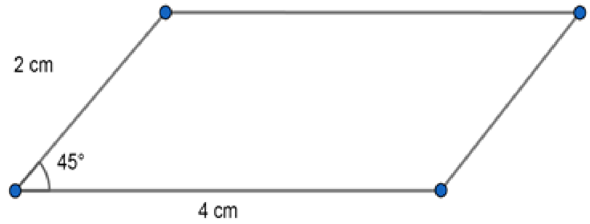
5. Trigonometry

Questions Pg-104

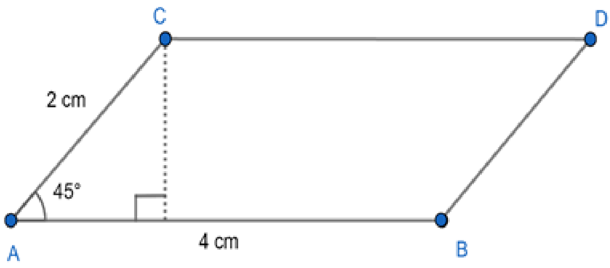
1 A. Question

Calculate the areas of the parallelograms shown below:



Answer

Let us consider a parallelogram ABDC.Let us draw a perpendicular on base AB from C say, CE

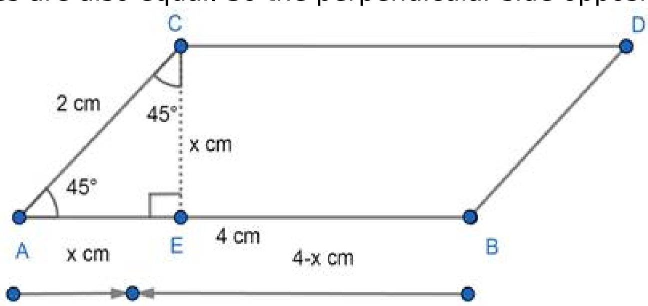


Let length of AE = x cm.

In Δ AEC:

∠CAE = 45° and ∠CEA = 90°⇒ ∠ACE = 45° (∵ ∠C + ∠E + ∠A = 180°)

We know that sides opposite to equal angles are also equal. So the perpendicular side opposite to the angle 45° is also equal to x cm∴ AE = CE = x cm.



Using the Pythagoras theorem in Δ AEC ,we get,

$AC^2 = AE^2 + EC^2 \Rightarrow (2)^2 = (x)^2 + (x)^2 \Rightarrow (2)^2 = 2(x)^2 \Rightarrow (2) = (x)^2 \Rightarrow x = \sqrt{2}$ cm

OR

We know that sides of any triangle of angles 45°, 45° and 90°are in the ratio 1:1:√2.

⇒ x:x:2 = 1:1: √2⇒ x = √2 cm

⇒ CE = √2 cm and also line segment CE is a height of parallelogram ABDC.Also base AB = 4 cm.

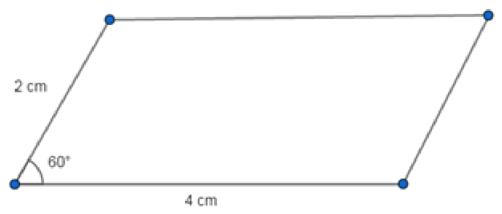
Thus area of parallelogram = base × height

∴ area (ABDC) = 4 × √2 cm^2 = 4√2 cm^2

Area = 4√2 cm^2

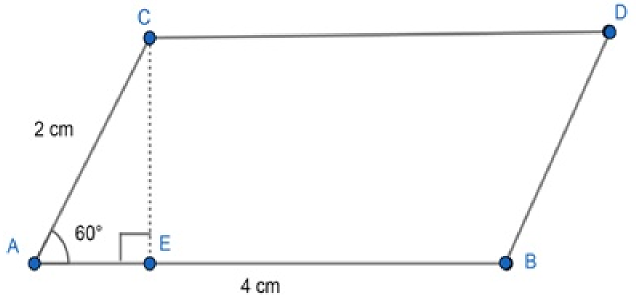
1 B. Question

Calculate the areas of the parallelograms shown below:



Answer

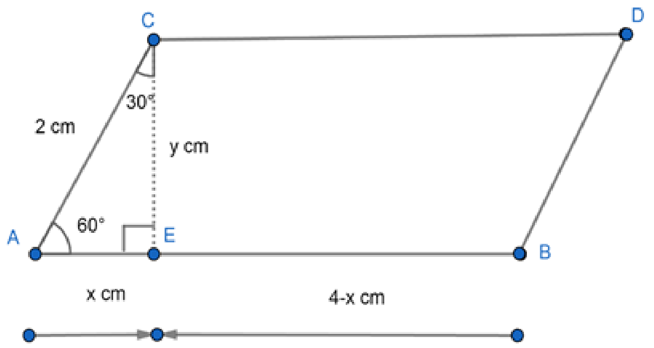
Consider a parallelogram ABDC.Let us draw a perpendicular on base AB from C say, CE.



Let length of AE = x cm and CE = y cm.

In Δ AEC:

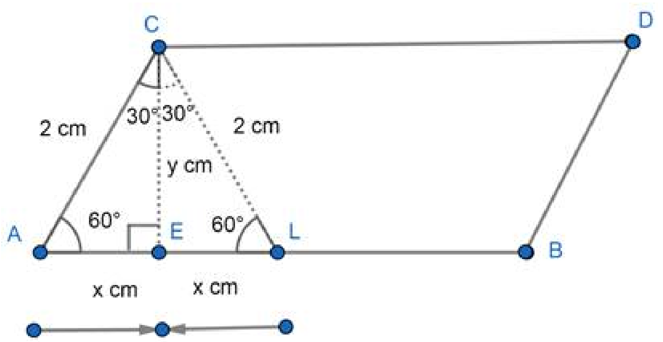
∠CAE = 60° and ∠CEA = 90°⇒ ∠ACE = 30° (∵ ∠C + E + ∠A = 180°)



We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1:√3:2.

⇒ x:y:2 = 1:√3:2⇒ x = 1 cm and y = √3 cm

ORWe can consider Δ AEC as a half of equilateral Δ ACL.



Since, Δ ACL is an equilateral triangle.We get,

AL = AC⇒ (x + x) = 2 cm⇒ 2x = 2 cm⇒ x = 1 cm

Using the Pythagoras theorem in Δ AEC ,we get,

$AC^2 = AE^2 + EC^2 \Rightarrow (2)^2 = (1)^2 + (y)^2 \Rightarrow 4 - 1 = (y)^2 \Rightarrow (3) = (y)^2 \Rightarrow y = \sqrt{3}$ cm

⇒ CE = √3 cm and also line segment CE is a height of parallelogram ABDC.Also base AB = 4 cm.

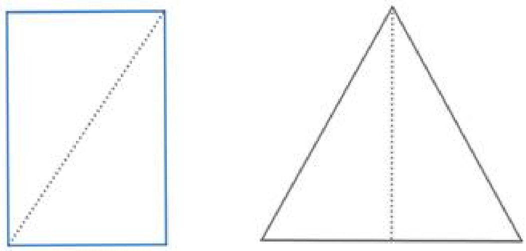
Thus area of parallelogram = base × height

∴ area (ABDC) = 4 × √3 cm^2 = 4√3 cm^2

Area = 4√3 cm^2

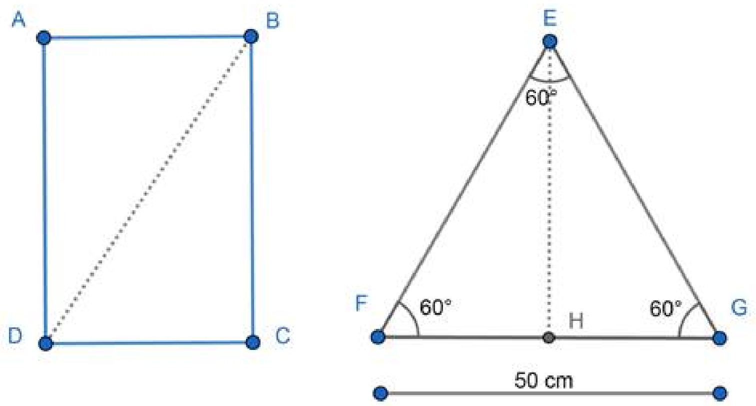
2. Question

A rectangular board is to be cut along the diagonal and the pieces rearranged to form an equilateral triangle; and the sides of the triangle must be 50 centimetres. What should be the length and breadth of the rectangle?



Answer

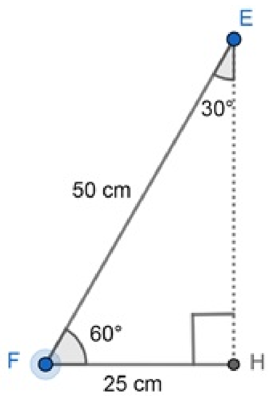
Let ABCD be the rectangle and BD be the diagonal and EFG be the derived equilateral triangle. Let EH be perpendicular to FG.



Given,Δ EFG is an equilateral triangle⇒ FG = FE = EG = 50 cm.⇒ FH = HG(∵ In equilateral triangle median is same as altitude)

⇒ FG = FH + GH = 50 cm⇒ 2(FH) = 2(GH) = 50 cm⇒ FH = GH = 25 cm

Also, ∠FEH = 2(∠FEH) = 2(∠GEH) = 60°⇒ ∠FEH = ∠GEH = 30°(∵ In equilateral triangle altitude bisects the angle)



We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1:√3:2.

⇒ 25 : EH : 50 = 1:√3:2⇒ 25 : EH = 1:√3⇒ EH = 25√3 cm

∴ IN Δ EFG and rect. ABCDWe get,

Δ DBC ≈ Δ EFG

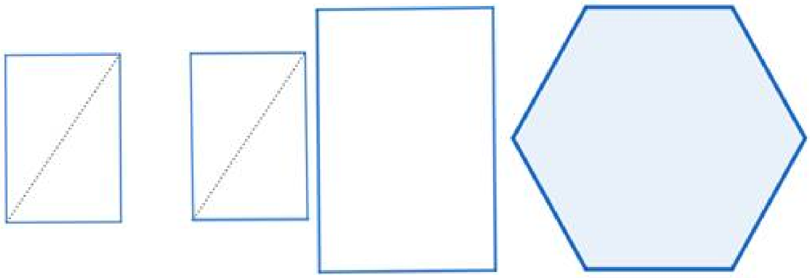
(∵ Δ EFG is same as Δ BDC because both are part of same rectangle divided be diagonal BD.)

∴ AB = CD = FH = HG = 25 cm and BC = EH = 25√3 cm

Length and Breadth of the rectangle is 25√3 cm and 25 cm.

3. Question

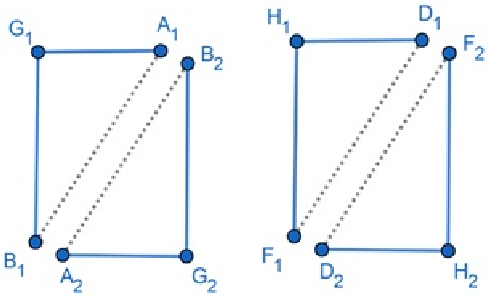
Two rectangles are cut along the diagonal and the triangles got are to be joined to another rectangle to make a regular hexagon as shown below:

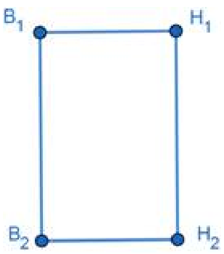


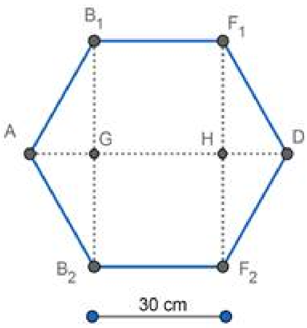
If the sides of the hexagon are 30 centimetres, what would be the length and breadth of the rectangles?

Answer

Let each polygon be assigned with name.







In regular hexagon $AB_1F_1DF_2B_2A$

$AB_1 = B_1F_1 = F_1D = DF_2 = F_2B_2 = B_2A = 30$ cm

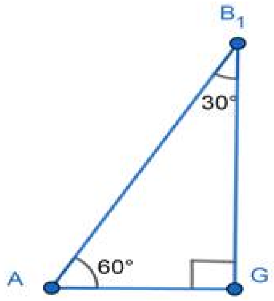
$\angle B_2AB_1 = \angle AB_1F_1 = \angle B_1F_1D = \angle F_1DF_2 = \angle DF_2B_2 = \angle F_2B_2A = 120°$

Δ AB_1G and Δ AB_2G are the two equal halves of one rectangle.and Δ F_1DH and Δ F_2DH are two equal halves of another rectangle.

⇒ $\angle B_1AG = \angle B_2AG$

Now, $\angle B_1AB_2 = \angle B_1AG + \angle B_2AG = 120°$ ⇒ $\angle B_1AG = \angle B_2AG = 60°$

In Δ AGB_1,



$\angle AGB_1 + \angle AB_1G + \angle GAB_1 = 180° \Rightarrow 90° + 60° + \angle GAB_1 = 180° \Rightarrow \angle GAB_1 = 30°$

We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1:√3:2.

$\Rightarrow AG : B_1G : AB_1 = 1 : \sqrt{3} : 2 \Rightarrow AG : B_1G : 30 = 1 : \sqrt{3} : 2 \Rightarrow AG = 15$ cm and $B_1G = 15\sqrt{3}$ cm

Similarly, In $\Delta AB_1B_2 \Rightarrow B_1B_2 = B_1G + B_2G = 2(B_1G) = 2(15\sqrt{3})$ cm $= 30\sqrt{3}$ cm

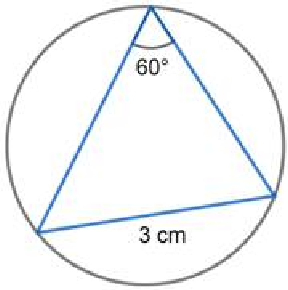
∴ For small rectangles :Length $= B_2G = B_1G = 15\sqrt{3}$ cmBreadth = AG = 15 cm

∴ For rectangle $B_1B_2\ H_2H_1$:Length $= B_1B_2 = 30\sqrt{3}$ cmBreadth $= B_1F_1 = 30$ cm

Length and Breadth of rectangles in cm are(15√3 ,15),(15√3, 15) and (30√3, 30).

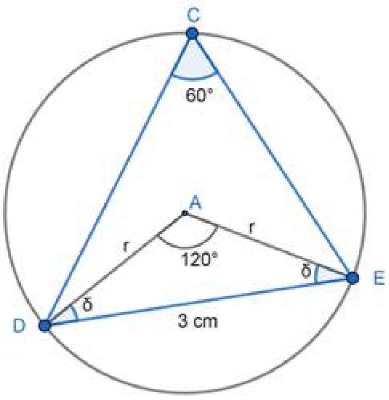
4. Question

The picture shows a triangle and its circumcircle. What is the radius of the circle?

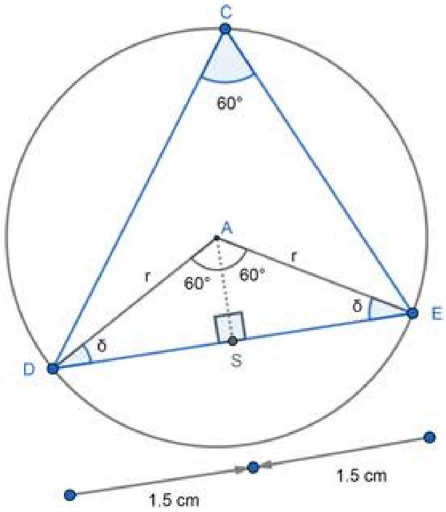


Answer

Construction, Let the centre of the circle be A and radius be r.Let the triangle be CDE. Also, join AD and AE.



Now, AD = AE = r (radius)We know that,The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. i.e. a = 2b.∴ $\angle OAE = 2\ (\angle OCE) \Rightarrow \angle OAE = 2\ (60°) = 120°$

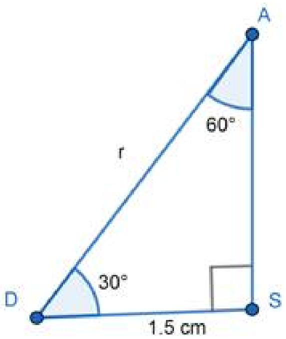
In Δ DAE :Draw a perpendicular bisector on DE from A say AS. 

AS is a perpendicular bisector of DE.∴ ∠DAS = ∠EAS , DS = DE and ∠ASD = ∠ASE = 90°

⇒ ∠DAE = ∠DAS + ∠EAS = 120°⇒ ∠DAE = 2(∠DAS) = 2(∠EAS) = 120° ⇒ ∠DAS = ∠EAS = 60°

In Δ ADE,ΔDAE is an isosceles triangle (∵ AD = AE)∴ ∠ADE = ∠AED = δ

Now, ∠ADE + ∠DEA + ∠DAE = 180°⇒ ∠DAE + δ + δ = 180° ⇒ 120° + δ + δ = 180°⇒ δ = ∠ADS = 30°We get,∠ADS = 30° , ∠DAS = 60° and ∠DAS = 90°



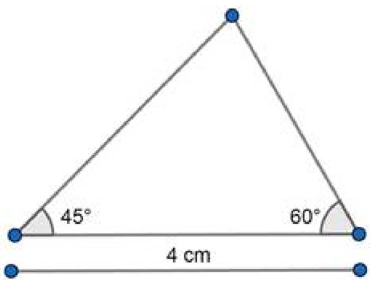
We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1:√3:2.

⇒ AS :DS: AD = 1: √3: 2⇒ AS :1.5 :AD = 1: √3: 2⇒ AD = √3 cm = r

Radius of circle is √3 cm.

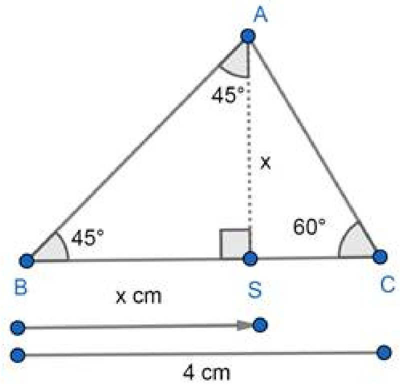
5. Question

Calculate the area of the triangle shown.



Answer

Let ABC be the required triangle.Draw perpendicular from AS on BC.



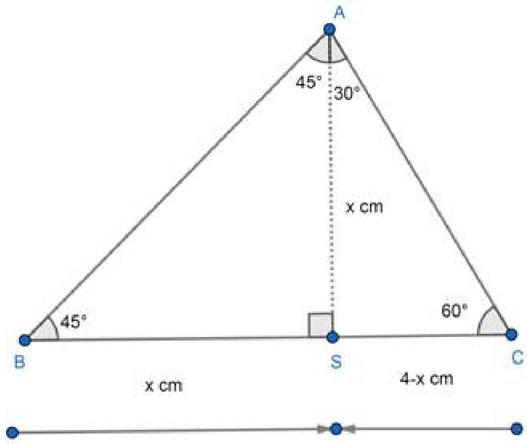
Let AS = x cm.

In Δ BAS,

∠ABS + ∠BAS + ∠ASB = 180°⇒ 45° + ∠BAS + 90° = 180° ⇒ ∠BAS = 45°⇒ Δ BAS is an isosceles triangle.

∴ AS = BS = x cm

BC = BS + CS = 4 cm⇒ x + CS = 4 cm⇒ CS = (4–x) cm



In Δ CAS,

∠ACS + ∠CAS + ∠ASC = 180°⇒ 60° + ∠CAS + 90° = 180° ⇒ ∠CAS = 30°

We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1: √3: 2 .

⇒ CS :AS: AC = 1: √3: 2⇒ (4-x) :x : AC = 1: √3 :2⇒ (4 - x) : x = 1 : √3⇒ $\frac{4-x}{x} = \frac{1}{\sqrt{3}}$⇒ √3(4 - x) = x⇒ 4√3 - x√3 = x⇒ x(√3 + 1) = 4√3

$$\Rightarrow x = \frac{4\sqrt{3}}{(\sqrt{3}+1)} \text{ cm}$$

Multiplying and Dividing by (√3 - 1)

$$\Rightarrow x = \frac{4}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} \text{ cm}$$

$$\Rightarrow x = \frac{4(\sqrt{3}-1)}{(\sqrt{3})^2 - (1)^2} \text{ cm}$$

$$\Rightarrow x = \frac{4(\sqrt{3}-1)}{2} \text{ cm}$$

⇒ x = 2(√3 - 1) cm

In Δ BAC,

Height = x = 2($\sqrt{3}$ - 1) cm and Base = 4 cm

$$\text{Area of }\Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area }(\Delta) = \frac{1}{2} \times 2\left(\sqrt{3}-1\right) \times 4\text{ cm}^2$$

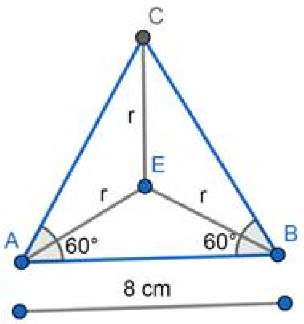
Area (Δ) = 4($\sqrt{3}$ - 1) cm^2 = 10.92 cm^2

6. Question

What is the circumradius of an equilateral triangle of sides 8 centimetres?

Answer

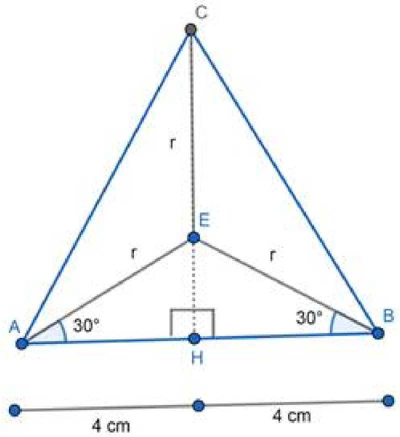
Let the equilateral triangle be Δ ABC, Circumcentre and circumradius be E and r respectively.



We know that in equilateral triangle median and internal angle bisector are same.Here AE, BE and CE are part of medians.⇒ EA acts as an internal angle bisector for ∠CAB.⇒ ∠CAE = ∠EAB

In Δ ABC,∠CAB = ∠CAE + ∠EAB = 60° ⇒ ∠CAB = 2(∠CAE) = 2(∠EAB) = 60°⇒ ∠CAE = ∠EAB = 30°

Extend CE to meet at AB at H.CH is a median as well as altitude.⇒ CH ⊥ AB and ∠CHA = ∠CHB = 90°



In Δ AEH,

∠EAH + ∠AHE + ∠HEA = 180° ⇒ 30° + 90° + ∠HEA = 180° ⇒ ∠HEA = 60°

We know that sides of any triangle of angles 30°, 60° and 90°are in the ratio 1: $\sqrt{3}$: 2 .

⇒ EH :AH :AE = 1: $\sqrt{3}$: 2⇒ EH :4 :r = 1: $\sqrt{3}$: 2⇒ 4 :r = $\sqrt{3}$: 2⇒ $r = \frac{4 \times 2}{\sqrt{3}} = \frac{8}{\sqrt{3}}\text{ cm} = 4.62\text{ cm}$

Questions Pg-110

1. Question

Without drawing pictures or looking up the tables, arrange the numbers

Sin 1°, cos 1°, sin 2°, cos 2°

in ascending order.

Answer

Consider the angles :

sin 0°, cos 0°, sin 30°, cos 30°

We know that angles 0° and 30° lies in the same part of the graph of sin and cos where 1° and 2° lies.

∴ we can take the reference of them to arrange the other trigonometry angles which are counterpart of them.

Taking the values of the given trigonometry angles :

sin 0° = 0

cos 0° = 1

sin 30° = 0.5

cos 30° = 0.866

Arranging in ascending order :

⇒ sin 0° < sin 30° < cos 30° < cos 0°

Similarly,

⇒ sin 1° < sin 2° < cos 2° < cos 1°

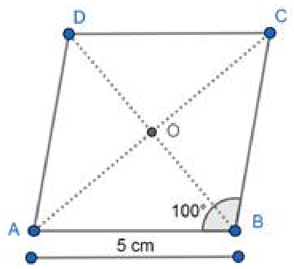
Remember that we have taken 0° and 30° for comparison with 1° and 2° as the properties of sin and cos does not change in this range and they behave same as 1° and 2°.

sin 1° < sin 2° < cos 2° < cos 1°

2. Question

The sides of a rhombus are 5 centimetres long and one of its angles in 100°. Compute is area.

Answer



Let ABCD be the required rhombus.

⇒ AB = BC = CD = DA = 5 cm and∠ ABC = 100°

In ⋄ ABCD,

Diagonals are the perpendicular bisector of each other

⇒ AO = OC and BO = OD and AC ⊥ BD

⇒ ∠ AOB = ∠ BOC = ∠ COD = ∠ DOA = 90°

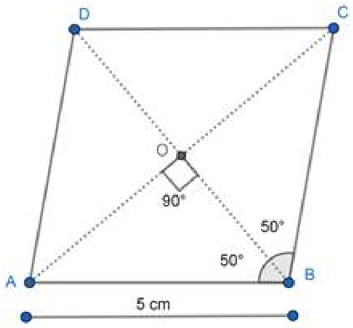
Diagonals are the internal angle bisector.

⇒ ∠ ABO = ∠ CBO

⇒ ∠ ABC = ∠ ABO + ∠ CBO

⇒ 100° = 2(∠ ABO) = 2(∠ CBO)

⇒ ∠ ABO = ∠ CBO = 50°



In Δ AOB,

AO = sin 50° × AB

⇒ AO = sin50° × 5 cm

(From table: sin 50° = 0.776)

⇒ AO = 0.776 × 5 = 3.88 cm

Again,

BO = cos 50° × AB

⇒ BO = cos 50° × 5 cm

(From table: cos 50° = 0.642)

⇒ BO = 0.642 × 5 = 3.21 cm

Now,

AC = AO + OC

⇒ AC = 2(AO) = 2(OC)

⇒ AC = 2 × 3.88 cm

⇒ AC = 7.76 cm

BC = BO + OD

⇒ BC = 2(BO) = 2(OD)

⇒ BC = 2 × 3.21 cm

⇒ BC = 6.42 cm

$\text{Area of Rhombus } = \frac{1}{2} \times d_1 \times d_2$

$\text{Area of Rhombus ABCD } = \frac{1}{2} \times 7.76 \times 6.42 \text{ cm}^2$

Area of Rhombus = 24.9096 cm^2 = 25 cm^2

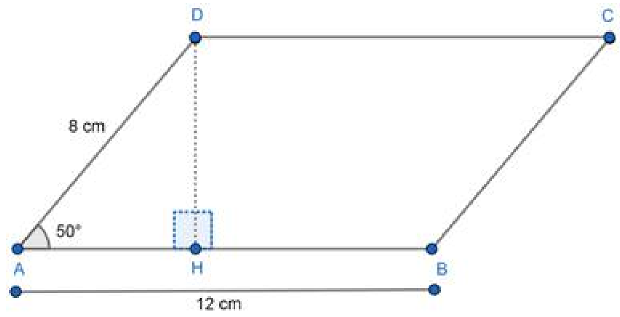
Area of Rhombus = 25 cm^2.

3. Question

The sides of a parallelogram are 8 centimetres and 12 centimetres and the angle between them is 50°. Calculate its area.

Answer

Let us draw a perpendicular from D on AB say DH ⇒ DH ⊥ AB



In Δ ADH,

DH = sin 50° × AD

(From table: sin 50° = 0.776)

⇒ DH = 0.776 × 8 cm

⇒ DH = 6.208 cm

Area of parallelogram = Base × Height

Area = 12 × 6.208 cm^2

Area = 74.496 cm^2

Area of parallelogram is 74.496 cm^2.

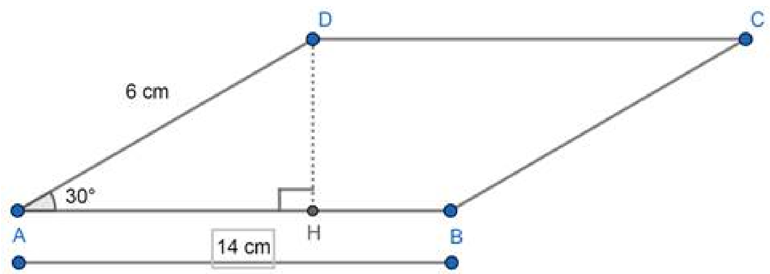
4. Question

The sides of a parallelogram are 6 centimetres and 14 centimetres and the angle between them is 30°. What are the

lengths of its diagonals?

Answer

Let us draw a perpendicular from D on AB say DH ⇒ DH ⊥ AB



In Δ ADH,

DH = sin 30° × AD

(From table: sin 30° = 0.5)

⇒ DH = 0.5 × 6 cm

⇒ DH = 3 cm

Area of parallelogram = Base × Height

Area = 14 × 3 cm^2

Area = 42 cm^2

Area of parallelogram is 42 cm^2.

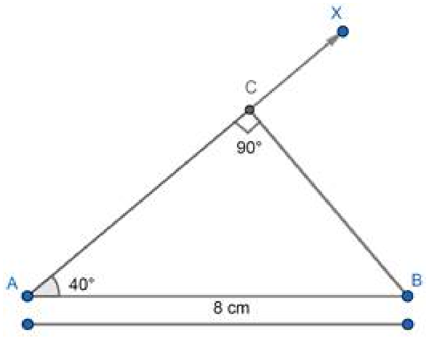
5. Question

A triangle is to be drawn with one side 8 centimetres and an angle on it 40°. What is the minimum length of the side opposite this angle?

Answer

We know that perpendicular length drawn between two lines is the smallest length.

Let AB = 8 cm and ∠ BAX = 40°



From B draw perpendicular on AX say BC.

In Δ ABC,

BC = AB × sin 40°

(From table, sin 40° = 0.64)

⇒ BC = 8 × 0.64 cm

⇒ BC = 5.12 cm

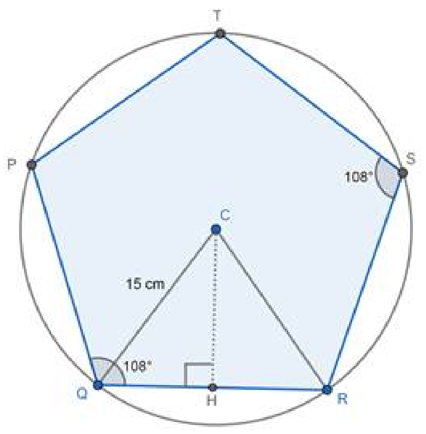
The minimum length of the side opposite to angle 40° is 5.12 cm.

6. Question

A regular pentagon is drawn with vertices on a circle of radius 15 centimetres. Calculate the length of its sides.

Answer

Consider a pentagon PQRST with circumcircle having centre C and radius r = 15 cm.



Join CQ and CR and draw a perpendicular CH on QR.

Since, PQRST is a regular pentagon.

We get,

Each internal angle = 108°

⇒ ∠ PQR = 108°

Also CQ acts as an internal angle bisector for each internal angle of regular pentagon.

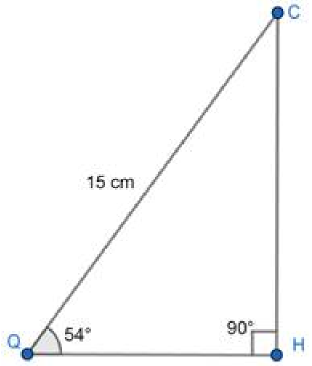
∴ ∠ PQC = ∠ RQC

⇒ ∠ PQR = ∠ PQC + ∠ RQC = 108°

⇒ ∠ PQR = 2(∠ PQC) = 2(∠ RQC) = 108°

⇒ ∠ PQC = ∠ RQC = 54°

Now in Δ QCH,



CD = sin 54° × QC

⇒ CD = sin 54° × 15 cm

(Frome table, sin 54° = 0.809)

⇒ CD = 0.809 × 15 = 12.135 cm

⇒ CD = 12.13 cm

We know that in regular pentagon all sides are equal.

∴ Length of each side = 12.13 cm

Length of each side = 12.13 cm.

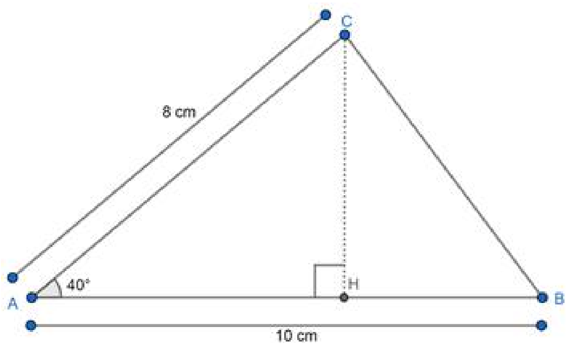
7. Question

The lengths of two sides of a triangle are 8 centimetres and 10 centimetres and the angle between them is 40°. Calculate its area. What is the area of the triangle with sides of the same length, but angle between them 140°?

Answer

Consider a triangle ABC with AB = 10 cm, AC = 8 cm and

∠ BAC = 40°



Draw perpendicular from C on AB, say CH.

From right angles triangle ABC,

CH = CA × sin 40°

⇒ CH = 8 × sin 40°

(From the table, sin 40° = 0.642)

⇒ CH = 5.136 cm

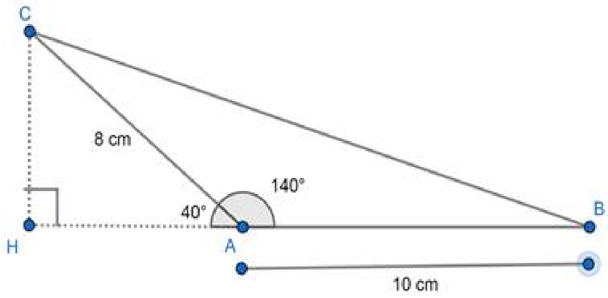
$$\text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{Height}$$

$$\text{Area } (\Delta\, ABC) = \frac{1}{2} \times AB \times CH$$

$$\text{Area } (\Delta\, ABC) = \frac{1}{2} \times 10 \times 5.136\ cm^2$$

Area (Δ ABC) = 25.68 cm^2

If given angle is 140° i.e. ∠ BAC = 140°



Draw perpendicular from C on AB and extend AB to meet it at H.

∴ CH ⊥ HB

HB is a straight line.

⇒ ∠ HAB = ∠ HAC + ∠ BAC = 180°

⇒ ∠ HAC + 140° = 180°

⇒ ∠ HAC = 40°

In right triangle CHA,

∠ HAC = 40° and AC = 8 cm

CH = CA × sin 40°

⇒ CH = 8 × sin 40°

(From the table, sin 40° = 0.642)

⇒ CH = 5.136 cm

$$\text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{Height}$$

$$\text{Area } (\Delta\, ABC) = \frac{1}{2} \times AB \times CH$$

$$\text{Area } (\Delta\, ABC) = \frac{1}{2} \times 10 \times 5.136\ cm^2$$

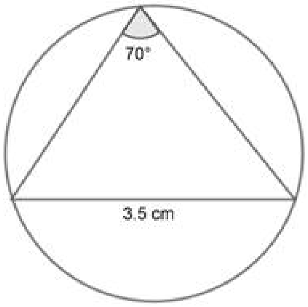
Area (Δ ABC) = 25.68 cm^2

Hence, Area is same only the position of perpendicular line was changed if the angle is considered as 140°.

Area of triangle (given angle is 40°) = 25.68 cm^2 and area of triangle (angle is taken as 140°) = 25.68 cm^2.

8. Question

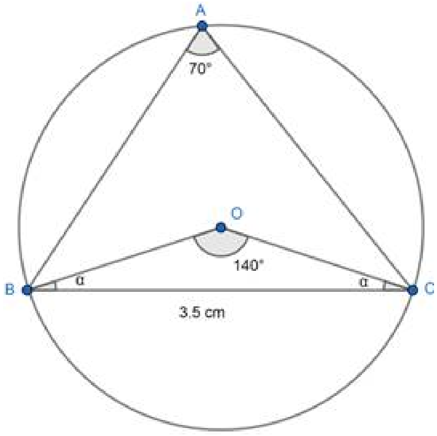
The picture shows a triangle a triangle and its circumcircle. What is the radius of the circle?



Answer

Construction, Let the centre of the circle be O and radius be r.

Let the triangle be ABC . Also, join OC and OB.



Now, OB = OC = r (radius)

We know that,

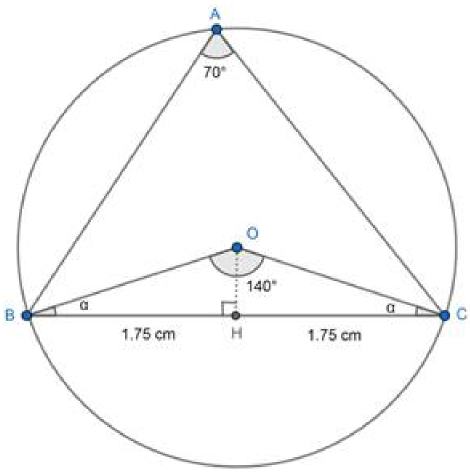
The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. i.e. a = 2b.

∴ ∠ BOC = 2 (∠ BAC)

⇒ ∠ BOC = 2 (70°) = 140°

In Δ BOC :

Draw a perpendicular bisector on BC from O say OH.



OH is a perpendicular bisector of BC.

∴ ∠ BOH = ∠ COH, OB = OC and ∠ OHB = ∠ OHC = 90°

⇒ ∠ BOC = ∠ BOH + ∠ COH = 140°

⇒ ∠ BOC = 2(∠ BOH) = 2(∠ COH) = 140°

⇒ ∠ BOH = ∠ COH = 70°

In Δ BOC,

Δ BOC is an isosceles triangle (∵ OB = OC)

∴ ∠ OBC = ∠ OCB = α

Now, ∠ OBC + ∠ OCB + ∠ BOC = 180°

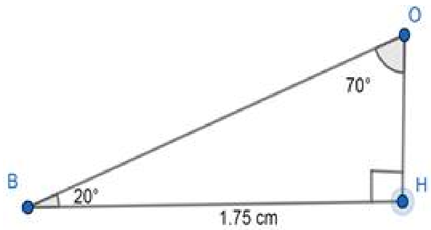
⇒ α + α + 140° = 180°

⇒ 2(α) + 140° = 180°

⇒ α = ∠ OBC = 20°

We get,

∠ OBC = 20° , ∠ BOH = 70° and ∠ OHB = 90°



In right triangle BOH,

BH = BO × sin 70°

$$\Rightarrow \mathrm{BO} = \frac{\mathrm{BH}}{\sin 70°}$$

$$\Rightarrow \mathrm{BO} = \frac{1.75}{\sin 70°}$$

(From table, sin 40° = 0.939)

$$\Rightarrow \mathrm{BO} = \frac{1.75}{0.939} = 1.86 \text{ cm}$$

BO = r = 1.86 cm

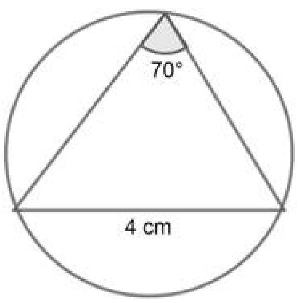
Radius of circle is 1.86 cm.

Questions Pg-114

1. Question

Using the sine and cosine tables, and if needed a calculator, do these problems.

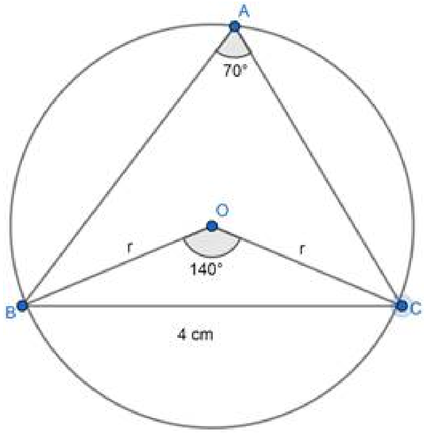
A triangle and its circumcircle are shown in the picture. Calculate the diameter of the circle.



Answer

Construction, Let the centre of the circle be O and radius be r.

Let the triangle be ABC . Also, join OC and OB.



Now, OB = OC= r (radius)

We know that,

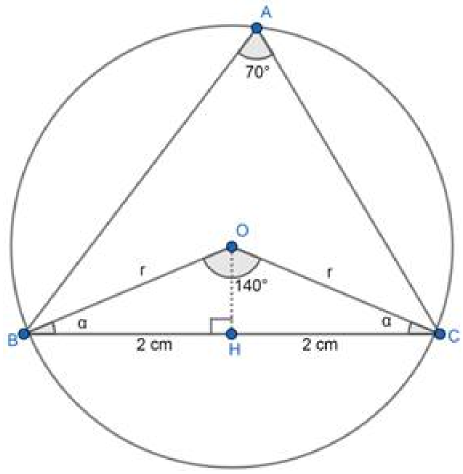
The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. i.e. a = 2b.

∴ ∠BOC = 2 (∠BAC)

⇒ ∠BOC = 2 (70°) = 140°

In Δ BOC :

Draw a perpendicular bisector on BC from O say OH.



OH is a perpendicular bisector of BC.

∴ ∠BOH = ∠COH , OB = OC and ∠OHB = ∠OHC = 90°

⇒ ∠BOC = ∠BOH + ∠COH = 140°

⇒ ∠BOC = 2(∠BOH) = 2(∠COH) = 140°

⇒ ∠BOH = ∠COH = 70°

In Δ BOC,

Δ BOC is an isosceles triangle (∵ OB = OC)

∴ ∠OBC = ∠OCB = α

Now, ∠OBC + ∠OCB + ∠BOC = 180°

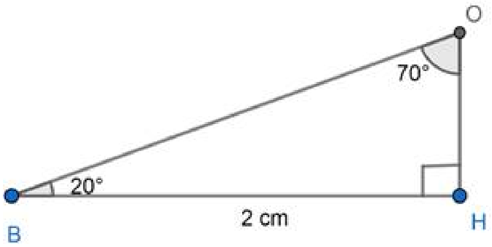
⇒ α + α + 140° = 180°

⇒ 2(α) + 140° = 180°

⇒ α = ∠OBC = 20°

We get,

∠OBC = 20° , ∠BOH = 70° and ∠OHB = 90°



In right triangle BOH,

BH = BO × sin 70°

$\Rightarrow \mathrm{BO} = \frac{\mathrm{BH}}{\sin 70°}$

$\Rightarrow \mathrm{BO} = \frac{2}{\sin 70°}$

(From table, sin 40° = 0.939)

$\Rightarrow \mathrm{BO} = \frac{2}{0.939} = 2.1299 \text{ cm} = 2.13 \text{ cm}$

BO = r = 2.13 cm

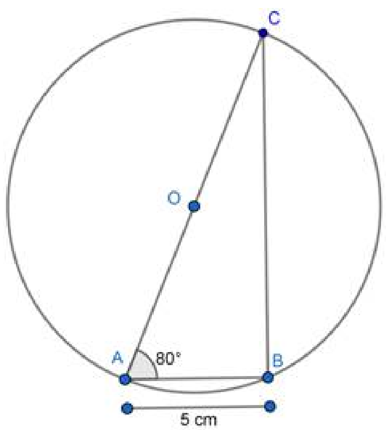
Radius of circle is 2.13 cm.

2. Question

Using the sine and cosine tables, and if needed a calculator, do these problems.

A circle is to be drawn, passing through the ends of a line, 5 centimetres long; and the angle on the circle on one side of the line should be 80°. What should be the radius of the circle?

Answer



Let AB = 5 cm and ∠BAC = 80°.

Now Let O be a midpoint of AB.

With O as centre and radius OA = OB,

Construct a circumcircle for Δ ABC.

We know that,

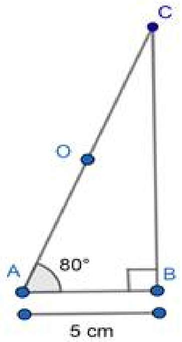
In circle, The angle formed by the diameter on its circumference

is always equal to 90° .

∴ ∠ABC = 90° (∵ AC is a diameter of circle O)

AC = AO + OC = r + r = 2r

In Δ ABC,



AB = AC × cos 80°

$$\Rightarrow AC = \frac{AB}{\cos\ 80°}$$

(From table, cos 80° = 0.173)

$$\Rightarrow AC = \frac{5}{0.173} = 28.9 \text{ cm}$$

⇒ AC = 2r

$$\Rightarrow r = \frac{AC}{2} = \frac{28.9}{2} = 14.45 \text{ cm}$$

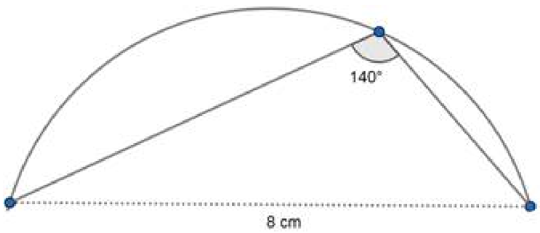
⇒ r = 14.45 cm

Radius of a circle is 14.45 cm

3. Question

Using the sine and cosine tables, and if needed a calculator, do these problems.

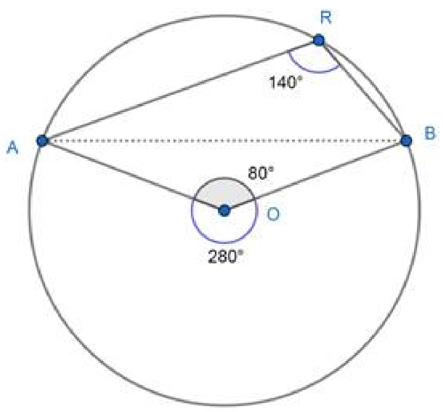
The picture below shows part of a circle:



What is the radius of the circle?

Answer

Let us draw a complete circle.



Let the centre be O. Join OA and OB.

We know that,

The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. i.e. a = 2b.

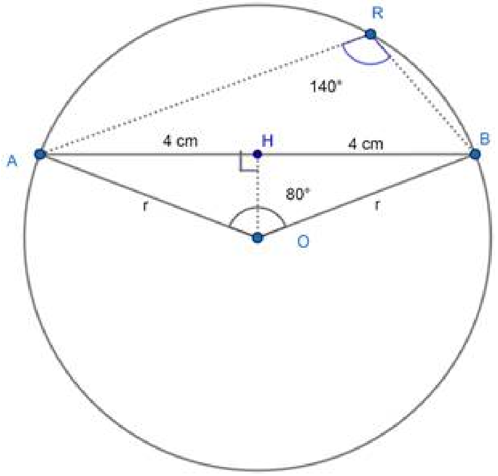
∴ ∠AOB (External) = 280°

⇒ ∠AOB (Internal) = 360° - 280° = 80°

Let radius be r.

⇒ OA = OB = r.

Draw perpendicular bisector OH on AB



In Δ OHB

OH is a perpendicular bisector of AB.

∴ ∠AOH = ∠BOH , OA = OB and ∠OHA = ∠OHB = 90°

⇒ ∠BOA = ∠BOH + ∠AOH = 80°

⇒ ∠BOA = 2(∠BOH) = 2(∠AOH) = 80°

⇒ ∠BOH = ∠AOH = 40°

In Δ BOA,

Δ BOA is an isosceles triangle (∵ OB = OA)

∴ ∠OBA = ∠OAB = α

Now, ∠OBA + ∠OAB + ∠BOA = 180°

⇒ α + α + 80° = 180°

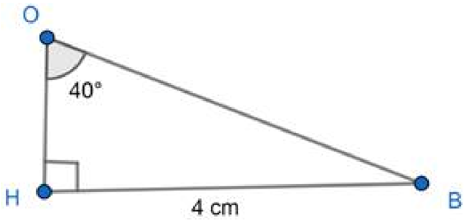
⇒ 2(α) + 80° = 180°

⇒ α = ∠OBA = 50°

We get,

∠OBA = 50° , ∠BOH = 40° and ∠OHB = 90°

In right triangle BHO,



BH = BO × sin 40°

$$\Rightarrow \ \mathrm{BO} = \frac{\mathrm{BH}}{\sin 40°}$$

(From table, sin 40° = 0.642)

$$\Rightarrow \mathrm{BO} = \frac{\mathrm{BH}}{0.642} = \frac{4}{0.642} = 6.23 \text{ cm}$$

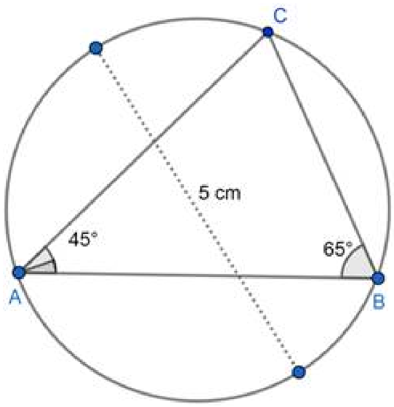
⇒ BO = r = 6.23 cm

Radius of a circle is 6.23 cm

4. Question

Using the sine and cosine tables, and if needed a calculator, do these problems.

Draw the picture shown in your notebook and explain how it was drawn.



Calculate the lengths of all three sides.

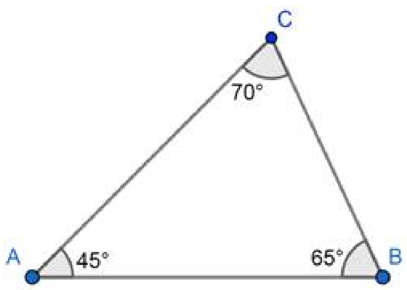
Answer

Let us draw a triangle ABC with all the given angles.

∠BAC + ∠ABC + ∠ACB = 180°

⇒ 45° + 65° + ∠ACB = 180°

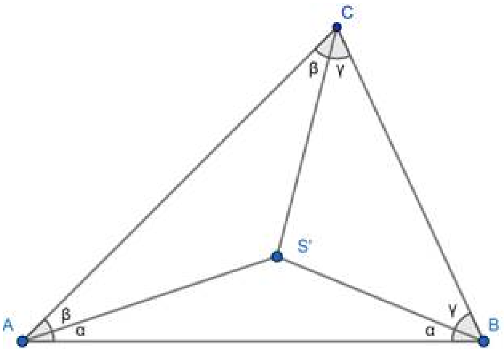
⇒ ∠ACB = 180° - (45° + 65°) = 70°



Since, We do not know the length of any side of triangle , So we cannot use the concept of perpendicular bisector to construct a circumcircle.

But we know that every triangle formed by a circumcentre with the two consecutive points is an isosceles triangle.

Let each internal angle of a triangle be divided into two angles by a point S’ as shown in fig.



Here,

$\alpha + \beta = 45°$ ...(i)

$\alpha + \gamma = 65°$ ...(ii)

$\beta + \gamma = 70°$ ...(iii)

Subtracting eq. (i) from (ii),

We get,

$(\alpha + \gamma) - (\alpha + \beta) = 65° - 45°$

$\Rightarrow \gamma - \beta = 20°$ ...(iv)

Adding eq. (iii) and (iv),

We get,

$(\beta + \gamma) + (\gamma - \beta) = 70° = 20°$

$\Rightarrow 2\gamma = 90°$

$\Rightarrow \gamma = 45°$

Substituting the value of γ in eq. (ii) and eq. (iii)

we get,

$\alpha + 45° = 65°$ and $\beta + 45° = 70°$

$\Rightarrow \alpha = 20°$ and $\beta = 25°$

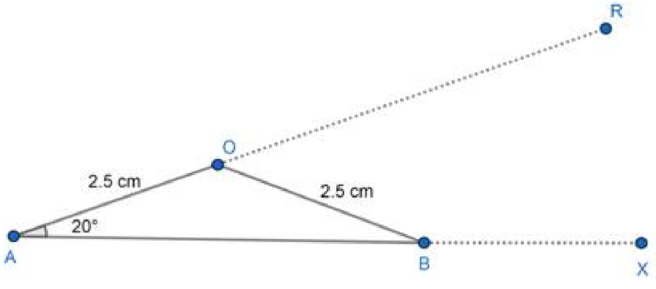
Steps of construction :

1.Mark the angle α on AX, such that ∠RAX = α = 20°.

2. Cut the arc of length 2.5 cm on AR say BO. (r = 2.5 cm)

3. With O as a centre and radius equal to 2.5 cm cut the arc

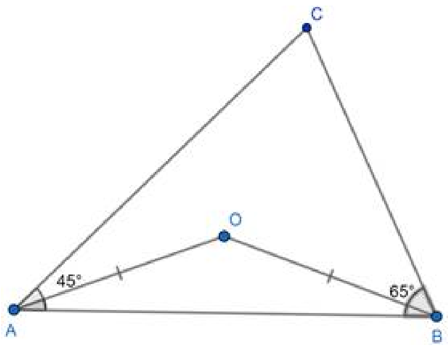
on AX say OB.



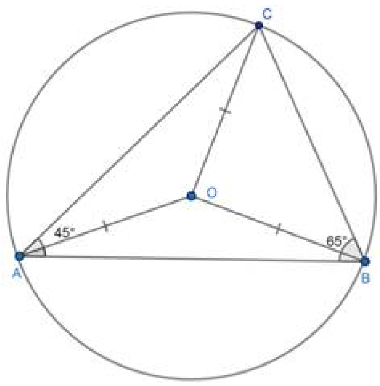
4. With base as AB construct ∠BAQ = 45° and ∠ABR = 65°.

Let them meet at C.

5. Join BC and AC.



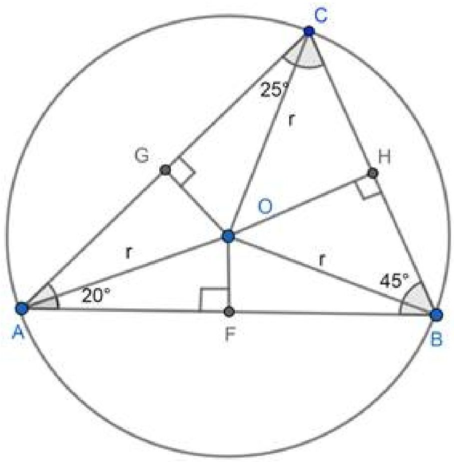
6. With O as a centre and radius = 2.5 cm draw a circle.



Thus we find that the circle with centre O is the circumcircle of Δ ABC.

We know that perpendicular from the circumcentre act as a perpendicular bisector.

Construct OF ⊥ AB, OG ⊥ AC and OH ⊥ BC.



In right Δ OFA,

∠OAF = 20° and OA = r = 2.5 cm

AF = OA × cos 20°

⇒ AF = 2.5 × 0.939 cm (From table, cos 20° = 0.939)

⇒ AF = 2.347 cm = 2.35 cm

In Δ AOB

AF = BF

⇒ AB = 2(AF) = 2(2.35) = 4.7 cm

In right Δ OGC,

∠OCG = 25° and OC = r = 2.5 cm

GC = OC × cos 25°

⇒ GC = 2.5 × 0.906 cm (From table, cos 25° = 0.906)

⇒ GC = 2.265 cm

In Δ AOC

AG = GC

⇒ AC = 2(GC) = 2(2.265) = 4.53 cm

In right Δ OHB,

∠OBH = 45° and OB = r = 2.5 cm

BH = OB × cos 45°

⇒ BH = 2.5 × 0.707 cm (From table, cos 45° = 0.707)

⇒ BH = 1.767 cm

In Δ BOC

BH = HC

⇒ BC = 2(BH) = 2(1.767) = 3.53 cm

Thus AB = 4.7 cm, BC = 3.53 cm and CA = 4.53 cm

Length of all sides of triangle are 4.7 cm, 3.53 cm and 4.53 cm.

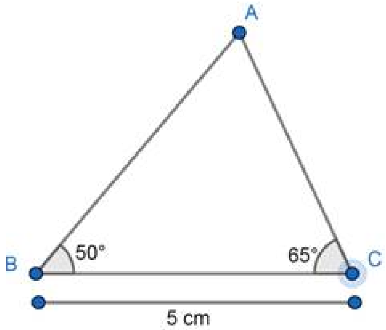
5. Question

Using the sine and cosine tables, and if needed a calculator, do these problems.

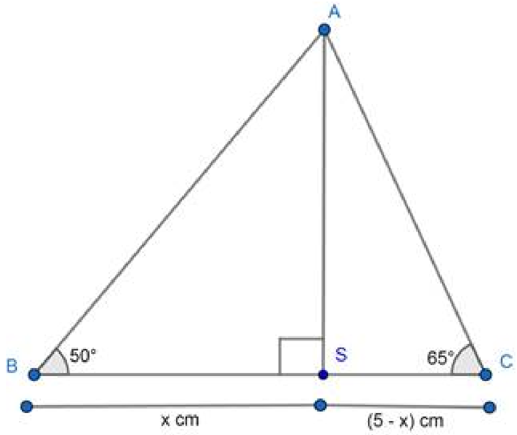
A triangle is made by drawing angles of 50° and 65° at the ends of a line 5 centimetres long. Calculate its area.

Answer

Let ABC be the required triangle.



Draw perpendicular from AS on BC.



Let BS = x cm

⇒ SC = (5 – x) cm

In Δ BAS,

AS = BS × tan 50°

⇒ AS = x × tan 50° cm ...(i)

In Δ CAS,

AS = CS × tan 65°

⇒ AS = (5 - x) × tan 65° cm ...(ii)

From eq. (i) and (ii), We get :

x × tan 50° = (5 - x) × tan 65° cm

⇒ x tan 50° = 5 tan 65° - x tan 65°

⇒ x tan 50° + x tan 65° = 5 tan 65°

⇒ x (tan 50° + tan 65°) = 5 tan 65°

$$\Rightarrow x = \frac{5 \times \tan 65°}{\tan 50° + \tan 65°}$$

(From table, tan 65° = 2.144 and tan 50° = 1.191)

$$\Rightarrow x = \frac{5 \times 2.144}{1.191 + 2.144} = 3.21 \text{ cm}$$

In Δ BAC,

Height = x = 3.21 cm and Base = 5 cm

$$\text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area } (\Delta) = \frac{1}{2} \times 3.21 \times 5 \text{ cm}^2$$

Area (Δ) = 8.025 cm^2

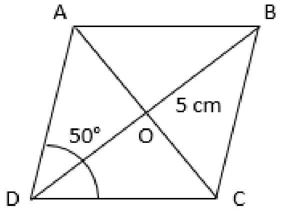
Questions Pg-117

1. Question

One angle of a rhombus is 50° and one diagonal is 5 centimetres. What is the area?

Answer

The rhombus ABCD is as shown in the figure shown below,



where,

DB = 5 cm

∠ADC = 50°

We know that,

Diagonals of a rhombus are perpendicular bisector to each other.

Also, diagonals bisect the corner angles of rhombus

So,

∠ODC = 25° [∵ DB bisects ∠D]

OD = 2.5 cm [∵ AC bisects BD]

Now OC is given by,

OC = OD × tan25°

OC = 2.5 × 0.466

OC = 1.166 cm

Also, AC = 2 × OC [∵ BD bisects AC]

AC = 2 × 1.166

AC = 2.332 cm

We know, Area of rhombus = $\frac{1}{2}(\mathrm{d1} \times \mathrm{d2})$

Area = $\frac{1}{2}(\mathrm{AC} \times \mathrm{BD})$

Area = $\frac{1}{2}(2.332 \times 5)$

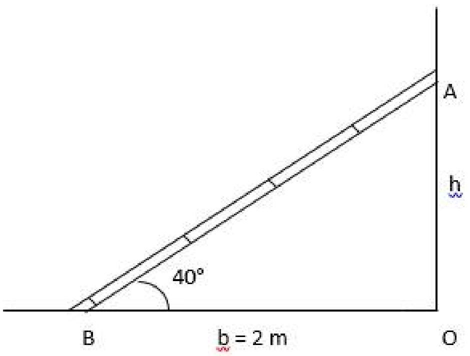
Area = $5.83\ \mathrm{cm}^2$

2. Question

A ladder leans against a wall, with its foot 2 metres away from the wall and the angle with the floor 40°. How high is the top end of the ladder from the ground?

Answer

The question can be figured out as:



Here ladder is forming a right angled triangle with wall and floor,

By using trigonometry,

$$\tan 40^\circ = \frac{\mathrm{h}}{\mathrm{b}}$$

⇒ h = b × tan40°

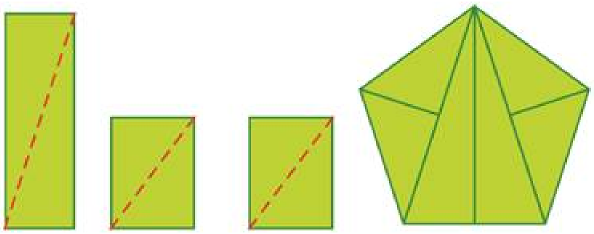
⇒ h = 2 × 0.839

⇒ h = 1.678 m

So, top end of ladder is 1.678 m high from ground.

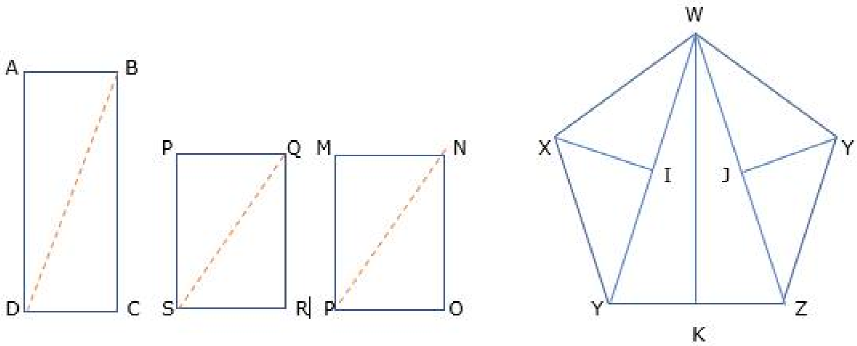
3. Question

Three rectangles are to be cut along the diagonals and the triangles so got rearranged to form a regular pentagon, as shown in the picture. If the sides of the pentagon are to be 30 centimetres, what should be the length and breadth of the rectangles?



Answer

For the figures,



Given, WX = XY = YZ = ZY = YW = 30 cm

So, It can be easily seen that,

SQ = PN = WX = 30 cm [∵ Side of pentagon given]

CD = AB = $\frac{1}{2} \times \mathrm{YZ}$

CD = AB = $\frac{1}{2} \times 30 \ = \ 15 \text{ cm}$

Let’s say in rectangle PQRS,

∠PQS = ∠RSQ

[∵ Alternate interior angles for PQ ∥ SR on rectangle PQRS]

Similarly,

∠MNP = ∠OPN

∠RQS = ∠PSQ

[∵ Alternate interior angles for SP ∥ RQ on rectangle PQRS]

∠ONP = ∠MPN

So ∠WYJ = ∠ZYJ = ∠WXI = ∠YXI

And,

∠WYJ = $\frac{1}{2} \times \angle \mathrm{WYZ}$

∠WYJ = $\frac{1}{2} \times 108°$ [∵ Interior angle of regular pentagon = 108°]

∠WYJ = 54°

So,

WJ = WY × sin54°

WJ = 30 × 0.81

WJ = 24.3 cm (i)

So, WZ = 2× WJ = 2× 24.3

WZ = 48.6

And, YJ = WY× cos54°

YJ = 30 × 0.59

YJ = 17.7 cm (ii)

Also, YZ = 2 × KZ

⇒ 30 = 2× KZ

⇒ KZ = 15 cm (iii)

By applying Pythagoras Theorem to ΔWKZ, we have,

$WK^2 = WZ^2 - KZ^2$

$\Rightarrow WK^2 = 48.6^2 - 15^2$

$\Rightarrow WK^2 = 2361.96 – 225$

$\Rightarrow WK^2 = 2136.96$

⇒ WK = 46.2 cm (iv)

Thus dimensions of rectangles are as follows:

In ABCD,

Length AD = BC = 46.2 cm [From (iv) WK = 46.2 cm]

Breadth AB = CD = 15 cm [From (iii) KZ = 15 cm]

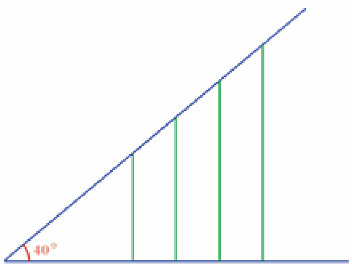
In PQRS and MNOP,

Length PS = QR = MP = NO = 24.3 cm [From (i) WJ = 24,3 cm]

Breadth PQ = SR = MN = PO = 17.7 cm [From (ii) YJ = 17.7 cm]

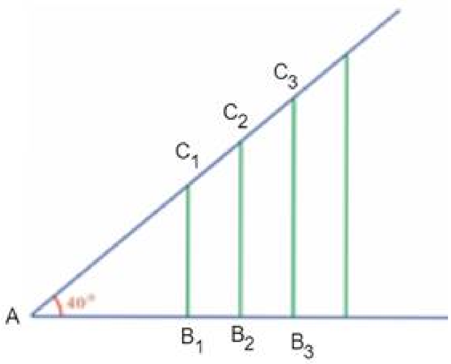
4. Question

In the picture, the vertical lines are equally spaced. Prove that their heights are in arithmetic sequence. What is the common difference?



Answer

Let us label the diagram



Let the distance between two consecutive vertical lines be 'x' and height of vertical lines be h_1, h_2, h_3, ..., and so on.

Let $AB_1 = x$

Now, we know

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

⇒ perpendicular = base × tanθ

Therefore, in consecutive triangles, we have

$B_1C_1 = AB_1 \times \tan 40°$

⇒ $h_1 = a\tan40°$

$B_2C_2 = AB_2 \times \tan 40°$

⇒ $h_2 = (a + x)\tan40°$

$B_3C_3 = AB_3 \times \tan 40°$

⇒ $h_3 = (a + 2x)\tan40°$

.

.

.

and so on.

Now, we have

$h_1, h_2, h_3, \ldots = a\tan40°, (a + x)\tan40°, (a + x)\tan40°, \ldots$

Let us calculate the common difference.

$h_2 - h_1 = (a + x)\tan40° - a\tan40° = x\tan40°$

$h_3 - h_2 = (a + 2x)\tan40° - (a + x)\tan40° = x\tan40°$

As, the common difference between the terms is same, terms are in AP and hence,

$h_1, h_2, h_3, \ldots$ are in AP.

Therefore, heights of vertical lines are in AP with common difference $x\tan40°$, where 'x' is the space between two consecutive lines. [tan 40° = 0.8391]

5. Question

One side of a triangle is 6 centimetres and the angles at its ends are 40° and 65°. Calculate its area.

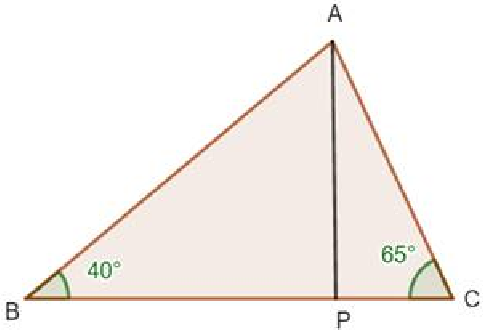
Answer

Let ABC be a triangle, with BC = 6 cm and angles at its ends are ∠ABC = 40° and ∠ACB = 65° respectively.

Draw AP ⊥ BC, such that APB and APC are right-angled triangles,

Let AP = 'h' cm

BP = 'x' cm and PC = (6 - x) cm



In ΔAPB,

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan\angle ABC = \frac{AP}{BP}$

$\Rightarrow \tan 40° = \frac{h}{x}$

$\Rightarrow 0.8391 = \frac{h}{x}$

$\Rightarrow x = \frac{h}{0.8391}$ ...[1]

In ΔAPC

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan\angle ACB = \frac{AP}{BP}$

$\Rightarrow \tan 65° = \frac{h}{6 - x}$

$\Rightarrow 2.1445 = \frac{h}{(6 - x)}$

⇒ 2.1445(6 - x) = h

$\Rightarrow 2.1445\left(6 - \frac{h}{0.8391}\right)$ = h [From 1]

⇒ 12.867 - 2.556h = h

⇒ 3.556h = 12.867

⇒ h = 3.618 cm

Also, area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

$\Rightarrow \text{area}(\Delta ABC) = \frac{1}{2} \times BC \times AP$

$\Rightarrow \text{area}(\Delta ABC) = \frac{1}{2} \times 6 \times h$

⇒ area(ΔABC) = 3 × 3.618

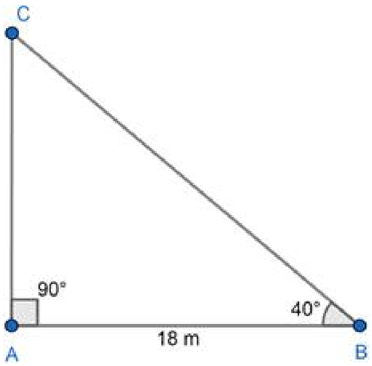
= 10.854 cm^2 [appx]

Questions Pg-122

1. Question

When the sun is at an elevation of 40°, the length of the shadow of a tree is 18 metres. What is the height of the tree.

Answer



Given :Angle of elevation = 40° and Length of shadow = 18 m.Let the required triangle be Δ ABC.∴ Angle of elevation = ∠ ABC = 40° and Length of shadow = AB = 18 m and Height of tree be AC.

In right Δ BAC,AC = AB × tan 40°⇒ AC = 18 × tan 40° (From table, tan 40° = 0.839)

⇒ AC = 18 × 0.839 = 15.102 m = 15.1 m

Height of tree is 15.1 metre.

2. Question

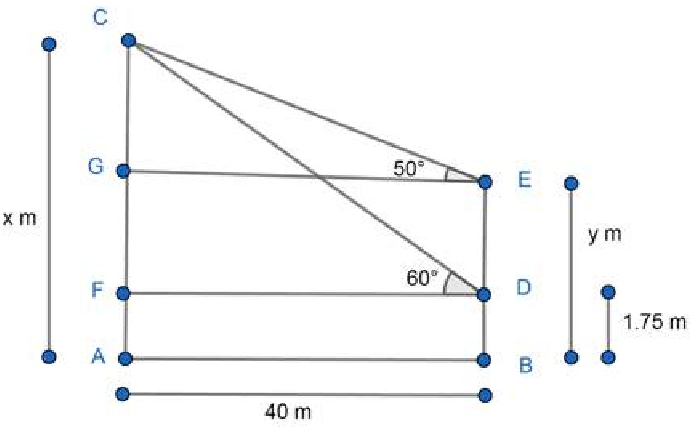
A 1.75 metre tall man, standing at the foot of a tower, sees the top of a hill 40 metres away at an elevation of 60°. Climbing to the top of the tower, he sees it at an elevation of 50°. Calculate the heights of the tower and the hill.

Answer

Let the height of the hill be x m and height of tower be y m.

Let BD be height of boy 1.75 m.

The below diagram shows the relation for the given condition.



In right Δ DFC,FC = AC – AF = (x – 1.75) m and FD = AB = 40 m

FC = tan 60° × FD⇒ (x – 1.75) = tan 60° × 40(From table, tan 60° = 1.732)⇒ (x – 1.75) = 1.732× 40 = 69.28⇒ x = 69.28 + 1.75 = 71.03 m

In right Δ EGC,GC = AC – AG = (71.03 – y) m and EG = AB = 40 m

GC = tan 50° × EG⇒ (71.03 – y) = tan 50° × 40(From table, tan 50° = 1.19)⇒ (71.03 – y) = 1.19× 40 =

47.6⇒ y = 71.03 – 47.6 = 23.43 m

The height of the hill is 71.03 m and height of tower is 23.43 m.

3. Question

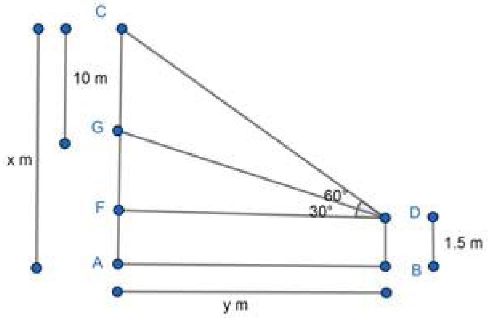
A 1.5 metre tall boy saw the top of a building under construction at an elevation of 30°. The completed building was 10 metres higher and the boy saw its top at an elevation of 60° from the same spot. What is the height of the building?

Answer

Let x m be actual height of building and BD be the height of

boy i.e. BD = 1.5 m.

Let the distance between the boy and the building be y m.



In right Δ DFC,FC = AC – AF = (x – 1.5) m and FD = AB = y m

FC = tan 60° × FD⇒ (x – 1.5) = tan 60° × y(From table, tan 60° = 1.732)⇒ (x – 1.75) = 1.732× y …(i)

In right Δ DFG,FG = AC – CG – AF = (x – 10 – 1.5) m = (x – 11.5) mand FD = AB = y m

FG = tan 30° × FD

⇒ (x – 11.5) = tan 30° × y(From table, tan 30° = 0.57)

⇒ (x – 11.5) = 0.57 × y …(ii)

Dividing eq. (i) from eq. (ii)

$$\Rightarrow \frac{x - 1.75}{x - 11.5} = \frac{1.732 \times y}{0.57 \times y}$$

⇒ 1.732(x – 11.5) = 0.57(x – 1.75)

⇒ 1.732(x) – 1.732(11.5) = 0.57(x) – 0.57(1.75)

⇒ 1.732(x) – 0.57(x) = 1.732(11.5) – 0.57(1.75)

⇒ (1.732 – 0.57)x = 19.918 – 0.9975

⇒ 1.162(x) = 18.92

$$\Rightarrow x = \frac{18.92}{1.162} = 16.28$$

Height of the building is 16.28 m.

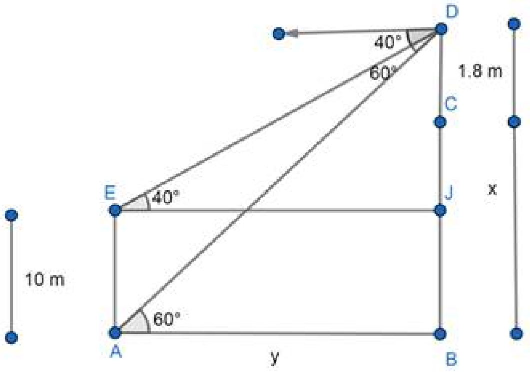
4. Question

A man 1.8 metre tall standing at the top of a telephone tower, saw the top of a 10 metre high building at a depression of 40° and the base of the building at a depression of 60°.What is the height of the tower? How far is it from the building?

Answer

Let the height of tower be x metre and distance between tower and building be y metre.

Let CD be the height of man i.e. is 1.8 m.



In right Δ ABD,BD = (x + 1.8) m and AB = y m

BD = tan 60° × AB⇒ (x + 1.8) = tan 60° × y(From table, tan 60° = 1.732)⇒ (x + 1.8) = 1.732× y ...(i)

In right Δ DEJ,DJ = (1.8 + x – 10) m = (x – 8.2) m and EJ = AB = y m

DJ = tan 40° × EJ⇒ (x – 8.2) = tan 40° × y(From table, tan 40° = 0.839 = 0.84)⇒ (x – 8.2) = 0.84× y ...(ii)

Dividing eq. (i) from eq. (ii),

$$\Rightarrow \frac{x\ +1.8}{x-8.2}=\frac{1.732\times y}{0.84\times y}$$

⇒ 1.732(x – 8.2) = 0.84(x + 1.82)

⇒ 1.732(x) – 1.732(8.2) = 0.84(x) + 0.84(1.82)

⇒ 1.732(x) – 0.84(x) = 1.732(8.2) + 0.84(1.82)

⇒ x(1.732 – 0.84) = 14.2024 + 1.5288

⇒ x(0.892) = 15.7312

$$\Rightarrow\ x=\frac{15.7312}{0.892}=17.63$$

⇒ x = 17.63 mSubstituting the value of x in eq. (i)

⇒ (x + 1.8) = 1.732× y

⇒ (17.63 + 1.8) = 1.732(y)

⇒ 19.43 = 1.732(y)

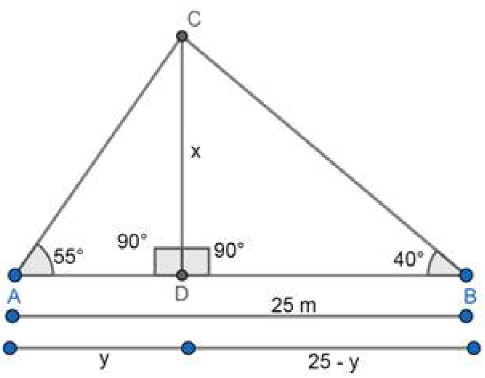
⇒ y = 11.21 m

Height of tower is 17.63 m and distance between tower and building is 11.21 m.

5. Question

From the top of an electric post, two wires are stretched to either side and fixed to the ground, 25 metres apart. The wires make angles 55° and 40° with the ground. What is the height of the post?

Answer



Let the height of the post be x metre and distance between post and one of the wire fixed point be y metre.

In right Δ ADC,

AD = y m and CD = x mCD = tan 55° × y

⇒ x = tan 55° × y

(From table, tan 55° = 1.428 = 1.43)

⇒ x = 1.43 × y ...(i)

In right Δ CDB,

BD = AB – AD = (25 –y) m and CD = x

CD = tan 40° × BD

⇒ CD = tan 40° × (25 – y)

(From table, tan 40° = 0.839 = 0.84)

⇒ x = tan 40° × (25 – y)

⇒ x = 0.84 × (25 – y) ...(ii)

Dividing eq. (i) from eq. (ii)

$$\Rightarrow \frac{x}{x} = \frac{1.43 \times y}{0.84 \times (25 - y)}$$

⇒ 0.84(25) – 0.84(y) = 1.43(y)

⇒ 1.43(y) + 0.84(y) = 0.84(25)

⇒ 2.27 (y) = 21

⇒ y = 9.25 mSubstituting value of y in eq. (i)

⇒ x = (1.43 × 9.25) m = 13.2275 m

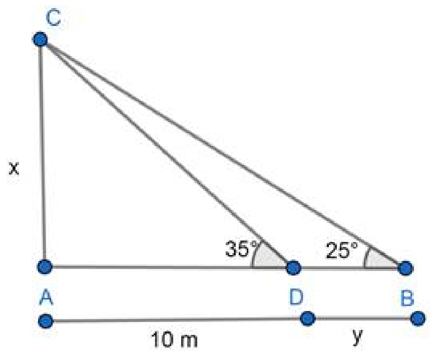
Height of post is 9.25 m

6. Question

When the sun is at an elevation of 35°, the shadow of a tree is 10 metres. What would be the length of the shadow, when the sun is at an elevation of 25°?

Answer

Let the height of the tree be x metre.



In right Δ BAC,

AC = x and AB = AD + DB = (10 + y) m

AC = tan 25° × AB

⇒ x = tan 25° × (10 + y)

(From table, tan 25° = 0.466)

⇒ x = 0.466 × (10 + y) ...(i)

In right Δ DAC,

AC = x and AD = y

AC = tan 35° × AD

⇒ x = tan 35° × (y)

(From table, tan 35° = 0.7)

⇒ x = 0.7 × y ...(ii)

Dividing eq. (i) from eq. (ii),

$$\Rightarrow \frac{x}{x} = \frac{0.466 \times (10 + y)}{0.7 \times y}$$

⇒ 0.7(y) = 0.466(10) + 0.466(y)

⇒ 0.7(y) – 0.466(y) = 0.466(10)

⇒ 0.234 (y) = 4.66

⇒ y = 19.91 m

AB = 10 + y = 10 + 19.91 = 29.91 m

Length of shadow at 25° = AB = 29.91 m

Length of shadow = 29.91 m