ICSE 2025 EXAMINATION

Sample Question Paper - 11

Mathematics

Time: 2 ½ Hours. **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) $\begin{bmatrix} 2 & 1 & 5 & 2 \\ 2 & 3 & 9 & 0 \\ 4 & 5 & 7 & 4 \\ 9 & 0 & 0 & 4 \\ 4 & 5 & 5 & 1 \\ 6 & 8 & 8 & 6 \end{bmatrix}$ is a matrix of order?

(a) 4×6
(b) 6×4
(c) 24
(d) 0

ii) Let the quadratic equation $4x^2 - (p - 2)x + 1 = 0$ has equal roots, then p is equal to

(a) -1
(b) 2
(c) -4
(d) 6

iii) A dealer in Meerut buys some goods worth Rs. 14,000. If the rate of GST is 28%, find how much will the dealer pay as CGST.

(a) Rs. 960
(b) Rs. 1960
(c) Rs. 17,920
(d) Rs. 16,960

iv) The roots of the equation $x^2 - \left(1+\sqrt{2}\right)x + \sqrt{2} = 0$ are

(a) $1, -\sqrt{2}$
(b) $1, \sqrt{2}$
(c) $-1, \sqrt{2}$
(d) $-1, -\sqrt{2}$

v) If the sum of first 'n' terms of an Arithmetic Progression 24, 21, 18,… is 78, then the value of 'n' is
(a) 4
(b) 12
(c) 14
(d) 16

vi) If $\frac{y-x}{x} = \frac{3}{8}$, find the value of $\frac{y}{x}$.

(a) $\frac{8}{11}$
(b) $\frac{11}{8}$
(c) $\frac{12}{7}$
(d) $\frac{7}{12}$

vii) Two figures are said to be similar if and only if they have…
(a) same shape and same size
(b) same shape but not necessarily same size
(c) same size but not necessary same shape
(d) either same shape or the same size

viii) If a cone has volume 154 cm^3 and the perpendicular height 12 cm, then the radius will be
(a) 3.5 cm
(b) 7 cm
(c) 3 cm
(d) 12 cm

ix) A card is drawn from a well-shuffled deck of 52 cards.

Statement 1: The probability of getting an ace card is $\frac{1}{13}$.

Statement 2: The probability of getting a black ace card is $\frac{2}{13}$.

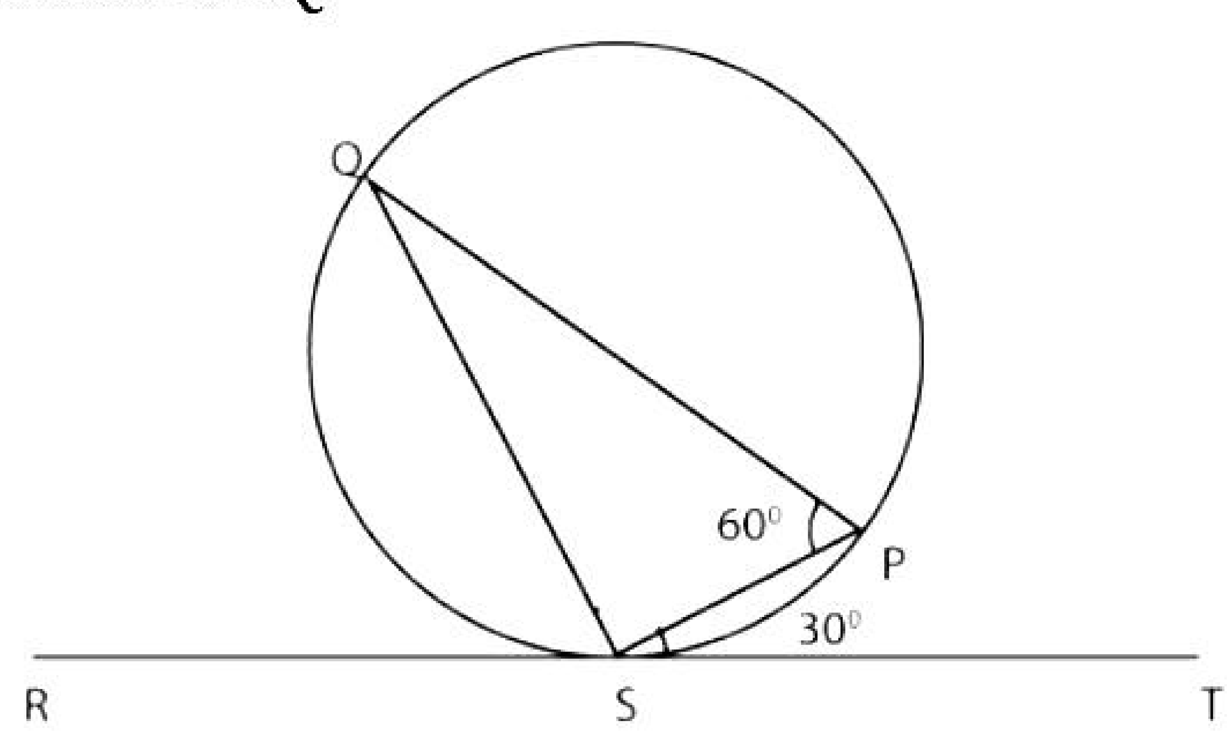
Which of the followings option is valid?

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

x) The locus of a stone dropped from the top of a tower will be a _______________ line through the point from which the stone is dropped.

(a) curved
(b) slant
(c) horizontal
(d) vertical

xi) In the given figure, RT is a tangent touching the circle at S. If $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$ then $\angle PSQ =$



(a) 30°
(b) 60°
(c) 90°
(d) 80°

xii) The shadow of a 7 m long stick is 4 m long. At the same time, the length of the shadow of a 17.5 m high tree is

(a) 8 m
(b) 10 m
(c) 12 m
(d) 14 m

xiii) The median class for the following distribution is

Class interval	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	10	4	8	11	5

(a) 30–40
(b) 40–50
(c) 50–60
(d) 60–70

xiv) A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P are
(a) (0, 4)
(b) (–10, 0)
(c) (4, 0)
(d) (0, –10)

xv) **Assertion (A):** If shares are available at a premium, then Market value > Nominal Value.
Reason (R): The market value of a share does not change with time.
(a) A is true, R is false
(b) A is false, R is true
(c) Both A and R are true, and R is the correct reason for A.
(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) If r_1, r_2 and r_3 are the radii of three spheres of gold, and they are melted into one solid sphere. Prove that the radius of the new sphere is $\sqrt[3]{r_1^3 + r_2^3 + r_3^3}$. [4]

ii) Mrs. Shah deposited Rs. 1500 per month in her recurring bank account for a period of 4 years. At the time of maturity, she got Rs. 86700. [4]
a) Find the rate of interest p.a.
b) Find the total interest earned by Mrs. Shah.

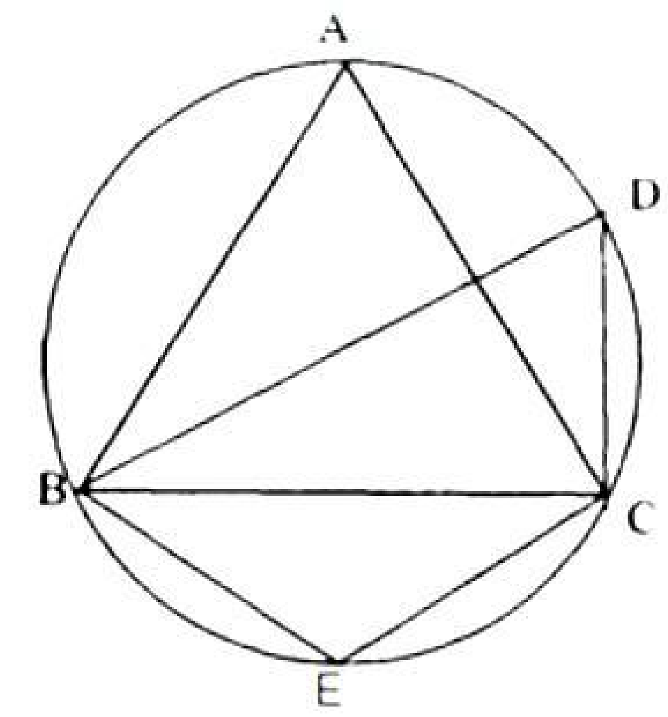
iii) If $x = \cot A + \cos A$ and $y = \cot A - \cos A$, then show that $\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$ [4]

Question 3

i) If $x = \frac{\sqrt{x+2y} + \sqrt{x-2y}}{\sqrt{x+2y} - \sqrt{x-2y}}$, prove that $y = x^2(1 - y)$. [4]

ii) In the figure, m∠DBC = 58°. BD is the diameter of the circle. Calculate: [4]

(i) m∠BDC
(ii) m∠BEC
(iii) m∠BAC



iii) Draw a cumulative frequency curve for the following distribution: [5]

Class Interval	Frequency
5–10	8
10–15	13
15–20	6
20–25	17
25–30	24

Section B

(Attempt any four questions from this Section.)

Question 4

i) If $A = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}$, find $(A - 2I)(A - 3I)$ [3]

ii) Solve the following equation and give your answer up to two decimal places:

$$3x - \frac{1}{x} = 6$$ [3]

iii) B_1 is a point on the side AC of ΔABC and B_1B is joined. A line is drawn through A parallel to B_1B, meeting CB produced in A_1 and another line is drawn through C parallel to B_1B, meeting AB produced in C_1. Prove that $\frac{1}{AA_1} + \frac{1}{CC_1} = \frac{1}{BB_1}$. [4]

Question 5

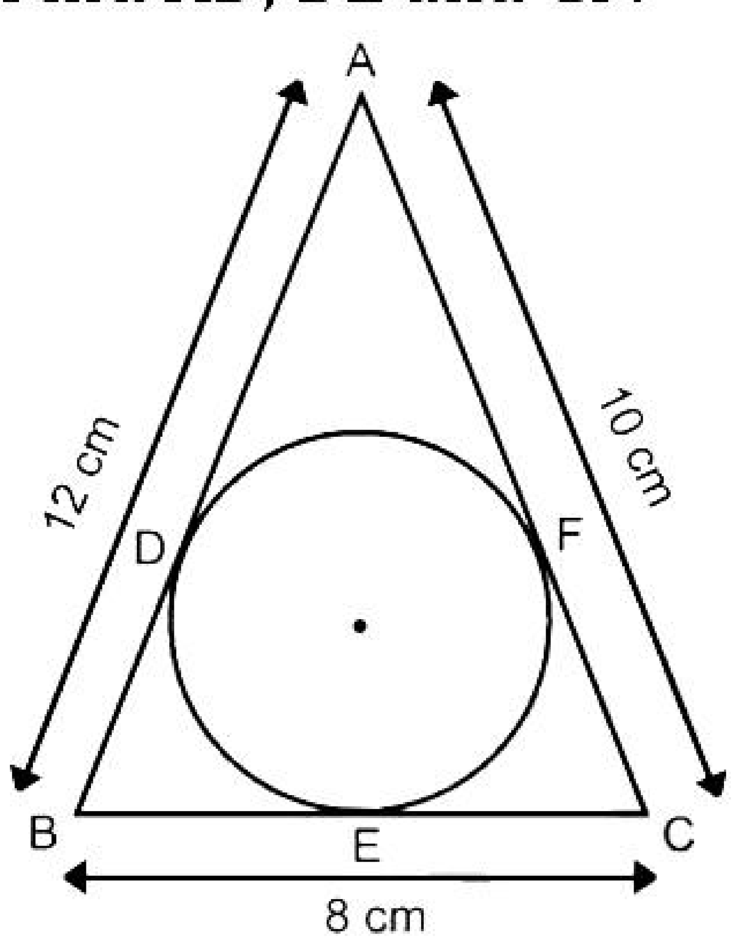
i) The mean of the following distribution using direct method is 40.6 [3]

Class interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	4	6	15	8	5	7

Verify the result using the short-cut method.

ii) For a dealer A, the list price of an article is Rs. 9000, which he sells to dealer B at some lower price. Further, dealer B sells the same article to a customer at its list price. If the rate of GST is 18% and dealer B paid a tax, under GST, equal to Rs. 324 to the government, find the amount (inclusive of GST) paid by dealer B. [3]

iii) A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in the figure. Find AD, BE and CF. [4]



Question 6

i) If x, 2x + 2, 3x + 3 are the first three terms of a geometric progression, find its fourth term. [3]

ii) The marks of a Mathematics test of 50 students were recorded as follows: [3]

Marks	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of Students	4	8	14	19	5

Draw a histogram from the above data using a graph paper and locate the mode.

iii) A conical tent is to accommodate 11 people. Each person must have 4 sq. metres of the space on the ground and 20 cubic metres of air to breathe. Find the height of the conical tent. [4]

Question 7

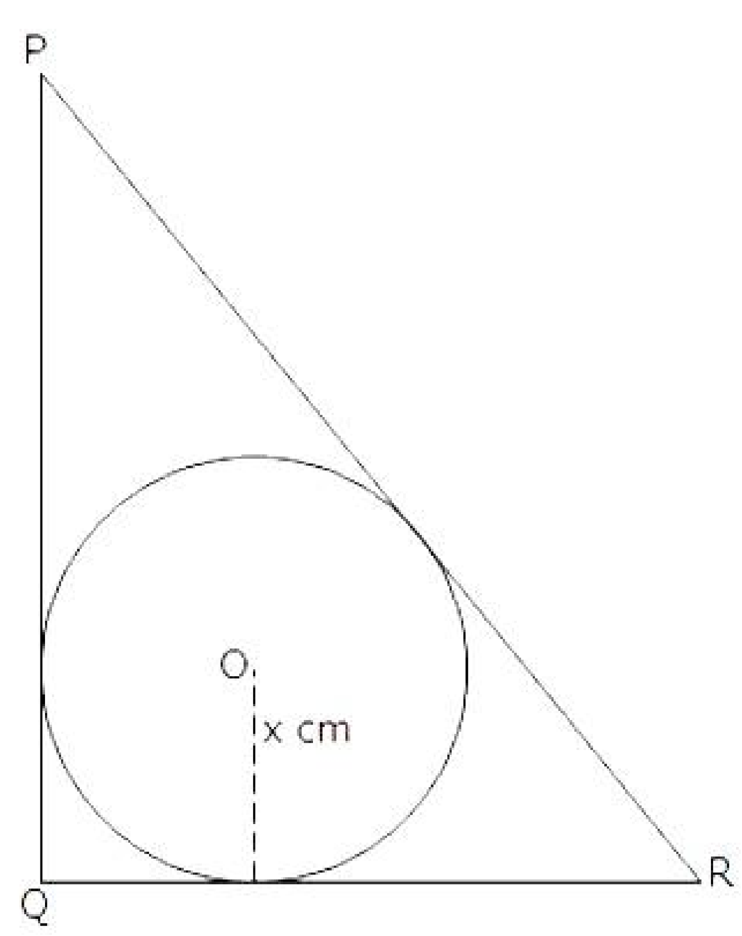
i) A(8, −6), B(−4, 2) and C(0, −10) are the vertices of triangle ABC. If P is the mid-point of AB and Q is the mid-point of AC, use co-ordinate geometry to show that PQ is parallel to BC. Give the special name of quadrilateral PBCQ. [5]

ii) A man stands 15 m away from a flagpole. He observes that the angle of elevation at the top of the pole is 34° and the angle of depression at the bottom of the pole is 23°. Calculate the height of the pole (Take $\tan 34^\circ = 0.6745$ and $\tan 23^\circ = 0.4244$). [5]

Question 8

i) If $A = \{x: 9x - 3 > 2x + 4, x \in R\}$ and $B = \{x: 12x - 7 \geq 5 + 6x, x \in R\}$, find the range of set $A \cap B$ and represent it on the number line. [3]

ii) In triangle PQR, PQ = 24 cm, QR = 7 cm and $\angle PQR = 90^\circ$. Find the radius of the inscribed circle. [3]



iii) ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Also, find the length of side BC. [4]

Question 9

i) Using properties of proportion, solve for x. Given that x is positive: [3]

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

ii) Rs. 600 is divided equally among 'x' children. If the number of children were 10 more, then each would have got Rs. 10 less. Find 'x'. [3]

iii) Use ruler and compass only for the following questions. All constructions lines and arcs must be clearly shown. [4]

Construct Δ ABC in which BC = 6.5 cm, ∠ ABC = 60°, AB = 5 cm.

(i) Construct the locus of points at a distance of 3.5 cm from A.

(ii) Construct the locus of points equidistant from AC and BC.

Question 10

i) What number must be subtracted from the polynomial $x^3 + 12x^2 + 17x - 21$ so that it becomes exactly divisible by $x + 10$? [3]

ii) A die is thrown twice. Find the probability that [3]

a) the sum of the numbers is 8.

b) the second throw shows a number 5.

iii) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1). [4]

(a) Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

(b) Write down the coordinates of A' and B'.

(c) Name two points which are invariant under the above reflection.

(d) Name the polygon A'B'CD.

Solution

Section A

Solution 1

i)

Correct option: (b)

Explanation:

The given matrix is of order 6×4, since it has 6 rows and 4 columns.

ii)

Correct option: (d)

Explanation:

As the quadratic equation $4x^2 - (p - 2)x + 1 = 0$ has equal roots,

Discriminant, $D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-(p - 2)]^2 - 4(4)(1) = 0$

$\Rightarrow p^2 - 4p + 4 - 16 = 0$

$\Rightarrow p^2 - 4p - 12 = 0$

$\Rightarrow (p - 6)(p + 2) = 0$

$\Rightarrow p = 6$ or $p = -2$

iii)

Correct option: (b)

Explanation:

It is a case of Intra–state transaction.

For the dealer, cost of goods = Rs. 14,000 and GST rate is 28%.

CGST = 14% of Rs. 14,000 = $14/100 \times 14000$ = Rs. 1960

iv)

Correct option: (b)

Explanation:

$x^2 - \left(1 + \sqrt{2}\right)x + \sqrt{2} = 0$

$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x(x-1) - \sqrt{2}(x-1) = 0$

$\Rightarrow (x-1)\left(x - \sqrt{2}\right) = 0$

$\Rightarrow (x-1) = 0$ or $x - \sqrt{2} = 0$

$\Rightarrow x = 1$ or $x = \sqrt{2}$

Hence, 1 and $\sqrt{2}$ are the roots of the given equation.

v)

Correct option: (a)

Explanation:

In the A.P. 24, 21, 18,…

First term, a = 24, common difference, d = 21 – 24 = –3

Let S_n be the sum of first n terms of this A.P.

$$\Rightarrow S_n = \frac{n}{2}\left[2\times 24+(n-1)(-3)\right]$$

$$\Rightarrow 78 = \frac{n}{2}\left[48-3n+3\right]$$

$$\Rightarrow 156 = -3n^2+51n$$

$$\Rightarrow 3n^2-51n+156=0$$

$$\Rightarrow n^2-17n+52=0$$

$$\Rightarrow (n-13)(n-4)=0$$

$$\Rightarrow n=13 \text{ or } n=4$$

vi)

Correct option: (b)

Explanation:

$$\frac{y-x}{x}=\frac{3}{8}$$

$$\Rightarrow \frac{\frac{y}{x}-\frac{x}{x}}{\frac{x}{x}}=\frac{3}{8}$$

$$\Rightarrow \frac{\frac{y}{x}-1}{1}=\frac{3}{8}$$

$$\Rightarrow \frac{y}{x}=\frac{3}{8}+1=\frac{11}{8}$$

vii)

Correct option: (b)

Explanation:

Two figures are said to be similar if and only if they have **same shape but not necessarily same size**.

viii)

Correct option: (a)

Explanation:

Volume of the cone = 154 cm^3

$$\Rightarrow \frac{1}{3} \times \pi r^2 h = 154$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 12 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 3 \times 7}{12 \times 22}$$

$$\Rightarrow r^2 = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2}$$

$$\Rightarrow r = 3.5 \text{ cm}$$

ix)

Correct option: (c)

Explanation:

Let S denote the sample space of this experiment, therefore, n(S) = 52.

Let A be an event getting an ace card.

Number of ace cards in a deck of 52 cards = 4 = n(A)

$\Rightarrow$Probability of getting an ace card = P(A) = $\frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

Hence, statement 1 is true.

Let B be an event getting a black ace card.

Number of black ace cards in a deck of 52 cards = 2 = n(B)

$\Rightarrow$Probability of getting a black ace card = P(B) = $\frac{n(B)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

x)

Correct option: (d)

Explanation:

The locus of a stone dropped from the top of a tower will be a **vertical** line through the point from which the stone is dropped.

xi)

Correct option: (c)

Explanation:

$\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$

By alternate segment theorem,

$\angle PST = \angle SQP$

$\Rightarrow \angle SQP = 30^\circ$

In ΔPSQ,

$\angle PSQ + \angle SQP + \angle SPQ = 180^\circ$

$\angle PSQ + 30^\circ + 60^\circ = 180^\circ$

$\angle PSQ = 90^\circ$

xii)

Correct option: (b)

Explanation:

Let the length of a shadow of 17.5 m high tree = 'y' m

Now,

Ratio of lengths of objects = Ratio of lengths of their shadow

$$\Rightarrow \frac{7}{17.5} = \frac{4}{y}$$

$$\Rightarrow y = \frac{4 \times 17.5}{7} = 10 \text{ m}$$

xiii)

Correct option: (c)

Explanation:

Class Interval	Frequency	Cumulative frequency (c.f.)
20–30	7	7
30–40	10	17
40–50	4	21
50–60	8	29
60–70	11	40
70–80	5	45
	N = 45	

Here, N/2 = $(22.5)^{th}$ term which lies in the class 50–60.

So, the median class is 50–60.

xiv)

Correct option: (d)

Explanation:

A line intersects the y-axis at point P.

∴ Coordinates of P are of the form $(0, y)$.

Also, the same line intersects the x-axis at point Q.

∴ Coordinates of Q are of the form (x, 0).

Let S be the midpoint of PQ.

∴ Coordinates of S are $(2, -5)$.

Now,

$$\frac{0 + x}{2} = 2 \text{ and } \frac{y + 0}{2} = -5$$

$$\Rightarrow x = 4 \text{ and } y = -10$$

Therefore, the coordinates of P are (0, −10).

xv)

Correct option: (a)

Explanation:

The statement given in the assertion is correct.

Hence, the assertion is true.

The market value of a share may change with time.

Hence, the reason is false.

Solution 2

i)

Radius of the first sphere = r_1

Radius of the second sphere = r_2

Radius of the third sphere = r_3

$\Rightarrow$ Sum of the volumes of 3 spheres

$$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$= \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right)$$

Let the radius of the new sphere be R.

Then,

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right)$$

$$\Rightarrow R^3 = r_1^3 + r_2^3 + r_3^3$$

$$\Rightarrow R = \sqrt[3]{r_1^3 + r_2^3 + r_3^3}$$

ii)

According to the given information,

a) Maturity value = Rs. 86700, P = Rs. 1500, n = 4 years = 4 × 12 months = 48 months

$$\text{Maturity value } = P \times n + P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$86700 = 1500 \times 48 + 1500 \times \frac{48 \times 49}{2 \times 12} \times \frac{r}{100}$$

86700 = 72000 + 1470r

1470r = 14700

r = 10% p.a.

b) $\text{Interest} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$$= 1500 \times \frac{48 \times 49}{2 \times 12} \times \frac{10}{100}$$

= Rs. 14700

iii)

$x = \cot A + \cos A$ and $y = \cot A - \cos A$

Thus, we have

$x + y = (\cot A + \cos A) + (\cot A - \cos A) = 2\cot A$

$x - y = (\cot A + \cos A) - (\cot A - \cos A) = 2\cos A$

$$\text{L.H.S.} = \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

$$= \left(\frac{2\cos A}{2\cot A}\right)^2 + \left(\frac{2\cos A}{2}\right)^2$$

$$= \left(\frac{\cos A}{\cot A}\right)^2 + (\cos A)^2$$

$$= \left(\frac{\cos A}{\cos A / \sin A}\right)^2 + (\cos A)^2$$

$$= (\sin A)^2 + (\cos A)^2$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

$$= \text{R.H.S.}$$

Solution 3

i)

$$x = \frac{\sqrt{x+2y} + \sqrt{x-2y}}{\sqrt{x+2y} - \sqrt{x-2y}}$$

$$\Rightarrow \frac{x}{1} = \frac{\sqrt{x+2y} + \sqrt{x-2y}}{\sqrt{x+2y} - \sqrt{x-2y}}$$

Using componendo and dividendo,

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{x+2y} + \sqrt{x-2y} + \sqrt{x+2y} - \sqrt{x-2y}}{\sqrt{x+2y} + \sqrt{x-2y} - \left(\sqrt{x+2y} - \sqrt{x-2y}\right)}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{x+2y}}{2\sqrt{x-2y}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{x+2y}}{\sqrt{x-2y}}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{x+2y}{x-2y}$$

Again using componendo and dividendo,

$$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{x+2y+x-2y}{x+2y-(x-2y)}$$

$$\Rightarrow \frac{2x^2+2}{4x} = \frac{2x}{4y}$$

$$\Rightarrow \frac{x^2+1}{x} = \frac{x}{y}$$

$$\Rightarrow yx^2+y = x^2$$

$$\Rightarrow y = x^2 - yx^2$$

$$\Rightarrow y = x^2(1-y)$$

ii) Consider the following figure.

(i) Given that BD is a diameter of the circle.

The angle in a semi circle is a right angle.

$\therefore \angle BCD = 90^\circ$

Also given that $\angle DBC = 58^\circ$

In ΔBDC, by angle sum property, we have

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow 58^\circ + 90^\circ + \angle BDC = 180^\circ$

$\Rightarrow 148^\circ + \angle BDC = 180^\circ$

$\Rightarrow \angle BDC = 180^\circ - 148^\circ$

$\Rightarrow \angle BDC = 32^\circ$

(ii) □BECD is a cyclic quadrilateral.

$\Rightarrow \angle BEC + \angle BDC = 180^\circ$ (Opposite angles are supplementary)

$\Rightarrow \angle BEC + 32^\circ = 180^\circ$

$\Rightarrow \angle BEC = 148^\circ$

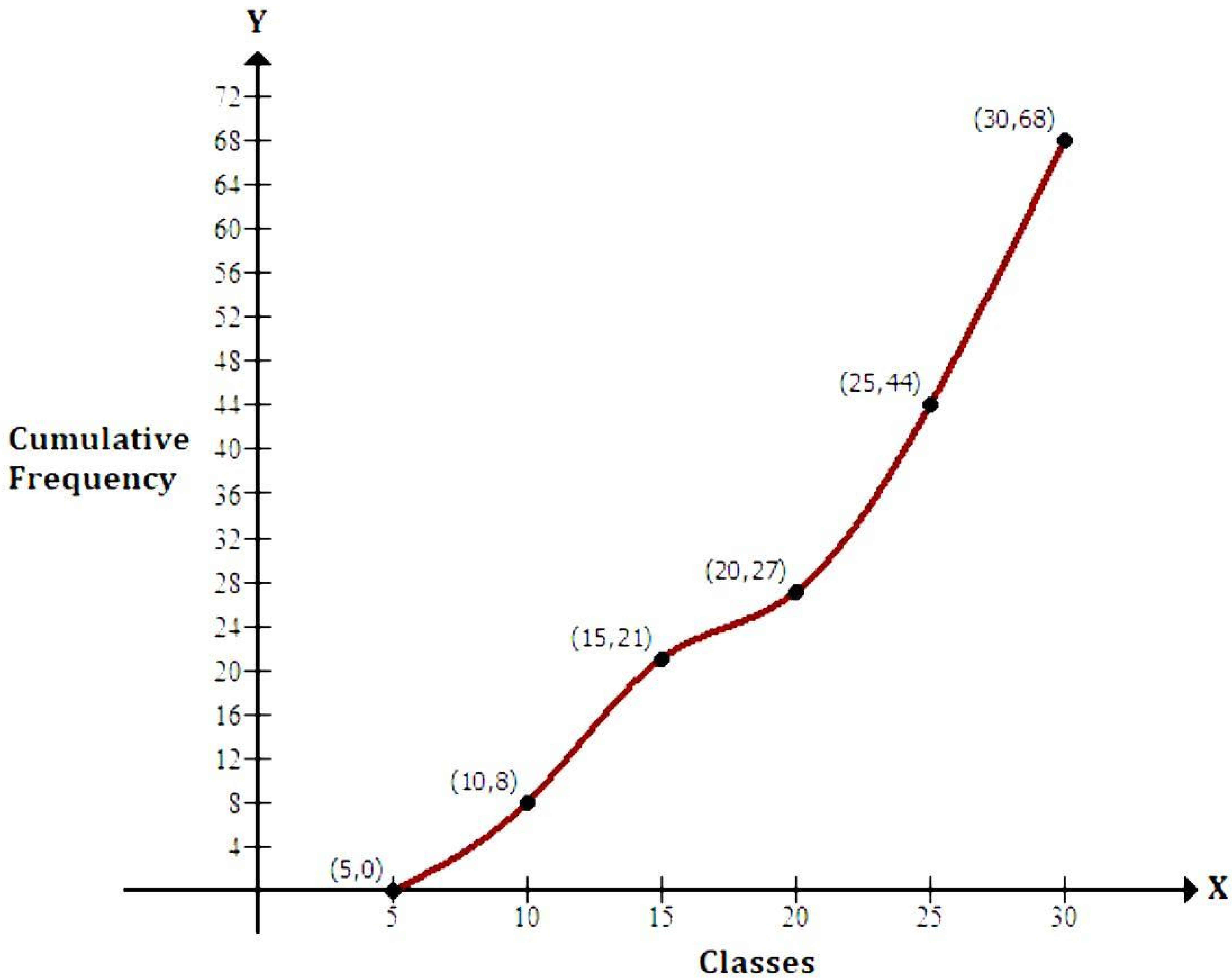
(iii) Angles in the same segment are equal.

$\Rightarrow \angle BAC = \angle BDC = 32^\circ$

iii)

Class Interval	Frequency	Cumulative frequency
0–5	0	0
5–10	8	8
10–15	13	21
15–20	6	27
20–25	17	44
25–30	24	68

Taking the upper-class limits along the x-axis and corresponding cumulative frequencies along the y-axis, mark the points (5, 0), (10, 8), (15, 21), (20, 27), (25, 44) and (30, 68). Join the points marked by a free-hand curve (as shown below).



Section B

Question 4

i) $A = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A - 2I = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A - 2I = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow A - 2I = \begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$

$A - 3I = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A - 3I = \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow A - 3I = \begin{bmatrix} -4 & 2 \\ 4 & -4 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 4 & -4 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 12+8 & -6-8 \\ -16-12 & 8+12 \end{bmatrix}$$

$$\Rightarrow (A - 2I)(A - 3I) = \begin{bmatrix} 20 & -14 \\ -28 & 20 \end{bmatrix}$$

ii) $3x - \frac{1}{x} = 6$

$\Rightarrow 3x^2 - 1 = 6x$

$\Rightarrow 3x^2 - 6x - 1 = 0$

Comparing $3x^2 - 6x - 1 = 0$ with $ax^2 + bx + c = 0$,

$a = 3$, $b = -6$ and $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3(-1)}}{2 \times 3}$$

$$= \frac{6 \pm \sqrt{36+12}}{6}$$

$$= \frac{6 \pm \sqrt{48}}{6}$$

$$= \frac{6 \pm 4\sqrt{3}}{6}$$

$$= 1 \pm \frac{2\sqrt{3}}{3}$$

$$= 1 \pm 1.15$$

$\Rightarrow x = 2.15$ or $x = -0.15$

iii) In ΔABB_1 and ΔAC_1C

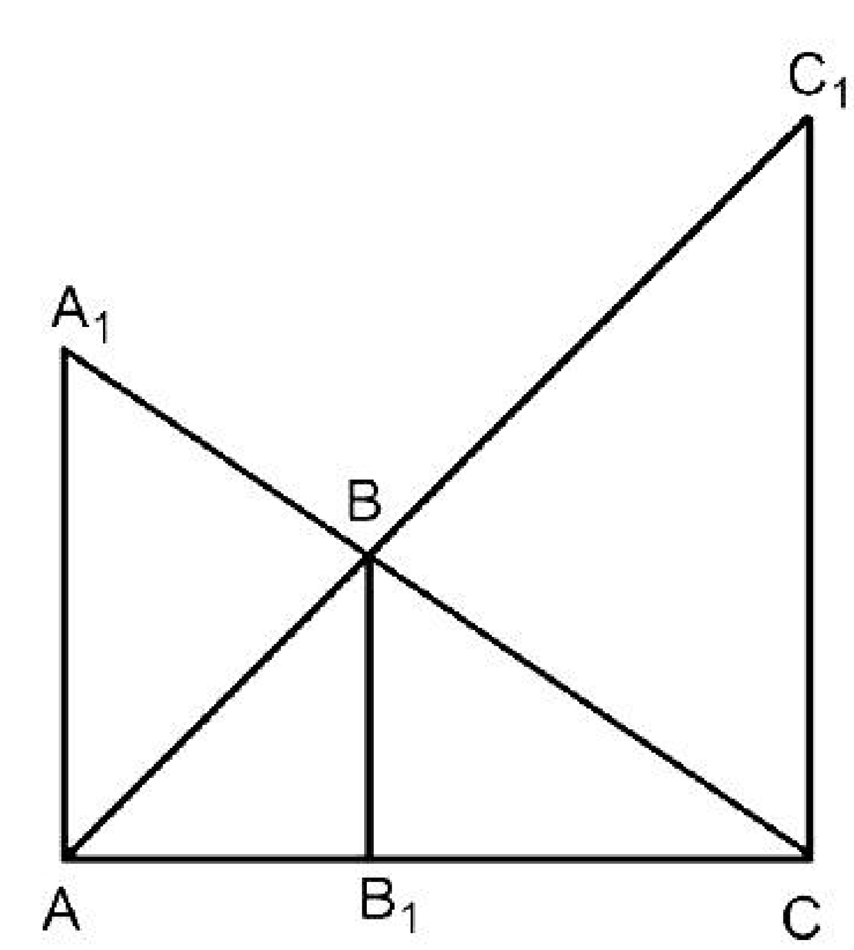
$\angle BAB_1 = \angle C_1AC$ (common angle)

$\angle AB_1B = \angle ACC_1$ (corresponding angles)

$\therefore \Delta ABB_1 \sim \Delta AC_1C$ (by AA similarity)

$\therefore \frac{BB_1}{CC_1} = \frac{AB_1}{AC}$ ... (i)

Also, $\Delta CBB_1 \sim \Delta CA_1A$ by AA similarity.



$$\therefore \frac{BB_1}{A_1A} = \frac{CB_1}{CA} \quad \dots \text{(ii)}$$

Adding (i) and (ii)

$$\frac{BB_1}{CC_1} + \frac{BB_1}{AA_1} = \frac{AB_1}{AC} + \frac{CB_1}{AC}$$

$$\therefore BB_1\left(\frac{1}{CC_1} + \frac{1}{AA_1}\right) = \frac{AB_1 + CB_1}{AC}$$

$$\therefore BB_1\left(\frac{1}{CC_1} + \frac{1}{AA_1}\right) = \frac{AC}{AC}$$

$$\therefore BB_1\left(\frac{1}{CC_1} + \frac{1}{AA_1}\right) = 1$$

$$\therefore \frac{1}{CC_1} + \frac{1}{AA_1} = \frac{1}{BB_1}$$

Question 5

i) Mean by direct method = 40.6

Using the shortcut method:

C.I.	f	Class mark (x)	Assumed mean A = 35 $\therefore d = x - A$	f × d
10−20	4	15	−20	−80
20−30	6	25	−10	−60
30−40	15	35	0	0
40−50	8	45	10	80
50−60	5	55	20	100
60−70	7	65	30	210
	$\sum f = 45$			$\sum fd = 250$

Here, $n = \sum f = 45$, $\sum fd = 250$ and assumed mean A = 35

$$\therefore \text{ Mean } = A + \frac{\sum fd}{n} = 35 + \frac{250}{45} = 35 + 5.6 = 40.6$$

Hence, verified.

ii) Let A sells an article to dealer B at Rs. x lower price.

According to the question,

Net Tax paid by dealer B is

Output tax – Input Tax = Rs. 324

$\Rightarrow$ 18% of 9000 – 18% of (9000 – x) = 324

$\Rightarrow$ 1620 – 1620 + 18% of x = 324

$\Rightarrow$ 18% of x = 324

$\Rightarrow$ x = 1800

Hence, selling price for B = Rs. (9000 – 1800) = Rs. 7200
The amount (inclusive of GST) paid by dealer B
= 7200 + 18% of 7200
= 7200 + 1296
= Rs. 8496

iii) Tangents drawn from an external point to a circle are equal.
AD = AF = x
BD = BE = y
CE = CF = z
AB = 12 cm, BC = 8 cm, AC = 10 cm
$x + y = 12$ ...(1)
$y + z = 8$ ...(2)
$x + z = 10$ ...(3)
Adding (1), (2) and (3),
$\therefore 2(x + y + z) = 30$
$\therefore x + y + z = 15$ ...(4)
$\therefore 12 + z = 15$ [From (1)]
$\therefore z = 3$
Similarly, using (2) and (3) in (4), we get y = 5, x = 7.
Therefore,
AD = AF = x = 7 cm
BE = BD = y = 5 cm
CF = CE = z = 3 cm

Question 6
i) x, 2x + 2, 3x + 3 are in G.P.

$$\Rightarrow \frac{2x+2}{x} = \frac{3x+3}{2x+2}$$

$$\Rightarrow (2x+2)^2 = x(3x+3)$$

$$\Rightarrow 4x^2 + 8x + 4 = 3x^2 + 3x$$

$$\Rightarrow x^2 + 5x + 4 = 0$$

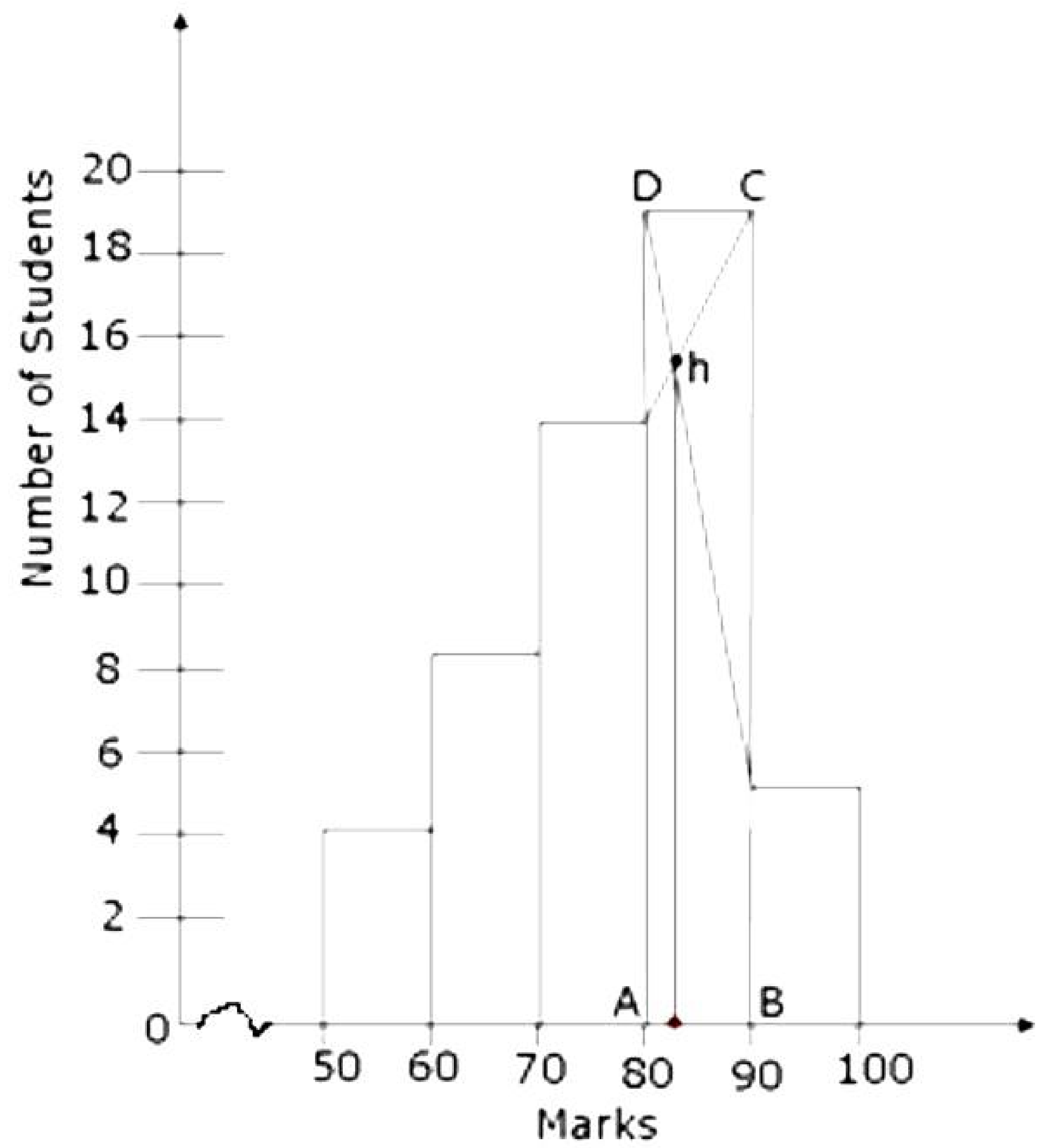
$$\Rightarrow (x+4)(x+1) = 0$$

$\Rightarrow x = -4$ or $x = -1$
If $x = -1$, then the second and third terms will be zero.
Hence, $x = -4$.
Then, G.P. becomes −4, −8 + 2, −12 + 3, i.e. −4, −6, −9.

$$\Rightarrow a = -4,\ r = \frac{-6}{-4} = \frac{3}{2}$$

$$\Rightarrow \text{Fourth term} = ar^3 = (-4)\left(\frac{3}{2}\right)^3 = (-4) \times \frac{27}{8} = -13.5$$

ii) The histogram for the given data can be drawn by taking the marks on the x-axis and the number of students on the y-axis.



To locate the mode from the histogram, we proceed as follows:

i) Find the modal class: Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. Hence, the modal class is 80 – 90.

ii) Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these lines intersect at point h.

iii) The x-value of the point h is the mode. Thus, the mode of the given data is approximately 83.

iii) Let the height of the conical tent be h metres and radius be r metres.
Number of people the tent can accommodate = 11
Space required by each person on the ground = 4 m^2
Volume of air required by each person = 20 m^3
Now, area of the base = 11 × 4 = 44 m^2

$\Rightarrow \pi r^2 = 44 \quad \ldots (i)$

$\Rightarrow$ Volume of the cone = 11 × 20 = 220 m^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 220 \quad \ldots (ii)$$

Dividing (ii) by (i),

$$\Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{220}{44}$$

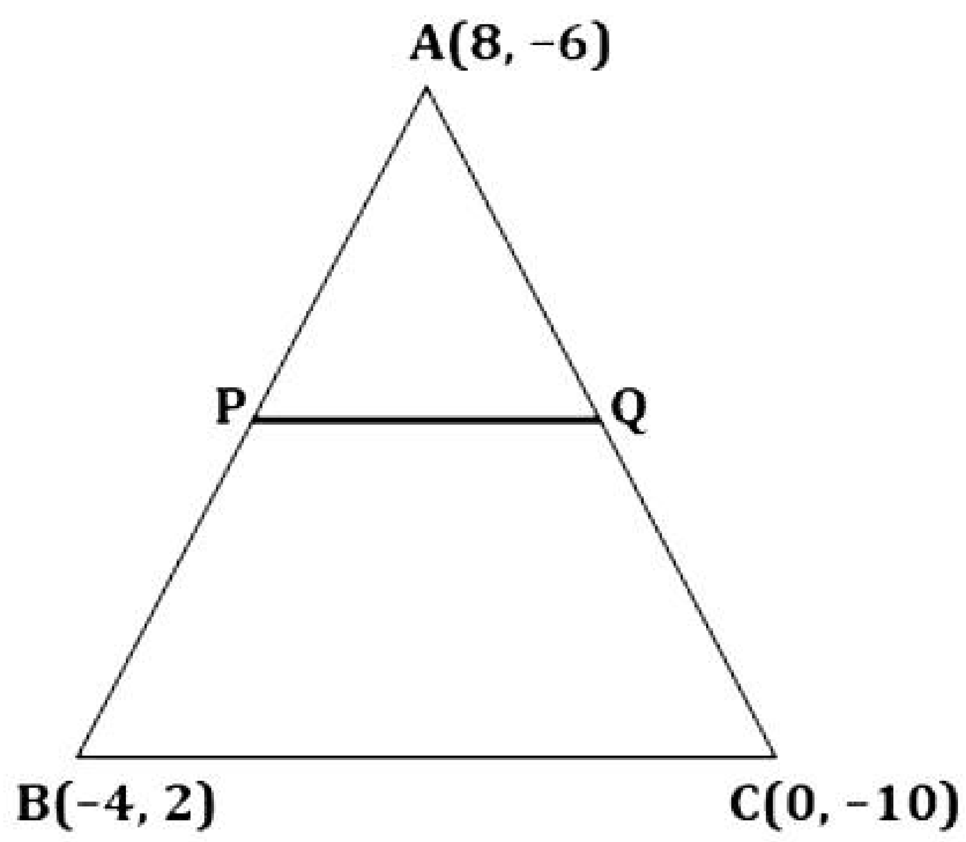
$$\Rightarrow \frac{h}{3} = 5$$

$\Rightarrow h = 15$

Therefore, the height of the cone is 15 m.

Question 7

i)



P is the mid-point of AB, where A(8, −6) and B(−4, 2).

∴ The co-ordinates of P are $\left(\frac{8-4}{2},\frac{-6+2}{2}\right)=(2,-2)$

Q is the mid-point of AC, where A(8, −6) and C(0, −10).

∴ The co-ordinates of Q are $\left(\frac{8}{2},\frac{-6-10}{2}\right)=(4,-8)$

To show that PQ is parallel to BC, we need to find their slopes, where P(2, −2) and Q(4, −8)

∴ Slope of PQ = $\frac{-8-(-2)}{4-2}=-3$

B(−4, 2) and C(0, −10)

∴ Slope of BC = $\frac{-10-2}{0-(-4)}=-3$

As the slopes of PQ and BC are equal, PQ is parallel to BC.

Now, P(2, −2) and B(−4, 2)

∴ Slope of PB = $\frac{2-(-2)}{-4-2}=-\frac{2}{3}$

C(0, −10) and Q(4, −8)

∴ Slope of CQ = $\frac{-8-(-10)}{4}=\frac{1}{2}$

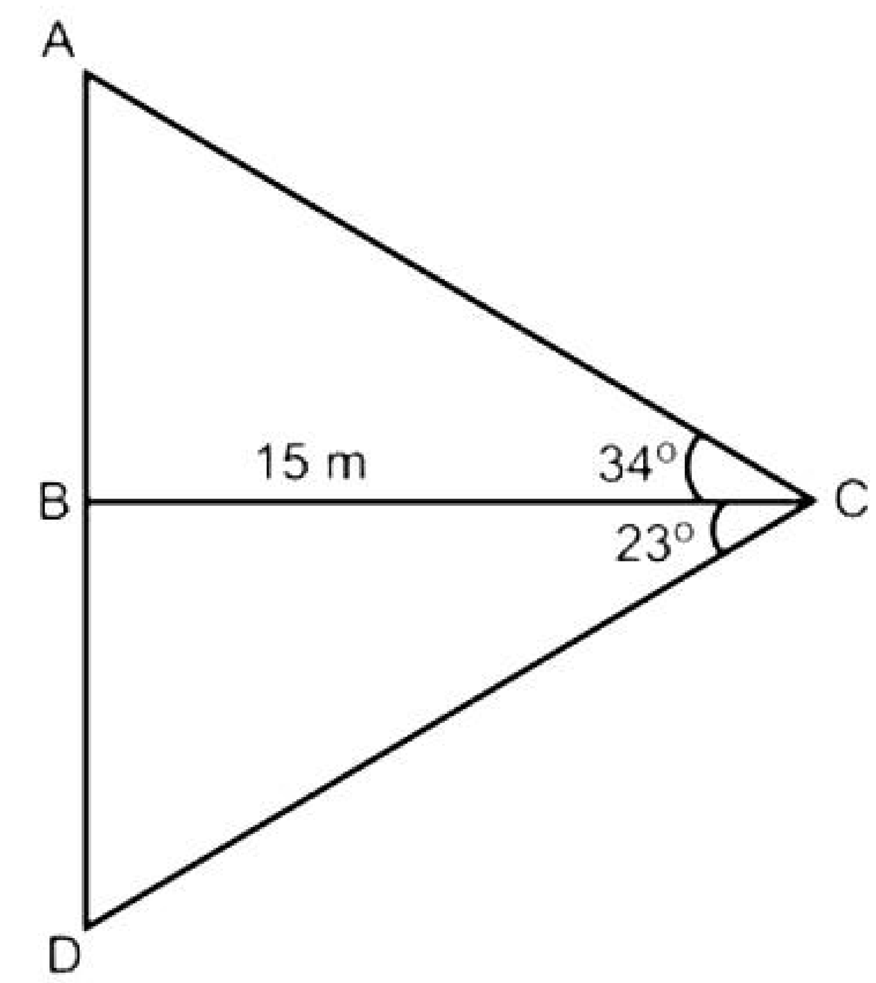
As the slopes of PB and CQ are not equal, they are not parallel.

Hence, PBCQ is a trapezium.

ii) Let AD be the flagpole, and C be the point where a man is standing.

Now, according to the question, the diagram will be as follows:

Given, BC = 15 m



Now, in triangle ABC,

$$\tan\angle ACB = \frac{AB}{BC}$$

$$\tan 34 = \frac{AB}{15}$$

$$\Rightarrow AB = 15\times\tan 34$$

$$\Rightarrow AB = 15\times 0.6745$$

$$\Rightarrow AB = 10.12 \text{ m} \text{ .... (i)}$$

In triangle CBD,

$$\tan\angle BCD = \frac{BD}{BC}$$

$$\tan 23 = \frac{BD}{15}$$

$$\Rightarrow BD = 15\times\tan 23$$

$$\Rightarrow BD = 15\times 0.4244$$

$$\Rightarrow BD = 6.37 \text{ m} \text{ .... (ii)}$$

Now, AD = AB + BD

$= (10.12 + 6.37)$ m

$= 16.49$ m

Hence, the height of the pole is 16.49 m.

Question 8

i) $9x - 3 > 2x + 4$

$\Rightarrow 9x - 2x > 4 + 3$

$\Rightarrow 7x > 7$

$\Rightarrow x > 1$

$A = \{x: 9x - 3 > 2x + 4, x \in R\} = \{x: x > 1, x \in R\}$

Also, $12x - 7 \geq 5 + 6x$

$\Rightarrow 12x - 6x \geq 5 + 7$

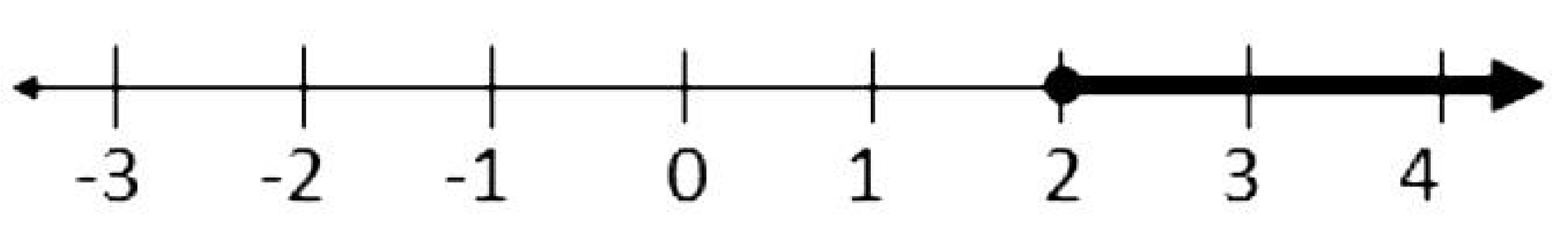
$\Rightarrow 6x \geq 12$

$\Rightarrow x \geq 2$

$\Rightarrow B = \{x: 12x - 7 \geq 5 + 6x, x \in R\} = \{x: x \geq 2, x \in R\}$

$\Rightarrow A \cap B = \{x: x \geq 2\ x \in R\}$

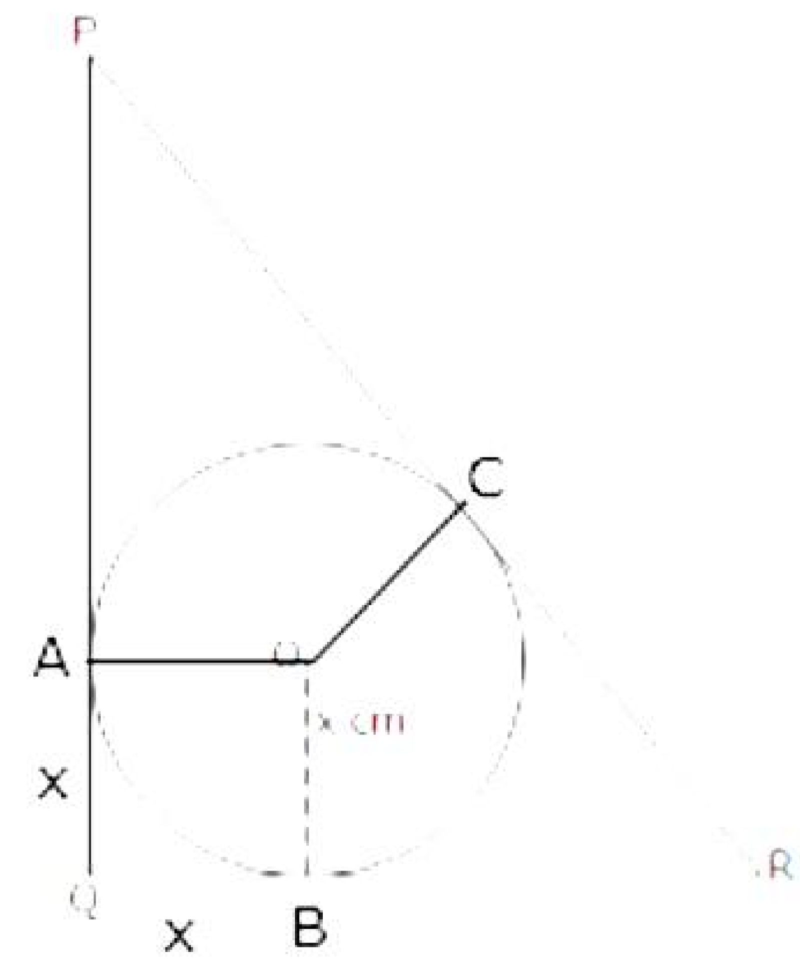
This can be represented on the number line as shown below:



ii) Since ΔPQR is a right-angled angle,

PR = $\sqrt{7^2+24^2}=\sqrt{49+576}=\sqrt{625}=25\,\text{cm}$

Let the given inscribed circle touches the sides of the given triangle at points A, B and C respectively.



Then, clearly, OAQB is a square.

$\Rightarrow$ AQ = BQ = x cm

PA = PQ – AQ = (24 – x) cm

RB = QR – BQ = (7 – x) cm

Since tangents from an exterior point to a circle are equal,

PC = PA = (24 – x) cm

And, RC = RB = (7 – x) cm

PR = PC + CR

$\Rightarrow$ 25 = (24 – x) + (7 – x)

$\Rightarrow$ 25 = 31 – 2x

$\Rightarrow$ 2x = 6

$\Rightarrow$ x = 3 cm

Hence, the radius of the inscribed circle is 3 cm.

iii) The coordinates of the vertices of ΔABC are A(1, 3), B(4, b) and C(a, 1).

Then, the coordinates of the centroid of ΔABC = $\left(\frac{1+4+a}{3},\frac{3+b+1}{3}\right)=\left(\frac{5+a}{3},\frac{4+b}{3}\right)$

It is given that the coordinates of the centroid are G(4, 3).

Therefore, we have

$\frac{5+a}{3}=4$ and $\frac{4+b}{3}=3$

$\Rightarrow 5+a=12 \qquad 4+b=9$

$\Rightarrow a=7 \qquad b=5$

Thus, the coordinates of B and C are (4, 5) and (7, 1) respectively.

Using distance formula, we have

$$BC = \sqrt{(7-4)^2 + (1-5)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

Question 9

i) $\dfrac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$

$\Rightarrow \dfrac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \dfrac{4+1}{4-1}$ (By componendo – dividendo)

$\Rightarrow \dfrac{4x}{2\sqrt{4x^2 - 1}} = \dfrac{5}{3}$

$\Rightarrow \dfrac{2x}{\sqrt{4x^2 - 1}} = \dfrac{5}{3}$

$\Rightarrow \dfrac{4x^2}{4x^2 - 1} = \dfrac{25}{9}$ (squaring both sides)

$\Rightarrow \dfrac{4x^2 - 4x^2 + 1}{4x^2 - 1} = \dfrac{25-9}{9}$ (By dividendo)

$\Rightarrow \dfrac{1}{4x^2 - 1} = \dfrac{16}{9}$

$\Rightarrow 9 = 64x^2 - 16$

$\Rightarrow 64x^2 = 25$

$\Rightarrow x^2 = \dfrac{25}{64}$

$\Rightarrow x = \pm\dfrac{5}{8}$

$\Rightarrow x = \dfrac{5}{8}$ (x is positive)

ii) When Rs. 600 is divided equally among 'x' children.

Each child gets = Rs. $\dfrac{600}{x}$

When the number of children were 10 more than the original number of children, each child gets = Rs. $\dfrac{600}{x+10}$

According to the question,

$$\frac{600}{x} - \frac{600}{x+10} = 10$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+10} = \frac{1}{60}$$

$$\Rightarrow \frac{x+10-x}{x(x+10)} = \frac{1}{60}$$

$$\Rightarrow 600 = x^2 + 10x$$

$$\Rightarrow x^2 + 10x - 600 = 0$$

$$\Rightarrow x^2 + 30x - 20x - 600 = 0$$

$$\Rightarrow x(x+30) - 20(x+30) = 0$$

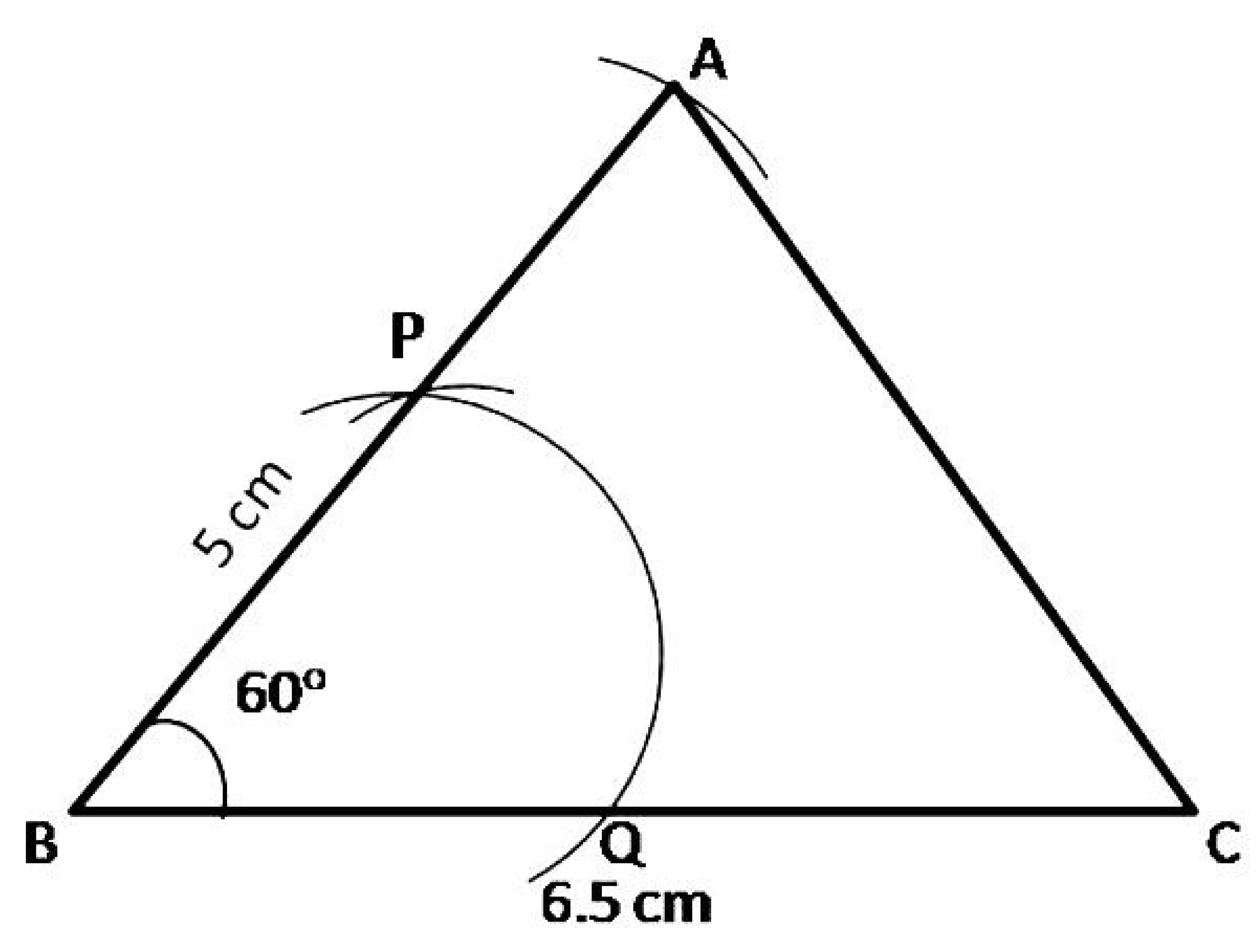
$$\Rightarrow (x+30)(x-20) = 0$$

$$\Rightarrow x = -30 \text{ or } x = 20$$

But x cannot be negative.
Hence, x = 20.

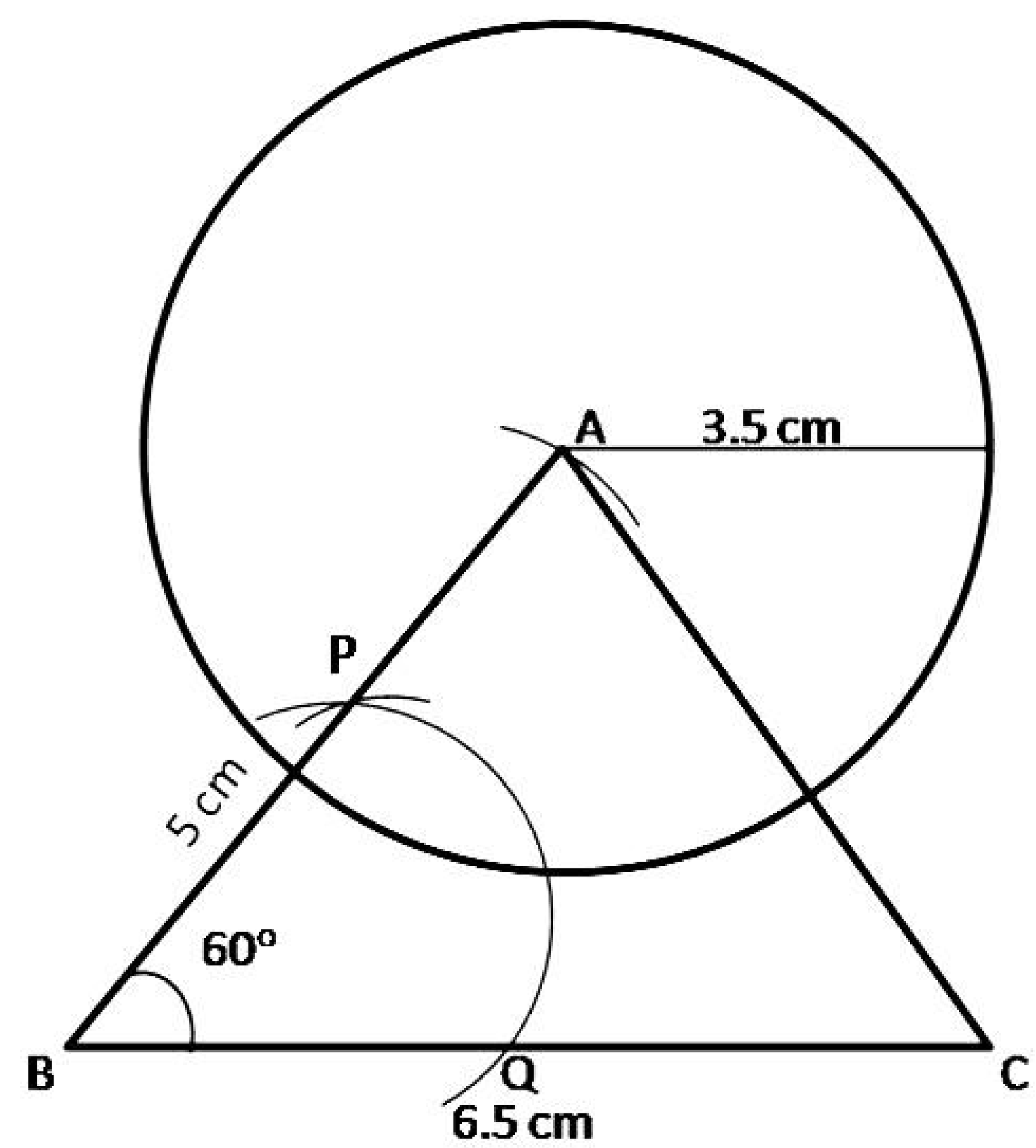
iii) Steps of construction:

1. Draw BC = 6.5 cm using a ruler.
2. With B as the centre and radius equal to approximately half of BC, draw an arc that cuts the segment BC at Q.
3. With Q as the centre, and same radius, cut the previous arc at P.
4. Join BP and extend it.
5. With B as the centre and radius 5 cm, draw an arc that cuts the arm PB to obtain point A.
6. Join AC to obtain ΔABC.



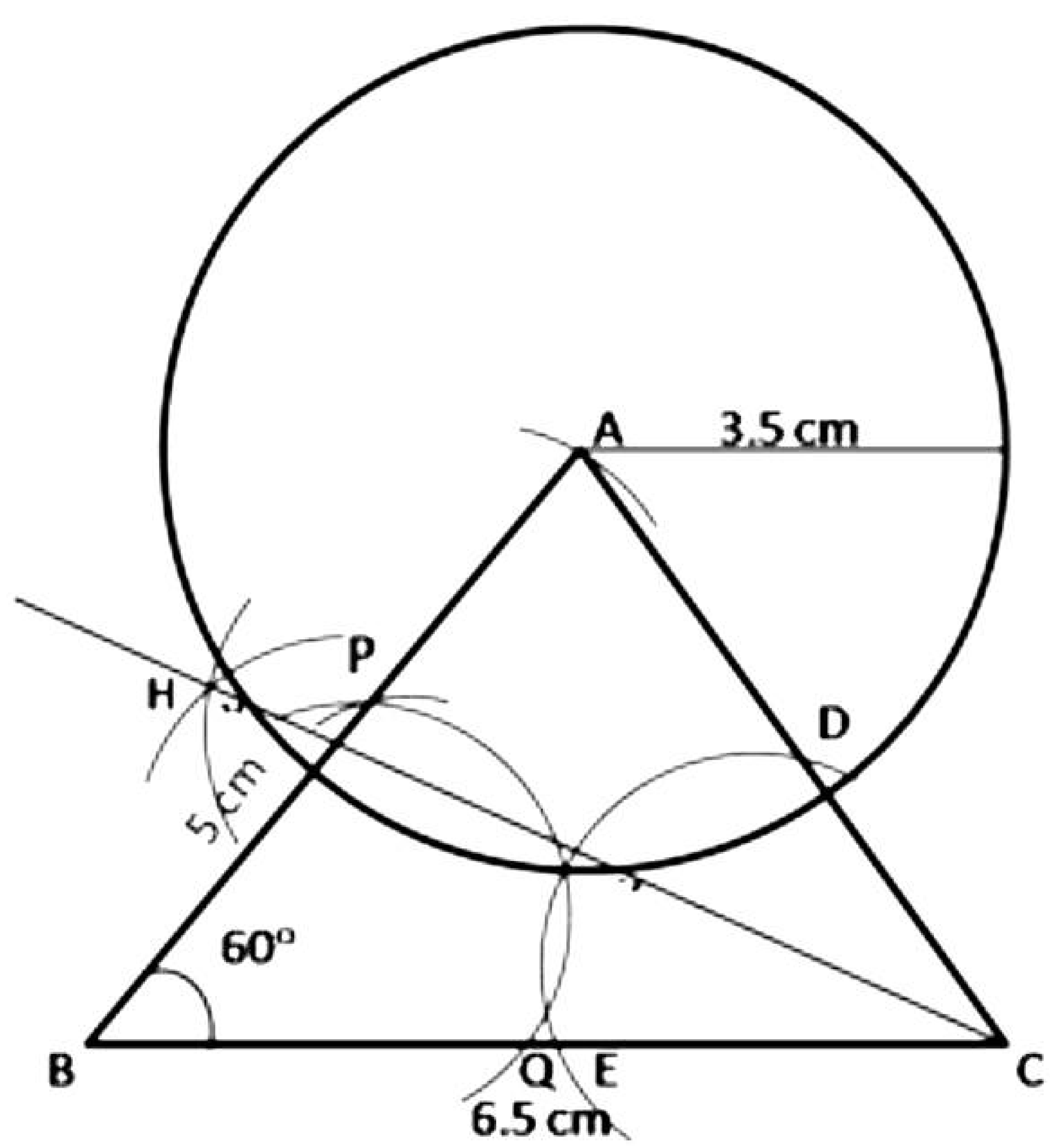
(i) Steps for construction :

1. With A as the centre and radius 3.5 cm, draw a circle.
2. The circumference of a circle is the required locus.



(ii) Steps for construction :

1. With C as the centre and with radius of a length less than CA or BC, draw an arc to cut the line segments AC and BC at D and E respectively.
2. With the same radius or a suitable radius and with D as the centre, draw an arc of a circle.
3. With the same radius and with E as the centre draw an arc such that the two arcs intersect at H.
4. Join C and H.
5. CH is the bisector of $\angle ACB$ and is the required locus.



Question 10

i) Let $p(x) = x^3 + 12x^2 + 17x - 21$ and $q(x) = x + 10$

When p(x) is divided by q(x), the remainder will be a number.

Let the number be k so that when it is subtracted from p(x), the result is exactly divisible by q(x).

$\Rightarrow f(x) = p(x) - k$

$\Rightarrow f(x) = x^3 + 12x^2 + 17x - 21 - k$

Since, x + 10 divides f(x),

$\Rightarrow f(-10) = 0$

$\Rightarrow (-10)^3 + 12 \times (-10)^2 + 17 \times (-10) - 21 - k = 0$

$\Rightarrow -1000 + 1200 - 170 - 21 - k = 0$

$\Rightarrow 9 - k = 0$

$\Rightarrow k = 9$

Thus, 9 must be subtracted from $x^3 + 12x^2 + 17x - 21$, so that it becomes exactly divisible by x + 10.

ii) Let S denote the sample space of this experiment.

A die is thrown twice, so the set of all possible outcomes is given as

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\Rightarrow$ Number of all possible outcomes = n(S) = 36

a) Let A be an event of getting the sum of the numbers on the dice as 8.

Then, the favourable outcomes = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$\Rightarrow n(A) = 5$

$\therefore$ Probability of getting the sum as 8 = $P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

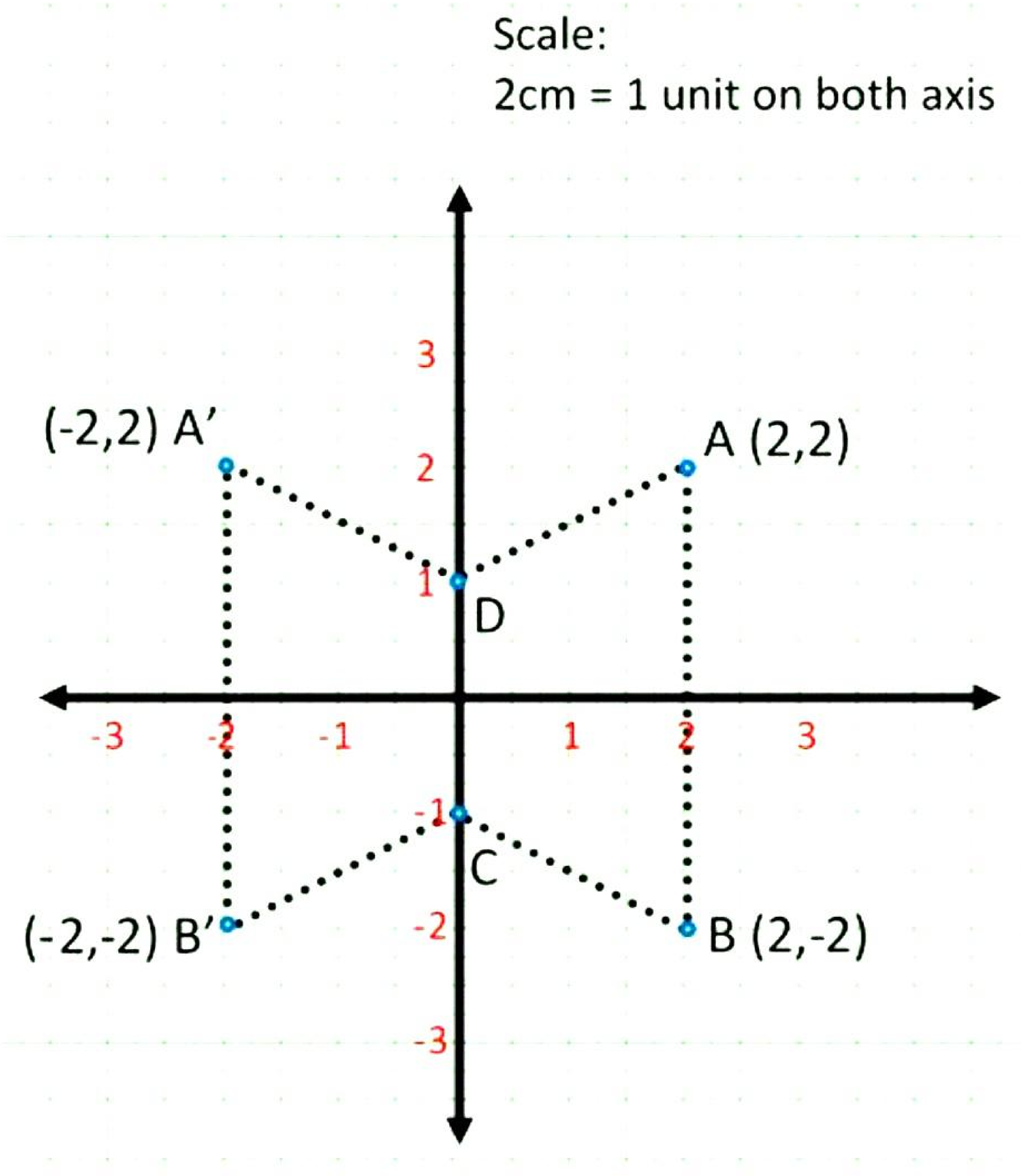
b) Let B be an event where the second throw shows a number 5.

Then, the favourable outcomes = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)}

$\Rightarrow n(B) = 6$

$\therefore$ Probability that the second dice shows 5 = $P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

iii) (a) and (b)



(c) D and C are invariant points.
(d) A'B'CD is a trapezium.