Triangles

Quick Revision

Similar Figures

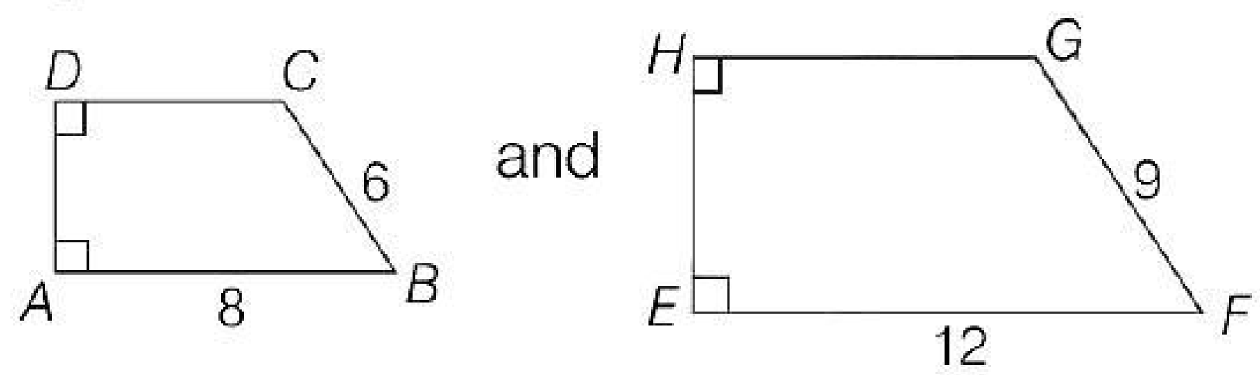
Two geometrical figures are said to be similar figures, if they have same shape but not necessarily the same size.

Similar Polygons

Two polygons of the same number of sides are similar, if

(i) all the corresponding angles are equal and

(ii) all the corresponding sides are in the same ratio (or proportion).



If only one condition from (i) and (ii) is true for two polygons, then they cannot be similar.

Scale Factor

The ratio that compares the measurements of two similar shapes, is called the scale factor or representative fraction. It is equal to the ratio of corresponding sides of two figures. We can use the ratio of corresponding sides to find unknown sides of similar shapes.

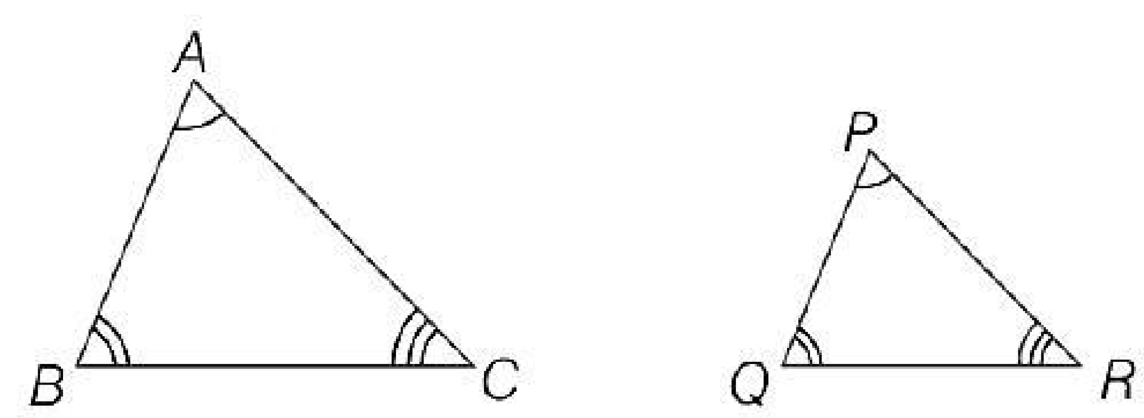
Similar Triangles

Two triangles are said to be similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are proportional.
(i.e. the ratios of the lengths of corresponding sides are same).

Symbolically it can be represented by the symbol '~'.



e.g. In ΔABC and ΔPQR, if

$$\angle A = \angle P, \quad \angle B = \angle Q, \ \angle C = \angle R$$

and $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$.

Then, ΔABC is similar to ΔPQR.

Conversely If ΔABC is similar to ΔPQR, then

$$\angle A = \angle P, \ \angle B = \angle Q, \angle C = \angle R$$

and $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

Basic Proportionality Theorem (BPT)

Theorem 1 (Thales Theorem) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

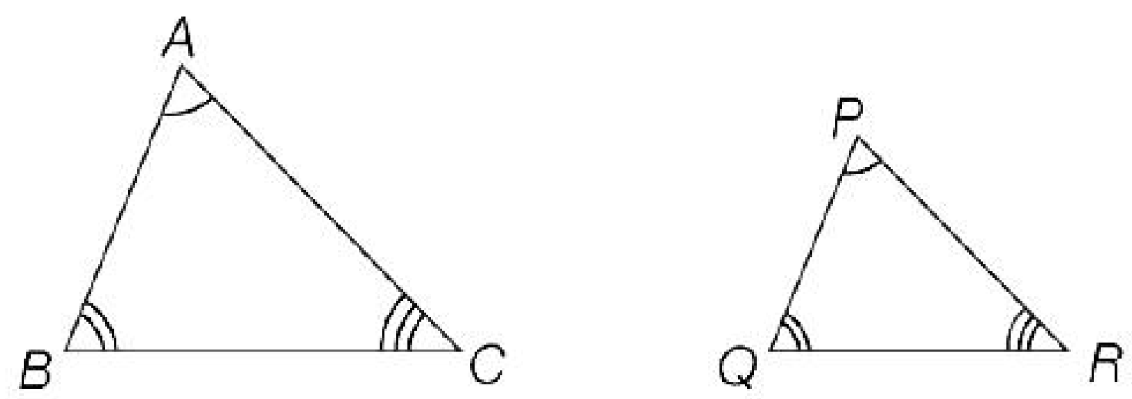
Theorem 2 (Converse of Basic Proportionality Theorem) If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Criteria for Similarity of Triangles

We have some criteria for congruency of two triangles involving only three pairs of corresponding parts (elements) of two triangles. Similarly, we have some criteria for similarity of two triangles, which are given below:

(i) AAA Similarity Criterion

In two triangles, if corresponding angles are equal, then their corresponding sides are proportional and hence the two triangles are similar.



Here, $\angle A = \angle P, \angle B = \angle Q$

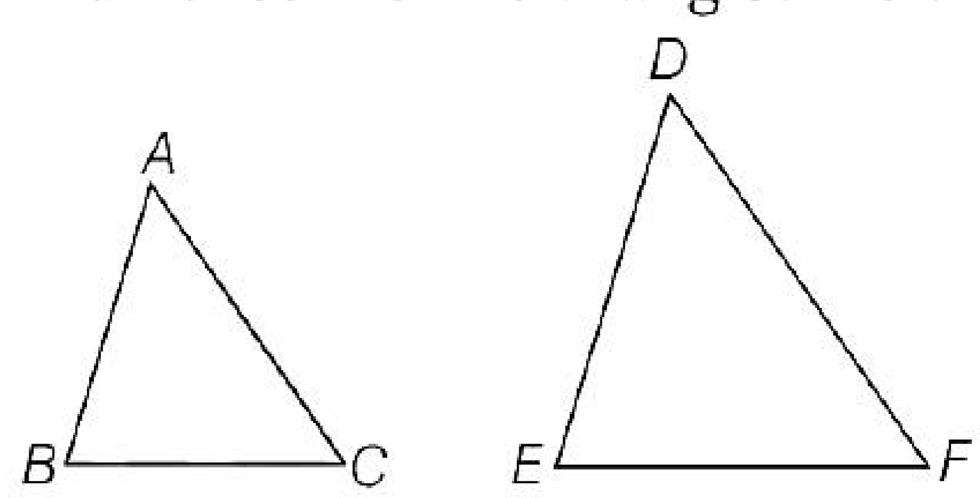
and $\angle C = \angle R$

$\therefore \quad \Delta ABC \sim \Delta PQR$

Note If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. AAA similarity criterion can be consider as **AA similarity criterion**.

(ii) SSS Similarity Criterion

If in two triangles, three sides of one triangle are proportional (i.e., in the same ratio) to the three sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

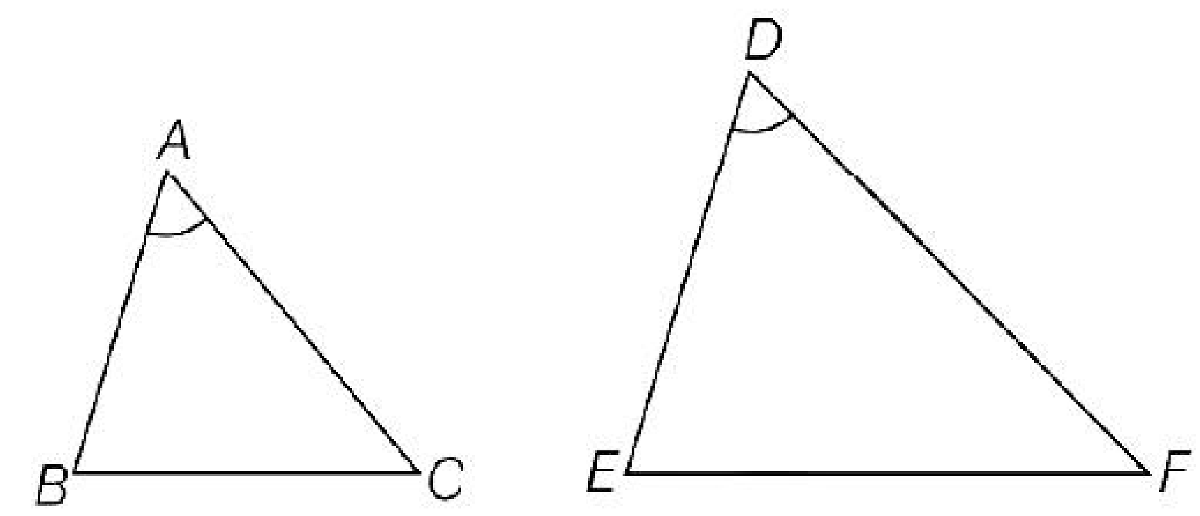


Here, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} (< 1)$

$\therefore \quad \Delta ABC \sim \Delta DEF$

(iii) SAS Similarity Criterion

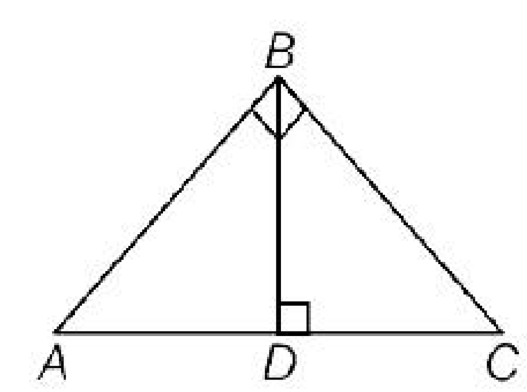
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



Here, $\frac{AB}{DE} = \frac{AC}{DF} (< 1)$ and $\angle A = \angle D$

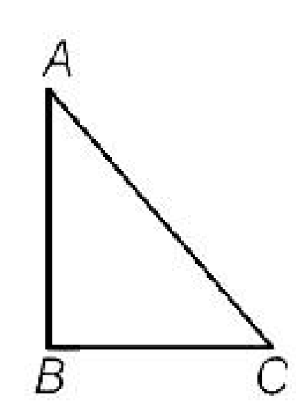
$\therefore \quad \Delta ABC \sim \Delta DEF$

Theorem 1 If a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.



i.e. $\Delta ADB \sim \Delta ABC$ and $\Delta BDC \sim \Delta ABC$

Theorem 2 (Pythagoras Theorem) In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



i.e. $AC^2 = AB^2 + BC^2$

Objective Questions

Multiple Choice Questions

1. Two figures having the same shape but not necessarily the same size are called similar figures.

(a) True (b) False
(c) Cannot say (d) Partially True/False

2. Two triangles are similar. If their corresponding angles are proportional.

(a) True (b) False
(c) Cannot say (d) Partially True/False

3. All .... triangles are similar.

(a) equilateral triangle (b) right triangle
(c) scalene triangle (d) None of these

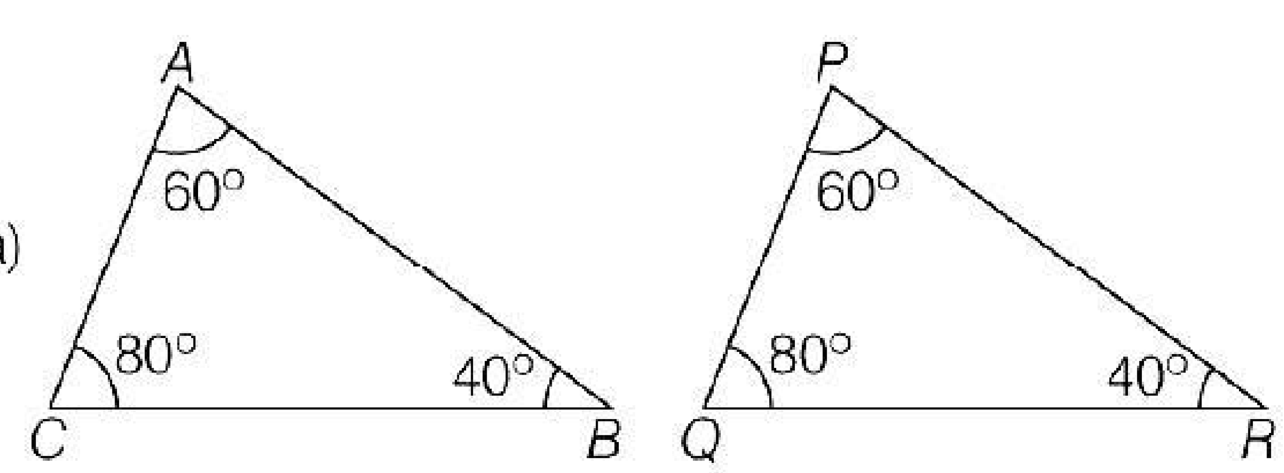
4. Two triangle are similar. If their corresponding sides are ....

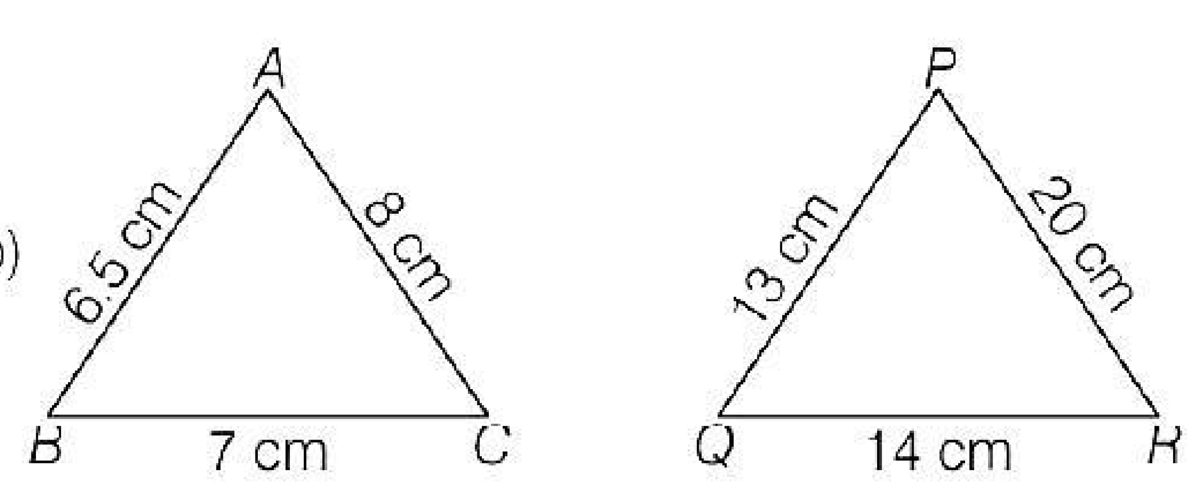
(a) equal
(b) proportional
(c) right angle
(d) None of the above

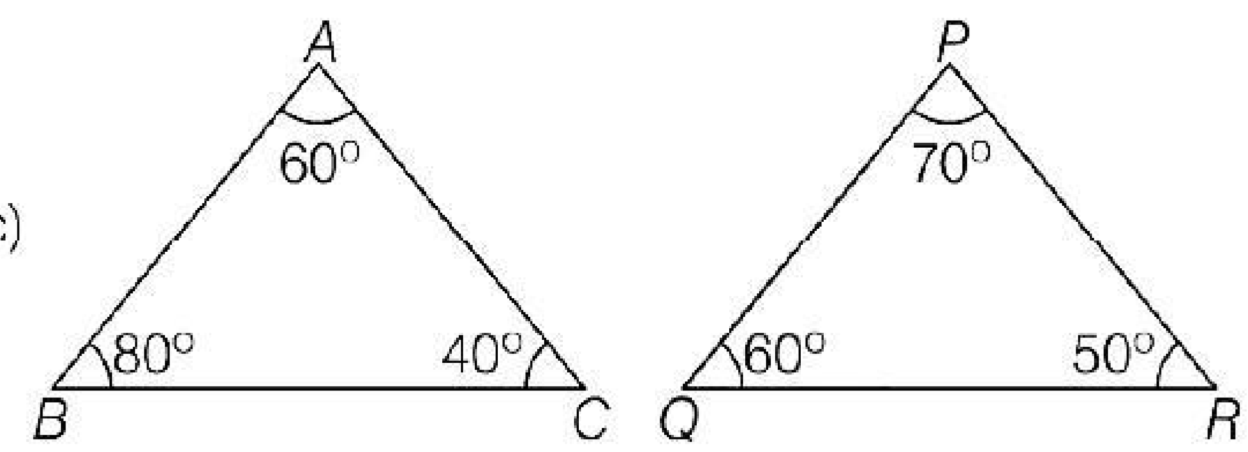
5. "Two quadrilaterals are similar, if their corresponding angles are equal".

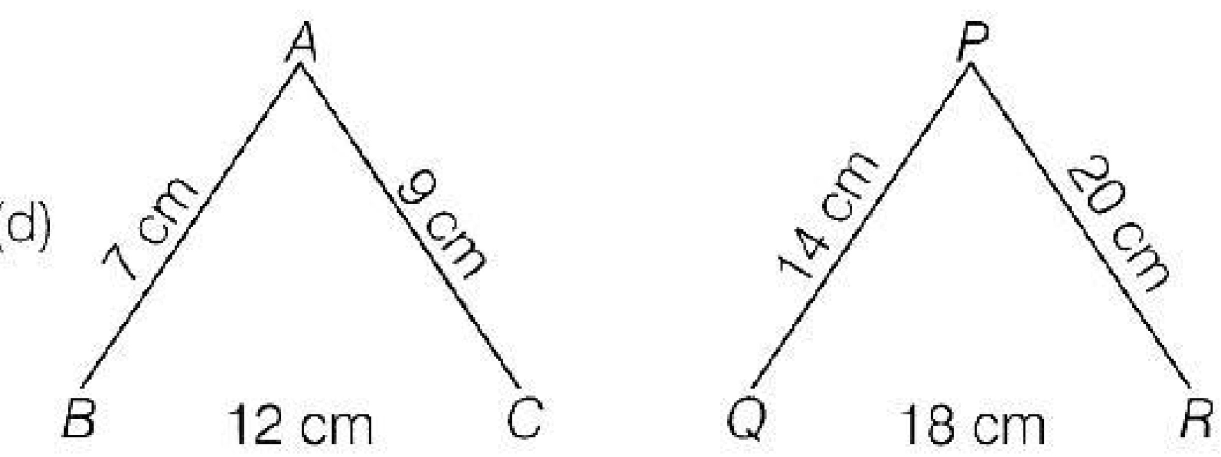
(a) True
(b) False
(c) Cannot say
(d) Partially true/false

6. Which pair of triangles are similar?

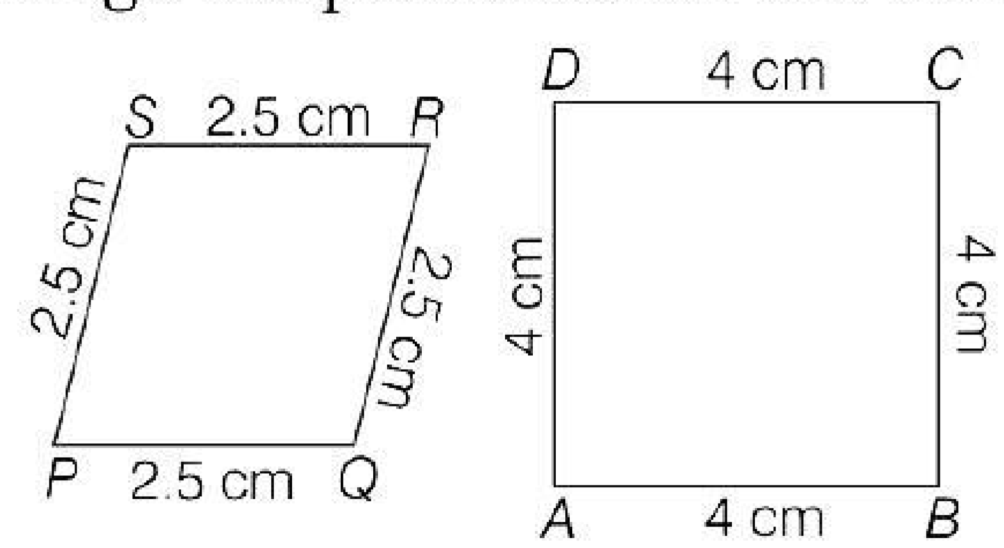
(a)


(b)


(c)


(d)


7. The given quadrilaterals are ....

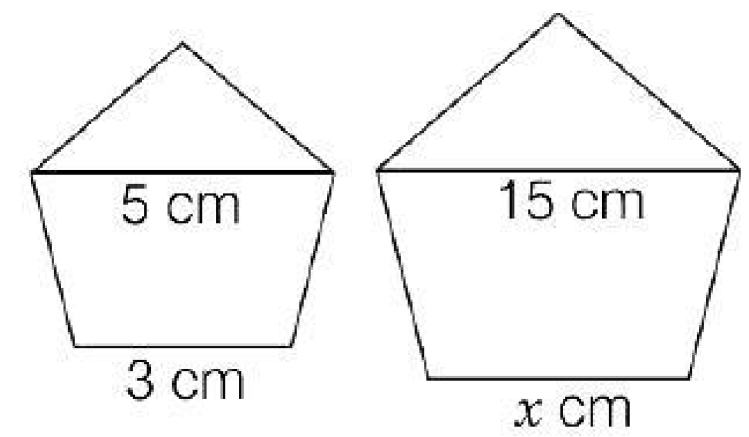


(a) similar
(b) not similar
(c) cannot say
(d) None of the above

8. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle then the two triangles are similar.

(a) True (b) False
(c) Cannot say (d) Partially true/false

9. The given shapes are mathematically similar. The unknown side (x) is



(a) 5 cm (b) 3 cm
(c) 9 cm (d) 15 cm

10. If $\Delta ABC \sim \Delta PQR$, $AB = 6.5$ cm, $PQ = 10.4$ cm and perimeter of $\Delta ABC = 60$ cm, the perimeter of ΔPQR is

(a) 65 cm (b) 96 cm
(c) 60 cm (d) 104 cm

11. It is given that $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm. The lengths of the remaining sides of the triangles is

(a) 16.8 cm, 6.25 cm (b) 16.8 cm, 12 cm
(c) 12 cm, 6.25 cm (d) None of these

12. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

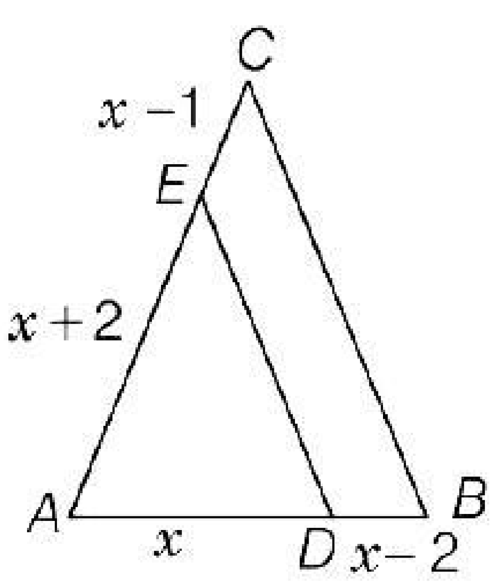
[NCERT Exemplar]

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

13. If a line divides any two sides of a triangle in the same ratio, then the line is ……… to the third sides.

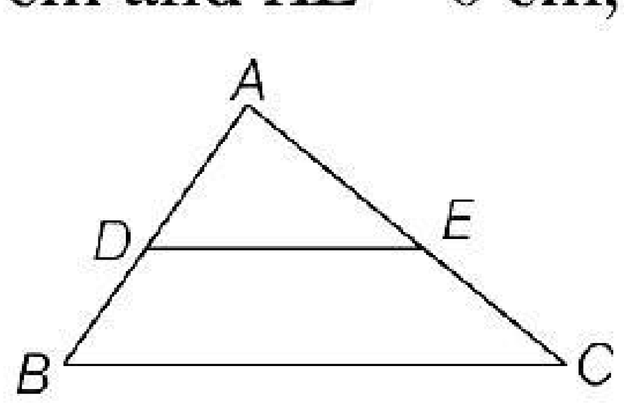
(a) perpendicular (b) parallel
(c) equal (d) None of these

14. In the given figure $DE \| BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, then the value of x is



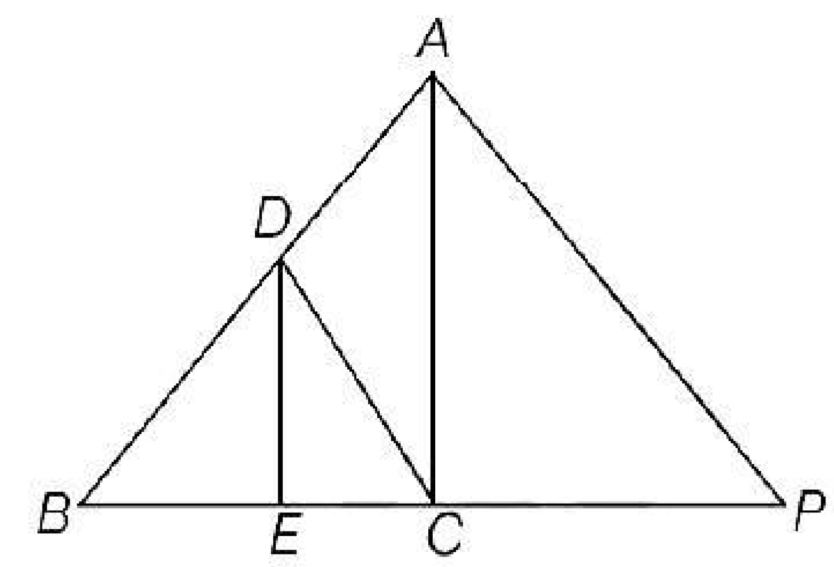
(a) 9 (b) 4 (c) 4.5 (d) 8

15. In the given figure, $DE \| BC$. If $AD = 3$ cm, $DB = 4$ cm and $AE = 6$ cm, then EC is



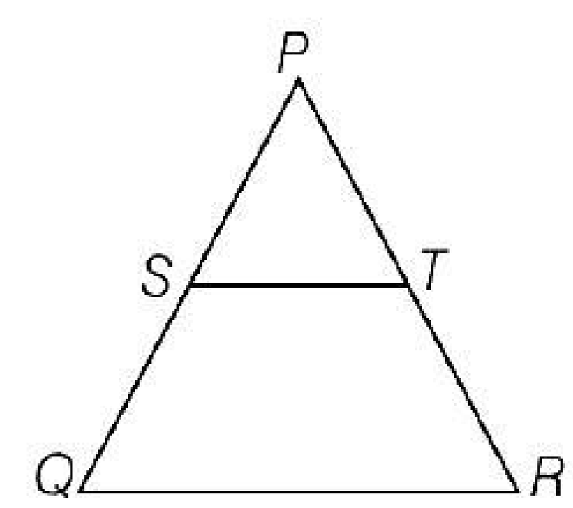
(a) 8 cm (b) 12 cm
(c) 6 cm (d) 4 cm

16. In the given figure of ΔABC, $DE \| AC$. If $DC \| AP$, where point P lies on BC produced, then $\frac{BE}{EC} =$



(a) $\frac{BD}{CP}$ (b) $\frac{BC}{CP}$
(c) $\frac{BC}{DA}$ (d) None of these

17. In ΔPQR, $ST \| QR$, $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, then the value of PT is

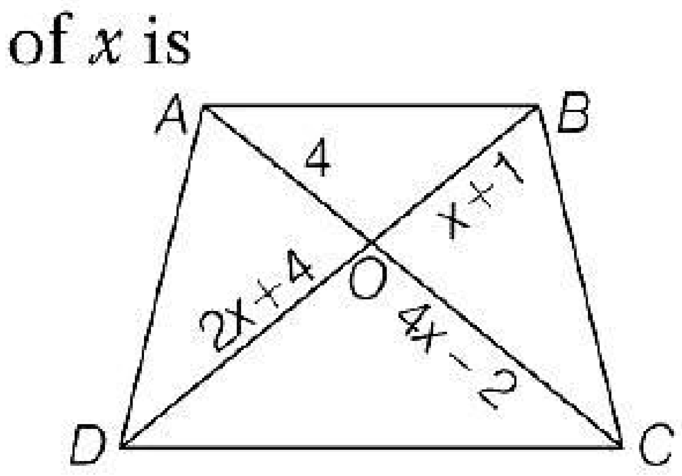


(a) 9.5 cm (b) 9 cm
(c) 10 cm (d) 10.5 cm

18. In ΔABC, D and E are points on the sides AB and AC respectively, such that $DE \| BC$. If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$ and $CE = 5x - 3$, then the value of x is

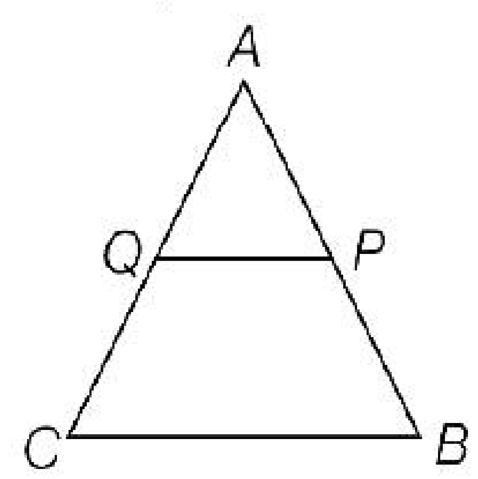
(a) $\frac{1}{2}$ (b) 4
(c) 1 (d) $\frac{2}{3}$

19. In the given figure, if $AB \| CD$, then the value of x is



(a) 6 (b) 8
(c) 3 (d) 9

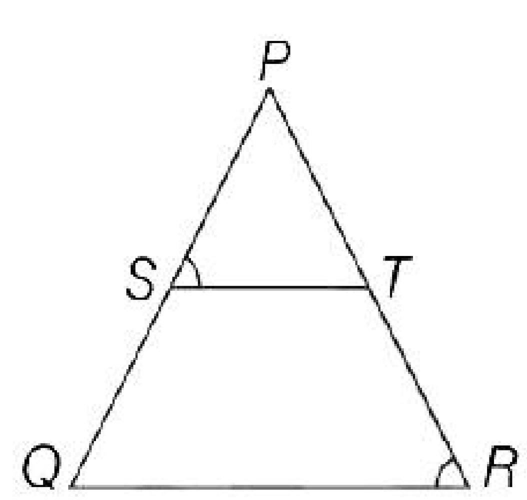
20. In the given figure P and Q are points on sides AB and AC respectively of ΔABC such that $AP = 3.5$ cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm.
If $PQ = 4.5$ cm, then the value of BC is



(a) 13.5 cm (b) 3.4 cm
(c) 2.6 cm (d) 1.6 cm

21. The line joining the mid-points of two sides of a triangle is
(a) bisector of the third side
(b) perpendicular to the third side
(c) parallel to the third side
(d) None of the above

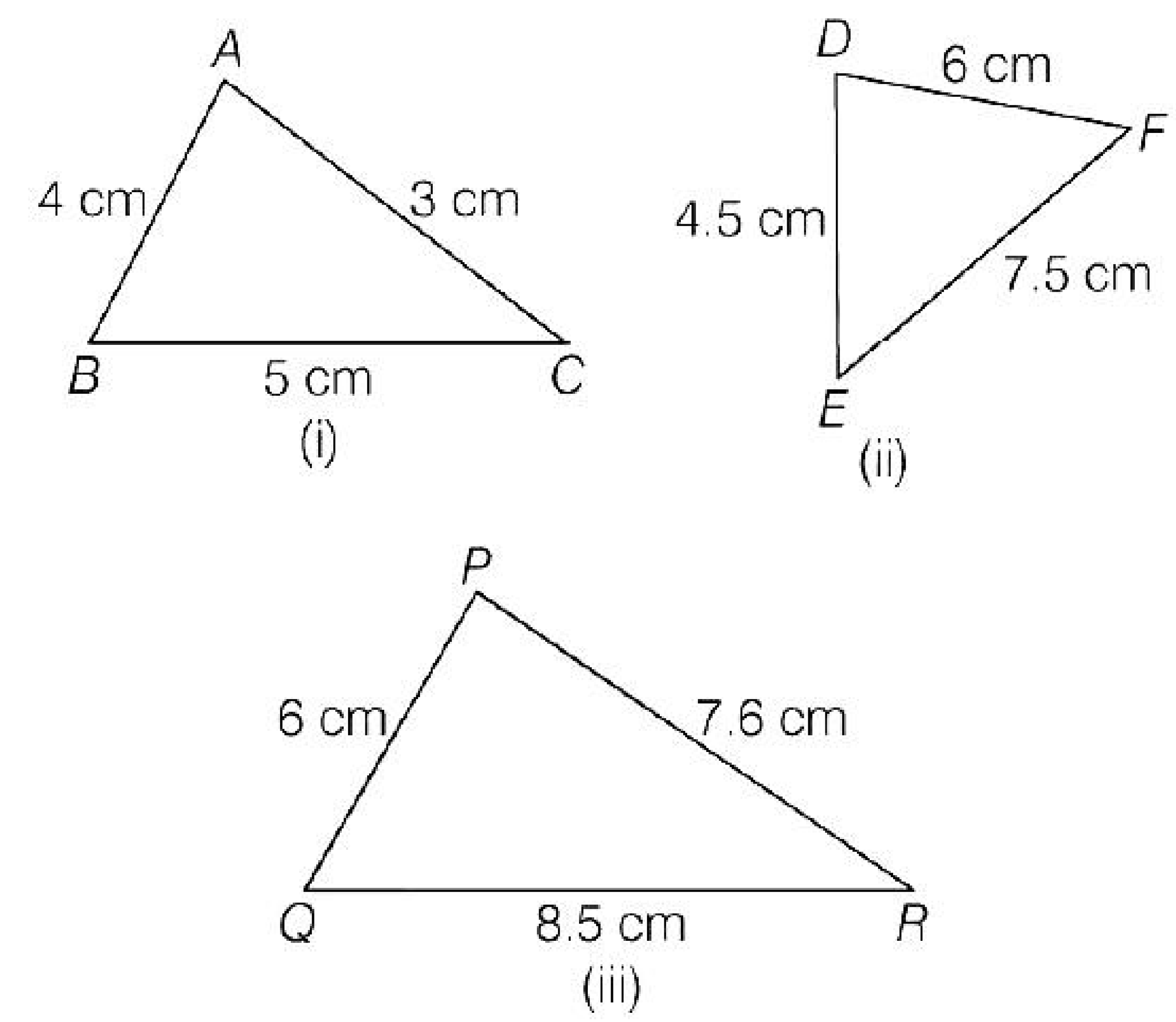
22. In the adjoining figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Then, ΔPQR is an



(a) equilateral triangle
(b) rightangle triangle
(c) isosceles triangle
(d) Cannot say

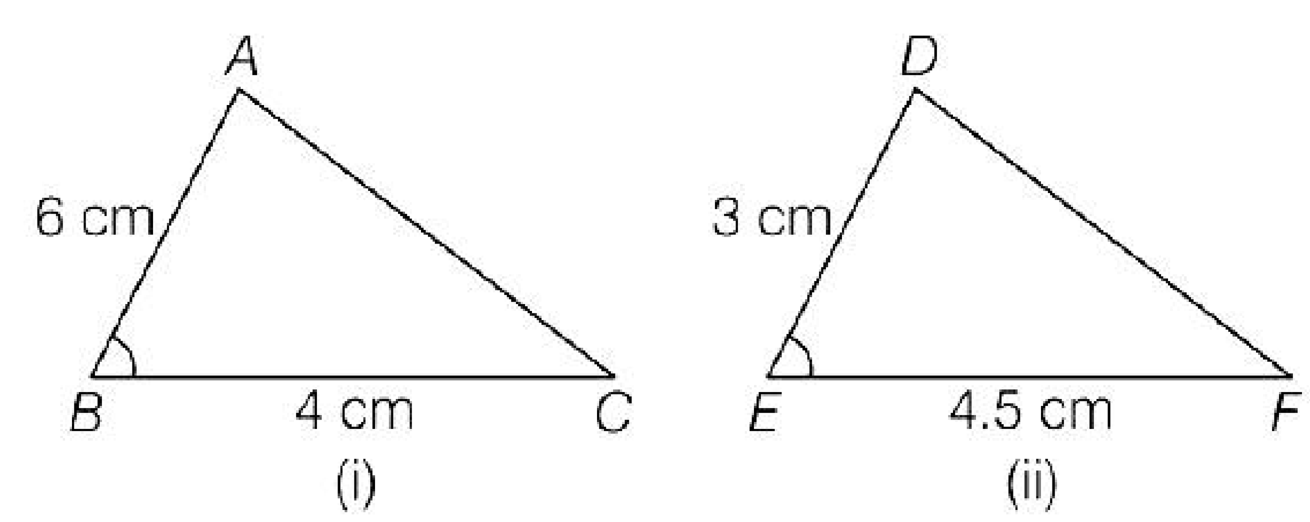
23. In ΔABC, points P and Q are on CA and CB, respectively such that $CA = 16$ cm, $CP = 10$ cm, $CB = 30$ cm and $CQ = 25$ cm. Then,
(a) $PQ \parallel AB$
(b) $PQ \nparallel AB$
(c) $\frac{QB}{CQ} = \frac{PA}{CP}$
(d) None of the above

24. Which pairs of triangles in the given figure are similar?



(a) (i) and (iii) (b) (ii) and (iii)
(c) (i) and (ii) (d) None of these

25. Two similar triangles are given in the figure the similarity criterion used is

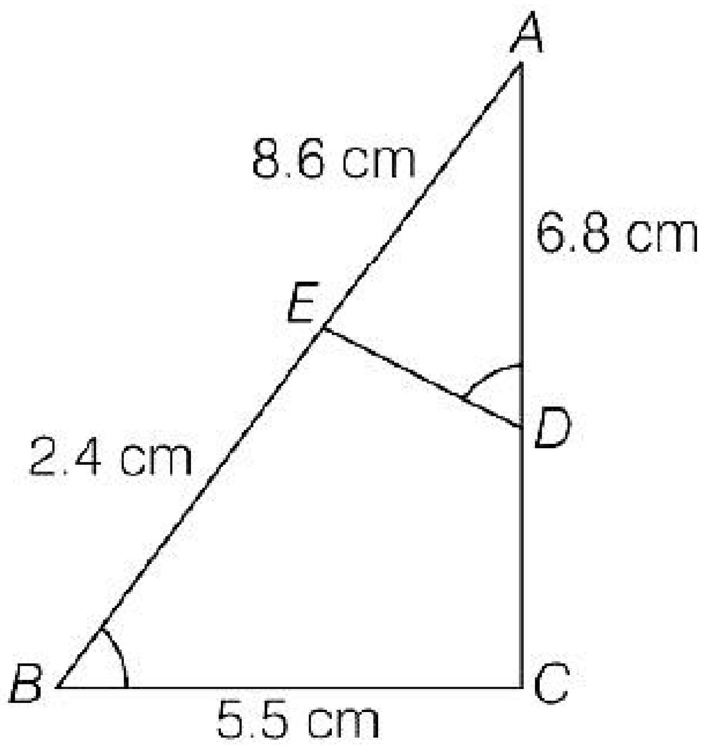


(a) SAS (b) SSS
(c) AAA (d) None of these

26. In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are ***[NCERT Exemplar]***
(a) congruent but not similar
(b) similar but not congruent
(c) neither congruent nor similar
(d) congruent as well as similar

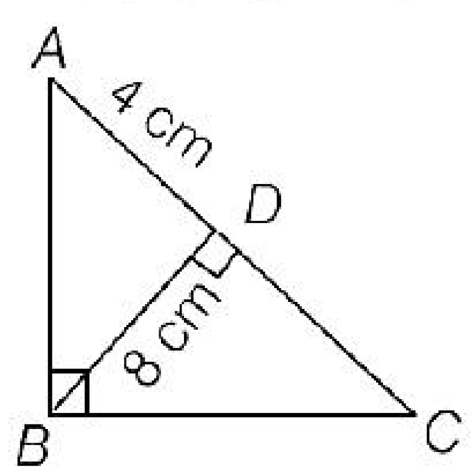
27. Diagonals AC and BD of a trapezium $ABCD$ with $AB \| DC$ intersect each other at the point O, then $\frac{OA}{OC} = \frac{OB}{OD}$
(a) True (b) False
(c) Cannot say (d) Partially True/False

28. In the given figure, if $\angle ADE = \angle B$, and $AD = 6.8$ cm, $AE = 8.6$ cm, $BE = 2.4$ cm and $BC = 5.5$ cm, then the value of DE is



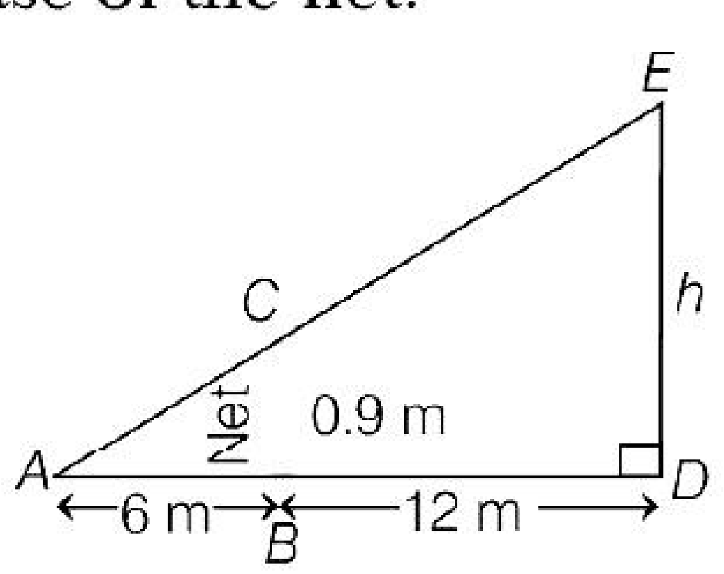
(a) 6.8 cm (b) 2.4 cm
(c) 3.4 cm (d) 4.8 cm

29. In the given figure, $\angle ABC = 90°$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, then the value of CD is



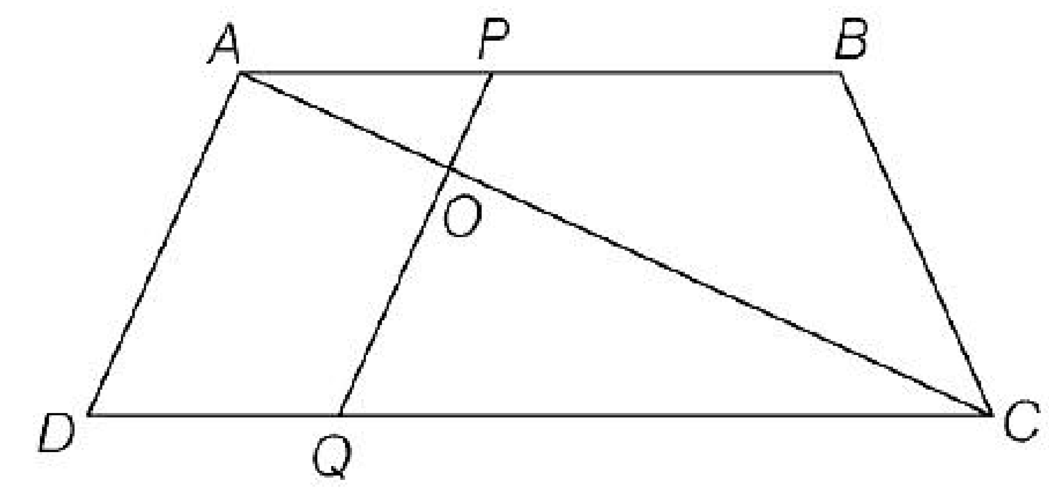
(a) 8 cm (b) 12 cm
(c) 14 cm (d) 16 cm

30. The value of the height 'h' in the adjoining figure is, at which the tennis ball must be hit, so that it will just pass over the net and land 6 m away from the base of the net.



(a) 3.6 m (b) 3 m
(c) 2.7 m (d) 0.27 m

31. In the figure given below, if $AB || DC$ and AC and PQ intersect each other at point O, then the value of $OA \cdot CQ$ is

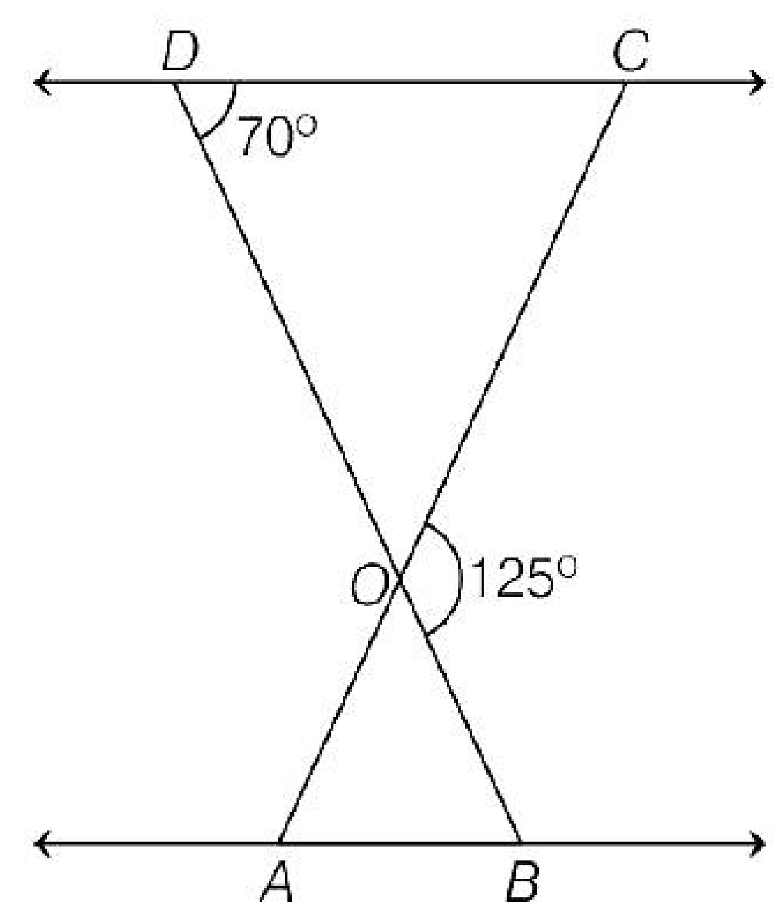


(a) $OC \cdot OQ$ (b) $OP \cdot OC$
(c) $OC \cdot AP$ (d) $OQ \cdot OP$

32. In ΔABC, $AD \perp BC$ and $AD^2 = BD \cdot CD$, then the value of $\angle BAC$ is

(a) 45° (b) 90°
(c) 180° (d) 60°

33. In the given figure, $\Delta ODC \sim \Delta OBA$. $\angle BOC = 125°$ and $\angle CDO = 70°$, then the value of $\angle OAB$ is



(a) 70° (b) 125°
(c) 65° (d) 55°

34. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is

(a) 100 m (b) 120 m
(c) 25 m (d) 200 m

35. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, then the value of length of her shadow after 4s is

(a) 3.2 m (b) 4.8 m
(c) 1.6 m (d) 3.6 m

36. $\Delta ABC \sim \Delta DEF$ and the perimeters of ΔABC and ΔDEF are 30 cm and 18 cm respectively. If $BC = 9$ cm, then EF =

(a) 6.3 cm (b) 5.4 cm
(c) 7.2 cm (d) 4.5 cm

37. .... state that in a right angle triangle; the square of hypotenuse is equal to the sum of the square of the other two sides.

(a) BPT theorem
(b) Converse of Pythagoras theorem
(c) Converse of BPT theorem
(d) Pythagoras theorem

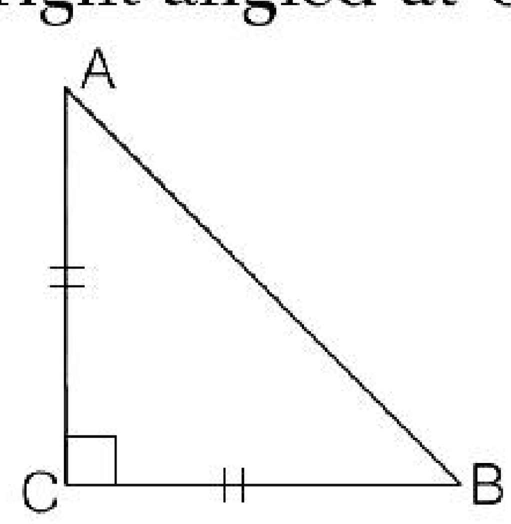
38. Right angle triangle whose hypotenuse is of length p cm, one side of length q cm and $p - q = 1$, then the length of third side of the triangle is

(a) $\sqrt{1+2q}$ cm (b) $\sqrt{p}+\sqrt{q}$ cm
(c) $\sqrt{p-q}$ cm (d) $\sqrt{p}-\sqrt{q}$ cm

39. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(a) 9 cm (b) 10 cm
(c) 8 cm (d) 20 cm

40. In given figure, ABC is an isosceles triangle, right-angled at C. Therefore



[CBSE 2020]

(a) $AB^2 = 2AC^2$ (b) $BC^2 = 2AB^2$
(c) $AC^2 = 2AB^2$ (d) $AB^2 = 4AC^2$

41. In ΔPQR, $PD \perp QR$ such that D lies on QR, if $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, then the value of $(a+b)(a-b)$ is

(a) $\frac{c+d}{c-d}$ (b) $\frac{c^2-d^2}{c-d}$
(c) $(c+d)(c-d)$ (d) $\frac{c^2+d^2}{c^2-d^2}$

42. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm
(c) 17 cm (d) data insufficient

43. A flag pole 18 m high casts a shadow 9.6 m long. Then, the distance of the top of the pole from the far end of the shadow is **[NCERT Exemplar]**

(a) 18 m (b) 26 m
(c) 21 m (d) 20.4 m

44.

	List I		List II
P.	In ΔABC and ΔPQR $\frac{AB}{PQ} = \frac{AC}{PR}$, $\angle A = \angle P$ $\Rightarrow \Delta ABC \sim \Delta PQR$	1.	AA similarity criterion
Q.	In ΔABC and ΔPQR $\angle A = \angle P$, $\angle B = \angle Q$ $\Rightarrow \Delta ABC \sim \Delta PQR$	2.	SAS similarity criterion
R.	In ΔABC and ΔPQR $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ $\Rightarrow \Delta ABC \sim \Delta PQR$	3.	SSS similarity criterion
S.	In ΔABC, $DE \parallel BC$ $\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$	4.	BPT

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	1	3	4
(c)	4	3	2	1
(d)	4	3	2	4

45. If in a ΔABC, $DE \parallel BC$ and intersects AB at D and AC at E, then match the lists

	List I		List II
P.	$\frac{AD}{DB}$	1.	$\frac{AC}{AE}$
Q.	$\frac{AB}{AD}$	2.	$\frac{AE}{EC}$
R.	$\frac{DB}{AB}$	3.	$\frac{AB}{AC}$
S.	$\frac{AD}{AE}$	4.	$\frac{EC}{AC}$

Codes

	P	Q	R	S		P	Q	R	S
(a)	1	2	3	4	(b)	4	3	2	1
(c)	2	1	4	3	(d)	1	3	2	4

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-55) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

46. **Assertion** (A) All regular polygons of the same number of sides such as equilateral triangle, squares etc. are similar.

Reason (R) Two polygons are said to be similar, if their corresponding angles are equal and lengths of corresponding sides are proportional.

47. **Assertion** (A) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to third side.

Reason (R) Line segment joining the mid-point of any two sides of a triangle is parallel to the third side.

48. **Assertion** (A) $ABCD$ is a trapezium with $DC \parallel AB$. E and F are points on AD and BC respectively such that $EF \parallel AB$. Then, $\frac{AE}{ED} = \frac{BF}{FC}$.

Reason (R) Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

49. **Assertion** (A) In a ΔABC, if D is a point on BC such that D divides BC in the ratio $AB : AC$, then AD is the bisector of $\angle A$.

Reason (R) The external bisector of an angle of a triangle divides the opposite sides internally in the ratio of the sides containing the angle.

50. **Assertion** (A) If in a ΔABC, a line $DE \parallel BC$, intersects AB at D and AC at E, then $\frac{AB}{AD} = \frac{AC}{AE}$.

Reason (R) If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

51. **Assertion** (A) In a rhombus of side 15 cm, one of the diagonals is 20 cm long. The length of the second diagonal is $10\sqrt{6}$ cm.

Reason (R) The sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

52. **Assertion** (A) In ΔABC, $\angle B = 90°$ and $BD \perp AC$. If $AD = 4$ cm and $CD = 5$ cm, then BD is $2\sqrt{5}$ cm.

Reason (R) If a line divides any two sides of a triangle in the same ratio, then the line must not be parallel to the third side.

53. **Assertion (A)** ΔABC is an isosceles, right triangle, right angled at C. Then, $AB^2 = 2AC^2$.

Reason (R) In a right angled triangle, the cube of the hypotenuse is equal to the sum of the squares of the other two sides.

54. **Assertion (A)** ΔABC is a right triangle right angled at B. Let D and E be any points on AB and BC respectively. Then, $AE^2 + CD^2 = AC^2 + DE^2$.

Reason (R) In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

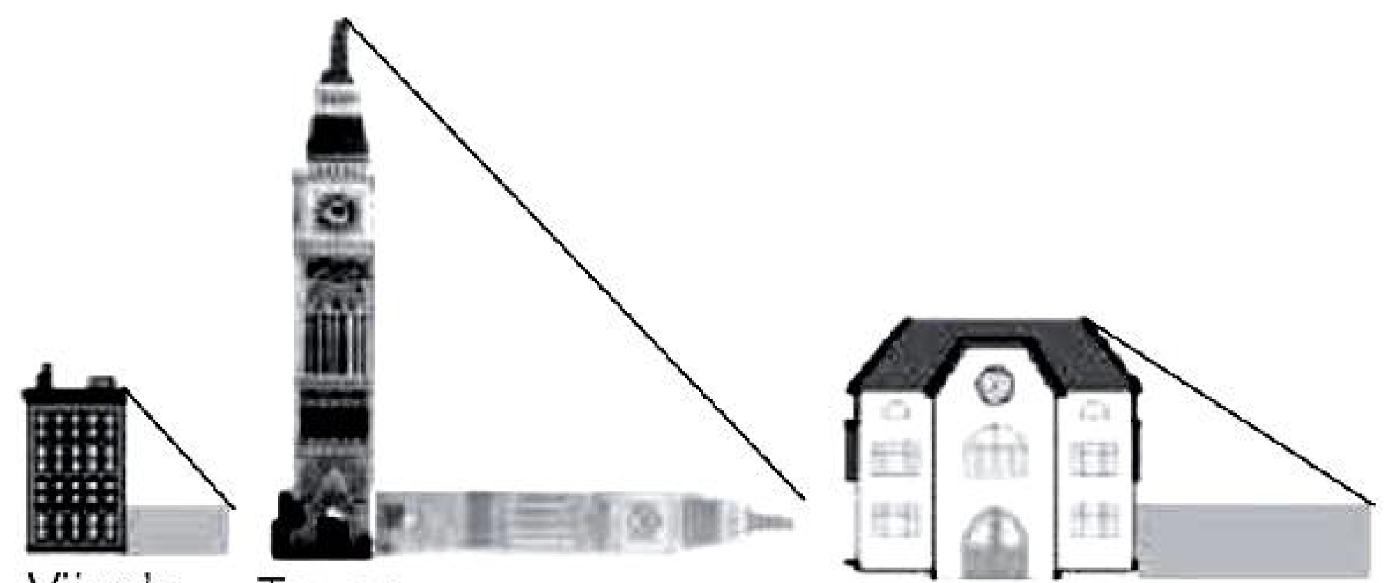
55. **Assertion (A)** In a ΔPQR, N is a point on PR such that $QN \perp PR$. If $PN \times NR = QN^2$, then $\angle PQR = 90°$.

Reason (R) In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two.

Case Based MCQs

56. Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house, if 20 m when Vijay's house casts a shadow 10m long on the ground.

At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.



Based on the above information, answer the following questions

(i) What is the height of the tower?

(a) 20 m (b) 50 m
(c) 100 m (d) 200 m

(ii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?

(a) 75 m (b) 50 m
(c) 45 m (d) 60 m

(iii) What is the height of Ajay's house?

(a) 30 m (b) 40 m
(c) 50 m (d) 20 m

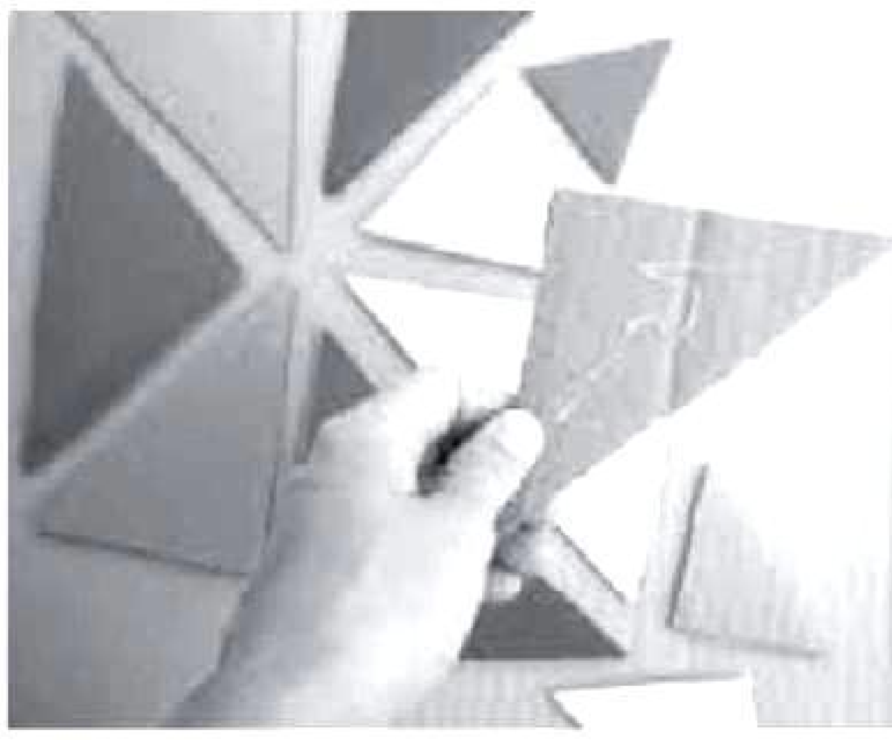
(iv) When the tower casts a shadow of 40 m, same time what will be the length of the shadow of Ajay's house?

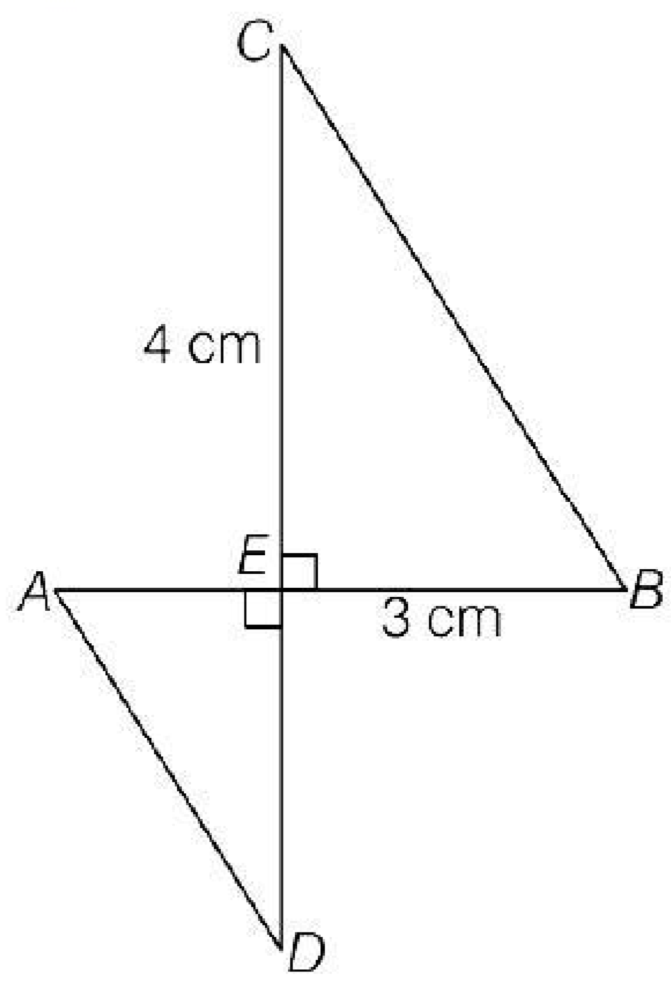
(a) 16 m (b) 32 m
(c) 20 m (d) 8 m

(v) When the tower casts a shadow of 40 m, same time what will be the length of the shadow of Vijay's house?

(a) 15 m (b) 32 m
(c) 16 m (d) 8 m

57. Aashi wants to make a toran for Home using some pieces of cardboard. She cuts some cardboard pieces as shown below. If perimeter of ΔADE and ΔBCE are in the ratio 4 : 3, then answer the following questions.





Based on the above information, answer the following questions

(i) If the two triangles here are similar by SAS similarity rule, then their corresponding proportional sides are

(a) $\frac{AE}{CE}=\frac{DE}{BE}$ (b) $\frac{BE}{AE}=\frac{CE}{DE}$

(c) $\frac{AD}{CE}=\frac{BE}{DE}$ (d) None of these

(ii) Length of BC =

(a) 20/3 cm (b) 4 cm

(c) 5 cm (d) None of these

(iii) Length of AD =

(a) 10/3 cm (b) 9/4 cm

(c) 5/3 cm (d) 20/3 cm

(iv) Length of ED =

(a) 4/3 cm (b) 8/3 cm

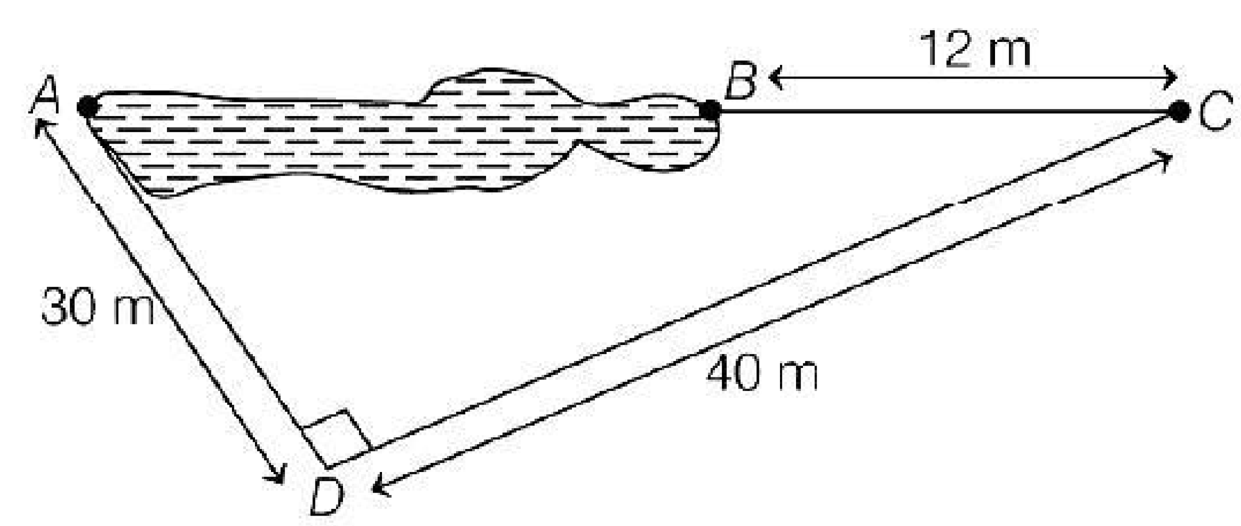
(c) 7/3 cm (d) 16/3 cm

(v) Length of AE =

(a) $\frac{2}{3}\times BE$ (b) $\sqrt{AD^2-DE^2}$

(c) $\frac{4}{3}\times\sqrt{BC^2-CE^2}$ (d) All of these

58. Rohan wants to measure the distance of a pond during the visit to his native. He marks points A and B on the opposite edges of a pond as shown in the figure below. To find the distance between the points, he makes a right-angled triangle using rope connecting B with another point C are a distance of 12 m, connecting C to point D at a distance of 40 m from point C and the connecting D to the point A which at distance of 30 m from D such that $\angle ADC = 90°$.



Based on the above information, answer the following questions

(i) Which property of geometry will be used to find the distance AC ?

(a) Similarity of triangles

(b) Thales Theorem

(c) Pythagoras Theorem

(d) Area of similar triangles

(ii) What is the distance AC ?

(a) 50 m (b) 12 m

(c) 100 m (d) 70 m

(iii) Which is the following does not form a Pythagoras triplet?

(a) (7,24,25)

(b) (15,8,17)

(c) (5,12,13)

(d) (21,20,28)

(iv) Find the length AB?

(a) 12 m (b) 38 m

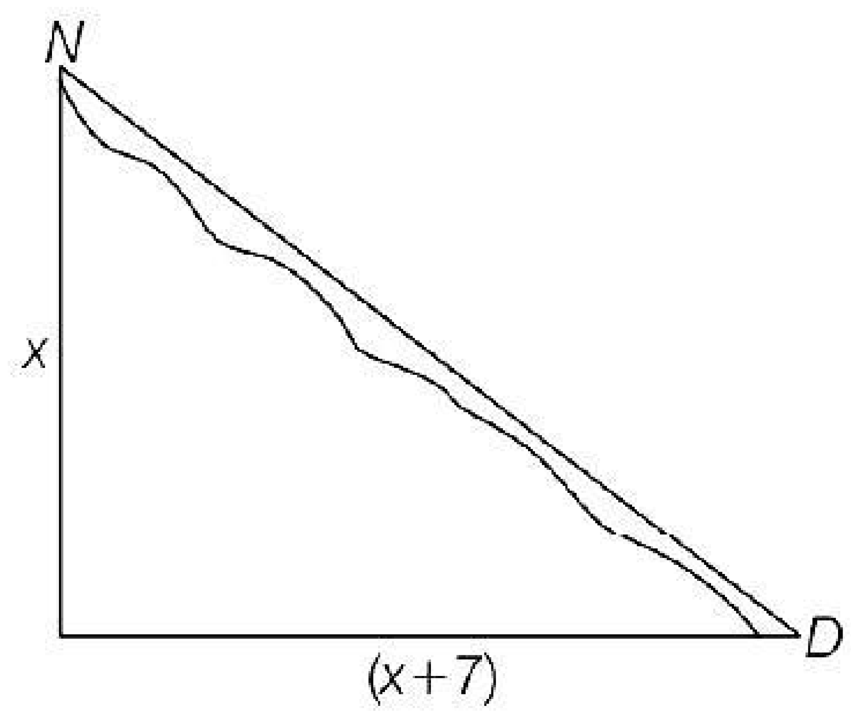
(c) 50 m (d) 100 m

(v) Find the length of the rope used.

(a) 120 m (b) 70 m

(c) 82 m (d) 22 m

59. D.M of a district went to town Noida from city Delhi. There is a route via town Ghaziabad such that $NG \perp GD$, $NG = x$ km and $GD = (x + 7)$ km. He noticed that there is proposal to construct a 17 km highway which directly connects the two towns Noida and Delhi.



Based on the above information, answer the following questions.

(i) Which concept can be used to get the value of x

(a) Thales theorem
(b) Pythagoras theorem
(c) Converse of thales theorem
(d) Converse of Pythagoras theorem

(ii) The value of x is

(a) 4 (b) 6
(c) 5 (d) 8

(iii) The value of NG is

(a) 10 km (b) 20 km
(c) 8 km (d) 25 km

(iv) The value of GD is

(a) 12 km (b) 24 km
(c) 16 km (d) 15 km

(v) How much distance will be saved in reaching city Delhi after the construction of highway?

(a) 6 km (b) 9 km
(c) 4 km (d) 8 km

60. A scale drawing of an object is the same shape at the object but a different size.

The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. The ratio of two corresponding sides in similar figures is called the scale factor.

Scale factor = length in image/ corresponding length in object

If one shape can become another using revising, then the shapes are similar. Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn. In the photograph below showing the side view of a train engine. Scale factor is 1 : 200.

This means that a length of 1 cm on the photograph above corresponds to a length of 200 cm or 2 m, of the actual engine. The scale can also be written as the ratio of two lengths.

Based on the above information, answer the following questions

(i) If the length of the model is 11cm, then the overall length of the engine in the photograph above, including the couplings(mechanism used to connect) is

(a) 22 cm
(b) 220 cm
(c) 220 m
(d) 22 m

(ii) What will affect the similarity of any two polygons?

(a) They are flipped horizontally
(b) They are dilated by a scale factor
(c) They are translated down
(d) They are not the mirror image of one another.

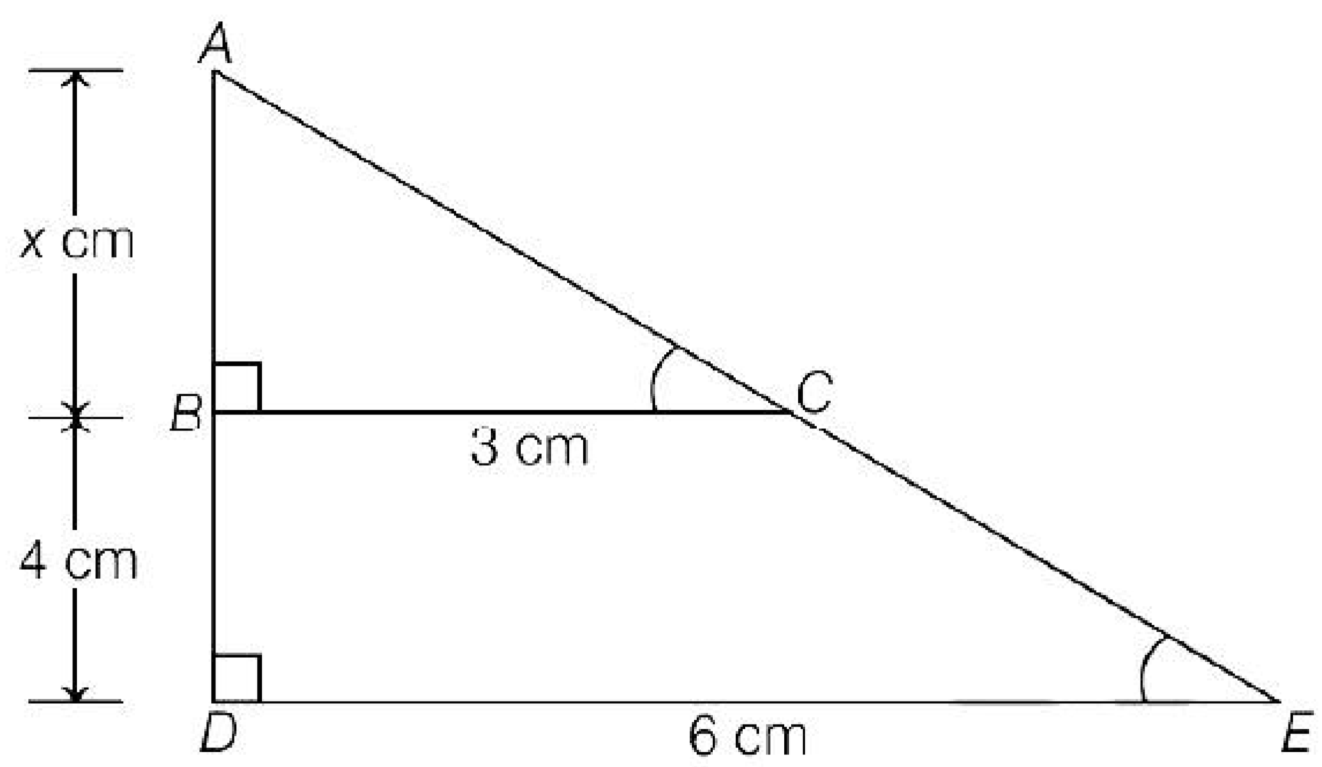
(iii) What is the actual width of the door, if the width of the door in photograph is 0.35 cm?

(a) 0.7 m
(b) 0.7 cm
(c) 0.07 cm
(d) 0.07 m

(iv) If two similar triangles have a scale factor 5 : 3 which statement regarding the two triangles is true?

(a) The ratio of their perimeters is 15 : 1
(b) Their altitudes have a ratio 25 :15
(c) Their medians have a ratio 10 : 4
(d) Their angle bisectors have a ratio 11 : 5

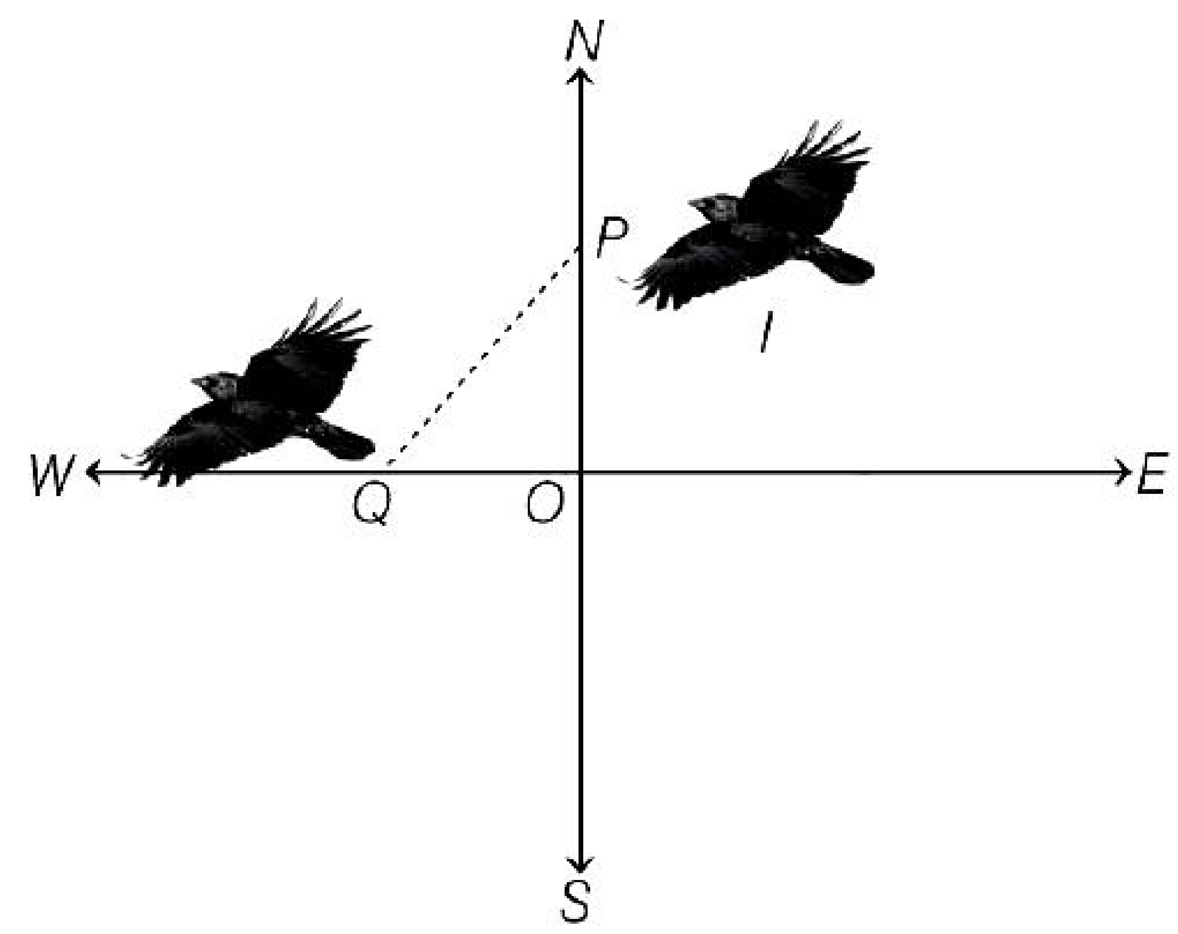
(v) The length of AB in the given figure



(a) 8 cm (b) 6 cm
(c) 4 cm (d) 10 cm

61. Application of Pythagoras Theorem

A crow leaves a tree and flies due north at a speed of 600 km/h. At the same time, another crow leaves the same place and flies due west at the speed 800 km/h as shown below. After $3\frac{1}{2}$ h both the crow reaches at point P and Q respectively.



Based on the above information, answer the following questions.

(i) Distance travelled by crow towards north after $3\frac{1}{2}$ h is

(a) 1800 km (b) 1500 km
(c) 1400 km (d) 2100 km

(ii) Distance travelled by crow towards west after $3\frac{1}{2}$ h is

(a) 1600 km (b) 2800 km
(c) 2250 km (d) 2625 km

(iii) In the given figure, $\angle POQ$ is

(a) 70° (b) 90°
(c) 80° (d) 100°

(iv) Distance between crow after $3\frac{1}{2}$ h is

(a) $450\sqrt{41}$ km (b) 3500 km
(c) $125\sqrt{12}$ km (d) $472\sqrt{41}$ km

(v) Area of ΔPOQ is

(a) 2940000 km^2 (b) 179000 km^2
(c) 186000 km^2 (d) 2025000 km^2

ANSWERS

Multiple Choice Questions

1. (a)	2. (b)	3. (a)	4. (b)	5. (c)	6. (a)	7. (b)	8. (a)	9. (c)	10. (b)
11. (a)	12. (c)	13. (b)	14. (b)	15. (a)	16. (b)	17. (d)	18. (c)	19. (c)	20. (a)
21. (c)	22. (c)	23. (b)	24. (c)	25. (a)	26. (b)	27. (a)	28. (c)	29. (d)	30. (c)
31. (c)	32. (b)	33. (d)	34. (a)	35. (c)	36. (b)	37. (d)	38. (a)	39. (b)	40. (b)
41. (c)	42. (b)	43. (d)	44. (b)	45. (c)					

Assertion-Reasoning MCQs

46. (a)	47. (b)	48. (a)	49. (c)	50. (a)	51. (d)	52. (c)	53. (c)	54. (a)	55. (b)

Case Based MCQs

56. (i) - (c); (ii) - (d); (iii) - (b); (iv) - (a); (v) - (d)
57. (i) - (b); (ii) - (c); (iii) - (d); (iv) - (d); (v) - (c)
58. (i) - (c); (ii) - (a); (iii) - (d); (iv) - (b); (v) - (c)
59. (i) - (b); (ii) - (d); (iii) - (c); (iv) - (d); (v) - (a)
60. (i) - (d); (ii) - (d); (iii) - (a); (iv) - (b); (v) - (c)
61. (i) - (d); (ii) - (b); (iii) - (b); (iv) - (b); (v) - (a)

SOLUTIONS

1. All congruent figures are called similar but all similar figures are not congruent.
2. Two triangles are similar, if their corresponding angles are equal not proportional.
3. All equilateral triangles are similar because all equilateral triangle have same shape but size can vary.
4. Two triangles are similar. If their corresponding sides are proportional.
5. Two quadrilaterals are similar, if their corresponding angles are equal and corresponding sides must also be proportional.
6. When two triangles are said to be similar, then corresponding angles are equal and corresponding sides are proportional.

   Option (a) : Corresponding angles are equal
   $\angle A = \angle P$; $\angle B = \angle R$; $\angle C = \angle Q$

   Option (b) : Corresponding sides are not proportional.
   $$\frac{AB}{PQ} = \frac{BC}{QR} \neq \frac{AC}{PR} \quad \text{[not correct]}$$

   Option (c) : Corresponding angles are not equal.
   $$\angle A \neq \angle P \quad \text{[not correct]}$$

   Option (d) : Corresponding sides are not proportional.
   $$\frac{AB}{PQ} \neq \frac{BC}{QR} \neq \frac{AC}{PR} \quad \text{[not correct]}$$

7. Here, $\frac{PQ}{AB} = \frac{2.5}{4} = \frac{25}{10 \times 4} = \frac{5}{8}$

   and $\frac{RQ}{BC} = \frac{2.5}{4} = \frac{25}{10 \times 4} = \frac{5}{8}$

   Clearly, the corresponding sides of quadrilaterals *ABCD* and *PQRS* are proportional but their corresponding angles are not equal. Hence, quadrilaterals *ABCD* and *PQRS* are not similar.

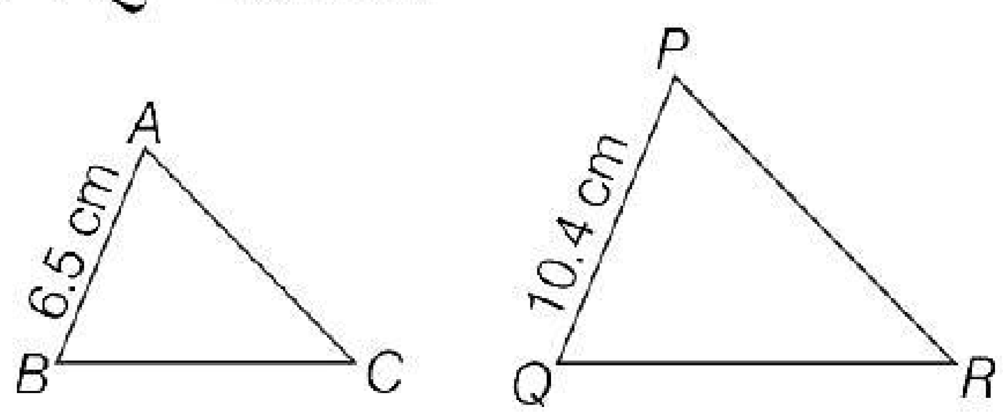
8. Here, the corresponding two sides and the perimeters of two triangles are proportional, then third side of both triangles will also in proportion.
9. Given, both figures are similar.

   So, the ratio of corresponding sides will be equal.

   $\therefore \quad \frac{3}{x} = \frac{5}{15}$

   $\Rightarrow \quad x = \frac{3 \times 15}{5} = 9 \text{ cm}$

10. Given, $\Delta ABC \sim \Delta PQR$ with $AB = 6.5$ cm and $PQ = 10.4$ cm



Since, $\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{6.5}{10.4} = \frac{65}{104}$

[$\because$ corresponding sides of similar triangles are proportional]

$\Rightarrow AB = \frac{65}{104} PQ, \; BC = \frac{65}{104} QR, AC = \frac{65}{104} PR$

Also given, perimeter of $\Delta ABC = 60$

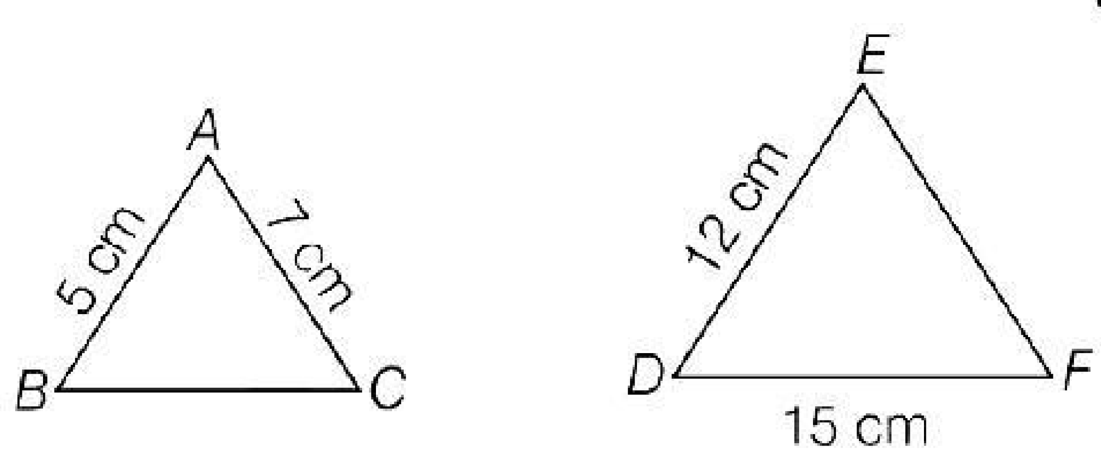
$\therefore \qquad AB + BC + AC = 60$

$\Rightarrow \qquad \frac{65}{104}(PQ + QR + PR) = 60$

$\Rightarrow \quad PQ + QR + PR = \frac{60 \times 104}{65} = 96$ cm

Hence, perimeter of ΔPQR is 96 cm.

11. Given, $\Delta ABC \sim \Delta EDF$ with $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm. ...(i)



Since, $\Delta ABC \sim \Delta EDF$,

$\therefore \quad \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$

[$\because$ corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \underset{\text{I}}{\frac{5}{12}} = \underset{\text{II}}{\frac{7}{EF}} = \underset{\text{III}}{\frac{BC}{15}}$ [from Eq. (i)]

On taking first I and II ratios, we get

$$\frac{5}{12} = \frac{7}{EF}$$

$\Rightarrow \quad EF = \frac{7 \times 12}{5} = 16.8$ cm

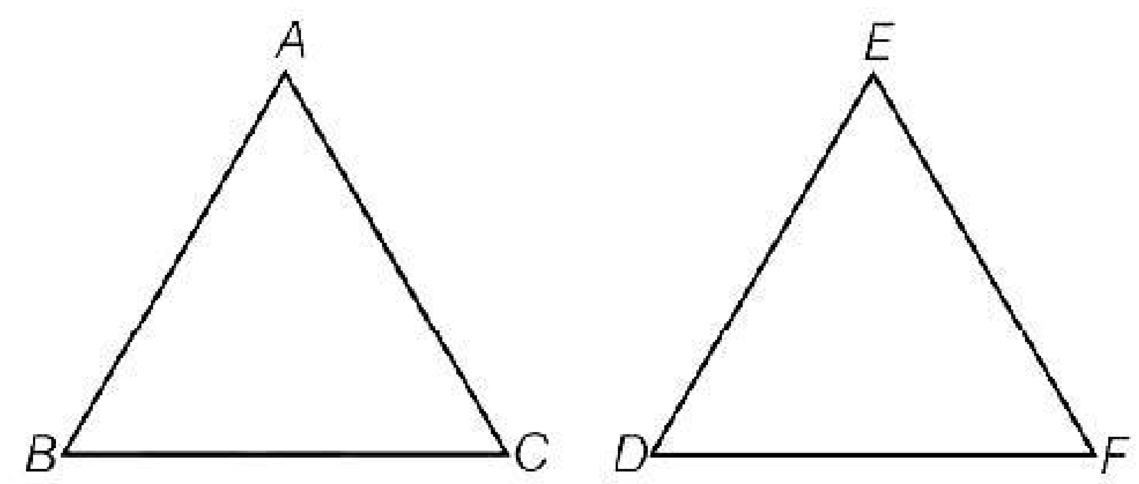
On taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15}$$

$\Rightarrow \qquad BC = \frac{5 \times 15}{12} = 6.25$ cm

Hence, lengths of the remaining sides of the triangles are $EF = 16.8$ cm and $BC = 6.25$ cm.

12. Given, in ΔABC and ΔEDF, $\frac{AB}{DE} = \frac{BC}{FD}$



By converse of basic proportionality theorem,

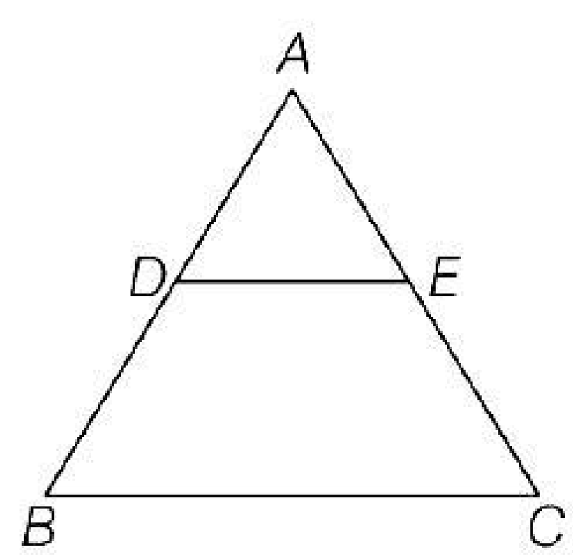
$$\Delta ABC \sim \Delta EDF$$

Then, $\quad \angle B = \angle D, \angle A = \angle E$ and $\angle C = \angle F$

$\therefore \qquad \angle B = \angle D$

13. (converse of BPT) If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{(given)}$$



then, $DE \parallel BC$

14. Given in $\Delta ABC \; DE \parallel BC$,

Now, $\quad \frac{AD}{DB} = \frac{AE}{EC}$ [by Thales theorem]

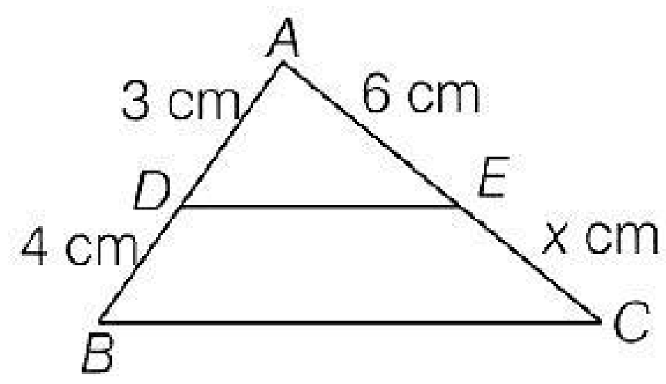
$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow \quad x(x-1) = (x-2)(x+2)$

$\Rightarrow \quad x^2 - x = x^2 - 4$

$\therefore \qquad x = 4$

15. In ΔABC, $DE \parallel BC$



Let $EC = x$ cm, then $\frac{AD}{DB} = \frac{AE}{EC}$

[by basic proportionality theorem]

$\Rightarrow \quad \frac{3}{4} = \frac{6}{x}$

$\Rightarrow \quad x = \frac{4 \times 6}{3} = 8$ cm

$\therefore \quad EC = 8$ cm

16. Given, in ΔABC, $DE \parallel AC$

So, $\frac{BE}{EC} = \frac{BD}{DA}$ ...(i)

[by basic proportionality theorem]

Also, in ΔABP,

$DC \parallel AP$ [given]

So, $\frac{BC}{CP} = \frac{BD}{DA}$ ...(ii)

[by basic proportionality theorem]

From Eqs. (i) and (ii), we get

$\frac{BE}{EC} = \frac{BC}{CP}$

17. Given, $ST \parallel QR$, $PS/SQ = 3/5$ and $PR = 28$ cm

By using basic proportionality theorem, we get

$\frac{PS}{SQ} = \frac{PT}{TR} \Rightarrow \frac{PS}{SQ} = \frac{PT}{PR - PT}$

$\Rightarrow \quad \frac{3}{5} = \frac{PT}{28 - PT}$

$\Rightarrow \quad 3(28 - PT) = 5PT$

$\Rightarrow \quad 84 = 5PT + 3PT$

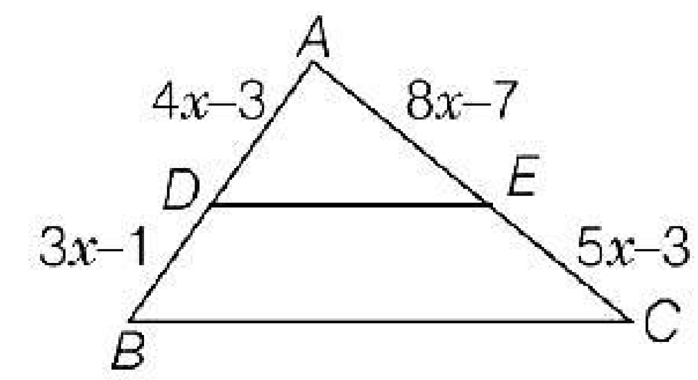
$\Rightarrow \quad PT = \frac{84}{8}$

$\Rightarrow \quad PT = 10.5$ cm

Hence, the length of PT is 10.5 cm.

18. Given, in ΔABC, $DE \parallel BC$

$\therefore$ By Thales theorem, we get



$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4x - 3}{3x - 1} = \frac{8x - 7}{5x - 3}$

[$\because AD = 4x - 3, DB = 3x - 1, AE = 8x - 7, EC = 5x - 3$]

$\Rightarrow \quad (4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$\Rightarrow \quad 20x^2 - 12x + 9 - 15x = 24x^2 - 21x - 8x + 7$

$\Rightarrow \quad 4x^2 - 2x - 2 = 0$

$\Rightarrow \quad 2x^2 - x - 1 = 0$

[dividing both sides by 2]

$\Rightarrow \quad 2x^2 - 2x + x - 1 = 0$

[by splitting the middle term]

$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$

$\Rightarrow \quad (2x + 1)(x - 1) = 0$

$\Rightarrow \quad x = -\frac{1}{2}$ or $x = 1$

If $x = -\frac{1}{2}$, then $AD = 4 \times \left(-\frac{1}{2}\right) - 3 = -5 < 0$

[not possible; since, length cannot be negative]

Hence, $x = 1$ is the required value.

19. Given, $AB \parallel CD$

$\therefore$ Quadrilateral $ABCD$ is a trapezium

$\therefore \quad \frac{AO}{CO} = \frac{BO}{DO}$

[diagonals of trapezium divides each other proportionally]

$\Rightarrow \quad \frac{4}{4x - 2} = \frac{x + 1}{2x + 4}$

$\Rightarrow \quad 4(2x + 4) = (x + 1)(4x - 2)$

$\Rightarrow \quad 8x + 16 = 4x^2 - 2x + 4x - 2$

$\Rightarrow \quad 4x^2 - 6x - 18 = 0$

$$\Rightarrow \quad 2x^2 - 3x - 9 = 0$$
$$\Rightarrow \quad 2x^2 - 6x + 3x - 9 = 0$$
$$\Rightarrow \quad 2x(x-3) + 3(x-3) = 0$$
$$\Rightarrow \quad (2x+3)(x-3) = 0$$
$$\Rightarrow \quad x = 3 \text{ and } x = -\frac{3}{2} \text{ (not possible)}$$

Hence, the value of $x = 3$.

20. In the given figure, $\frac{AQ}{AC} = \frac{AP}{AB} = \frac{1}{3}$

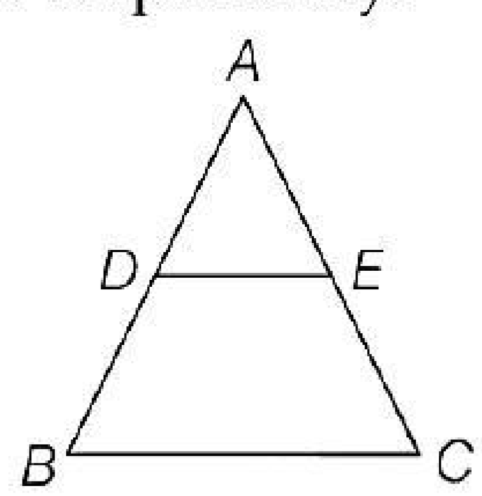
Therefore, by converse of basic proportionality theorem, we have

$$QP \parallel CB$$

Hence, $\Delta AQP \sim \Delta ACB$

$$\frac{AQ}{AC} = \frac{AP}{AB} = \frac{QP}{CB}$$
$$\Rightarrow \quad \frac{1}{3} = \frac{4.5}{BC}$$
$$BC = 13.5 \text{ cm}$$

21. Let ΔABC and D and E be mid-points of AB and AC respectively.



$$AD = DB \text{ and } AE = EC$$
$$\Rightarrow \quad \frac{AD}{DB} = 1 \text{ and } \frac{AE}{EC} = 1$$
$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$
$$\Rightarrow \quad DE \parallel BC$$

[by the converse of basic proportionality theorem]

22. Given, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$

Since, $\frac{PS}{SQ} = \frac{PT}{TR}$

$\therefore \quad ST \parallel QR$

[by converse of basic proportionality theorem]

Then, $\angle PST = \angle PQR$ [corresponding angles] ...(i)

Also, $\angle PST = \angle PRQ$ [given] ...(ii)

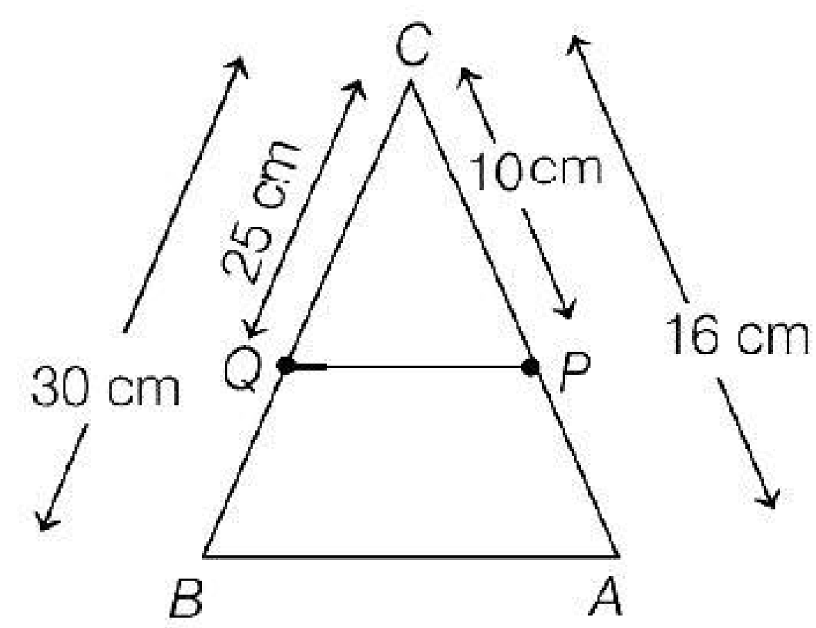
From Eqs. (i) and (ii), we get

$$\angle PRQ = \angle PQR$$
$$\Rightarrow \quad PQ = PR$$

[since, sides opposite to equal angles of a triangle are also equal]

Hence, ΔPQR is an isosceles triangle.

23. Given, $CQ = 25$ cm, $CB = 30$ cm, $CP = 10$ cm and $CA = 16$ cm



Here, $\frac{CQ}{CB} = \frac{25}{30} = \frac{5}{6}$

and $\frac{CP}{CA} = \frac{10}{16} = \frac{5}{8} \Rightarrow \frac{CQ}{CB} \neq \frac{CP}{CA}$

$$\Rightarrow \quad \frac{CB}{CQ} \neq \frac{CA}{CP} \Rightarrow \frac{CB}{CQ} - 1 \neq \frac{CA}{CP} - 1$$
$$\Rightarrow \frac{CB - CQ}{CQ} \neq \frac{CA - CP}{CP}$$
$$\Rightarrow \quad \frac{QB}{CQ} \neq \frac{PA}{CP} \text{ or } \frac{CQ}{QB} \neq \frac{CP}{PA}$$

Hence, by converse of basic proportionality theorem, PQ is not parallel to AB.

24. Here, $\frac{AB}{DF} = \frac{4}{6} = \frac{2}{3}$, $\frac{BC}{EF} = \frac{5}{7.5} = \frac{2}{3}$,
$\frac{AC}{DE} = \frac{3}{4.5} = \frac{2}{3}$

As, $\frac{AB}{DF} = \frac{BC}{EF} = \frac{AC}{DE}$

So, $\Delta ABC \sim \Delta DFE$

[by SSS similarity criterion]

Hence, figures (i) and (ii) are similar triangles, but no other pairs of triangles in the given figure are similar.

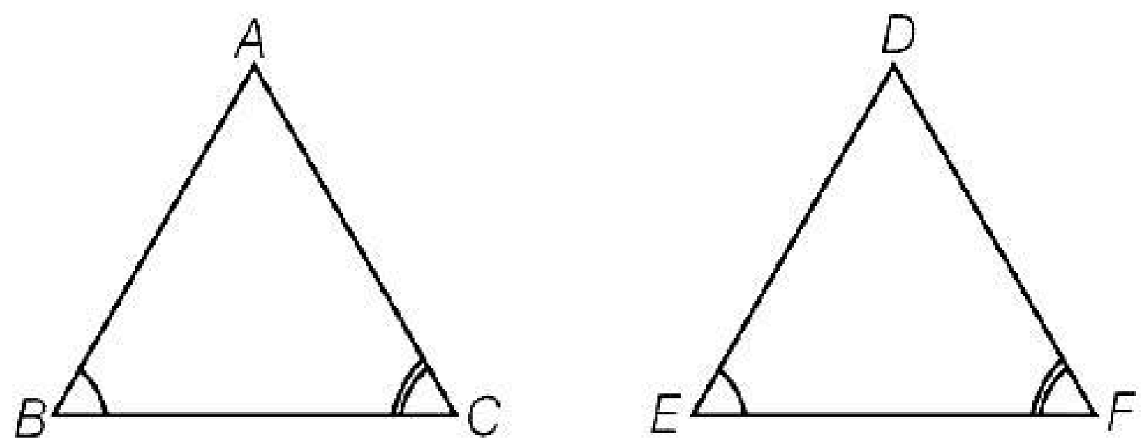
25. Here, $\frac{AB}{EF}=\frac{6}{4.5}=\frac{60}{45}=\frac{4}{3}$, $\frac{BC}{DE}=\frac{4}{3}$

$\Rightarrow \frac{AB}{EF}=\frac{BC}{DE}$ and $\angle ABC=\angle FED$ [given]

So, $\Delta ABC \sim \Delta FED$

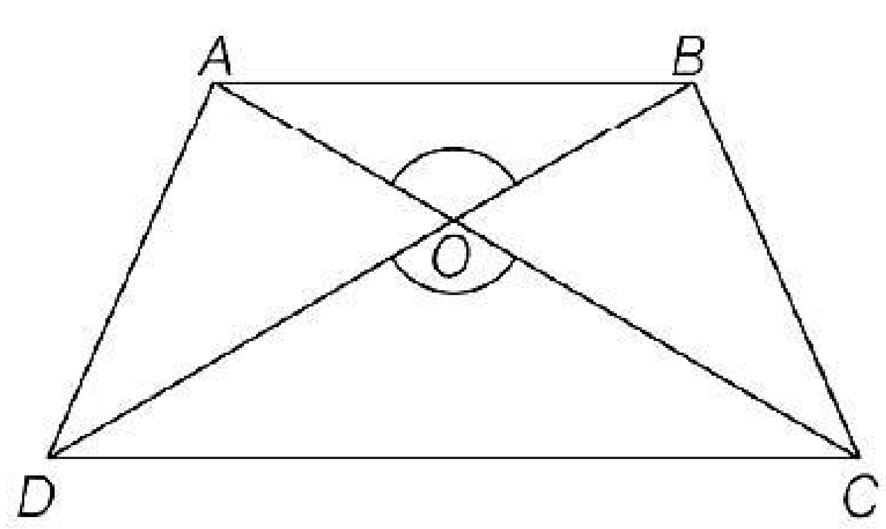
[by SAS similarity criterion]

26. In ΔABC and ΔDEF, $\angle B=\angle E$, $\angle F=\angle C$ and $AB=3DE$



We know that, if in two triangles corresponding two angles are same, then they are similar by AAA similarity criterion. Also, ΔABC and ΔDEF do not satisfy any rule of congruency, (SAS, ASA, SSS), so both are not congruent.

27.



In ΔAOB and ΔOCD,

$\angle AOB=\angle COD$ [vertically opposite angle]

$\angle ABO=\angle CDO$ [$AB||CD$, alternate angle]

$\angle BAO=\angle OCD$ [$AB||CD$, alternate angle]

$\Delta OAB \sim \Delta OCD$ (AAA similarity)

Then, $\frac{OA}{OC}=\frac{OB}{OD}$

[corresponding sides are proportional].

28. In ΔADE and ΔABC,

$\angle ADE=\angle B$ [given]

and $\angle A=\angle A$ [common]

$\Delta ADE \sim \Delta ABC$ [by AA corollary]

$\therefore \frac{AD}{AB}=\frac{DE}{BC}$

[sides of similar triangles are proportional]

$\Rightarrow \frac{AD}{AE+EB}=\frac{DE}{BC}$

$\Rightarrow \frac{6.8}{8.6+2.4}=\frac{DE}{5.5}$

$\Rightarrow DE=\frac{6.8\times 5.5}{8.6+2.4}$ cm $=3.4$ cm

Hence, $DE=3.4$ cm

29. Given, $BD=8$ cm and $AD=4$ cm

In ΔADB and ΔBDC,

$\angle BDA=\angle CDB$ [each 90°]

$\angle DBA=\angle DCB$ [each $(90°-\angle A)$]

$\therefore \Delta ADB \sim \Delta BDC$

[by AA similarity criterion]

$\Rightarrow \frac{BD}{CD}=\frac{AD}{BD}$

[since, corresponding sides of similar triangles are proportional]

$\Rightarrow CD=\frac{BD^2}{AD}$

$\therefore CD=\frac{8^2}{4}=\frac{64}{4}=16$ cm

30. Since, the height h is measured vertically, so $\angle EDA$ is a right angle. We assume that the net (i.e. CB) is vertical.

Here, ΔADE and ΔABC are similar

[by AA similarity criterion]

$\therefore \frac{DE}{BC}=\frac{AD}{AB}$

[since, corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{h}{0.9}=\frac{18}{6} \Rightarrow \frac{h}{0.9}=3$

$\Rightarrow h=0.9\times 3=2.7$ m

Hence, the height at which the ball should be hit, is 2.7 m.

31. Given, $AB||DC$ and AC and PQ intersect each other at point O.

Now, in ΔAOP and ΔCOQ

$\angle AOP=\angle COQ$ [Vertical opposite angles]

$\angle APO=\angle CQO$

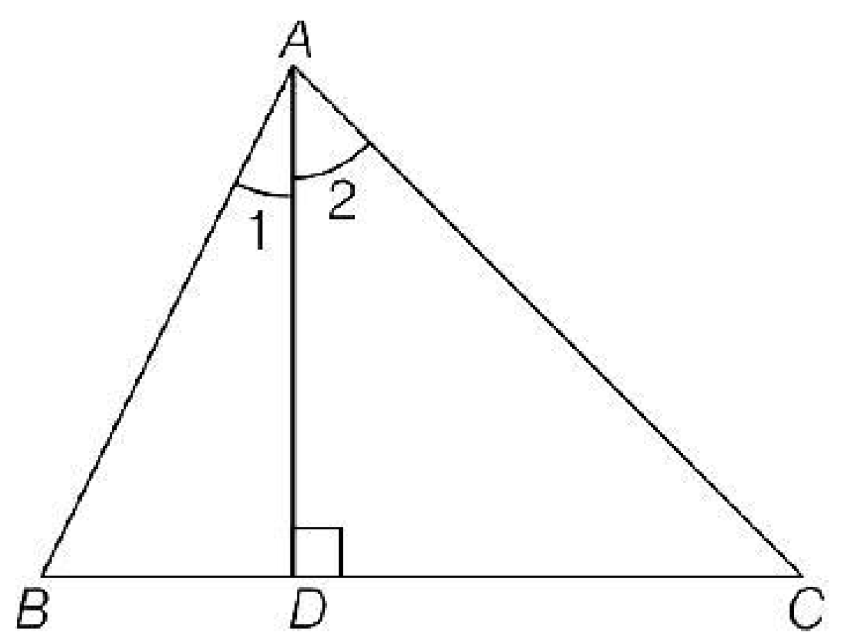
[$\because$ alternate angle, $AB||DC$ and PQ is a transversal]

$\therefore \Delta AOP \sim \Delta COQ$ [from AAA similarity] then, $\frac{OA}{OC} = \frac{AP}{CQ}$

[$\because$ corresponding sides are proportional]

$\Rightarrow OA \cdot CQ = OC \cdot AP$

32. Given, ΔABC in which $AD \perp BC$ and $AD^2 = BD \cdot CD$.



Now, in ΔDBA and ΔDAC, we have

$$\angle BDA = \angle ADC = 90^\circ$$

$$\frac{BD}{AD} = \frac{AD}{CD}$$

$\Delta DBA \sim \Delta DAC$ [by SAS similarity]

$\angle B = \angle 2$ and $\angle 1 = \angle C$

$\angle 1 + \angle 2 = \angle B + \angle C$

$\angle A = \angle B + \angle C$

$2\angle A = \angle A + \angle B + \angle C = 180^\circ$

$\angle A = \frac{180^\circ}{2} = 90^\circ$

33. $\angle DOC + \angle COB = 180^\circ$ [linear pair]

$\Rightarrow \angle DOC + 125^\circ = 180^\circ$

Hence, $\angle DOC = 180^\circ - 125^\circ = 55^\circ$

Again, $\angle DCO + \angle CDO + \angle DOC = 180^\circ$

[angles of a triangle]

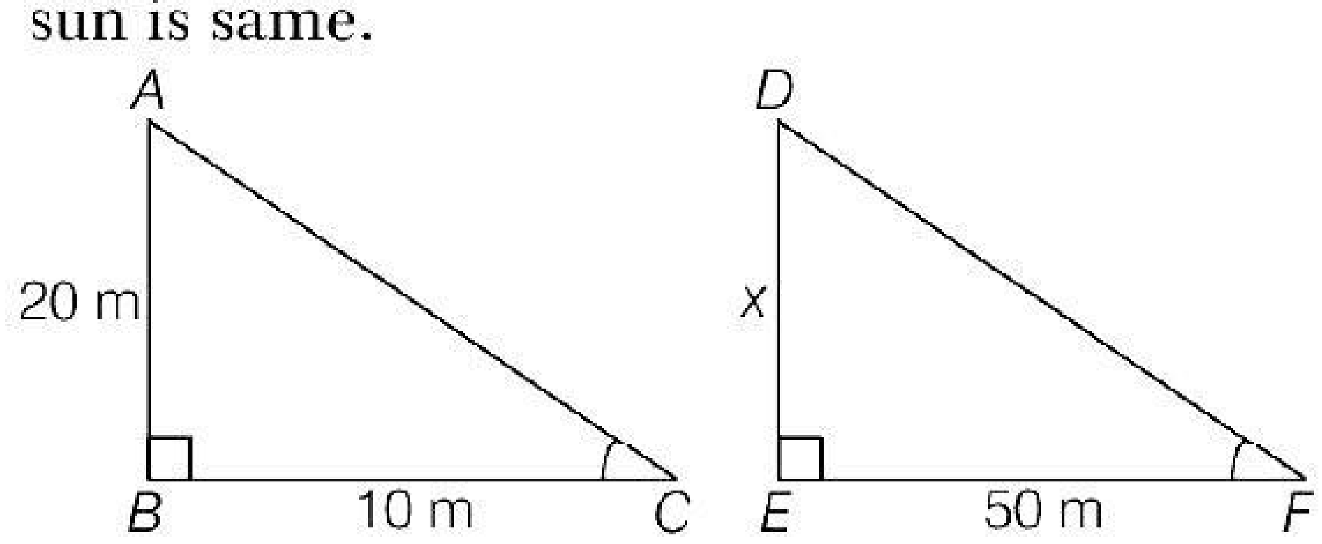
$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$

Hence, $\angle DCO = 180^\circ - 125^\circ = 55^\circ$ ...(i)

$\because$ $\Delta ODC \sim \Delta OBA$ [given]

$\therefore$ $\angle OAB = \angle OCD = 55^\circ$ [from Eq. (i)]

34. Let the height of the tower be x. As $\angle B = \angle E = 90^\circ$ and angle of elevation of sun is same.



In ΔABC and ΔDEF,

$\angle B = \angle E$ [right angle]

$\angle C = \angle F$ [angle of elevation]

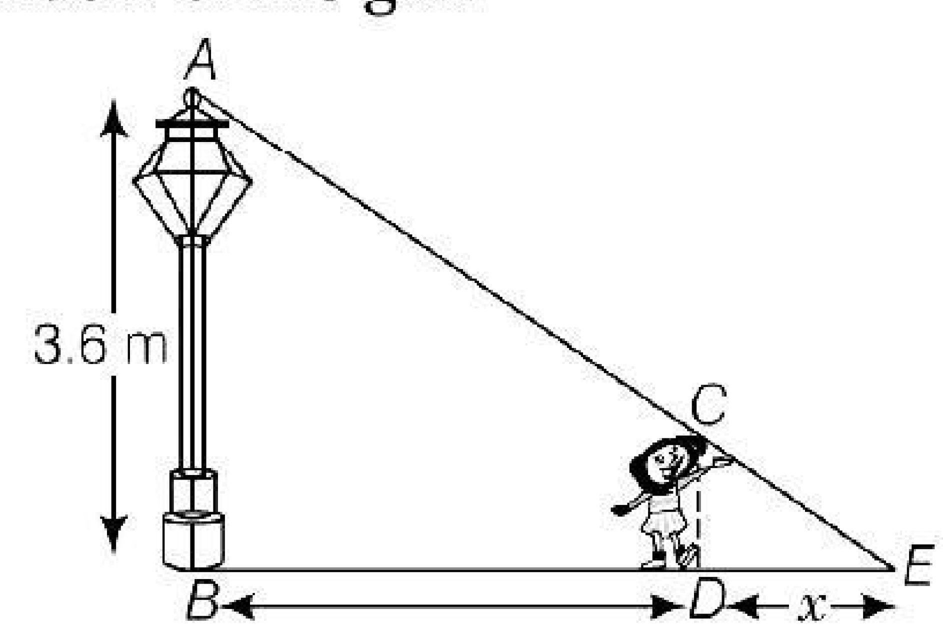
$\Delta ABC \sim \Delta DEF$ [from AA similarity]

$\therefore$ $\frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{20}{x} = \frac{10}{50}$

$\Rightarrow$ $x = \frac{20 \times 50}{10} = 100$ m

35. Let AB be the lamp-post, CD be the girl and D be the position of girl after 4s.

Again, let $DE = x$ m be the length of shadow of the girl.



Given, $CD = 90$ cm $= 0.9$ m, $AB = 3.6$ m

and speed of the girl $= 1.2$ m/s

$\therefore$ Distance of the girl from lamp-post after 4s,

$BD = 1.2 \times 4 = 4.8$ m

[$\because$ distance = speed $\times$ time]

In ΔABE and ΔCDE,

$\angle B = \angle D$ [each 90°]

$\angle E = \angle E$ [common angle]

$\therefore$ $\Delta ABE \sim \Delta CDE$

[by AA similarity criterion]

$\Rightarrow$ $\frac{BE}{DE} = \frac{AB}{CD}$ ...(i)

[since, corresponding sides of similar triangles are proportional]

On substituting all the values in Eq. (i), we get

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9}$$

[$\because BE = BD + DE = 4.8 + x$]

$\Rightarrow$ $\frac{4.8 + x}{x} = 4$

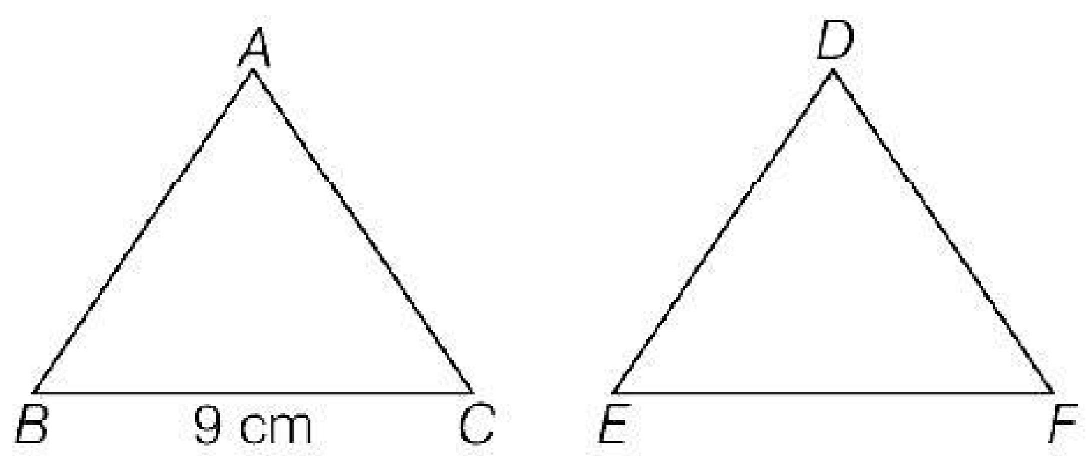
$\Rightarrow$ $4.8 + x = 4x$

$\Rightarrow$ $3x = 4.8$

$\Rightarrow \quad x = \frac{4.8}{3} = 1.6$ m

Hence, the length of her shadow after 4s is 1.6 m.

36.



Given,

$$\Delta ABC \sim \Delta DEF$$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \text{...(i)}$$

From eq. (i),

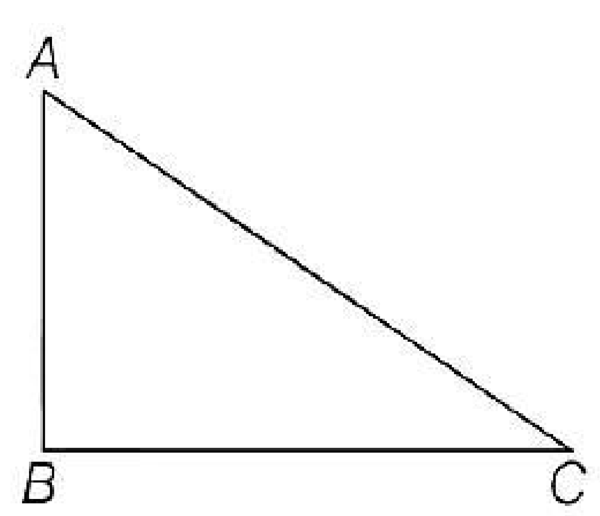
$$\frac{AB + BC + CA}{DE + EF + DF} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{30}{18} \quad \text{...(ii)}$$

Therefore, $\frac{\text{Perimeter } (\Delta ABC)}{\text{Perimeter } (\Delta DEF)} = \frac{BC}{EF}$

$$\Rightarrow \quad \frac{30}{18} = \frac{9}{EF}$$

$$\Rightarrow \quad EF = \frac{18 \times 9}{30} = 5.4 \text{ cm}$$

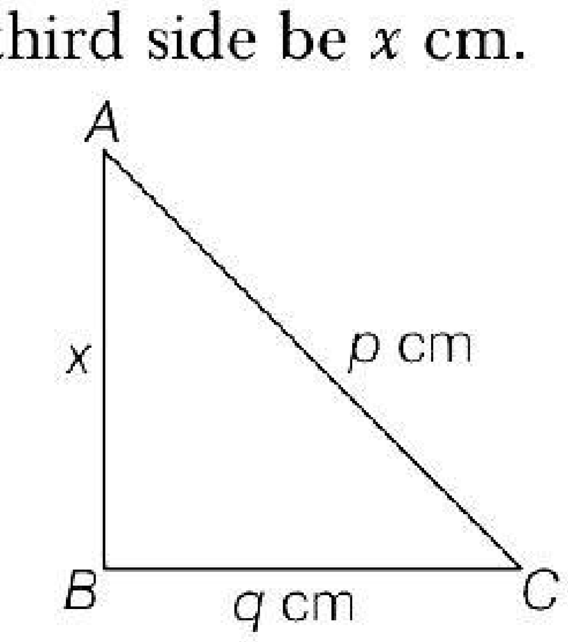
37.



$$AC^2 = AB^2 + BC^2$$

[by Pythagoras theorem]

38. Let the third side be x cm.



$p^2 = q^2 + x^2$ [by Pythagoras theorem]

$$\Rightarrow \quad x^2 = p^2 - q^2$$

$$\Rightarrow \quad x^2 = (p - q)(p + q)$$

$$\Rightarrow \quad x = \sqrt{(p + q)} \quad [\because p - q = 1]$$

$$\Rightarrow \quad x = \sqrt{(1 + q) + q} \quad [p = 1 + q]$$

$$\Rightarrow \quad x = \sqrt{2q + 1} \text{ cm}$$

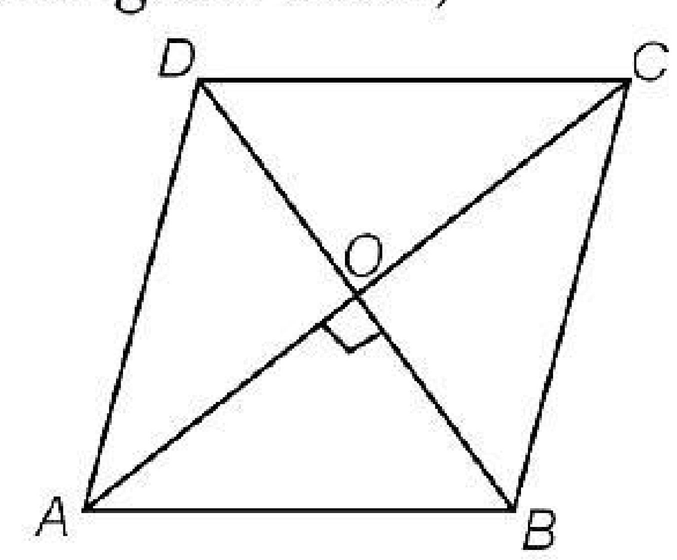
39. We know that, the diagonals of a rhombus are perpendicular bisector of each other.

Given, $AC = 16$ cm and $BD = 12$ cm [let]

$\therefore \quad AO = 8$ cm, $BO = 6$ cm

and $\angle AOB = 90°$

In right angled ΔAOB,



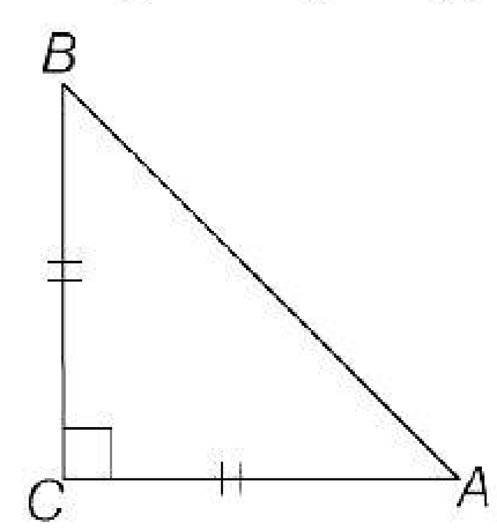
$$AB^2 = AO^2 + OB^2$$

[by Pythagoras theorem]

$$\Rightarrow \quad AB^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$\therefore \quad AB = 10 \text{ cm}$$

40. In right ΔACB, use Pythagoras theorem



$$AB^2 = AC^2 + BC^2$$

$$= AC^2 + (AC)^2 \quad [\because BC = AC = \text{given}]$$

$$= 2AC^2$$

41. Given, in ΔPQR, $PD \perp QR$, $PQ = a$, $PR = b$, $QD = c$ and $DR = d$

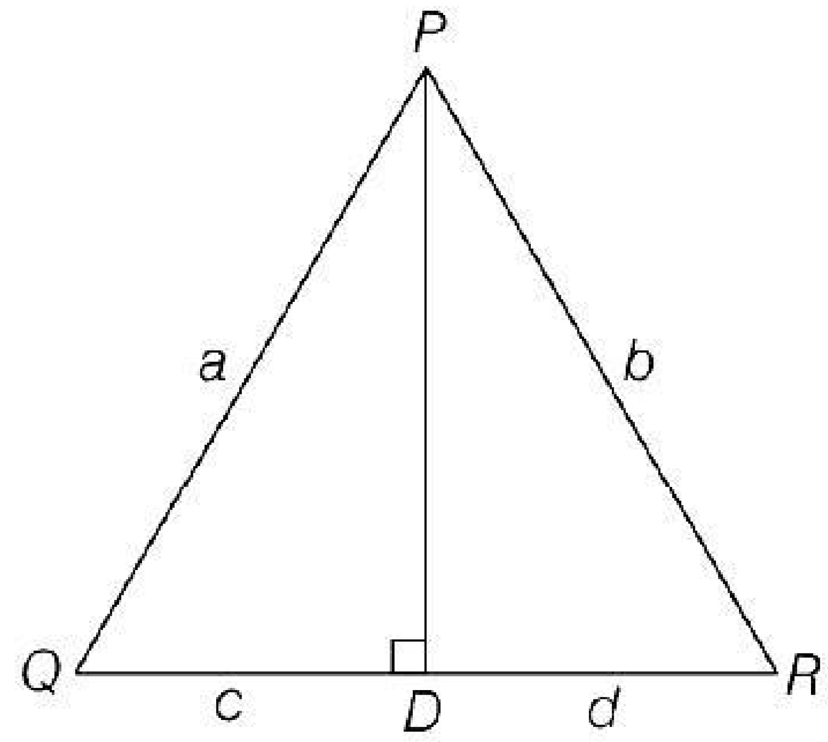
In right angled ΔPOQ,

$$PQ^2 = PD^2 + QD^2$$

[by Pythagoras theorem]

$$\Rightarrow \quad a^2 = PD^2 + c^2$$

$\Rightarrow \quad PD^2 = a^2 - c^2 \quad \ldots(i)$



In right angled ΔPDR,

$$PR^2 = PD^2 + DR^2$$

[by Pythagoras theorem]

$\Rightarrow \quad b^2 = PD^2 + d^2$

$\Rightarrow \quad PD^2 = b^2 - d^2 \quad \ldots(ii)$

From Eqs. (i) and (ii),

$$a^2 - c^2 = b^2 - d^2$$

$\Rightarrow \quad a^2 - b^2 = c^2 - d^2$

$\Rightarrow \quad (a - b)(a + b) = (c - d)(c + d)$

42. Suppose hypotenuse of the triangle is c and other sides are a and b, obviously.

$$c = \sqrt{a^2 + b^2} \quad \ldots(i)$$

We have,

$$a + b + c = 40$$

and $\quad \frac{1}{2} ab = 40 \Rightarrow ab = 80$

or $\quad 40 - (a + b) = \sqrt{a^2 + b^2}$

$\Rightarrow \quad c = 40 - (a + b)$ and $ab = 80$

or $\quad 40 - (a + b) = \sqrt{a^2 + b^2}$ [from Eq. (i)]

By squaring the above equation both sides

$\Rightarrow (a + b)^2 - 2 \times 40(a + b) + 1600 = a^2 + b^2$

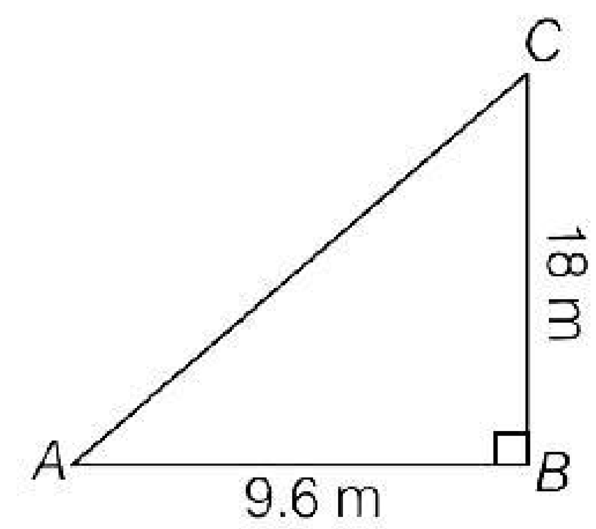
$\Rightarrow a^2 + b^2 + 2 \times 80 - 80(a + b) + 1600 = a^2 + b^2$

$\Rightarrow \quad 80[(a + b) - 2] = 1600$

$\Rightarrow \quad a + b = 20 + 2 = 22$

$\therefore \quad c = 40 - (a + b) = 40 - 22 = 18$ cm

43. Let $BC = 18$ m be the flag pole and its shadow be $AB = 9.6$ m. The distance of the top of the pole, C from the far end i.e. A of the shadow is AC.



In right angled ΔABC, $AC^2 = AB^2 + BC^2$

[by Pythagoras theorem]

$\Rightarrow \quad AC^2 = (9.6)^2 + (18)^2$

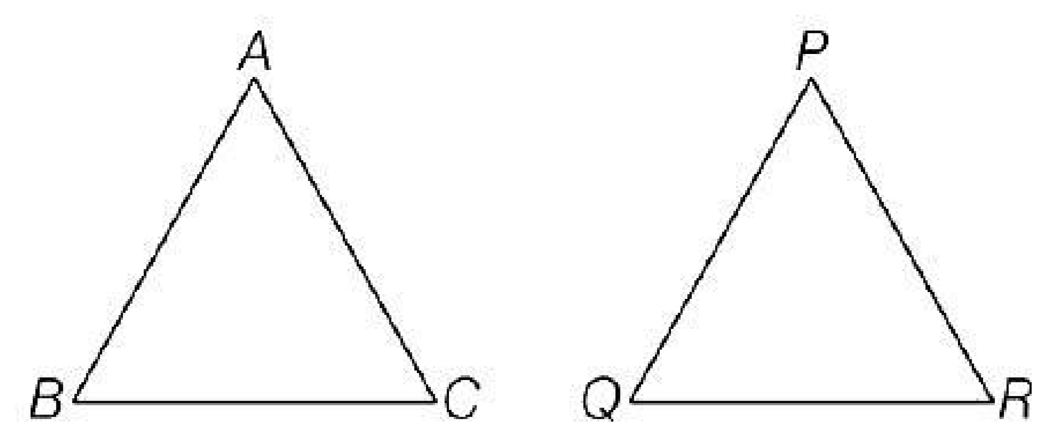
$\quad AC^2 = 92.16 + 324$

$\Rightarrow \quad AC^2 = 416.16$

$\therefore \quad AC = \sqrt{416.16} = 20.4$ m

Hence, the required distance is 20.4 m.

44. (P)



Given, $\frac{AB}{PQ} = \frac{AC}{PR}$,

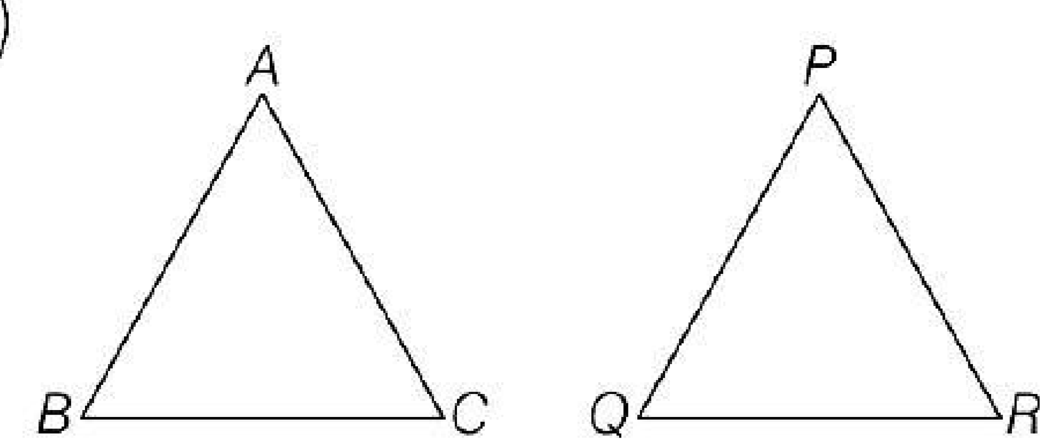
$\angle A = \angle P$

$\because$ $\angle A$ is containing the sides AB and AC and $\angle P$ is containing the sides PQ and PR.

$\therefore \quad \Delta ABC - \Delta PQR$

[by SAS criterion of similarity]

(Q)

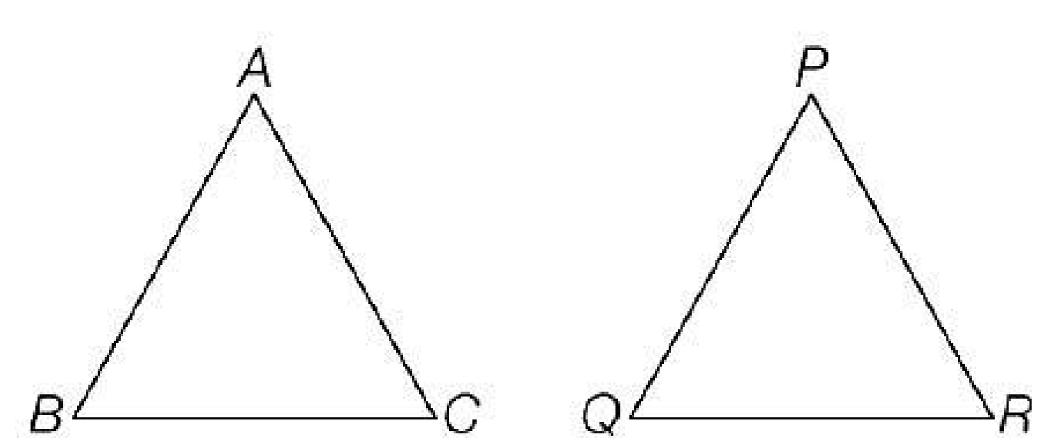


Given, $\angle A = \angle P$, $\angle B = \angle Q$

$\therefore \quad \Delta ABC - \angle PQR$

[by AA criterion of similarity]

(R)



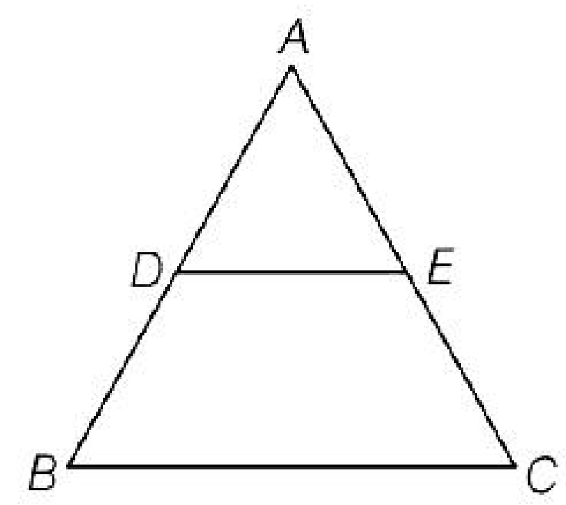
Given, $\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{BC}{QR}$

$\because$ Sides of the ΔABC and ΔPQR are in proportion

$\therefore$ $\Delta ABC \sim \Delta PQR$

[by SSS criterion of similarity]

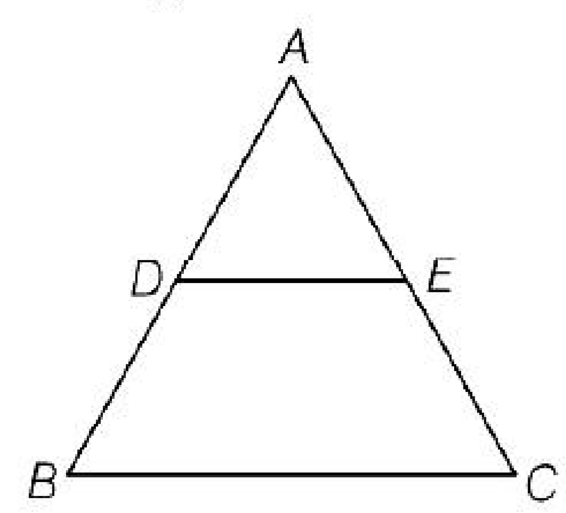
(S)



Given, $DE \parallel BC$

$\therefore \quad \dfrac{AD}{BD} = \dfrac{AE}{EC}$ [by BPT]

45. In ΔABC, $BC \parallel DE$



(P) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (BPT)

(Q) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \dfrac{DB}{AD} = \dfrac{EC}{AE} \Rightarrow \dfrac{DB}{AD} + 1 = \dfrac{EC}{AE} + 1$

$\Rightarrow \dfrac{BD + AD}{AD} = \dfrac{EC + AE}{AE} \Rightarrow \dfrac{AB}{AD} = \dfrac{AC}{AE}$

(R) Similarly, $\dfrac{DB}{AB} = \dfrac{EC}{AC}$

(S) $\dfrac{AD}{AB} = \dfrac{AE}{AC} \Rightarrow \dfrac{AD}{AE} = \dfrac{AB}{AC}$

46. Two polygons are similar, if their corresponding angles are equal and sides are proportional.

$\because$ In equilateral triangle and square, each angle are equal and sides are also equal therefore, regular polygons are similar.

Statement I is True and Statement II is True and Statement II is the correct explanation of Statement I.

47. Statement I is a basic proportionality theorem and Statement II is a mid-point theorem. But mid-point theorem is not the correct explanation of BPT.

Statement I is True and Statement II is True but statement II is not the correct explanation of Statement I.

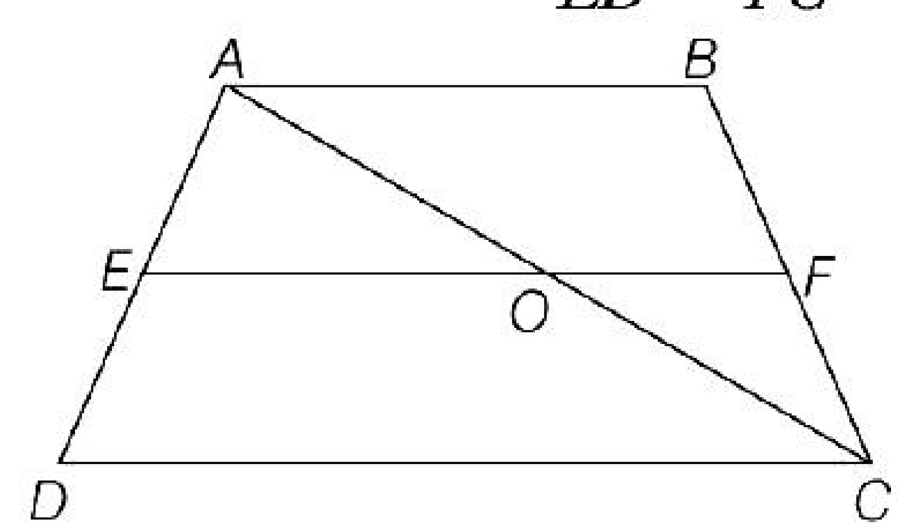
48. Given, $\quad EF \parallel AB$

$\therefore \quad OE \parallel AB \parallel CD \quad [\because AB \parallel CD]$

In ΔACD, $\dfrac{AE}{ED} = \dfrac{AO}{OC}$ [by BPT] ... (i)

Similarly in ΔABC, $\dfrac{AO}{OC} = \dfrac{BF}{FC}$ ... (ii)

From Eqs. (i) and (ii), $\dfrac{AE}{ED} = \dfrac{BF}{FC}$



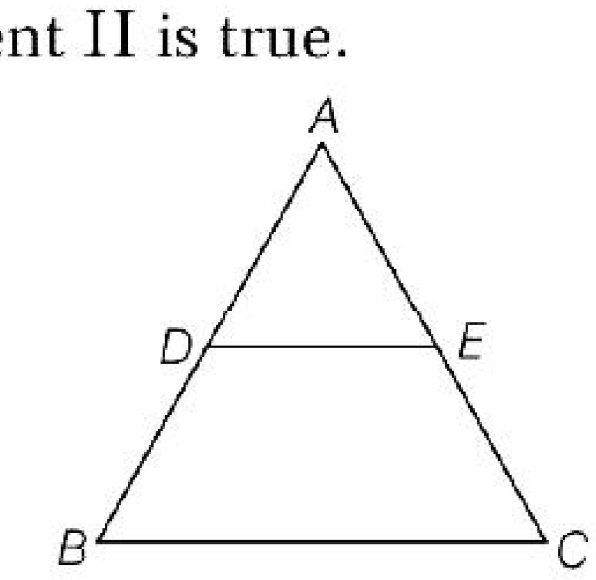
$\therefore$ Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Both Statement I and II are True and Statement II is the correct explanation of Statement I.

49. The internal bisector of an angle of a triangle divides the opposite sides internally in the ratio of sides containing the angle.

Statement I is True and Statement II is False.

50. Statement II is true.



For Assertion,

Since, $DE \parallel BC$

$\therefore$ By Thales Theorem

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

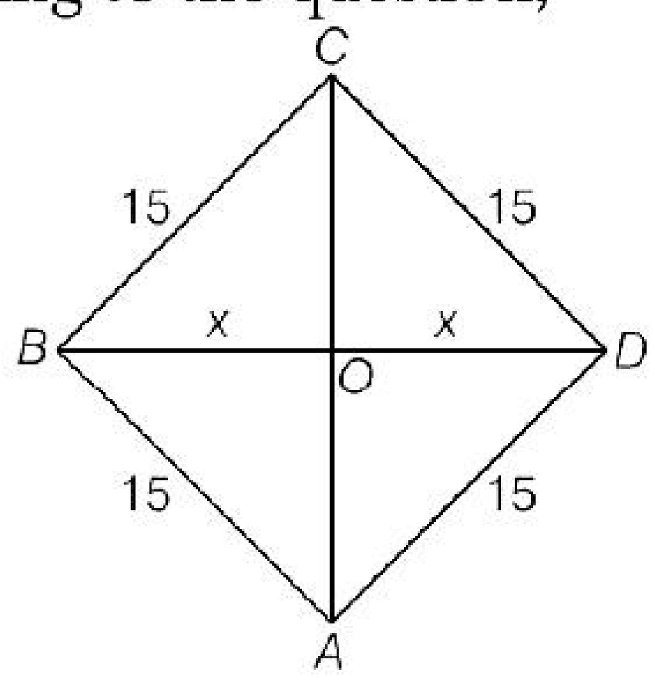
$$\Rightarrow \quad 1+\frac{DB}{AD}=1+\frac{EC}{AE}$$

$$\Rightarrow \quad \frac{AD+DB}{AD}=\frac{AE+EC}{AE}$$

$$\Rightarrow \quad \frac{AB}{AD}=\frac{AC}{AE}$$

$\therefore$ Both Statement I and II are true and Statement II is correct explanation of statement I.

51. According to the question,



Given, $AB = BC = CD = DA = 15$ cm

$AC = 20$ cm [let]

Let $BO = OD = x$ cm

$\therefore$ In ΔBOC, $x^2 + 10^2 = 15^2$

[by Pythagoras theorem]

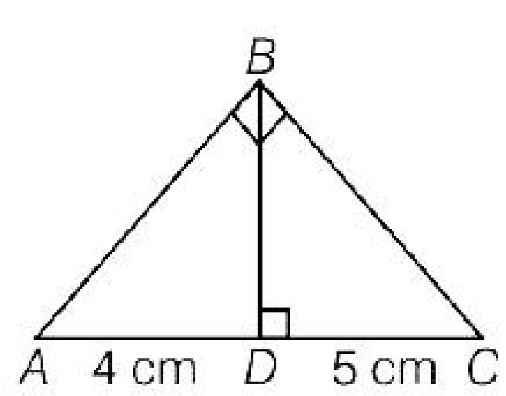
$$x^2 = 125$$

$$x = 5\sqrt{5}$$

$$BD = 2x = 10\sqrt{5} \text{ cm}$$

Hence, Statement I is false and Statement II is true.

52.



ΔABC is similar to ΔADB

$$\therefore \quad \frac{AB}{AD}=\frac{AC}{AB}$$

$$AB^2 = AD \times AC$$

$$AB^2 = 4 \times 9$$

$$AB = 6 \text{ cm}$$

In ΔADB, $AB^2 = AD^2 + BD^2$

$$36 = 16 + BD^2$$

$$BD^2 = 20$$

$$BD = 2\sqrt{5} \text{ cm}$$

Hence, Statement I is true and Statement II is false.

53. In right angled ΔABC,

$$AB^2 = AC^2 + BC^2$$

[by Pythagoras theorem]

$$= AC^2 + AC^2 \quad [\because BC = AC]$$

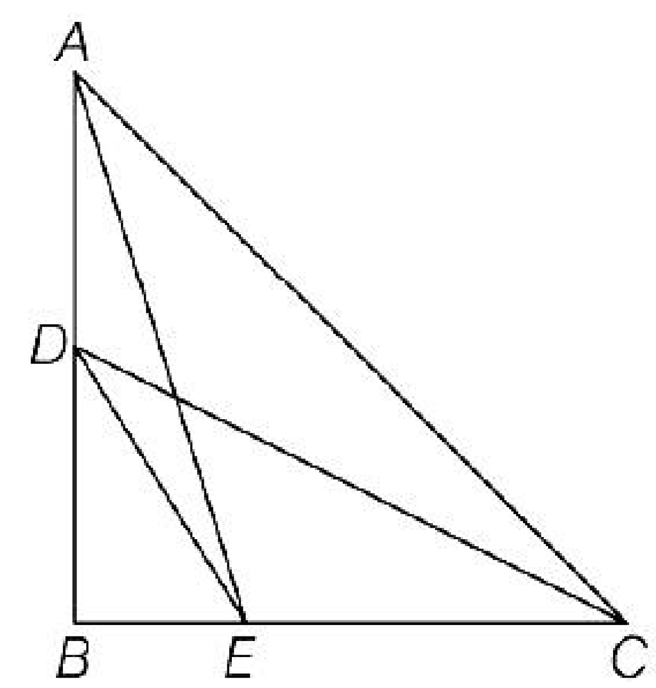
$$= 2AC^2$$

$$\therefore \quad AB^2 = 2AC^2$$

Hence, Statement I is true and Statement II is false.

54. Since, ΔABE is right triangle right-angled at B.

$$\therefore \quad AE^2 = AB^2 + BE^2 \quad \ldots\text{(i)}$$



Again, ΔDBC is right triangle right angled at B.

$$CD^2 = BD^2 + BC^2 \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$AE^2 + CD^2 = (AB^2 + BE^2) + (BD^2 + BC^2)$$

$$\Rightarrow AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$$

Using Pythagoras theroem in ΔABC and ΔDBE, we have

$$AC^2 = AB^2 + BC^2$$

and $\quad DE^2 = BE^2 + BD^2$

$$\therefore AE^2 + CD^2 = AC^2 + DE^2$$

Hence, $AE^2 + CD^2 = AC^2 + DE^2$

Both Statement I and II are true and Statement II is the correct explanation of Statement I.

55. In ΔPQR,

$QN \perp PR$ and $PN \times RN = QN^2$

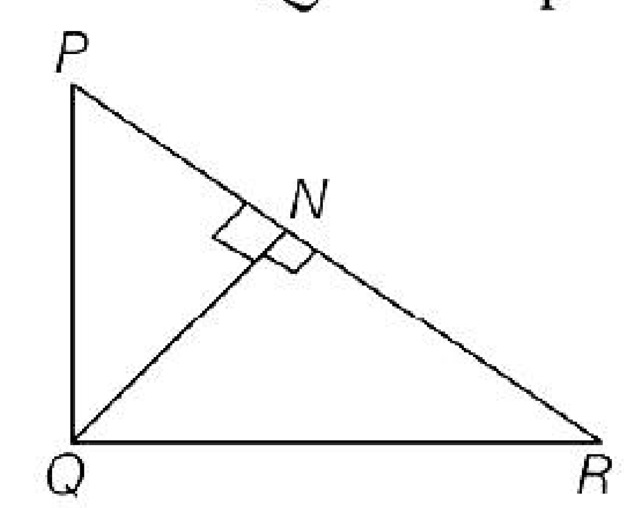
$$\frac{PN}{QN} = \frac{QN}{NR}$$

In ΔPQN and ΔRQN,

$$\angle QNP = \angle QNR$$
$$\Delta QPN \sim \Delta RQN$$

[by SAS similarity]

$\therefore \Delta QPN$ and ΔRQN are equiangular.



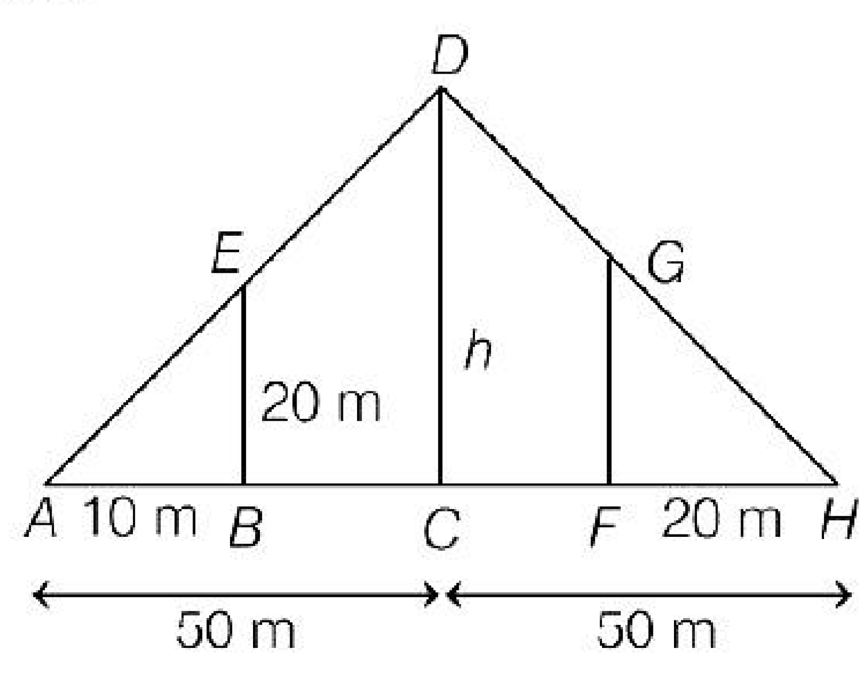
$$\angle 1 = \angle R \text{ and } \angle 2 = \angle P$$
$$\angle 1 + \angle 2 = \angle R + \angle P$$
$$\angle Q = \angle R + \angle P$$

Now, $\angle Q + \angle R + \angle P = 180^\circ$

$$2\angle Q = 180^\circ \; [\angle Q = \angle R + \angle P]$$
$$\angle Q = 90^\circ$$

Both Statements are true.

56. (i) Let $CD = h$ m be the height of the tower. Let $BE = 20$ m be the height of Vijay's house and GF be the height of Ajay's house.



$$\Delta ACD \sim ABE$$

$\therefore$ $$\frac{AC}{AB} = \frac{CD}{EB}$$

$\Rightarrow$ $$\frac{50}{10} = \frac{h}{20}$$

$\Rightarrow$ $$h = 100 \text{ m}$$

(ii) Given $AB = 12$ m, let $AC = h$

In similar ΔABE and ΔACD,

$$\frac{AB}{AC} = \frac{BE}{CD} \Rightarrow \frac{12}{h} = \frac{20}{100}$$

$\Rightarrow$ $$h = \frac{12 \times 100}{20} = 12 \times 5 = 60 \text{ m}$$

(iii) Let height of Ajay's house be $GF = h_1$

Since, $\Delta HFG \sim \Delta HCD$

$\therefore$ $$\frac{HF}{HC} = \frac{FG}{CD} \Rightarrow \frac{20}{50} = \frac{h_1}{100}$$

$\Rightarrow$ $$h_1 - \frac{20 \times 100}{50} = 40 \text{ m}$$

(iv) Given, $HC = 40$ cm

Let length of the shadow of Ajay's house be $HF = l$ m

Since, $\Delta HFG \sim \Delta HCD$

$\therefore$ $$\frac{HF}{HC} = \frac{FG}{CD}$$

$\Rightarrow$ $$\frac{l}{40} = \frac{40}{100}$$

$\Rightarrow$ $$l = \frac{40 \times 40}{100} = 16 \text{ m}$$

(v) Given, $AC = 40$ cm

Let length of the shadow of Vijay's house be $AB = l$ m

Since, $\Delta ABE \sim ACD$

$\therefore$ $$\frac{AB}{AC} = \frac{EB}{CD}$$

$\Rightarrow$ $$\frac{l}{40} = \frac{20}{100}$$

$\Rightarrow$ $$h = \frac{20 \times 40}{100} = 8 \text{ m}$$

57. (i) $\Delta AED \sim \Delta BEC$ [by SAS rule]

$$\frac{AD}{BC} = \frac{ED}{CE} = \frac{AE}{BE}$$
$$\Rightarrow \frac{BE}{AE} = \frac{CE}{ED}$$

(ii) $BC = \sqrt{BE^2 + CE^2}$

[Pythagoras theorem Apply]

$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

(iii) $\frac{AD}{BC} = \frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE}$

$\frac{AD}{5} = \frac{4}{3}$

$AD = 20/3$ cm

(iv) $\frac{ED}{CE} = \frac{4}{3}$ [similarly as (iii)]

$ED = \frac{4}{3} \times CE = \frac{4}{3} \times 4 = 16/3$ cm

(v) $\frac{AE}{BE} = \frac{4}{3}$

$AE = \frac{4}{3} \times BE = \frac{4}{3} \times 3 = 4$ cm

$\frac{4}{3}\sqrt{BC^2 - CE^2} = \frac{4}{3}\sqrt{25-16}$

$= \frac{4}{3} \times 3 = 4$cm

58. (i) To find the distance AC in the given figure, we use Pythagoras theorem.

(ii) In right ΔADC, use Pythagoras theorem,

$AC = \sqrt{AD^2 + CD^2}$

$= \sqrt{(30)^2 + (40)^2}$

$= \sqrt{900 + 1600} = \sqrt{2500} = 50$ m

(iii) (a) Now, $24^2 + 7^2 = 576 + 49$

$= 625 = 25^2$,

which forms a Pythagoras triplet

(b) $15^2 + 8^2 = 225 + 64 = 289 = 17^2$,

which forms a Pythagoras triplet

(c) $12^2 + 5^2 = 144 + 25 = 169 = 13^2$,

which form a Pythagoras triplet.

(d) $20^2 + 21^2 = 400 + 441 = 881 \neq 28^2$,

which does not form a Pythagoras triplet.

(iv) Since, $AC = 50$ m

$\therefore \quad AB = AC - BC = 50 - 12 = 38$ m

(v) The length of the rope used

$= BC + CD + DA$

$= 12 + 40 + 30 = 82$ m

59. (i) Pythagoras theorem concept can be used to get the value of x.

(ii) $NG^2 + GD^2 = ND^2$

[by Pythagoras theorem]

$\Rightarrow x^2 + (x+7)^2 = 17^2$

$\Rightarrow x^2 + x^2 + 49 + 14x = 289$

$\Rightarrow 2x^2 + 14x - 240 = 0$

$\Rightarrow x^2 + 7x - 120 = 0$

$\Rightarrow x^2 + 15x - 8x - 120 = 0$

$\Rightarrow x(x+15) - 8(x+15) = 0$

$\Rightarrow (x-8)(x+15) = 0$

$x = 8$

$x = -15$ (Not possible)

(iii) Value of $NG = 8$ km

(iv) Value of $GD = 8 + 7 = 15$ km

(v) Distance will be save after the construction $= (NG + GD) - ND$

$= (8 + 15) - 17 = 23 - 17 = 6$ km

60. Given, scale factors = 1 : 200

It means that length of 1 cm on the photograph above corresponds to a length of 200 cm (or 2 m) of the actual engine.

(i) Since, length of the model is 11 cm.

Therefore, the overall length of the engine $= 11 \times 200 = 2200$ cm = 22 m

(ii) The similarity of any two polygons will affect that they are not the mirror image of one another.

(iii) The actual width of the door

$= 0.35 \times 200$ cm = 70 cm = 0.7 m

(iv) If two similar triangles have a scale factor 5 : 3, then their altitudes have a ratio 25 : 15.

(v) In the given $BC \parallel DE$.

$\therefore \Delta ABC \sim ADE$,

$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE} \Rightarrow \frac{x}{x+4} = \frac{3}{6}$

$\Rightarrow \frac{x}{x+4} = \frac{1}{2} \Rightarrow 2x = x + 4 \Rightarrow x = 4$ cm

61. (i) Distance travelled by crow towards north after $3\frac{1}{2}$ h

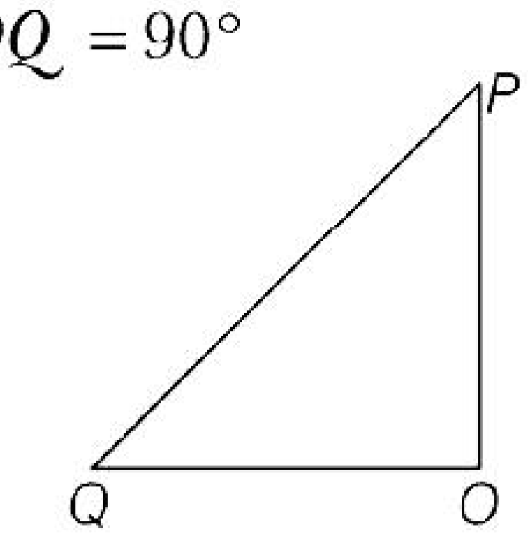
Distance = Speed × time

$$= 600 \times \frac{7}{2} = 2100 \text{ km}$$

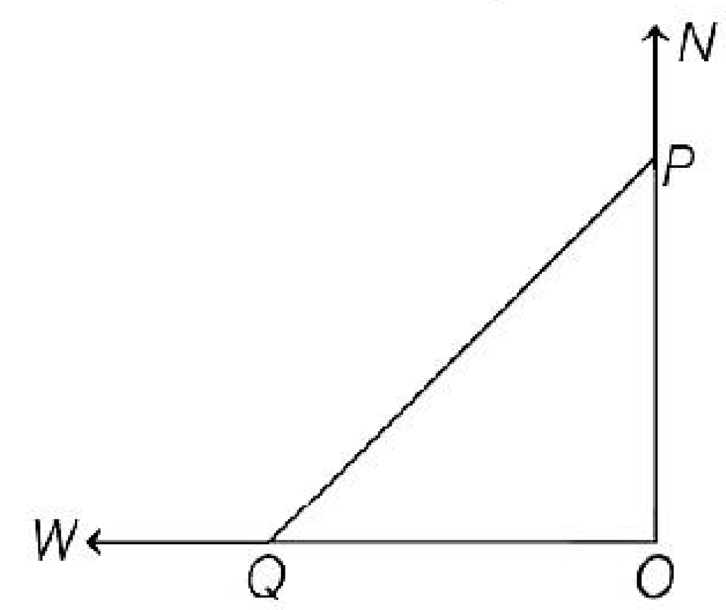
(ii) Distance travelled by crow towards west after $3\frac{1}{2}$ h

$$\text{Distance} = 800 \times \frac{7}{2} = 2800 \text{ km}$$

(iii) $\angle POQ = 90°$



(iv) $OP = 2100$ km, $OQ = 2800$ km



$$PQ = \sqrt{OP^2 + OQ^2}$$
$$= \sqrt{(2100)^2 + (2800)^2}$$
$$= 100\sqrt{441 + 784}$$
$$= 100\sqrt{1225}$$
$$= 100 \times 35 = 3500 \text{ km}$$

(v) Area of $\Delta POQ = \frac{1}{2} \times OP \times OQ$

$$= \frac{1}{2} \times 2100 \times 2800 = 2940000 \text{ km}^2$$