EXERCISE: 10.2 VECTOR ALGEBRA.

Q No 1 Compute the magnitude of following vectors.

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$; $\vec{b} = 2\hat{i}-7\hat{j}-3\hat{k}$; $\vec{c} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$.

Sol. As we know that magnitude $|\vec{a}|$ of $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ is given by $|\vec{a}| = \sqrt{a_1^2+a_2^2+a_3^2}$

$\therefore$ $|\vec{a}| = |\hat{i}+\hat{j}+\hat{k}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ units.

$|\vec{b}| = |2\hat{i}-7\hat{j}-3\hat{k}| = \sqrt{(2)^2+(-7)^2+(-3)^2} = \sqrt{4+49+9} = \sqrt{62}$ units.

$|\vec{c}| = \left|\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}\right| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(-\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \sqrt{\frac{1+1+1}{3}} = \sqrt{\frac{3}{3}} = 1$

Q No 2 Write two different vectors having same magnitude.

Sol: Vectors $\vec{a} = \hat{i}+\hat{j}$ and $\vec{b} = \hat{i}-\hat{j}$ are different vectors having same magnitude $\sqrt{1+1} = \sqrt{2}$.

Note: – Infinite number of pairs of such vectors are possible.

Q No 3. Write two different vectors having same direction.

Sol. Vector $\vec{a} = \hat{i}$ and $\vec{b} = 10\hat{i}$ are different vectors having same direction.

Q No 4. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.

Sol.

$$x\hat{i}+y\hat{j} = 2\hat{i}+3\hat{j}$$

$$\Rightarrow \quad x = 2 \text{ and } y = 3.$$

Q No 5 Find the scalar and vector components of vectors with initial point $(2,1)$ and terminal point $(-5,7)$

Sol. As vector joining the point $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $\vec{PQ} = (x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}$.

Let Initial point be $P(2,1)$ and terminal point be $Q(-5,7)$

Then $\vec{PQ} = (-5-2)\hat{i}+(7-1)\hat{j} = -7\hat{i}+6\hat{j}$.

$\Rightarrow$ Scalar Components of vector $\vec{PQ}$ are -7 and 6 and Vector components are $-7\hat{i}$ and $6\hat{j}$.

Q No 6 find the sum of vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$; $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

Sol: Required Sum $\vec{a} + \vec{b} + \vec{c} = (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$

$$= (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k}$$

$$= 0\hat{i} - 4\hat{j} - \hat{k}.$$

Q No. 7. Find the unit vector in direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Sol. Given Vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{1+1+4} = \sqrt{6}$$

$\therefore$ Unit Vector in direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$

$$= \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}.$$

Q No. 8. Find the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points P(1,2,3) and Q(4,5,6) respectively.

Sol. Position Vector $\vec{OP} = \hat{i} + 2\hat{j} + 3\hat{k}$

Position Vector of $Q = \vec{OQ} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\therefore \vec{PQ} = \vec{OQ} - \vec{OP} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$$= 3\hat{i} + 3\hat{j} + 3\hat{k}.$$

$$|\vec{PQ}| = \sqrt{(3)^2 + (3)^2 + (3)^2} = \sqrt{9+9+9} = \sqrt{27} = \sqrt{3 \times 9} = 3\sqrt{3}.$$

$\therefore$ Unit Vector in direction of vector $\vec{PQ} = \dfrac{\vec{PQ}}{|\vec{PQ}|}$

$$= \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Q No. 9 For given vector $\vec{a} = 2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}-\hat{k}$ find the unit vector in direction of $\vec{a}+\vec{b}$.

Soln:

$$\vec{a}+\vec{b} = (2\hat{i}-\hat{j}+2\hat{k}) + (-\hat{i}+\hat{j}-\hat{k})$$

$$= \hat{i}+0\hat{j}+\hat{k}$$

$$\Rightarrow |\vec{a}+\vec{b}| = \sqrt{(1)^2+(0)^2+(1)^2} = \sqrt{2}.$$

$\therefore$ Unit Vector in direction of $\vec{a}+\vec{b} = \dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$$= \frac{1}{\sqrt{2}}(\hat{i}+0\hat{j}+\hat{k}) = \frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$$

Q No 10 Find a vector in direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 8 units.

Sol.

Let $\vec{a} = 5\hat{i}-\hat{j}+2\hat{k}$

$$\therefore |\vec{a}| = \sqrt{(5)^2+(-1)^2+(2)^2} = \sqrt{25+1+4} = \sqrt{30}.$$

$\therefore$ Unit Vector in direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

$\therefore$ Vector in direction of $\vec{a}$ having magnitude 8

$$= 8\frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{8}{\sqrt{30}}(5\hat{i}-\hat{j}+2\hat{k})$$

$$= \frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}.$$

Q No. 11. Show that vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

Sol.

Since $-4\hat{i}+6\hat{j}-8k = -2(2\hat{i}-3\hat{j}+4\hat{k})$

$\therefore$ the given vectors are collinear.

Q No 12. Find the direction cosines of vector $\hat{i}+2\hat{j}+3\hat{k}$.

Sol: Since direction cosines of any vector are the

Components of its unit Vector.

Now Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$|\vec{a}| = \sqrt{(1)^2+(2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{14}}(\hat{i}+2\hat{j}+3\hat{k}) = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

$\therefore$ Direction Cosines of given vector are.

$$\left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

Q No. 13. Find the direction cosines of vector joining the points $A(1, 2, -3)$ and $B(-1, -2, 1)$ directed from A to B.

Sol:

Position Vector of $A = \vec{OA} = \hat{i} + 2\hat{j} - 3\hat{k}$

Position Vector of $B = \vec{OB} = -\hat{i} - 2\hat{j} + \hat{k}$

$$\therefore \vec{AB} = \vec{OB} - \vec{OA}$$
$$= (-\hat{i} - 2\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})$$
$$= -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\vec{AB}| = \sqrt{(-2)^2 + (-4)^2 + (4)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$\therefore \text{Unit Vector along } \vec{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{1}{6}(-2\hat{i} - 4\hat{j} + 4\hat{k})$$
$$= -\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$\therefore$ Direction Cosines of $\vec{AB} = \langle -1/3, -2/3, 2/3 \rangle$

Q No. 14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to axes OX, OY, OZ

Sol:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}.$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$\therefore$ Direction Cosines of $\vec{a}$ are $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

If $\vec{a}$ makes angles α, β, γ respectively with +ve. OX, OY and OZ

then $\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

i.e $\alpha = \beta = \gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore$ The vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ is equally inclined with OX, OY and OZ.

Q No. 15 Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ resp. in ratio (i) internally (ii) externally.

Sol.

Let $\vec{a} = \hat{i}+2\hat{j}-\hat{k}$

$\vec{b} = -\hat{i}+\hat{j}+\hat{k}$

(i) Using Section formula P.V of R which divides PQ in ratio 2:1 internally

$$= \frac{2\vec{b}+1.\vec{a}}{2+1} = \frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$= \frac{1}{3}\left[-\hat{i}+4\hat{j}+\hat{k}\right] = -\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) Using Section formula. P.V of R which divides PQ in ration 2:1 externally

$$= \frac{2\vec{b}-1.\vec{a}}{2-1} = \frac{2(-\hat{i}+\hat{j}+\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{1}$$

$$= -3\hat{i}+0\hat{j}+3\hat{k}$$

Q No. 16. Find the position vector of mid-points of vector joining the points P(2, 3, 4) and Q(4, 1, -2)

Sol. Using Mid-Point formula

$$\text{P.V of mid point of } \vec{PQ} = \frac{\text{P.V of P} + \text{P.V. of Q}}{2}$$

$$= \frac{1}{2}\left[(2\hat{i}+3\hat{j}+4\hat{k}) + (4\hat{i}+\hat{j}-2\hat{k})\right]$$

$$= \frac{1}{2}\left[6\hat{i}+4\hat{j}+2\hat{k}\right] = 3\hat{i}+2\hat{j}+\hat{k}.$$

QNo 17. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i}-4\hat{j}-4\hat{k}$; $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$; $\vec{c} = \hat{i}-3\hat{j}-5\hat{k}$ resp. form the vertices of right angled triangle.

Sol. Here $\overrightarrow{AB} = \vec{b}-\vec{a} = -\hat{i}+3\hat{j}+5\hat{k}$

$\therefore |AB| = |\overrightarrow{AB}| = \sqrt{(-1)^2+(3)^2+(5)^2} = \sqrt{35}$

$\overrightarrow{BC} = \vec{c}-\vec{b} = -\hat{i}-2\hat{j}-6\hat{k}$

$\therefore |BC| = |\overrightarrow{BC}| = \sqrt{(-1)^2+(-2)^2+(-6)^2} = \sqrt{1+4+36} = \sqrt{41}.$

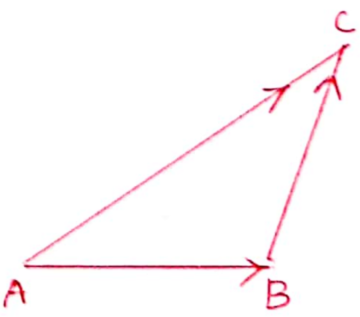
$\overrightarrow{CA} = \vec{a}-\vec{c} = 2\hat{i}-\hat{j}+\hat{k}$

$\therefore |CA| = |\overrightarrow{CA}| = \sqrt{(2)^2+(-1)^2+(1)^2} = \sqrt{6}.$

Now $AB^2 + CA^2 = (\sqrt{35})^2 + (\sqrt{6})^2 = 35+6 = 41 = (\sqrt{41})^2 = BC^2$

$\therefore$ ΔABC is right angled triangle with right angle at A.

QNo.18. In ΔABC (shown in fig) which of following is not true.



(i) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \vec{0}$

(ii) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{AC} = \vec{0}$

(iii) $\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA} = 0$

(iv) $\overrightarrow{AB}-\overrightarrow{CB}+\overrightarrow{CA} = \vec{0}$

Sol. In given ΔABC

$$\overrightarrow{AB}+\overrightarrow{BC}-\overrightarrow{CA} = \overrightarrow{AC}-\overrightarrow{CA}$$
$$= \overrightarrow{AC}+\overrightarrow{AC} = 2\overrightarrow{AC} \neq \vec{0}$$

$\therefore$ (C) is right option.

Q.No. 19 If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of following are incorrect.

(A) $\vec{b} = \lambda\vec{a}$ for some scalar λ.

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional.

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction but different magnitude.

Sol. Statements (A), (B), (C) are true.

(D) is not true as collinear vectors may have opposite direction.

eg $2\hat{i}$ and $-3\hat{i}$ are collinear but have opposite direction.

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