# RATIONAL NUMBER

**6**
**CHAPTER**

## CONTENTS



### ➤ DEFINITION

A rational number is a number of the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

p is called numerator ($N^r$) and q is called denominator ($D^r$)

❖ **Results**

Since every number is divisible by 1, we can say that :

(i) Every natural number is a rational number, but every rational number need not be a natural number.

For example, $3 = \frac{3}{1}, 5 = \frac{5}{1}, 9 = \frac{9}{1}$ and so on.

but, $\frac{7}{9}, \frac{11}{13}, \frac{5}{7}$ are rational numbers but not natural numbers.

(ii) Zero is a rational number because $\left(0 = \frac{0}{1} = \frac{0}{2} = ...\right)$

(iii) Every integer is a rational number, but every rational number may not be an integer.

For example, $\frac{-2}{1}, \frac{-5}{1}, \frac{0}{1}, \frac{3}{1}, \frac{5}{1}$ etc. are all rationals, but rationals like $\frac{3}{2}, \frac{-5}{2}$, etc. are not integers.

(iv) Rational numbers can be positive and negative.

**Eg :** $\frac{2}{3}, \frac{-7}{-8}, \frac{8}{11}, \frac{-9}{-3}$ etc. are positive rational numbers

**&** $\frac{-2}{3}, \frac{7}{8}, \frac{-8}{11}, \frac{11}{-20}$ etc. are negative rational numbers.

(v) Every positive rational number is greater than zero.

(vi) Every negative rational number is less than zero.

(vii) Every positive rational number is greater than every negative rational number.

(viii) Every negative rational number is smaller than every positive rational number.

### ❖ EXAMPLES ❖

**Ex.1** Write down the numerator of each of the following. Also find which is positive and negative rational number.

(i) $\frac{-1}{-2}$ (ii) $\frac{-3}{5}$

(iii) $\frac{7}{-5}$

**Sol.** (i) We have $\frac{-1}{-2}$ in which numerator is –1. It is a positive rational number.

(ii) Numerator is –3 and negative rational number.

(iii) Numerator is 7 and negative rational number.

**Ex.2** Write the denominator of each of the following rational numbers :

(i) $\frac{-8}{3}$ (ii) $\frac{5}{-2}$ (iii) $\frac{5}{4}$

**Sol.** Denominator are 3, –2, 4 respectively.

### ◈ Equivalent Rational Numbers

Θ Rational no. can be written with different $N^r$ and $D^r$.

**Eg :** $\frac{-5}{7} = \frac{-5\times2}{7\times2} = \frac{-10}{14}$ $\therefore \frac{-5}{7}$ is same as $\frac{-10}{14}$

$\frac{-5}{7} = \frac{-5\times3}{7\times3} = \frac{-15}{21}$ $\frac{-5}{7}$ is same as $\frac{-15}{21}$

$\frac{-5}{7} = \frac{-5\times-1}{7\times-1} = \frac{5}{-7}$ $\frac{-5}{7}$ is same as $\frac{5}{-7}$

Such rational number that are equal to each other are said to be equivalent to each other.

**❖ EXAMPLES ❖**

**Ex.3** Express $\frac{-5}{6}$ as a rational number with numerator –15.

**Sol.** By multiplying both the numerator and denominator of $\frac{-5}{6}$ by 3,

we get, $\frac{-5}{6} = \frac{(-5)\times3}{6\times3} = \frac{-15}{18}$

Therefore, the required rational number is $\frac{-15}{18}$.

**Ex.4** Write $\frac{2}{5}$ in an equivalent form so that the numerator is equal to –56.

**Sol.** Multiplying both the numerator and denominator of $\frac{2}{5}$ by –28, we have

$$\frac{2\times(-28)}{5\times(-28)} = \frac{-56}{-140}.$$

### ◈ Lowest Form of a Rational Number

A rational number is said to be in lowest form if the numerator and the denominator have no common factor other than 1.

**❖ EXAMPLE ❖**

**Ex.5** Write the following rational numbers in the lowest form :

(i) $\frac{-36}{180}$ (ii) $\frac{-64}{256}$

**Sol.** (i) Here, HCF of 36 and 180 is 36, therefore, we divide the numerator and denominator of $\frac{-36}{180}$ by 36, we have

$$\frac{-36\div36}{180\div36} = \frac{-1}{5}$$

So, the lowest form of $\frac{-36}{180}$ is $\frac{-1}{5}$

(ii) Here, HCF of 64 and 256 is 64.

Dividing the numerator and denominator of $\frac{-64}{256}$ by 64, we have

$$\frac{-64\div64}{256\div64} = \frac{-1}{4}$$

So, the lowest form of $\frac{-64}{256}$ is $\frac{-1}{4}$.

### ◈ Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in its standard form if

(i) its denominator 'q' is positive

(ii) the numerator and denominator have no common factor other than 1.

For example : $\frac{2}{3}, \frac{-1}{2}, \frac{5}{7}$, etc.

## ❖ EXAMPLES ❖

**Ex.6** Write the rational number $\frac{5}{-7}$ with a positive denominator.

**Sol.** $\frac{5}{-7} = \frac{5 \times (-1)}{(-7) \times (-1)} = \frac{-5}{7}$

which is the required answer.

**Ex.7** Express the rational number $\frac{14}{-21}$ in standard form.

**Sol.** The given rational number is $\frac{14}{-21}$.

1. **Its denominator is negative.** Multiply both the numerator and denominator by –1 to change it to **positive**, i.e.,

$$\frac{14}{-21} = \frac{14 \times (-1)}{(-21) \times (-1)} = \frac{-14}{21}$$

2. **The greatest common divisor of 14 and 21 is 7.** Dividing both numerator and denominator by 7, we have

$$\frac{-14}{21} = \frac{(-14) \div 7}{21 \div 7} = \frac{-2}{3}$$

Which is the required answer.

**Ex.8** Express in the standard form : $\frac{-144}{-504}$.

**Sol.** To change in the lowest form, divide both numerator and denominator by g.c.d. of 144 and 504 i.e., 72

$\frac{144 \div 72}{504 \div 72} = \frac{2}{7}$, which is the required answer.

**Ex.9** Find x such that :

(i) $\frac{-1}{5} = \frac{8}{x}$

(ii) $\frac{-48}{x} = 2$.

**Sol.** (i) Given that, $\frac{-1}{5} = \frac{8}{x}$, by cross multiplication, we get

$(-1) \times x = 5 \times 8$

$\Rightarrow -x = 40$

Multiplying both sides by (–1), we get

$(-x) \times (-1) = 40 \times (-1)$

$x = -40$.

(ii) Given that $\frac{-48}{x} = 2$ or $\frac{-48}{x} = \frac{2}{1}$, by cross multiplication, we get

$-48 \times 1 = 2 \times x$ or $x = \frac{-48}{2} = -24$.

## ➤ EQUALITY OF RATIONAL NUMBERS

**Method-1 :** If two or more rational numbers have the same standard form, we say that the given rational numbers are equal.

**Ex.10** Are the rational numbers $\frac{8}{-12}$ and $\frac{-50}{75}$ equal?

**Sol.** We first express these given rational numbers in the standard form.

The **first** rational number is $\frac{8}{-12}$.

(i) Multiplying both the numerator and denominator by –1.

We have, $\frac{8}{-12} = \frac{8 \times (-1)}{(-12) \times (-1)} = \frac{-8}{12}$

(ii) Dividing both the numerator and denominator by the greatest common divisor of 8 and 12, which is 4.

We have, $\frac{8}{-12} = \frac{(-8) \div 4}{12 \div 4} = \left[\frac{-2}{3}\right]$

Again, the second rational number is $\frac{-50}{75}$

(i) The denominator is positive.

(ii) Dividing both numerator and denominator by the greatest common divisor of 50 and 75, which is 25.

We have, $\frac{-50}{75} = \frac{(-50) \div 25}{75 \div 25} = \left[\frac{-2}{3}\right]$

Clearly, both the rational numbers have the same standard form.

Therefore, $\frac{8}{-12} = \frac{-50}{75}$

**Method-2 :** In this method, to test the equality of two rational numbers, say $\frac{a}{b}$ and $\frac{c}{d}$, we use cross multiplication in the following way :

$$\frac{a}{b} = \frac{c}{d}$$

Then $a \times d = b \times c$

If $a \times d = b \times c$, we say that the two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

**Ex.11** Check the equality of the rational numbers $\frac{-7}{21}$ and $\frac{3}{-9}$.

**Sol.** The given rational numbers are

$\frac{-7}{21}$ and $\frac{3}{-9}$.

By cross multiplication, we get

$(-7) \times (-9) = 21 \times 3$

i.e., $63 = 63$.

Clearly, both sides are same. Thus, we can say that

$$\frac{-7}{21} = \frac{3}{-9}.$$

## COMPARISON OF RATIONAL NUMBERS

- **Comparing fraction.** We compare two unequal fractions, each is written as another equal fraction so that both have the same denominators. Then the fraction with greater numerator is greater.

Example : To compare $\frac{7}{6}$ and $\frac{5}{8}$, find the L.C.M. of 6 and 8 (it is 24) and

$$\frac{7}{6} = \frac{7 \times 4}{6 \times 4} = \frac{28}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

As $\frac{28}{24} > \frac{15}{24}$ (as $28 > 15$)

$\Rightarrow \frac{7}{6} > \frac{5}{8}$

> Quicker method of comparison of $\frac{a}{b}$ and $\frac{c}{d}$ is that $\frac{a}{b} > \frac{c}{d}$ if $ad > bc$.
>
> $\frac{7}{6} > \frac{5}{8}$ as $(7 \times 8 > 6 \times 5)$

To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

For example,

$\frac{-9}{13}$ and $\frac{-5}{3}$,

we first compare $\frac{9}{13}$ and $\frac{5}{3}$.

$\frac{9}{13} < \frac{5}{3}$ $(\because 9 \times 3 < 13 \times 5 \Rightarrow 27 < 65)$

and conclude that $\frac{-9}{13} > \frac{-5}{3}$.

**Note :**

Every positive rational number is greater than negative rational number.

- **Comparing integers.** There are two methods for comparing the given integers.

(i) Represent the two integers by the points (say A and B) on the number line. The point to the right of the other represents the greater.

(ii) We write the integers in ascending order of infinity many integers.

$\ldots -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, \ldots$

Clearly, every positive integer (+1, +2, +3, +4 etc.) is greater than every negative integer (…–4, –3, –2, –1) and 0.

Every non-negative integer i.e., whole number (0, +1, +2, +3, +….) is greater than every negative integer (…–3, –2, –1,…)

> Let x and y be integers and if $x > y \Rightarrow -x < -y$.
>
> e.g. $8 > 5 \Rightarrow -8 < -5$.

## ❖ EXAMPLES ❖

**Ex.12** Which of the two rational numbers $\frac{5}{-12}$ and $\frac{-4}{9}$ is greater ?

**Sol.** The given rational numbers are $\frac{5}{-12}$ and $\frac{-4}{9}$ or $\frac{-5}{12}$ and $\frac{-4}{9}$.

Now, we write the rational numbers $\frac{-5}{12}$ and $\frac{-4}{9}$ in the **form of common denominator.**

$\frac{-5}{12} = \frac{(-5)\times 9}{12\times 9} = \frac{-45}{108}$ (multiplying by the denominator of $\frac{-4}{9}$)

Again, $\frac{-4}{9} = \frac{(-4)\times 12}{9\times 12} = \frac{-48}{108}$ (multiplying by the denominator of $\frac{-5}{12}$)

Since, $-48 < -45$, we have

$\frac{-48}{108} < \frac{-45}{108}$ or $\frac{-4}{9} < \frac{-5}{12}$

or $\frac{-4}{9} < \frac{5}{-12}$

**Ex.13** Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{3}{-4}$ in ascending order.

**Sol.** The given rational numbers are :

$$\frac{-7}{10}, \frac{5}{-8}, \frac{3}{-4}$$

The given rational numbers with **positive denominators** are

$$\frac{-7}{10}, \frac{-5}{8}, \frac{-3}{4}$$

We now convert these numbers so that they have **common denominator.**

For this, we take the **LCM of 10, 8 and 4.** The LCM is 40.

Therefore, we have

$$\frac{-7}{10} = \frac{-7\times 4}{10\times 4} = \frac{-28}{40}$$

$$\frac{-5}{8} = \frac{-5\times 5}{8\times 5} = \frac{-25}{40}$$

$$\frac{-3}{4} = \frac{-3\times 10}{4\times 10} = \frac{-30}{40}$$

Comparing the numerators of these numbers, we find that

$-30 < -28 < -25.$

Therefore, the ascending order is :

$$\frac{-30}{40} < \frac{-28}{40} < \frac{-25}{40}$$

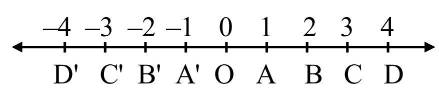
or $\frac{-3}{4} < \frac{-7}{10} < \frac{-5}{8}$

or $\frac{3}{-4} < \frac{-7}{10} < \frac{5}{-8}$.

## ➢ REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

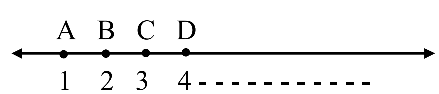
We know that the **natural numbers, whole numbers** and **integers** can be represented on a number line. For representing an integer on a number line, we draw a line and choose a point O on it to represent '0'.

**We can represent this point 'O' by any other alphabet also.** Then we mark points on the number line at equal distances on **both sides** of O. Let A, B, C, D be the points on the right hand side and A', B', C', D' be the points on the left of O as shown in the figure.

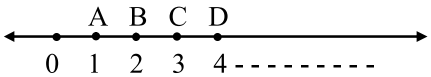



The points on the **left side of O,** i.e., A', B', C', D', etc. represent negative integers –1, –2, –3, –4 whereas, points on the **right side of O,** i.e., A, B, C, D represent positive integers 1, 2, 3, 4 etc. Clearly, the points A and A' representing the integers 1 and –1 respectively are on **opposite sides of O,** but at equal **distance from O.** Same is true for B and B' ; C and C' and other points on the number line.

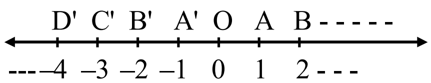
**(1) Natural Numbers**



**(2) Whole Numbers**




**(3) Integers**




Negative numbers are in left side of zero (0) & positive numbers are in right side.

$\Theta$ negative numbers are less than positive numbers

$\therefore$ If we move on number line from right to left we are getting smaller numbers.

Also OA = distance of 1 from 0

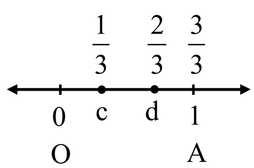
OD′ = distance of –4 from 0

D'A = distance between –4 and 1. etc.

**(4) Rational Numbers**

(a) If $N^r < D^r$ : We divide line segment OA (i.e. distance between 0 & 1) in equal parts as denominator ($D^r$).

**Eg :** $\frac{1}{3}$ and $\frac{2}{3}$ =




$\Theta$ $D^r$ is 3, so we divide OA in three equal parts by points c and d.

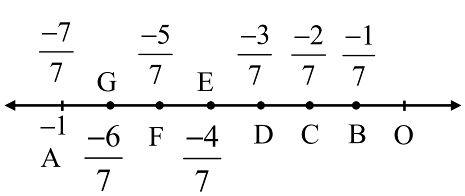
$\therefore$ $c = \frac{1}{3}$, $d = \frac{2}{3}$ and $A = \frac{3}{3} = 1$

**Eg :** $\frac{-1}{7}$ and $\frac{-4}{7}$

$\Theta$ $D^r$ is 7 $\therefore$ we divide OA in 7 equal parts by points B, C, D, E, F, G. So, these points represent

$\frac{-1}{7}, \frac{-2}{7}, \frac{-3}{7}, \frac{-4}{7}, \frac{-5}{7}, \frac{-6}{7}, \frac{-7}{7}$ respectively.

$\therefore$ $\frac{-1}{7}$ by B and $\frac{-4}{7}$ by E.

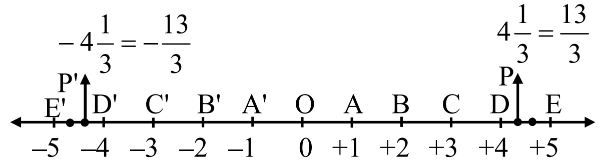



(b) If $N^r > D^r$

## ❖ EXAMPLES ❖

**Ex.14** Represent $\frac{13}{3}$ and $-\frac{13}{3}$ on number line.

**Sol.**




Draw a line *l* and mark zero on it

$$\frac{13}{3} = 4\frac{1}{3} = 4 + \frac{1}{3} \quad \text{and} \quad \frac{-13}{3} = -\left(4 + \frac{1}{3}\right)$$

Therefore, from O mark OA, AB, BC, CD and DE to the right of O such that

OA = AB = BC = CD = DE = 1 unit.

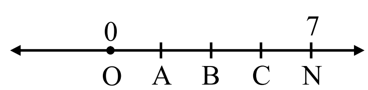
Clearly,

Point A,B,C,D,E represents the Rational numbers 1, 2, 3, 4, 5 respectively.

Since we have to consider 4 complete units and a part of the fifth unit, therefore divide the fifth unit DE into 3 equal parts. Take 1 part out of these 3 parts. Then point P is the representation of number $\frac{13}{3}$ on the number line. Similarly, take 4 full unit lengths to the left of 0 and divide the fifth unit D'E' into 3 equal parts. Take 1 part out of these three equal parts. Thus, P' represents the rational number $-\frac{13}{3}$.

**Ex.15** Represent the rational number $\frac{7}{4}$ on the number line.

**Sol.** Let N is the point that represents the integer 7 on the number line and is on the right hand side of the point O. Divide the segment ON into **four** (Denominator of $\frac{7}{4}$) equal parts (with the help of a ruler). Let A, B, C be the points of division as shown in the figure.



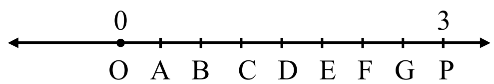

Then OA = AB = BC = CN.

Therefore, the point A represents the rational number $\frac{7}{4}$. Similarly, $\frac{-7}{4}$ can be represented on the number line on the left hand side of 'O'.

**Ex.16** Draw the number line and represent the following rational numbers on it.

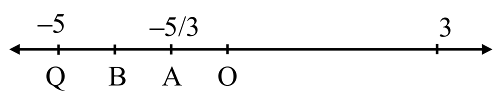
(i) $\frac{3}{8}$ (ii) $-\frac{5}{3}$

**Sol.** (i) In order to represent $\frac{3}{8}$ on number line, we first draw a number line and mark the point O on it representing '0' (zero), we find the point P on the number line representing the positive integer 3 as shown in figure.




Now divide the segment OP into 8 equal parts. Let A and B be points of division so that
OA = AB = BC = CD = DE = EF = FG = GP.
By construction, OA is $\frac{1}{8}$ of OP. Therefore, A represents the rational number $\frac{3}{8}$.

(ii)




## ➤ RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

If x and y are any two rational numbers such that x < y, then $x < \frac{x+y}{2} < y$ or that the rational number $\frac{x+y}{2}$ lies between x and y.

**Note** : There are countless rational numbers between any two rational numbers between any two distinct rational numbers, there is always a rational number.

### ❖ EXAMPLES ❖

**Ex.17** Write five rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

**Sol.** To find five rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$ take higher common multiple of 2 and 3 that is 36 then $\frac{1}{3} = \frac{1}{3} \times \frac{12}{12} = \frac{12}{36}$ and $\frac{1}{2} = \frac{1}{2} \times \frac{18}{18} = \frac{18}{36}$ So five rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$ are $\frac{13}{36}, \frac{14}{36}, \frac{15}{36}, \frac{16}{36}$ and $\frac{17}{36}$.

**Ex.18** Write three more numbers in the following pattern :

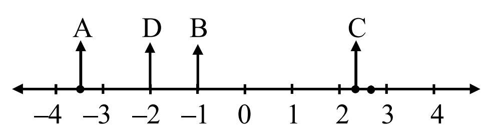
$$\frac{-2}{5}, \frac{-4}{10}, \frac{-6}{15}, \frac{-8}{20}, \ldots\ldots$$

**Sol.** $\frac{-4}{10} = \frac{-2\times 2}{5\times 2}$, $\frac{-6}{15} = \frac{-2\times 3}{5\times 3}$, $\frac{-8}{20} = \frac{-2\times 4}{5\times 4}$

Thus, we observe a pattern in these numbers. The other numbers would be

$$\frac{-2\times 5}{5\times 5} = \frac{-10}{25},\ \frac{-2\times 6}{5\times 6} = \frac{-12}{30},\ \frac{-2\times 7}{5\times 7} = \frac{-14}{35}.$$

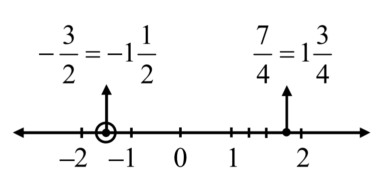
**Ex.19** Name the rational numbers denoted by A, B, C, D.




**Sol.** $A \to -3\frac{1}{2},\ B \to -1,\ C \to 2\frac{1}{3},\ D \to -2.$

**Ex.20** Represent $\frac{-3}{2}, \frac{+7}{4}$ on number line.

**Sol.** We can write, $\frac{-3}{2} = -\left(1+\frac{1}{2}\right), \frac{7}{4} = \left(1+\frac{3}{4}\right)$




**Ex.21** Identify the positive and negative rational numbers :

$$\frac{7}{8}, \frac{5}{-4}, \frac{-8}{3}, \frac{-5}{-7}, \frac{-3}{8}, \frac{7}{-4}, \frac{-1}{-22}, \frac{-9}{+4}.$$

**Sol.** Negative rational numbers

$= \frac{5}{-4}, \frac{-8}{3}, \frac{-3}{8}, \frac{7}{-4}, \frac{-9}{4}.$

Positive rational numbers $= \frac{7}{8}, \frac{-5}{-7}, \frac{-1}{-22}.$

**Ex.22** Reduce $\frac{-2}{-5}, \frac{2}{16}, \frac{64}{-128}$ in standard form.

**Sol.** $\frac{-2}{-5} = \frac{-2\times -1}{-5\times -1} = \frac{2}{5}$

$\frac{2}{16} = \frac{2\div 2}{16\div 2} = \frac{1}{8}$ ($\because$ H.C.F. of 2 and 16 = 2)

$\frac{64}{-128} = \frac{64\div(-64)}{(-128)\div(-64)} = \frac{-1}{2}.$

($\because$ H.C.F. of 64 and 128 = 64)

**Ex.23** Give two rational numbers equivalent to

(i) $\frac{-8}{9}$ (ii) $\frac{2}{11}$ (iii) $\frac{5}{-6}$ (iv) $\frac{-3}{7}$

**Sol.** (i) Rational numbers equivalent to :

$\frac{-8}{9}$ are $\frac{-16}{18}, \frac{-24}{27}$ etc.

(ii) Rational numbers equivalent to :

$\frac{2}{11}$ are $\frac{4}{22}, \frac{6}{33}$ etc.

(iii) Rational numbers equivalent to :

$\frac{5}{-6}$ are $\frac{10}{-12}, \frac{15}{-18}$ etc.

(iv) Rational numbers equivalent to :

$\frac{-3}{7}$ are $\frac{-6}{14}, \frac{-9}{21}$ etc.

**Ex.24** Fill in the blank boxes with the correct symbol out of >, < or =

(i) 0 $\boxed{\phantom{<}}$ $\frac{8}{9}$ (ii) 2 $\boxed{\phantom{<}}$ $\frac{-4}{7}$

(iii) 0 $\boxed{\phantom{<}}$ $+\frac{1}{7}$

**Sol.** (i) 0 $\boxed{<}$ $\frac{8}{9}$ (ii) 2 $\boxed{>}$ $\frac{-4}{7}$

(iii) 0 $\boxed{<}$ $+\frac{1}{7}$

**Ex.25** Write two rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$.

**Sol.** Two rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$

Means two rational numbers between $\frac{5}{20}, \frac{4}{20}$

Means two rational numbers between $\frac{50}{200}, \frac{40}{200}.$

i.e., $\frac{41}{200}, \frac{42}{200}, ........ \frac{49}{200}$ (any two)

**Ex.26** Write the following rational numbers in ascending and descending order :

$\frac{-6}{5}, \frac{1}{5}, \frac{0}{5}, \frac{-3}{-5}.$

**Sol.** Ascending order : $\frac{-6}{5}, \frac{0}{5}, \frac{1}{5}, \frac{-3}{-5}.$

Descending order : $\frac{-3}{-5}, \frac{1}{5}, \frac{0}{5}, \frac{-6}{5}.$

**Ex.27** Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

**Sol.** Given rational numbers are $\frac{-2}{3}$ and $\frac{1}{4}$.

Then a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ can be obtained by the following method :

(Addition of two rational numbers) ÷ 2

$\left(\frac{-2}{3}+\frac{1}{4}\right)\div 2 = \left(\frac{-8+3}{12}\right)\div 2 = \frac{-5}{12}\div 2$

$= \frac{-5}{12}\times\frac{1}{2} = \frac{-5\times 1}{12\times 2} = \frac{-5}{24}$

So, the rational number between

$\frac{-2}{3}$ and $\frac{1}{4}$ is $\frac{-5}{24}$.

**Ex.28** Find a rational number between $\frac{-7}{6}$ and $\frac{-6}{7}$.

**Sol.** (Addition of two rational numbers) ÷ 2

$\left[\frac{-7}{6}+\left(\frac{-6}{7}\right)\right]\div 2 = \left(\frac{-49+(-36)}{42}\right)\div 2$

[LCM of 6 and 7 is 42]

$$=\left(\frac{-49-36}{42}\right)\div 2=\frac{-85}{42}\div 2=\frac{-85}{42}\times\frac{1}{2}=\frac{-85}{84}.$$

Therefore, a rational number between $\frac{-7}{6}$ and $\frac{-6}{7}$ is $\frac{-85}{84}$.

**Ex.29** Find ten rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$.

**Sol.** We know that

$-3 < -2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8$

This gives us 10 integers between –3 and 8.

So, $\frac{-2}{11},\frac{-1}{11},0,\frac{1}{11},\frac{2}{11},\frac{3}{11},\frac{4}{11},\frac{5}{11},\frac{6}{11},\frac{7}{11}$ are the ten desired rational numbers.

## OPERATION ON RATIONAL NUMBERS

There are four basic operations on rational numbers :

(i) Addition (ii) Subtraction

(iii) Multiplication (iv) Division.

**(i) Addition :** If two rational numbers are to be added, we first express each one of them as rational number with positive denominator.

There are two possibilities :

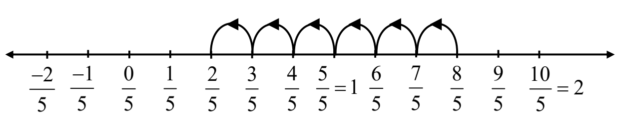
(1) Either they have same denominators, or

(2) They have different denominators.

### ❖ Rational Numbers with Same Denominator

Let us add $\frac{8}{5}$ and $\frac{-6}{5}$.

Represent the numbers on the number line.




Here, the distance between two consecutive points is $\frac{1}{5}$. For $\frac{-6}{5}$, move 6 steps to the left of $\frac{8}{5}$ and we reach at $\frac{2}{5}$.

So, $$\frac{8}{5}+\left(\frac{-6}{5}\right)=\frac{8+(-6)}{5}=\frac{2}{5}$$

### ❖ EXAMPLES ❖

**Ex.30** Add : $\frac{-5}{9}$ and $\frac{-17}{9}$.

**Sol.** Given rational numbers are $\frac{-5}{9}$ and $\frac{-17}{9}$.

Adding these two numbers, we have

$$\frac{(-5)}{9}+\frac{(-17)}{9}=\frac{(-5)+(-17)}{9}=\frac{-22}{9}$$

Which is the required answer.

**Ex.31** Add : $\frac{-23}{28}$ and $\frac{5}{-28}$

**Sol.** We first express $\frac{5}{-28}$ as a rational number with positive denominator.

We have, $\frac{5\times(-1)}{(-28)\times(-1)}=\frac{-5}{28}$

Now, $\frac{-23}{28}+\left(\frac{-5}{28}\right)=\frac{-23}{28}-\frac{5}{28}$

[since (+) × (–) = –]

$$=\frac{-23-5}{28}=\frac{-28}{28}=-1$$

### ❖ Addition of Rational numbers with Different Denominators

In this case, **we convert the** given rational numbers **to a common denominator and then add.**

**Ex.32** Add $\frac{8}{-5}$ and $\frac{4}{-3}$.

**Sol.** The given rational numbers are $\frac{8}{-5}$ and $\frac{4}{-3}$.

Clearly, they have different denominators. Here, first we express the given rational numbers into standard forms.

i.e., $\frac{8}{-5}=\frac{8\times(-1)}{(-5)\times(-1)}=\frac{-8}{5}$

And, $\frac{4}{-3}=\frac{4\times(-1)}{(-3)\times(-1)}=\frac{-4}{3}$

Now, $\frac{8}{-5}+\frac{4}{-3}=\frac{(-8)}{5}+\frac{(-4)}{3}$

Converting to the same denominators, we have

$$\frac{-8}{5} = \frac{(-8)\times 3}{5\times 3} = \frac{-24}{15}$$

And $$\frac{-4}{3} = \frac{(-4)\times 5}{3\times 5} = \frac{-20}{15}$$

So, $\frac{-8}{5} + \left(\frac{-4}{3}\right) = \frac{-24}{15} + \left(\frac{-20}{15}\right)$

$= \frac{(-24)+(-20)}{15} = \frac{-44}{15}$

Which is the required answer.

**Ex.33** Add : $\frac{8}{10}$, 3.

**Sol.** We have, 3 which can be written as $\frac{3}{1}$.

Multiplying both the numerator and denominator of $\frac{3}{1}$ by 10,

we get $\frac{3}{1} = \frac{3\times 10}{1\times 10} = \frac{30}{10}$

Therefore, $\frac{8}{10} + 3 = \frac{8}{10} + \frac{3}{1} = \frac{8}{10} + \frac{30}{10}$

$= \frac{8+30}{10} = \frac{38}{10} = \frac{19}{5}$

Which is the required answer.

**Ex.34** Simplify : $\frac{4}{3} + \frac{3}{5} + \frac{-2}{5} + \frac{-11}{3}$.

**Sol.** $\frac{4}{3} + \frac{3}{5} + \frac{-2}{5} + \frac{-11}{3} = \left(\frac{4}{3} + \frac{-11}{3}\right) + \left(\frac{3}{5} + \frac{-2}{5}\right)$

$= \frac{4-11}{3} + \frac{3-2}{5} = \frac{-7}{3} + \frac{1}{5} = \frac{-7\times 5}{3\times 5} + \frac{1\times 3}{5\times 3}$

(changing them to same denominator)

$= \frac{-35}{15} + \frac{3}{15} = \frac{-32}{15}$.

**Ex.35** Add $\frac{7}{9}$ and $\frac{-5}{9}$.

**Sol.** $\frac{7}{9} + \left(\frac{-5}{9}\right) = \frac{7+(-5)}{9} = \frac{7-5}{9} = \frac{2}{9}$.

> In case, if denominator of the rational number is negative, first we make it (denominator) Positive and then add.

**Ex.36** Add : $\frac{6}{-5}$ and $\frac{4}{5}$.

**Sol.** $\frac{6}{-5} + \frac{4}{5} = \frac{-6}{5} + \frac{4}{5} = \frac{-6+4}{5} = \frac{-2}{5}$.

**Ex.37** Find the sum of $\frac{-8}{5}$ and $\frac{-5}{3}$.

**Sol.** LCM of 5 and 3 is 15.

$\frac{-8}{5} = \frac{-8\times 3}{5\times 3} = \frac{-24}{15}$

$\Rightarrow \frac{-5}{3} = \frac{-5\times 5}{3\times 5} = \frac{-25}{15}$

So, $\frac{-8}{5} + \left(\frac{-5}{3}\right) = \frac{-24}{15} + \left(\frac{-25}{15}\right) = \frac{-24-25}{15}$

$\frac{-8}{5} + \left(\frac{-5}{3}\right) = \frac{-49}{15}$

**Note :** Addition of rational numbers is closure (the sum is also rational) commutative (a + b = b + a) and associative(a + (b + c)) = ((a + b) + c)

### ◈ Additive inverse

The negative of a rational number is called additive inverse of the given number.

| Rational number | Additive inverse |
|---|---|
| $\frac{x}{y}$ | $\frac{-x}{y}$ |
| $\frac{-3}{7}$ | $\frac{3}{7}$ |
| $\frac{13}{17}$ | $\frac{-13}{17}$ |

**Note :** Zero is the only rational no. which is its even negative or inverse.

**(ii) Subtraction :** If we add the additive inverse of a rational number and other rational number then this is called subtraction of two rational numbers. So the subtraction is inverse process of addition and the term add the negative of use for subtraction.

## ❖ EXAMPLES ❖

**Ex.38** Find value of $\frac{2}{3}-\frac{4}{5}$.

**Sol.** $\frac{2}{3}$ + additive inverse of $\frac{4}{5}$.

$= \frac{2}{3}+\left(\frac{-4}{5}\right)=\frac{2\times 5}{3\times 5}+\left(\frac{-4\times 3}{5\times 3}\right)$

$= \frac{10}{15}+\frac{-12}{15}=\frac{10+(-12)}{15}$

$= \frac{-2}{5}$.

**Ex.39** Find value of $\frac{2}{7}-\left(\frac{-5}{3}\right)$.

**Sol.** $\frac{2}{7}-\left(\frac{-5}{3}\right)=\frac{2}{7}+$ additive inverse of $\left(\frac{-5}{3}\right)$

$= \frac{2}{7}+\frac{5}{3}$

$= \frac{2\times 3}{7\times 3}+\frac{5\times 7}{3\times 7}$

$= \frac{6}{21}+\frac{35}{21}=\frac{6+35}{21}=\frac{41}{21}$.

**Ex.40** Subtract $\frac{-5}{8}$ from $\frac{-3}{7}$.

**Sol.** The given rational numbers are $\frac{-5}{8}$ and $\frac{-3}{7}$.

Therefore,

$\frac{-3}{7}-\left(\frac{-5}{8}\right)=\frac{-3}{7}+\frac{5}{8}$

$=\frac{-24}{56}+\frac{35}{56}=\frac{-24+35}{56}=\frac{11}{56}$

**Ex.41** Simplify : $\frac{1}{6}+\frac{-2}{5}-\frac{-2}{15}$

**Sol.** We have,

$\frac{1}{6}+\frac{-2}{5}-\frac{-2}{15}=\frac{1}{6}-\frac{2}{5}+\frac{2}{15}$

$\left[\text{Since}-\left(\frac{-2}{15}\right)=\frac{2}{15}\right]$

$= \frac{1\times 5-2\times 6+2\times 2}{30}$ [LCM of 6, 5, 15 is 30]

$= \frac{5-12+4}{30}=\frac{9-12}{30}=\frac{-3}{30}=\frac{-1}{10}$

**Ex.42** What number should be added to $\frac{-5}{8}$ so that the sum is $\frac{5}{9}$ ?

**Sol.** The number will be obtained by subtracting $\frac{-5}{8}$ from $\frac{5}{9}$.

So, $\frac{5}{9}-\left(\frac{-5}{8}\right)=\frac{5}{9}+\frac{5}{8}=\frac{5\times 8+5\times 9}{72}$

$=\frac{40+45}{72}=\frac{85}{72}$

Therefore, required number is $\frac{85}{72}$.

**Ex.43** What number should be subtracted from $\frac{27}{11}$ so as to get $\frac{-5}{33}$ ?

**Sol.** We have, difference of the given number and the required number = $\frac{-5}{33}$

Given number = $\frac{27}{11}$

Therefore, $\frac{27}{11}-\frac{-5}{33}=\frac{27}{11}+\frac{5}{33}$

$= \frac{27\times 3+5\times 1}{33}=\frac{81+5}{33}=\frac{86}{33}$

Therefore, required number is $\frac{86}{33}$.

**Ex.44** The sum of two rational numbers is $\frac{-3}{5}$. If one of them is $\frac{-9}{10}$. Find the other.

**Sol.** Given, sum of the numbers = $\frac{-3}{5}$

One of the numbers = $\frac{-9}{10}$

The other number

= Sum of the numbers – One of the numbers

$= \frac{-3}{5} - \frac{-9}{10} = \frac{-3}{5} + \frac{9}{10}$

$= \frac{-3\times 2 + 9\times 1}{10}$

$= \frac{-6+9}{10} = \frac{3}{10}$

Therefore, required number is $\frac{3}{10}$.

**(iii) Multiplication :**

(a) Let $\frac{a}{b}$ and c are two rational numbers, then $\frac{a}{b}\times c = \frac{ac}{b}$.

**Eg :** Find product of $\frac{-5}{7}$ and 9.

$\frac{-5}{7}\times 9 = \frac{-5}{7}\times\frac{9}{1} = \frac{-5\times 9}{7\times 1} = \frac{-45}{7}$

(b) When we multiply two rational numbers :

i.e., $\frac{a}{b}\times\frac{c}{d} = \frac{a\times c}{b\times d} = \frac{ac}{bd}$

e.g., $\frac{1}{2}\times\frac{-3}{4} = \frac{1\times(-3)}{2\times 4} = \frac{-3}{8}$.

(i) On multiplying two rational numbers, we get result as a rational number.

e.g., $\frac{-3}{4}\times\frac{5}{7} = \frac{-3\times 5}{4\times 7} = \frac{-15}{28}$

(closure property)

(ii) $\frac{1}{3}\times\frac{1}{4} = \frac{1}{4}\times\frac{1}{3}$ (commutative i.e., on changing the order the result remains same)

(iii) $\frac{1}{3}\times\left(\frac{1}{4}\times\frac{1}{5}\right) = \left(\frac{1}{3}\times\frac{1}{4}\right)\times\frac{1}{5}$ (associative)

(iv) If 0 is multiplied to any rational number, the result is always zero.

e.g.,

(a) $0\times\frac{4}{5} = \frac{0\times 4}{5} = \frac{0}{5} = 0$

(b) $\frac{-2}{3}\times 0 = \frac{-2\times 0}{3} = \frac{0}{3} = 0.$

**(iv) Division :**

(a) Let $\frac{a}{b}$ be a rational number then its reciprocal will be $\frac{b}{a}$.

(i) The product of a rational number with its reciprocal is always 1.

For example,

(a) $\frac{5}{7}\times\frac{7}{5} = 1$ (b) $\frac{-3}{8}\times\frac{-8}{3} = 1$

(ii) Zero has no reciprocal as reciprocal of 0 $\left(\frac{0}{1}\right)$ is $\frac{1}{0}$ (which is not defined)

(iii) The reciprocal of a rational number is called the multiplicative inverse of rational number.

(iv) 1 and –1 are the only rational numbers which are their own reciprocal.

Reciprocal of 1 = $\frac{1}{1}$ = 1.

Reciprocal of –1 = $\frac{1}{-1}$ = –1.

(v) Reciprocal of a (+ve) rational number is (+ve) and reciprocal of (–ve) rational number is (–ve).

To divide one rational number by other rational numbers we multiply the rational number by the reciprocal of the other i.e., $\frac{a}{b}\div\frac{c}{d}$

$= \frac{a}{b}\times$ reciprocal of $\frac{c}{d} = \frac{a}{b}\times\frac{d}{c} = \frac{ad}{bc}$

For example, $\frac{5}{-6}\div\frac{-3}{+2}$

$\frac{5}{-6}\div\frac{-3}{2} = \frac{5}{-6}\times$ reciprocal of $\left(\frac{-3}{2}\right)$

$= \frac{5}{-6}\times\frac{+2}{-3}$

$= \frac{5\times 2}{-6\times(-3)} = \frac{10}{18} = \frac{5}{9}.$

(vi) Zero divided by any rational number is always equal to zero.

For example, $0 \div \frac{2}{5} = 0$ ; $0 \div \frac{-3}{8} = 0$

**Note :**

(i) When a rational number (except zero) is divided by another rational number (except 0) the quotient is always a rational number. (closed under division)

i.e., $\frac{a}{b} \div \frac{c}{d} = \left(\frac{ad}{bc}\right)$ is also rational number.

(ii) Division of any rational number by itself gives the quotient 1.

For example, $\frac{4}{5} \div \frac{4}{5} = 1$.

(iii) When a rational number is divided by 1, the quotient is a rational number itself.

For example, $\frac{3}{4} \div 1 = \frac{3}{4}$.

## ❖ EXAMPLES ❖

**Ex.45** Find the sum :

(i) $\frac{6}{4} + \left(\frac{-11}{4}\right)$ (ii) $\frac{7}{3} + \frac{3}{7}$

(iii) $\frac{-4}{3} + 0$ (iv) $-3\frac{1}{3} + 2\frac{4}{5}$

(v) $\frac{-4}{19} + \frac{(-3)}{57}$.

**Sol.** (i) $\frac{6}{4} + \left(\frac{-11}{4}\right) = \frac{6 + (-11)}{4} = \frac{6 - 11}{4} = \frac{-5}{4}$.

(ii) $\frac{7}{3} + \frac{3}{7}$

$\frac{7}{3} = \frac{7 \times 7}{3 \times 7} = \frac{49}{21}$

($\Theta$ LCM of 3 and 7 = 21)

$\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

So, $\frac{7}{3} + \frac{3}{7} = \frac{49}{21} + \frac{9}{21} = \frac{49 + 9}{21} = \frac{58}{21}$.

**Alternative Method :**

$\frac{7}{3} + \frac{3}{7}$

$= \frac{7 \times (7) + 3 \times (3)}{21}$

($\Theta$ LCM of 3 and 7 = 21)

$= \frac{49 + 9}{21} = \frac{58}{21}$.

(iii) $\frac{-4}{3} + 0 = \frac{-4}{3} + \frac{0}{3}$

$= \frac{(-4) + 0}{3} = \frac{-4}{3}$.

(iv) $-3\frac{1}{3} + 2\frac{4}{5}$

$= \frac{-10}{3} + \frac{14}{5} = \frac{-10 \times 5 + 14 \times 3}{15}$

($\Theta$ LCM of 3 and 5 = 15)

$= \frac{-50 + 42}{15} = \frac{-8}{15}$.

(v) $\frac{-4}{19} + \frac{(-3)}{57}$

$= \frac{-4 \times 3 + (-3) \times 1}{57} = \frac{-12 + (-3)}{57}$

($\Theta$ LCM of 19 and 57 = 57)

$= \frac{-12 - 3}{57} = \frac{-15}{57}$.

**Ex.46** Find :

(i) $\frac{5}{24} - \frac{13}{36}$ (ii) $\frac{4}{63} - \left(\frac{-5}{21}\right)$

(iii) $\frac{-7}{13} - \left(\frac{-6}{25}\right)$ (iv) $\frac{-5}{8} - \frac{8}{11}$

(v) $-3\frac{1}{7} - 3$

**Sol.** (i) $\frac{5}{24} - \frac{13}{36} = \frac{5 \times (3) - 13 \times 2}{72}$

($\Theta$ LCM of 24 and 36 = 72)

$= \frac{15 - 26}{72} = \frac{-11}{72}$

(ii) $\frac{4}{63}-\left(\frac{-5}{21}\right)=\frac{4}{63}+\frac{5}{21}$

($\because$ LCM of 21 and 63 = 63)

$=\frac{4\times1+5\times3}{63}=\frac{4+15}{63}=\frac{19}{63}.$

(iii) $\frac{-7}{13}-\left(\frac{-6}{25}\right)=\frac{-7}{13}+\frac{6}{25}$

($\because$ LCM of 13 and 25 = 325)

$=\frac{-7\times25+6\times13}{325}=\frac{-175+78}{325}=\frac{-97}{325}$

(iv) $\frac{-5}{8}-\frac{8}{11}=\frac{-5\times11-8\times8}{88}$

($\because$ LCM of 8 and 11 = 88)

$=\frac{-55-64}{88}=\frac{-119}{88}$

(v) $-3\frac{1}{7}-3=-\frac{22}{7}-\frac{3}{1}$ ($\because$ LCM of 7 & 1 = 7)

$=\frac{-22-3\times7}{7}=\frac{-22-21}{7}=\frac{-43}{7}.$

**Ex.47** Find the product :

(i) $\frac{7}{2}\times\left(\frac{-5}{4}\right)$

(ii) $\frac{7}{10}\times(-9)$

(iii) $\frac{4}{-5}\times\frac{-5}{4}$

(iv) $\frac{4}{7}\times\left(\frac{-2}{5}\right)$

**Sol.** (i) $\frac{7}{2}\times\left(\frac{-5}{4}\right)=\frac{7\times(-5)}{2\times4}=\frac{-35}{8}$

(ii) $\frac{7}{10}\times(-9)=\frac{7}{10}\times\frac{(-9)}{1}=\frac{7\times(-9)}{10\times1}=\frac{-63}{10}$

(iii) $\frac{4}{-5}\times\frac{-5}{4}=\frac{4\times(-5)}{(-5)\times4}=\frac{-20}{-20}=1$

(iv) $\frac{4}{7}\times\left(\frac{-2}{5}\right)=\frac{4}{7}\times\left(\frac{-2}{5}\right)=\frac{4\times(-2)}{7\times5}=\frac{-8}{35}$

**Ex.48** Find the value of :

(i) $(-6)\div\frac{2}{3}$

(ii) $\frac{-4}{5}\div2$

(iii) $\frac{3}{13}\div\left(\frac{-4}{65}\right)$

(iv) $\frac{-2}{8}\div\frac{-2}{8}$

(v) $\left(\frac{-6}{9}\right)\div1$

**Sol.** (i) $(-6)\div\frac{2}{3}=\frac{-6}{1}\times\frac{3}{2}=\frac{-6\times3}{1\times2}=\frac{-18}{2}=-9$

So, $(-6)\div\frac{2}{3}=-9.$

(ii) $\frac{-4}{5}\div2=\frac{-4}{5}\times\frac{1}{2}=\frac{-4\times1}{5\times2}$

$=\frac{-2\times1}{5}=\frac{-2}{5}=\frac{-2}{5}$

So, $\frac{-4}{5}\div2=\frac{-2}{5}.$

(iii) $\frac{3}{13}\div\left(\frac{-4}{65}\right)=\frac{3}{13}\times\frac{65}{-4}=\frac{3\times65}{13\times(-4)}$

$=\frac{15}{-4}=\frac{-15}{4}$

So, $\frac{3}{13}\div\left(\frac{-4}{65}\right)=\frac{-15}{4}.$

(iv) $\frac{-2}{8}\div\frac{-2}{8}=\frac{-2}{8}\times\frac{8}{-2}=\frac{(-2)\times8}{8\times(-2)}=\frac{1\times1}{1\times1}=1$

So, $\frac{-2}{8}\div\frac{-2}{8}=1.$

(v) $\left(\frac{-6}{9}\right)\div1=\left(\frac{-6}{9}\right)\div\frac{1}{1}=\frac{-6}{9}\times\frac{1}{1}$

$=\frac{-6\times1}{9\times1}=\frac{-2\times1}{3\times1}=\frac{-2}{3}.$

So, $\left(\frac{-6}{9}\right)\div1=\frac{-2}{3}.$

**Ex.49** Multiply :

(i) $\left(\frac{-8}{25}\right)$ by $\left(\frac{-5}{16}\right)$

(ii) $\left(\frac{9}{-11}\right)$ by $\left(\frac{22}{-27}\right)$

**Sol.** (i) Multiplication of $\left(\frac{-8}{25}\right)$ by $\left(\frac{-5}{16}\right)$

$$= \frac{-8}{25} \times \frac{-5}{16} = \frac{-8 \times -5}{25 \times 16} = \frac{40}{400}$$

Dividing both the numerator and denominator by the greatest common divisor of 40 and 400 which is 40.

$$= \frac{40 \div 40}{400 \div 40} = \frac{1}{10}$$

(ii) Multiplication of $\left(\frac{9}{-11}\right)$ by $\left(\frac{22}{-27}\right)$

$$= \frac{9}{-11} \times \frac{22}{-27}$$

$$= \frac{9 \times 22}{-11 \times -27} = \frac{198}{297}$$

$$= \frac{198 \div 99}{297 \div 99} = \frac{2}{3}$$

**Ex.50** Simplify : $\left(\frac{-2}{3} \times \frac{9}{5}\right) + \left(\frac{2}{3} \times \frac{-6}{7}\right)$.

**Sol.** We have, $\left(\frac{-2}{3} \times \frac{9}{5}\right) + \left(\frac{2}{3} \times \frac{-6}{7}\right)$

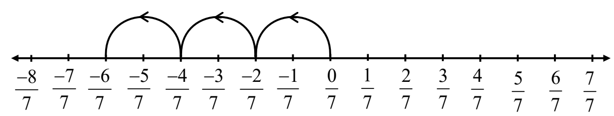
$$= \frac{-18}{15} + \left(\frac{-12}{21}\right) = \frac{-18 \times 7 + (-12 \times 5)}{105}$$

$$= \frac{-126 + (-60)}{105} = \frac{-126 - 60}{105} = \frac{-186}{105} = \frac{-62}{35}$$

## ➢ MULTIPLICATION OF RATIONAL NUMBERS ON A NUMBER LINE

The product of two rational numbers on the number line can be calculated in the following way.

When we multiply $\frac{-2}{7}$ by 3 on a number line, it means 3 jumps of $\frac{-2}{7}$ to the left from zero. Now we reach at $\frac{-6}{7}$. Thus we find $\frac{-2}{7} \times 3 = \frac{-6}{7}$, i.e., $\frac{-2}{7} \times 3 = \frac{-2}{7} \times \frac{3}{1} = \frac{-2 \times 3}{7 \times 1} = \frac{-6}{7}$.




This result reconfirms that the product of two rational numbers is rational number whose numerator is the product of the numerators of the given rational numbers and the denominator is the product of the denominators of the given numbers.

∴ Multiplication is closure (product is rational), commutative (ab = ba) and associative (a(bc) = (ab)c) for rational number.

# EXERCISE # 1

**Q.1** Draw a number line to represent the following rational numbers :

(i) $\frac{1}{3}$ (ii) $\frac{2}{3}$

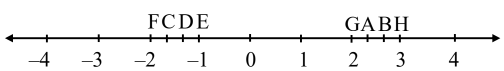
(iii) $\frac{5}{8}$ (iv) $\frac{7}{3}$

**Q.2** Draw the number line and represent the following rational numbers on it :

(i) $\frac{7}{5}$ (ii) $\frac{-5}{7}$

(iii) $-3\frac{1}{3}$ (iv) $\frac{12}{-5}$

**Q.3** The points A, B, C, D, E, F, G and H on the number line are such that ED = DC = CF and GA = AB = BH. Name the rational numbers represented by A, B, C and D.




**Q.4** Give four rational numbers equivalent to

(i) $\frac{3}{8}$ (ii) $\frac{7}{-4}$ (iii) $\frac{5}{6}$

(iv) $\frac{-3}{4}$ (v) $\frac{-2}{-3}$

**Q.5** List five rational numbers between

(i) –2 and 0 (ii) –3 and –2

(iii) $\frac{-3}{5}$ and $\frac{-2}{3}$ (iv) $\frac{1}{3}$ and $\frac{4}{5}$

**Q.6** Write three more rational numbers in each of the following patterns :

(i) $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots..$

(ii) $\frac{-1}{6}, \frac{-2}{12}, \frac{-3}{18}, \ldots..$

(iii) $\frac{-3}{5}, \frac{-9}{15}, \frac{-15}{25}, \ldots...$

(iv) $\frac{-2}{5}, \frac{2}{-5}, \frac{-4}{10}, \frac{4}{-10}, \ldots..$

**Q.7** Draw the number line and represent the following rational numbers on it :

(i) $\frac{4}{5}$ (ii) $\frac{-3}{5}$

(iii) $\frac{-8}{3}$ (iv) $\frac{9}{2}$

**Q.8** Give four rational numbers equivalent to

(i) $\frac{-1}{8}$ (ii) $\frac{6}{-5}$

(iii) $\frac{7}{9}$ (iv) $\frac{4}{7}$

**Q.9** Rewrite the following rational numbers in the simplest form :

(i) $\frac{-6}{4}$ (ii) $\frac{35}{40}$

(iii) $\frac{-55}{75}$ (iv) $\frac{14}{-28}$

**Q.10** Write the following rational numbers in ascending order :

(i) $\frac{-4}{5}, \frac{-2}{5}, \frac{-3}{5}$ (ii) $\frac{2}{3}, \frac{-2}{9}, \frac{-5}{3}$

(iii) $\frac{-4}{7}, \frac{-4}{3}, \frac{-4}{2}$

**Q.11** Write the following rational numbers in descending order :

(i) $\frac{7}{9}, \frac{-3}{5}, \frac{-2}{4}$ (ii) $\frac{3}{-5}, \frac{5}{7}, \frac{-2}{5}$

(iii) $\frac{3}{-5}, \frac{-13}{15}, \frac{2}{-5}$

**Q.12** Which is greater in each of the following :

(i) 0, $\frac{4}{7}$ (ii) $\frac{-5}{7}$, 0

(iii) $\frac{-2}{3}, \frac{-3}{2}$ (iv) $\frac{-1}{2}, \frac{-3}{9}$

(v) $\frac{-1}{4}, \frac{-4}{1}$ (vi) $\frac{-1}{40}, \frac{3}{-80}$

**Q.13** The product of two rational numbers is $\frac{-8}{9}$.

If one of the numbers is $\frac{-4}{15}$, find the other.

**Q.14** By what number should we multiply $\frac{-1}{6}$, so that the product is $\frac{-23}{9}$ ?

**Q.15** By what number should we multiply $\frac{-15}{28}$, so that the product is $\frac{-5}{7}$?

**Q.16** Find (x + y) ÷ (x – y) if :

(i) $x = \frac{2}{3}$, $y = \frac{3}{2}$ (ii) $x = \frac{1}{4}$, $y = \frac{3}{2}$

**Q.17** Find the sum of the following rational numbers :

(i) $\frac{6}{13}$ and $\frac{-2}{13}$ (ii) $\frac{5}{7}$ and $\frac{-3}{7}$

(iii) $\frac{1}{12}+\frac{-5}{8}+\frac{-2}{4}$ (iv) $\frac{-3}{5}$ and $\frac{5}{3}$

**Q.18** Subtract :

(i) $\frac{6}{9}$ from $\frac{6}{9}$ (ii) $\frac{7}{8}$ from $\frac{-2}{8}$

(iii) $\frac{-5}{7}$ from $\frac{-3}{21}$ (iv) $\frac{3}{4}$ from $\frac{19}{12}$

**Q.19** Verify the following :

(i) $\frac{-13}{5}+\frac{5}{7}=\frac{5}{7}+\frac{-13}{5}$

(ii) $\frac{+4}{(-9)}+\left(\frac{-6}{13}\right)=\left(\frac{-6}{13}\right)+\frac{4}{(-9)}$

**Q.20** Multiply :

(i) $\frac{6}{20}$ and $\frac{30}{18}$

(ii) $\frac{17}{4}$ and $\left(\frac{-4}{9}\right)$

(iii) $\left(\frac{36}{5}\right)$ and $\left(\frac{35}{-12}\right)$

(iv) $\frac{2}{5}\times\frac{6}{4}\times\frac{8}{3}$

(v) $-\frac{5}{2}\times\frac{7}{8}\times\frac{16}{7}$

(vi) $\frac{(-8)}{9}\times\frac{27}{32}\times\frac{(-8)}{35}$

**Q.21** Verify :

(i) $\frac{-12}{17}\times\frac{3}{6}=\frac{3}{6}\times\frac{-12}{17}$

(ii) $\frac{4}{-23}\times\frac{-7}{25}=\frac{-7}{25}\times\frac{4}{-23}$

**Q.22** Divide :

(i) $\frac{5}{10}$ by $\frac{-20}{35}$

(ii) $\frac{-6}{17}$ by $\frac{-17}{6}$

**Q.23** Simplify :

(i) $\frac{14}{18}\div\left(\frac{-4}{6}\right)$

(ii) $\frac{17}{2}\div\left(\frac{40}{-2}\right)$

**Q.24** State true or false for each of the following :

(i) Addition of two rational numbers is also a rational number.

(ii) $\left(\frac{1}{2}-\frac{1}{4}\right)=\left(\frac{1}{4}-\frac{1}{2}\right)$

(iii) $\frac{4}{5}\div\frac{6}{7}=\frac{6}{7}\div\frac{4}{5}$

(iv) $0\div\frac{6}{5}$ = meaningless

(v) $\frac{-8}{3}\div 0 = 0$

(vi) $\frac{-9}{14}\div\frac{14}{-9}=1$

(vii) $\left(\frac{9}{4}\times\frac{5}{7}\right)=\left(\frac{5}{7}\times\frac{9}{4}\right)$

# ANSWER KEY

**3.** A $\rightarrow 2\frac{1}{3}=\frac{7}{3}$, $B=2\frac{2}{3}$, $C=-1\frac{2}{3}$, $D=-1\frac{1}{3}$

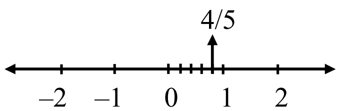
**4.** (i) $\frac{6}{16},\frac{9}{24},\frac{12}{32},\frac{15}{40}$ (ii) $\frac{14}{-8},\frac{21}{-12},\frac{28}{-16},\frac{35}{-20}$ (iii) $\frac{10}{12},\frac{15}{18},\frac{20}{24},\frac{25}{30}$ (iv) $\frac{-6}{8},\frac{-9}{12},\frac{-12}{16},\frac{-15}{20}$

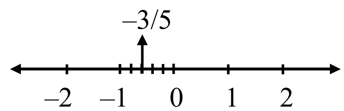
(v) $\frac{4}{6},\frac{6}{9},\frac{8}{12},\frac{10}{15}$

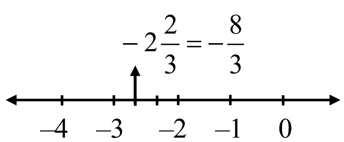
**5.** (i) $\frac{-19}{10},\frac{-18}{10},\frac{-17}{10},\frac{-10}{10}...\frac{-3}{10},\frac{-2}{10},\frac{-1}{10}$ etc. (ii) $\frac{-29}{10},\frac{-28}{10},\frac{-27}{10},\frac{-26}{10},\frac{-19}{10}$

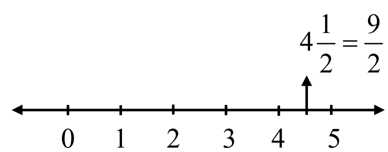
(iii) $\frac{-91}{150},\frac{-92}{150},\frac{-93}{150},....$ (iv) Any 5 rational number between $\frac{51}{150}$ to $\frac{119}{150}$.

**6.** (i) $\frac{4}{12},\frac{5}{15},\frac{6}{18}$ (ii) $\frac{-4}{24},\frac{-5}{30},\frac{-6}{36}$ (iii) $\frac{-21}{35},\frac{-27}{45},\frac{-33}{55}$ etc. (iv) $\frac{-6}{15},\frac{6}{-15},\frac{-8}{20},\frac{8}{-20}$

**7.** (i) 


(ii) 


(iii) 


(iv) 


**8.** (i) $\frac{-2}{16},\frac{-3}{24},\frac{-4}{32},\frac{-5}{40}$ (ii) $\frac{12}{-10},\frac{18}{-15},\frac{24}{-20},\frac{30}{-25}$ etc. (iii) $\frac{14}{18},\frac{21}{27},\frac{28}{36},\frac{35}{45}$ (iv) $\frac{8}{14},\frac{12}{21},\frac{16}{28},\frac{20}{35}$

**9.** (i) $\frac{-3}{2}$ (ii) $\frac{7}{8}$ (iii) $\frac{-11}{15}$ (iv) $\frac{-1}{2}$

**10.** (i) $\frac{-4}{5},\frac{-3}{5},\frac{-2}{5}$ (ii) $\frac{-5}{3},\frac{-2}{9},\frac{2}{3}$ (iii) $\frac{-4}{2},\frac{-4}{3},\frac{-4}{7}$

**11.** (i) $\frac{7}{9},\frac{-2}{4},\frac{-3}{5}$ (ii) $\frac{5}{7},\frac{-2}{5},\frac{-3}{5}$ (iii) $\frac{-2}{5},\frac{-3}{5},\frac{-13}{15}$

**12.** (i) $\frac{4}{7}>0$ (ii) $0>\frac{-5}{7}$ (iii) $\frac{-2}{3}>\frac{-3}{2}$ (iv) $\frac{-3}{9}>\frac{-1}{2}$ (v) $\frac{-1}{4}>-4$ (vi) $\frac{-1}{40}>\frac{3}{-80}$

**13.** $\frac{10}{3}$ **14.** $\frac{46}{3}$ **15.** $\frac{4}{3}$

**16.** (i) $-2\frac{3}{5}$ (ii) $-1\frac{2}{5}$

**17.** (i) $\frac{4}{13}$ (ii) $\frac{2}{7}$ (iii) $\frac{-25}{24}$ (iv) $\frac{16}{15}$

**18.** (i) 0 (ii) $\frac{-9}{8}$ (iii) $\frac{-4}{7}$ (iv) $\frac{5}{6}$

**20.** (i) $\frac{1}{2}$ (ii) $\frac{-17}{9}$ (iii) $-21$ (iv) $\frac{8}{5}$ (v) $-5$ (vi) $\frac{6}{35}$

**22.** (i) $\frac{-7}{8}$ (ii) $\frac{36}{289}$

**23.** (i) $\frac{-7}{6}$ (ii) $\frac{-17}{40}$

**24.** (i) T (ii) F (iii) F (iv) F (v) F (vi) F (vii) T

# EXERCISE # 2

**Q.1** Write True (T) or False (F) for the following statements :

(i) $\frac{-4}{6}$ is a fraction.

(ii) $\frac{26}{53}$ is equivalent to $\frac{2}{4}$.

(iii) $1\frac{1}{3}$ is a mixed number.

(iv) $\frac{170}{1700}$ is equivalent to $\frac{1}{10}$.

(v) Rational number $\frac{1}{7}$ is in the lowest form, but $\frac{7}{1}$ is not in the lowest form.

(vi) Equation 7x + 7 = 0 can be solved in integers.

(vii) Equation 6x + 5 = 0 can be solved in a fraction.

(viii) $\frac{2}{0}$ is not a rational number.

**Q.2** Write 'true' (T) or 'false' (F) for each of the following :

(i) The rational number $\frac{1}{4}$ lies to the left of the rational number $-\frac{1}{4}$.

(ii) Zero is greater than every positive rational number.

(iii) Every positive rational number is greater than every negative rational number.

(iv) Every negative rational number is smaller than zero.

**Q.3** Which of the following statements are true (T) or false (F) :

(i) Every integer is a rational number.

(ii) Every rational number is an integer.

(iii) If $\frac{a}{b}$ is a rational number and m is an integer, then $\frac{a}{b} = \frac{a \times m}{b \times m}$.

(iv) If $\frac{a}{b}$ is a rational number and m is the greatest common divisor of a and b, then $\frac{a}{b} = \frac{a \div m}{b \div m}$.

(v) Two rational numbers with different numerators cannot be equal.

(vi) The rational number $\frac{7}{-4}$ lies on the right of '0' on the number line.

(vii) The rational numbers $\frac{1}{2}$ and –1 lie on opposite sides of '0' on the number line.

**Q.4** Write True (T) or False (F) for the following statements :

(i) $-3 < \frac{-13}{6} - \frac{11}{7}$.

(ii) $\left(\frac{-3}{4}\right) - \left(\frac{-6}{5}\right) > \frac{1}{5}$.

(iii) The negative of a negative rational number is a positive rational number.

(iv) If x and y are two given rational numbers such that x > y, then (x – y) is always a positive rational number.

(v) If x and y are two given rational numbers such that x < y, then (x – y) is always a negative rational number.

**Q.5** Write the following rational numbers in the lowest form :

(i) $\frac{3}{15}$ (ii) $\frac{-35}{150}$ (iii) $\frac{64}{-256}$

**Q.6** Write the rational number whose numerator and denominator are given below :

(i) 1 and 64 (ii) –2 and –17

**Q.7** Express the rational number $\frac{-4}{5}$ with numerator :

(i) 8 (ii) –12 (iii) –20

**Q.8** Express the rational number $\frac{6}{-7}$ whose denominator is :

(i) 7 (ii) –21

**Q.9** Express $\frac{-48}{60}$ as rational number with denominator 5.

**Q.10** Fill in the blanks by the correct symbols >, < or = :

(i) $\frac{-7}{9} \square \frac{1}{9}$ (ii) $\frac{-3}{7} \square \frac{-5}{8}$

(iii) $\frac{-6}{-7} \square \frac{18}{21}$ (iv) $-8\frac{1}{3} \square -3\frac{6}{7}$

**Q.11** Verify that, x + y = y + x for the following :

(i) $x = \frac{1}{6}$ ; $y = \frac{2}{3}$ (ii) $x = \frac{-8}{9}$ ; $y = \frac{5}{7}$

**Q.12** Evaluate the following :

(i) $\frac{2}{5} + \frac{8}{3} + \frac{4}{5} + \frac{-2}{3}$

(ii) $\frac{-7}{4} + 0 + \frac{-9}{5} + \frac{19}{10} + \frac{11}{14}$

**Q.13** The sum of two rational numbers is –8. If one of the numbers is $\frac{-10}{7}$, find the other number.

**Q.14** What number should be subtracted from $\frac{1}{3}$ so as to get $\frac{-5}{12}$ ?

**Q.15** Subtract $\frac{-8}{9}$ from $\frac{11}{24}$. Also subtract $\frac{11}{24}$ from $\frac{-8}{9}$ and compare both the results.

**Q.16** The sum of two numbers is $\frac{-1}{3}$. If one of the numbers is $\frac{-12}{3}$, find the other number.

**Q.17** What should be added to $\frac{-7}{9}$ so as to get $\frac{5}{9}$ ?

# ANSWER KEY

**1.** (i) F (ii) F (iii) T (iv) T (v) F (vi) T (vii) F (viii) T

**2.** (i) F (ii) F (iii) T (iv) T **3.** (i) T (ii) F (iii) T (iv) T (v) F (vi) F (vii) T

**4.** (i) F (ii) T (iii) T (iv) T (v) T **5.** (i) $\frac{1}{5}$ (ii) $\frac{-7}{30}$ (iii) $\frac{-1}{4}$

**6.** (i) $\frac{1}{64}$ (ii) $\frac{-2}{-17}$ **7.**(i) $\frac{8}{-10}$ (ii) $\frac{-12}{15}$ (iii) $\frac{-20}{25}$

**8.** (i) $\frac{-6}{7}$ (ii) $\frac{18}{-21}$ **9.** $\frac{-4}{5}$

**10.** (i) $\frac{-7}{9} < \frac{1}{9}$ (ii) $\frac{-3}{7} > \frac{-5}{8}$ (iii) $\frac{-6}{-7} = \frac{18}{21}$ (iv) $-8\frac{1}{3} < -3\frac{6}{7}$

**12.** (i) $3\frac{1}{5}$ (ii) $\frac{121}{140}$ **13.** $\frac{-46}{7}$ **14.** $\frac{3}{4}$ **15.** $\frac{97}{72} > \frac{-97}{72}$ **16.** $\frac{11}{3}$ **17.** $\frac{4}{3}$