dots\dots)$
- (viii) $2t + r - p - q + s = 2t + r - (\dots\dots\dots)$

Solution:

- (i) $2a + b - c = 2a + (b - c)$
- (ii) $3x - z + y = 3x - (z - y)$
- (iii) $6p - 5x + q = 6p - (5x - q)$
- (iv) $a + b - c + d = a + (b - c + d)$
- (v) $5a + 4b + 4x - 2c = 4x - (2c - 5a - 4b)$
- (vi) $7x + 2z + 4y - 3 = -3 + 4y + (7x + 2z)$
- (vii) $3m - 2n + 6 = 6 - (2n - 3m)$
- (viii) $2t + r - p - q + s = 2t + r - (p + q - s)$

Question 2.

Insert the bracket as indicated :

- (i) $x - 2y = - (\dots\dots\dots)$
- (ii) $m + n - p = - (\dots\dots\dots)$
- (iii) $a + 4b - 4c = a + (\dots\dots\dots)$
- (iv) $a - 3b + 5c = a - (\dots\dots\dots)$
- (v) $x^2 - y^2 + z^2 = x^2 - (\dots\dots\dots)$
- (vi) $m^2 + x^2 - p^2 = - (\dots\dots\dots)$
- (vii) $2x - y + 2z = 2z - (\dots\dots\dots)$
- (viii) $ab + 2bc - 3ac = 2bc - (\dots\dots\dots)$

Solution:

- (i) $x - 2y = -(2y - x)$
- (ii) $m + n - p = -(p - m - n)$
- (iii) $a + 4b - 4c = a + (4b - 4c)$
- (iv) $a - 3b + 5c = a - (3b - 5c)$
- (v) $x^2 - y^2 + z^2 = x^2 - (y^2 - z^2)$
- (vi) $m^2 + x^2 - p^2 = -(p^2 - m^2 - x^2)$
- (vii) $2x - y + 2z = 2z - (y - 2x)$
- (viii) $ab + 2bc - 3ac = 2bc - (3ac - ab)$

REVISION EXERCISE

Question 1.

Find the value of $3ab + 10bc - 2abc$ when $a = 2$, $b = 5$ and $c = 8$.

Solution:

$$a = 2, b = 5, c = 8$$

$$\begin{aligned}\therefore 3ab + 10bc - 2abc \\ &= 3 \times 2 \times 5 + 10 \times 5 \times 8 - 2 \times 2 \times 5 \times 8 \\ &= 30 + 400 - 160 = 430 - 160 \\ &= 270\end{aligned}$$

Question 2.

If $x = 2$, $y = 3$ and $z = 4$, find the value of $3x^2 - 4y^2 + 2z^2$.

Solution:

$$x = 2, y = 3, z = 4$$

$$\begin{aligned}\therefore 3x^2 - 4y^2 + 2z^2 &= 3(2)^2 - 4(3)^2 + 2(4)^2 \\ &= 3 \times 4 - 4 \times 9 + 2 \times 16 \\ &= 12 - 36 + 32 \\ &= 12 + 32 - 36 = 44 - 36 = 8\end{aligned}$$

Question 3.

If $x = 3$, $y = 2$ and $z = 1$; find the value of:

- (i) xy
- (ii) y^x
- (iii) $3x^2 - 5y^2$
- (iv) $2x - 3y + 4z + 5$
- (v) $y^2 - x^2 + 6z^2$
- (vi) $xy + y^2z - 4zx$

Solution:

$$x = 3, y = 2, z = 1$$

$$(i) x^y = 3^2 = 3 \times 3 = 9$$

$$(ii) y^x = 2^3 = 2 \times 2 \times 2 = 8$$

$$(iii) 3x^2 - 5y^2 = 3(3)^2 - 5(2)^2 \\ = 3 \times 9 - 5 \times 4 = 27 - 20 = 7$$

$$(iv) 2x - 3y + 4z + 5 = 2 \times 3 - 3 \times 2 + 4 \times 1 + 5 \\ = 6 - 6 + 4 + 5 = 15 - 6 = 9$$

$$(v) y^2 - x^2 + 6z^2 \\ = (2)^2 - (3)^2 + 6(1)^2 \\ = 4 - 9 + 6 \times 1 = 4 - 9 + 6 \\ = 10 - 9 = 1$$

$$(vi) xy + y^2z - 4zx \\ = 3 \times 2 + (2)^2 \times 1 - 4 \times 1 \times 3 \\ = 6 + 4 - 12 = 10 - 12 = -2$$

Question 4.

If $P = -12x^2 - 10xy + 5y^2$, $Q = 7x^2 + 6xy + 2y^2$, and $R = 5x^2 + 2xy + 4y^2$; find :

(i) $P - Q$

(ii) $Q + P$

(iii) $P - Q + R$

(iv) $P + Q + R$

Solution:

$$P = -12x^2 - 10xy + 5y^2$$

$$Q = 7x^2 + 6xy + 2y^2$$

$$R = 5x^2 + 2xy + 4y^2$$

$$(i) P - Q = (-12x^2 - 10xy + 5y^2) - (7x^2 + 6xy + 2y^2)$$

$$= -12x^2 - 10xy + 5y^2 - 7x^2 - 6xy - 2y^2$$

$$= -12x^2 - 7x^2 - 10xy - 6xy + 5y^2 - 2y^2$$

$$= -19x^2 - 16xy + 3y^2$$

$$(ii) Q + P = (7x^2 + 6xy + 2y^2) + (-12x^2 - 10xy + 5y^2)$$

$$= 7x^2 + 6xy + 2y^2 - 12x^2 - 10xy + 5y^2$$

$$= 7x^2 - 12x^2 + 6xy - 10xy + 2y^2 + 5y^2$$

$$= -5x^2 - 4xy + 7y^2$$

$$(iii) P - Q + R = (-12x^2 - 10xy + 5y^2) - (7x^2 + 6xy + 2y^2) + (5x^2 + 2xy + 4y^2)$$

$$= -12x^2 - 10xy + 5y^2 - 7x^2 - 6xy - 2y^2 + 5x^2 + 2xy + 4y^2$$

$$= -12x^2 - 7x^2 + 5x^2 - 10xy - 6xy + 2xy + 5y^2 - 2y^2 + 4y^2$$

$$= -14x^2 - 14xy + 7y^2$$

$$(iv) P + Q + R = -12x^2 - 10xy + 5y^2 + 7x^2 + 6xy + 2y^2 + 5x^2 + 2xy + 4y^2$$

$$= -12x^2 + 7x^2 + 5x^2 - 10xy + 6xy + 2xy + 5y^2 + 2y^2 + 4y^2$$

$$= 0 - 2xy + 11y^2$$

$$= -2xy + 11y^2$$

Question 5.

If $x = a^2 - bc$, $y = b^2 - ca$ and $z = c^2 - ab$; find the value of :

(i) $ax + by + cz$

(ii) $ay - bx + cz$

Solution:

$$x = a^2 - bc, y = b^2 - ca, z = c^2 - ab$$

$$(i) \quad ax + by + cz = a(a^2 - bc) + b(b^2 - ca) + c(c^2 - ab)$$

$$= a^3 - abc + b^3 - abc + c^3 - abc$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$(ii) \quad ay - bx + cz = a(b^2 - ca) - b(a^2 - bc) + c(c^2 - ab)$$

$$= ab^2 - ca^2 - a^2b + b^2c + c^3 - abc$$

Question 6.

Multiply and then evaluate :

$$(i) \quad (4x + y) \text{ and } (x - 2y); \text{ when } x = 2 \text{ and } y = 1.$$

$$(ii) \quad (x^2 - y) \text{ and } (xy - y^2); \text{ when } x = 1 \text{ and } y = 2.$$

$$(iii) \quad (x - 2y + z) \text{ and } (x - 3z); \text{ when } x = -2, y = -1 \text{ and } z = 1.$$

Solution:

$$(i) \quad (4x + y) \times (x - 2y)$$

$$= 4x(x - 2y) + y(x - 2y)$$

$$= 4x^2 - 8xy + xy - 2y^2$$

$$= 4x^2 - 7xy - 2y^2$$

Verification :

$$\text{When } x = 2, y = 1$$

$$\text{L.H.S.} = (4x + y)(x - 2y)$$

$$= (4 \times 2 + 1)(2 - 2 \times 1)$$

$$= (8 + 1)(2 - 2) = 9 \times 0 = 0$$

$$\begin{aligned}\text{R.H.S.} &= 4x^2 - 7xy - 2y^2 \\ &= 4(2)^2 - 7 \times 2 \times 1 - 2(1)^2 \\ &= 4 \times 4 - 14 - 2 = 16 - 16 = 0\end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\begin{aligned}\text{(ii)} \quad (x^2 - y) \times (xy - y^2) \\ &= x^2(xy - y^2) - y(xy - y^2) \\ &= x^3y - x^2y^2 - xy^2 + y^3\end{aligned}$$

Verification :

$$\text{When } x = 1, y = 2$$

$$\begin{aligned}\therefore \text{L.H.S.} &= (x^2 - y)(xy - y^2) \\ &= [(1)^2 - 2][1 \times 2 - (2)^2] \\ &= (1 - 2)(2 - 4) = -1 \times -2 = 2\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= x^3y - x^2y^2 - xy^2 + y^3 \\ &= (1)^3 \times 2 - (1)^2(2)^2 - 1(2)^2 + (2)^3 \\ &= 1 \times 2 - 1 \times 4 - 1 \times 4 + 8 \\ &= 2 - 4 - 4 + 8 = 10 - 8 = 2\end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$\begin{aligned}\text{(iii)} \quad (x - 2y + z) \times (x - 3z) \\ &= x(x - 3z) - 2y(x - 3z) + z(x - 3z) \\ &= x^2 - 3zx - 2xy + 6yz + zx - 3z^2 \\ &= x^2 - 2zx - 2xy + 6yz - 3z^2\end{aligned}$$

Verification :

$$\text{When } x = -2, y = -1, z = 1$$

$$\begin{aligned}\text{L.H.S.} &= (x - 2y + z) \times (x - 3z) \\ &= [-2 - 2 \times (-1) + 1] \times [-2 - 3 \times 1] \\ &= (-2 + 2 + 1) \times (-2 - 3) = 1 \times (-5) = -5\end{aligned}$$

$$\begin{aligned}\text{R.H.S.} &= x^2 - 2zx - 2xy + 6yz - 3z^2 \\ &= (-2)^2 - 2(1)(-2) - 2(-2)(-1) + 6(-1)(1) - 3(1)^2 \\ &= 4 + 4 - 4 - 6 - 3 \\ &= 8 - 13 = -5\end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Question 7.

Simplify :

$$\text{(i)} \quad 5(x + 3y) - 2(3x - 4y)$$

$$\text{(ii)} \quad 3x - 8(5x - 10)$$

$$\text{(iii)} \quad 6\{3x - 8(5x - 10)\}$$

$$\text{(iv)} \quad 3x - 6\{3x - 8(5x - 10)\}$$

$$\text{(v)} \quad 2(3x - 4x - 8) - (3 - 5x - 2x)$$

$$(vi) 8x - (3x - 2x - 3)$$

$$(vii) 12x^2 - (7x - 3x^2 + 15)$$

Solution:

$$(i) 5(x + 3y) - 2(3x - 4y)$$

$$= 5x + 15y - 6x + 8y$$

$$= 5x - 6x + 15y + 8y = -x + 23y$$

$$(ii) 3x - 8(5x - 10)$$

$$= 3x - 40x + 80$$

$$= -37x + 80$$

$$(iii) 6\{3x - 8(5x - 10)\}$$

$$= 6\{3x - 40x + 80\}$$

$$= 18x - 240x + 480$$

$$= -222x + 480$$

$$(iv) 3x - 6\{3x - 8(5x - 10)\}$$

$$= 3x - 6\{3x - 40x + 80\}$$

$$= 3x - 18x + 240x - 480$$

$$= 243x - 18x - 480 = 225x - 480$$

$$(v) 2(3x^2 - 4x - 8) - (3 - 5x - 2x^2)$$

$$= (6x^2 - 8x - 16) - (3 - 5x - 2x^2)$$

$$= 6x^2 - 8x - 16 - 3 + 5x + 2x^2$$

$$= 6x^2 + 2x^2 - 8x + 5x - 16 - 3$$

$$= 8x^2 - 3x - 19$$

$$(vi) 8x - (3x - \overline{2x - 3})$$

$$= 8x - (3x - 2x + 3) = 8x - 3x + 2x - 3$$

$$= 10x - 3x - 3 = 7x - 3$$

$$(vii) 12x^2 - (7x - \overline{3x^2 + 15})$$

$$= 12x^2 - (7x - 3x^2 - 15)$$

$$= 12x^2 - 7x + 3x^2 + 15$$

$$= 12x^2 + 3x^2 - 7x + 15 = 15x^2 - 7x + 15$$

Question 8.

If $x = -3$, find the value of : $2x^3 + 8x^2 - 15$.

Solution:

$$x = -3$$

$$\therefore 2x^3 + 8x^2 - 15$$

$$= 2(-3)^3 + 8(-3)^2 - 15$$

$$= 2(-27) + 8(9) - 15$$

$$= -54 + 72 - 15 = -54 - 15 + 72$$

$$= -69 + 72 = 3$$