Chapter 12. Limits and Derivatives

Question-1

Find the indicated limit: $\lim_{x \to 1} \frac{x^2 + 2x + 5}{x^2 + 1}$

Solution:

$\lim_{x \to 1} \frac{x^2 + 2x + 5}{x^2 + 1} = \frac{1 + 2 + 5}{1 + 1}$ by direct substitution

$= \frac{8}{2} =$

Question-2

Find the indicated limit: $\lim_{x \to 2} \frac{x - 2}{\sqrt{2 - x}}$

Solution:

$\lim_{x \to 2} \frac{x - 2}{\sqrt{2 - x}} = \lim_{x \to 2} \frac{-(2 - x)}{\sqrt{2 - x}} = \lim_{x \to 2} -(\sqrt{2 - x}) = 0$

Question-3

Find the indicated limit: $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$

Solution:

$\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x + h)}{h} = \lim_{h \to 0} (2x + h) = 2x$

Question-4

Find the indicated limit: $\lim_{x \to 1} \frac{x^m - 1}{x - 1}$

Solution:

$$\lim_{x \to 1} \frac{x^m - 1}{x - 1} = \lim_{x \to 1} \frac{x^m - 1^m}{x - 1} = m(1)^{m-1} = m$$

Question-5

Find the indicated limit: $\lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{\sqrt{x-1} - \sqrt{2}}$

Solution:

$$\lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{\sqrt{x-1} - \sqrt{2}} = \lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{\sqrt{x-1} - \sqrt{2}} \times \frac{\sqrt{x-2} + \sqrt{2}}{\sqrt{x-2} + \sqrt{2}} \times \frac{\sqrt{2x+1} + 3}{\sqrt{2x+1} + 3}$$

$$= \lim_{x \to 4} \frac{(2x+1-9)}{(x-2-2)} \frac{\sqrt{x-2} + \sqrt{2}}{\sqrt{2x+1} + 3}$$

$$= \lim_{x \to 4} \frac{2x-8}{x-4} \frac{\sqrt{x-2} + \sqrt{2}}{\sqrt{2x+1} + 3}$$

$$= \lim_{x \to 4} 2\left\{\frac{\sqrt{x-2} + \sqrt{2}}{\sqrt{2x+1} + 3}\right\}$$

$$= 2\,\frac{2\sqrt{2}}{6}$$

$$= \frac{2\sqrt{2}}{3}$$

Question-6

Find the indicated limit: $\lim_{x \to 0} \frac{\sqrt{x^2 + p^2} - p}{\sqrt{x^2 + q^2} - q}$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{x^2 + p^2} - p}{\sqrt{x^2 + q^2} - q}$$

$$= \lim_{x \to 0} \frac{\sqrt{x^2 + p^2} - p}{\sqrt{x^2 + q^2} - q} \times \frac{\sqrt{x^2 + q^2} + q}{\sqrt{x^2 + q^2} + q} \times \frac{\sqrt{x^2 + p^2} + p}{\sqrt{x^2 + p^2} + p}$$

$$= \lim_{x \to 0} \frac{x^2 + p^2 - p^2}{x^2 + q^2 - q^2} \times \frac{\sqrt{x^2 + p^2} + q}{\sqrt{x^2 + q^2} + p}$$

$$= \lim_{x \to 0} \frac{2q}{2p} = \frac{q}{p}$$

Question-7

Find the indicated limit: $\lim_{x \to a} \frac{\sqrt[m]{x} - \sqrt[m]{a}}{x - a}$

Solution:

$$\lim_{x \to a} \frac{\sqrt[m]{x} - \sqrt[m]{a}}{x - a} = \lim_{x \to a} \frac{x^{1/m} - a^{1/m}}{x - a} = \frac{1}{m} a^{1/m - 1}$$ (na^{n-1} formula)

Question-8

Find the indicated limit: $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$

Solution:

$$\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{x^{1/3} - 1}{x^{1/2} - 1}$$

$$= \lim_{x \to 1} \frac{x^{1/3} - 1}{x - 1} \times \frac{x - 1}{x^{1/2} - 1} = \lim_{x \to 1} \frac{x^{1/3} - x^{1/3}}{x - 1} \times \frac{x - 1}{x^{1/2} - 1^{1/2}}$$

$$= \frac{1}{3}(1)^{1/3 - 1} \times \frac{1}{\frac{1}{2}(1)^{1/2 - 1}} = \frac{1/3}{1/2} = \frac{2}{3}$$

Question-9

Find the indicated limit: $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x} = \lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x} \times \frac{\sqrt{1 + x + x^2} + 1}{\sqrt{1 + x + x^2} + 1} = \lim_{x \to 0} \frac{1 + x + x^2 - 1}{x\left[\sqrt{1 + x + x^2} + 1\right]}$$

$$= \lim_{x \to 0} \frac{x(x + 1)}{x(\sqrt{1 + x + x^2} + 1)}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

Question-10

Find the indicated limit: $\lim_{x \to 0} \frac{\sin^2\left(\frac{x}{3}\right)}{x^2}$

Solution:

$$\lim_{x \to 0} \frac{\sin^2\left(\frac{x}{3}\right)}{x^2} = \lim_{x \to 0} \left(\frac{\sin x/3}{x/3}\right)^2 \frac{(x/3)^2}{x^2} = 1 \times \frac{1}{9} = \frac{1}{9}$$

Question-11

Find the indicated limit: $\lim_{x \to 0} \frac{\sin(a+x)-\sin(a-x)}{x}$

Solution:

$$\lim_{x \to 0} \frac{\sin(a+x)-\sin(a-x)}{x} = \lim_{x \to 0} \frac{2\cos a \sin x}{x} = \lim_{x \to 0} 2\cos a\left(\frac{\sin x}{x}\right) = 2\cos$$

Question-12

Find the indicated limit: $\lim_{x \to 0} \frac{\log(1+\alpha x)}{x}$

Solution:

$$\lim_{x \to 0} \frac{\log(1+\alpha x)}{x} = \lim_{x \to 0} \frac{\alpha x - \frac{(\alpha x)^2}{2} + \frac{(\alpha x)^3}{3}}{x} \ldots\ldots = \lim_{x \to 0} \alpha - \frac{\alpha^2}{2}x + \frac{\alpha^3}{3}x^2 \ldots\ldots$$

Question-13

Find the indicated limit: $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^{n+5}$

Solution:

$$\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^{n+5} = \lim_{n \to \infty} \left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}\right)^{5} = e.\,(1)^5 = e$$

Question-14

Evaluate the left and right limits of f(x) = $\frac{x^3 - 27}{x - 3}$ at x = 3. Does the limit of f(x) = x → 3 exist? Justify your answer.

Solution:

$\lim_{x \to 3} \left(\frac{x^3 - 27}{x - 3}\right)$

Let x = 3 + h

Then $\lim_{h \to 0} \left(\frac{(3+h)^3 - 27}{(3+h) - 3}\right) = \left(\frac{9h^2 + 27h + h^3}{h}\right) = \lim_{h \to 0} 27 + 9h + h^2 = 27$

$\lim_{h \to 3} \frac{x^3 - 27}{x - 3}$

Let x = 3 – h

Then $\lim_{h \to 0} \frac{(3-h)^3 - 27}{(3-h-3)} = \lim_{h \to 0} \frac{-27h^2 + 9h^2 - h^3}{-h} = 27$

Also, $\lim_{x \to 3} \frac{x^3 - 27}{x - 3} = \lim_{x \to 3} \frac{(x-3)(x^2 + 3x + 9)}{x - 3} = 9 + 9 + 9 = 27$

Question-15

Find the positive integer n such that $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$.

Solution:

$\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$

$n\,3^{n-1} = 108$

Put n = 4, then $4.3^3 = 4 \times 27 = 108$

$\therefore n = 4$

Question-16

Evaluate $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$ [Hint: Take e^x or $e^{\sin x}$ as common factor in numerator]

Solution:

$$\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x} = \lim_{x \to 0} \frac{e^x\left(1 - \frac{e^{\sin x}}{e^x}\right)}{(x - \sin x)}$$

$$= \frac{e^x(1 - e^{\sin x - x})}{(x - \sin x)} = \frac{e^x(1 - e^{-(x - \sin x)})}{(x - \sin x)}$$

$$= e^x \left[\frac{1 - (1 - (x - \sin x) + x - \sin x)^2 + \ldots\ldots}{(x - \sin x)}\right]$$

$$= e^x \left[\frac{(x - \sin x) - (x - \sin x)^2 + \ldots\ldots}{(x - \sin x)}\right]$$

$$= \lim_{x \to 0} e^x [1 - (x - \sin x) \ldots\ldots] = e^0 = 1$$

Question-17

If $f(x) = \frac{ax^2 + b}{x^2 - 1}$, $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to \infty} f(x) = 1$

Solution:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{ax^2 + b}{x^2 - 1} = b/-1 = -b = 1$$

$\therefore b = -1$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{a + \frac{b}{x^2}}{1 - \frac{1}{x^2}} = a = 1$$

$\therefore a = 1$

$$\therefore f(x) = \frac{x^2 - 1}{x^2 - 1} = 1$$

f(2) = 1 and f(-2) = 1

Question-18

Evaluate $\lim_{x \to 0^+} \frac{|x|}{x}$ and $\lim_{x \to 0^-} \frac{|x|}{x}$. What can you say about $\lim_{x \to 0} \frac{|x|}{x}$?

Solution:

Let $f(x) = \frac{x}{|x|}$ where $|x| = x \quad x \geq 0$

$= -x \quad x < 0$

Then, $\underset{x \to 0^+}{Lt} f(x) = \underset{x \to 0^+}{Lt} \frac{x}{|x|} = \underset{x \to 0^+}{Lt} \frac{x}{x} = 1$

$\underset{x \to 0^-}{Lt} f(x) = \underset{x \to 0^-}{Lt} \frac{x}{|x|} = \underset{x \to 0^-}{Lt} \frac{x}{-x} = -1$

$\therefore \underset{x \to 0^+}{Lt} f(x) \neq \underset{x \to 0^-}{Lt} f(x)$

$\therefore \underset{x \to 0}{Lt} \frac{x}{|x|}$ does not exist.

Question-19

Compute $\lim_{x \to 0} \frac{a^x - b^x}{x}, a, b > 0$. Hence evaluate $\lim_{x \to 0} \frac{5^x - 6^x}{x}$.

Solution:

$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{(a^x - 1) - (b^x - 1)}{x} = \lim_{x \to 0} \left(\frac{a^x - 1}{x}\right) - \lim_{x \to 0} \left(\frac{b^x - 1}{x}\right) = \log a - \log b = \log \left(\frac{a}{b}\right)$

$\therefore \lim_{x \to 0} \frac{5^x - 6^x}{x} = \log\left(\frac{5}{6}\right)$

Question-20

Without using the series expansion of $\log(1 + x)$, prove that $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

Solution:

$\lim_{x \to 0} \frac{\log(1 + x)}{x}$

Let $y = \log(1 + x)$ Then as $x \to 0$, $y \to 0$

$\lim_{y \to 0} \frac{y}{e^y - 1} = \lim_{y \to 0} \frac{y}{1 + y + \frac{y^2}{2} + \ldots\ldots - 1} = \lim_{y \to 0} \frac{y}{y\left(1 + \frac{y}{2} + \frac{y^2}{3} + \ldots\infty\right)} = \frac{1}{1} = 1$

Question-21

Differentiate the following with respect to x:

(i) $x^{7} + e^{x}$

(ii) $\log_7 x + 200$

(iii) $3 \sin x + 4 \cos x - e^{x}$

(iv) $e^{x} + 3 \tan x + \log x^{6}$

(v) $\sin 5 + \log_{10} x + 2 \sec x$

(vi) $x^{-3/2} + 8e + 7\tan x$

(vii) $\left(x+\frac{1}{x}\right)^{3}$

(viii) $\frac{(x-3)(2x^{2}-4)}{x}$

Solution:

(i) $y = x^{7} + e^{x}$

$\frac{dy}{dx} = 7x^{6} + e^{x}$

(ii) $y = \log_7 x + 200$

$= \log_e x \cdot \log_{10} e + 200$

$\frac{dy}{dx} = \log_{10} e\left(\frac{1}{x}\right)$

(iii) $y = 3 \sin x + 4 \cos x - e^x$

$\frac{dy}{dx} = 3 \cos x - 4 \sin x - e^x$

(iv) $y = e^x + 3 \tan x + 6 \log x$

$\frac{dy}{dx} = e^x + 3 \sec^2 x + \frac{6}{x}$

(v) $y = \sin 5 + \log_{10} x + 2 \sec x$

$= \sin 5 + \log_e x \log_{10} e + 2 \sec x$

$\frac{dy}{dx} = 0 + \frac{\log_{10} e}{x} + 2 \sec x \tan x$

(vi) $y = x^{-3/2} + 8e + 7\tan x$

$\frac{dy}{dx} = -\frac{3}{2} x^{-5/2} + 7 \sec^2 x$

(vii) $y = \left(x + \frac{1}{x}\right)^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = x^3 + 3x + 3x^{-1} + x^{-3}$

$\frac{dy}{dx} = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$

(viii) $\frac{(x-3)(2x^2-4)}{x} = \frac{2x^3 - 6x^2 - 4x + 12}{x}$

$y = 2x^2 - 6x - 4 + \frac{12}{x}$

$\frac{dy}{dx} = 4x - 6 - \frac{12}{x^2}$

Question-22

Differentiate the following function using quotient rule.

$$\frac{1}{ax^2 + bx + c}$$

Solution:

Let $y = \frac{1}{ax^2 + bx + c}$

$$\frac{dy}{dx} = \frac{(ax^2 + bx + c).0 - 1(2ax + b)}{(ax^2 + bx + c)^2} = -\frac{(2ax + b)}{(ax^2 + bx + c)^2}$$

Question-23

Differentiate the following function using quotient rule.

$\frac{\tan x + 1}{\tan x - 1}$

Solution:

Let y = $\frac{\tan x + 1}{\tan x - 1}$

$$\frac{dy}{dx} = \frac{(\tan x - 1)(\sec^2 x) - (\tan x + 1)\sec^2 x}{(\tan x - 1)^2} = \frac{-2\sec^2 x}{(\tan x - 1)^2}$$

Question-24

Differentiate the following function using quotient rule.

$\frac{\sin x + x\cos x}{x\sin x - \cos x}$

Solution:

Let y = $\frac{\sin x + x\cos x}{x\sin x - \cos x}$

$$\frac{dy}{dx} = \frac{(x\sin x - \cos x)(\cos x - x\sin x + \cos x) - (\sin x + x\cos x)(x\cos x + \sin x + \sin x)}{(x\sin x - \cos x)^2}$$

$$= \frac{x\sin x\cos x - \cos^2 x - x^2\sin^2 x + x\sin x\cos x + x\sin\cos x - \cos^2 x - x\sin x\cos x - x^2\cos^2 x - \sin^2 x + x\sin x\cos x - \sin^2 x - x\sin x\cos x)}{(x\sin x - \cos x)^2}$$

$$= \frac{-2\cos^2 x - 2\sin^2 x - x^2\sin^2 x + x^2\cos^2 x}{(x\sin x - \cos x)^2}$$

$$= \frac{-(2 + x^2)}{(x\sin x - \cos x)^2}$$

Question-25

Differentiate the following function using quotient rule.

Solution:

Let $y = \frac{\log x^2}{e^x} = \frac{2\log x}{e^x}$

$$\frac{dy}{dx} = \frac{e^x \frac{2}{x} - 2\log x . e^x}{(e^x)^2} = \frac{2e^x\left(\frac{1}{x} - \log x\right)}{(e^x)^2} = \frac{2\left(\frac{1}{x} - \log x\right)}{e^x} = e^{-x}\left(\frac{2}{x} - 2\log x\right)$$

Question-26

Differentiate the following function with respect to x.
log(sinx)

Solution:

y = log(sinx)

Let u = sinx

$\frac{du}{dx} = \cos x$

y = log u

$\frac{dy}{du} = \frac{1}{u} = \frac{1}{\sin x}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u}.\cos x = \frac{1}{\sin x}\cos x = \cot x$

Question-27

Differentiate the following function with respect to x.
$\mathbf{e^{\sin x}}$

Solution:

$y = e^{\sin x}$

Put u = sinx

$\frac{du}{dx} = \cos x$

$y = e^u$

$\frac{dy}{du} = e^u = e^{\sin x}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^{\sin x} \cdot \cos x$

Question-28

Differentiate the following function with respect to x.
$\sqrt{1 + \cot x}$

Solution:

$y = \sqrt{1 + \cot x}$

Put u = 1 + cotx

$\frac{du}{dx} = -\text{cosec}^2 x$

$y = u^{1/2}$

$\frac{dy}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2}(1 + \cot x)^{-1/2}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2}(1 + \cot x)^{1/2}(-\text{cosec}^2 x)$

Question-29

Differentiate the following function with respect to x.
tan(logx)

Solution:

y = tan(logx)

Put u = logx

$$\frac{du}{dx} = \frac{1}{x}$$

y = tan u

$$\frac{dy}{du} = \sec^2 u$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx} = \sec^2 u . \frac{1}{x} = \frac{\sec^2(\log x)}{x}$$

Question-30

Differentiate the following function with respect to x.

$$\frac{e^{bx}}{\cos(a+b)}$$

Solution:

$$y = \frac{e^{bx}}{\cos(a+b)}$$

$$\frac{dy}{dx} = \frac{\cos(ax+b).\frac{d}{dx}(e^{bx}) - e^{bx}\frac{d}{dx}(\cos(ax+b))}{\cos^2(ax+b)}$$

$$= \frac{\cos(ax+b).e^{bx}.b + e^{bx}.\sin(ax+b)a}{\cos^2(ax+b)}$$

$$= \frac{e^{bx}(b\cos(ax+b) + a\sin(ax+b))}{\cos^2(ax+b)}$$

Question-31

Differentiate the following function with respect to x.

log sec $\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Solution:

y = log sec $\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Put u = $\frac{\pi}{4}+\frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

y = log sec u

y = log v

Put v = sec u

$\frac{dv}{du}$ = sec u tan u

$$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{v} \cdot \sec\left(\frac{\pi}{4}+\frac{x}{2}\right) \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)} \sec\left(\frac{\pi}{4}+\frac{x}{2}\right) \cdot \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2} \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

Question-32

Differentiate the following function with respect to x.
log sin(e^x + 4x + 5)

Solution:

y = log sin(e^x + 4x + 5)

$$\frac{dy}{dx} = \frac{1}{\sin(e^x + 4x + 5)} \cos(e^x + 4x + 5)(e^x + 4)$$

$$= (e^x + 4) \frac{\cos(e^x + 4x + 5)}{\sin(e^x + 4x + 5)}$$

$$= (e^x + 4)\cot(e^x + 4x + 5)$$

Question-33

Differentiate the following function with respect to x.
sin($x^{3/2}$)

Solution:

y = sin($x^{3/2}$)

Put u = $x^{3/2}$

y = sin u

$$\frac{du}{dx} = \frac{3}{2} x^{1/2}$$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u . \frac{3}{2} x^{1/2} = \cos x^{3/2} \left(\frac{3}{2} x^{1/2}\right)$$

Question-34

Differentiate the following function with respect to x.
cos($\sqrt{x}$)

Solution:

y = cos u

Put u = $\sqrt{x}$

$\frac{dy}{du}$ = -sin u

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx} = -\sin u . \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

Question-35

Differentiate the following function with respect to x.
$e^{\sin(\log x)}$

Solution:

$y = e^{\sin(\log x)}$

Put u = logx

$\frac{du}{dx} = \frac{1}{x}$

$y = e^{\sin u}$

Put v = sin u

$\frac{dv}{du}$ = cos u = cos(logx)

Put $y = e^{v}$

$\frac{dy}{dv} = e^{v}$

Hence $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx} = e^{v} . \cos(\log x) . \frac{1}{x} = e^{\sin(\log x)} \cos^{(\log s)} . \frac{1}{x}$

Question-36

Find the indicated limit: $\lim_{x \to 1} \frac{x^2 + 2x + 5}{x^2 + 1}$

Solution:

$\lim_{x \to 1} \frac{x^2 + 2x + 5}{x^2 + 1} = \frac{1 + 2 + 5}{1 + 1}$ by direct substitution

$= \frac{8}{2} = 4$

Question-37

Find the indicated limit: $\lim_{x \to 2} \frac{x-2}{\sqrt{2-x}}$

Solution:

$$\lim_{x \to 2} \frac{x-2}{\sqrt{2-x}} = \lim_{x \to 2} \frac{-(2-x)}{\sqrt{2-x}} = \lim_{x \to 2} -(\sqrt{2-x}) = 0$$

Question-38

Find the indicated limit: $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

Solution:

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} (2x+h) = 2x$$

Question-39

Find the indicated limit: $\lim_{x \to 1} \frac{x^m - 1}{x-1}$

Solution:

$$\lim_{x \to 1} \frac{x^m - 1}{x-1} = \lim_{x \to 1} \frac{x^m - 1^m}{x-1} = m(1)^{m-1} = m$$

Question-40

Find the indicated limit: $\lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{\sqrt{x-1} - \sqrt{2}}$

Solution:

$$\lim_{x \to 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-1}-\sqrt{2}} = \lim_{x \to 4} \frac{\sqrt{2x+1}-3}{\sqrt{x-1}-\sqrt{2}} \times \frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{x-2}+\sqrt{2}} \times \frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3} = \lim_{x \to 4} \frac{(2x+1-9)}{(x-2-2)} \frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{2x+1}+3}$$

$$= \lim_{x \to 4} \frac{2x-8}{x-4} \frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{2x+1}+3}$$

$$= \lim_{x \to 4} 2\left\{\frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{2x+1}+3}\right\}$$

$$= 2\,\frac{2\sqrt{2}}{6}$$

$$= \frac{2\sqrt{2}}{3}$$

Question-41

Find the indicated limit: $\lim_{x \to 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q}$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q}$$

$$= \lim_{x \to 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q} \times \frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+q^2}+q} \times \frac{\sqrt{x^2+p^2}+p}{\sqrt{x^2+p^2}+p}$$

$$= \lim_{x \to 0} \frac{x^2+p^2-p^2}{x^2+q^2-q^2} \times \frac{\sqrt{x^2+p^2}+q}{\sqrt{x^2+q^2}+p}$$

$$= \lim_{x \to 0} \frac{2q}{2p} = \frac{q}{p}$$

Question-42

Find the indicated limit: $\lim_{x \to a} \frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a}$

Solution:

$$\lim_{x \to a} \frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a} = \lim_{x \to a} \frac{x^{1/m}-a^{1/m}}{x-a} = \frac{1}{m} a^{1/m-1}$$ (na^{n-1} formula)

Question-43

Find the indicated limit: $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$

Solution:

$$\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{x^{1/3} - 1}{x^{1/2} - 1}$$

$$= \lim_{x \to 1} \frac{x^{1/3} - 1}{x - 1} \times \frac{x - 1}{x^{1/2} - 1} = \lim_{x \to 1} \frac{x^{1/3} - x^{1/3}}{x - 1} \times \frac{x - 1}{x^{1/2} - 1^{1/2}}$$

$$= \frac{1}{3}(1)^{1/3 - 1} \times \frac{1}{1/2(1)^{1/2 - 1}} = \frac{1/3}{1/2} = \frac{2}{3}$$

Question-44

Find the indicated limit: $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$

Solution:

$$\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x} = \lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x} \times \frac{\sqrt{1 + x + x^2} + 1}{\sqrt{1 + x + x^2} + 1} = \lim_{x \to 0} \frac{1 + x + x^2 - 1}{x\left[\sqrt{1 + x + x^2} + 1\right]}$$

$$= \lim_{x \to 0} \frac{x(x + 1)}{x(\sqrt{1 + x + x^2} + 1)}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

Question-45

Find the indicated limit: $\lim_{x \to 0} \frac{\sin^2\left(\frac{x}{3}\right)}{x^2}$

Solution:

$$\lim_{x \to 0} \frac{\sin^2\left(\frac{x}{3}\right)}{x^2} = \lim_{x \to 0} \left(\frac{\sin x/3}{x/3}\right)^2 \frac{(x/3)^2}{x^2} = 1 \times \frac{1}{9} = \frac{1}{9}$$

Question-46

Find the indicated limit: $\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x}$

Solution:

$$\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x} = \lim_{x \to 0} \frac{2\cos a \sin x}{x} = \lim_{x \to 0} 2\cos a \left(\frac{\sin x}{x}\right) = 2\cos a$$

Question-47

Find the indicated limit: $\lim_{x \to 0} \frac{\log(1+\alpha x)}{x}$

Solution:

$$\lim_{x \to 0} \frac{\log(1+\alpha x)}{x} = \lim_{x \to 0} \frac{\alpha x - \frac{(\alpha x)^2}{2} + \frac{(\alpha x)^3}{3}}{x} \ldots\ldots = \lim_{x \to 0} \alpha - \frac{\alpha^2}{2} x + \frac{\alpha^3}{3} x^2 \ldots\ldots$$

Question-48

Find the indicated limit: $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n+5}$

Solution:

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n+5} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n} \left(1 + \frac{1}{n}\right)^{5} = e.\,(1)^5 = e$$

Question-49

Evaluate the left and right limits of f(x) = $\frac{x^3 - 27}{x - 3}$ at x = 3. Does the limit of f(x) = x › 3 exist? Justify your answer.

Solution:

$\lim_{x \to 3} \left(\frac{x^3 - 27}{x - 3}\right)$

Let x = 3 + h

Then $\lim_{h \to 0} \left(\frac{(3 + h)^3 - 27}{(3 + h) - 3}\right) = \left(\frac{9h^2 + 27h + h^3}{h}\right) = \lim_{h \to 0} 27 + 9h + h^2 = 27$

$\lim_{h \to 3} \frac{x^3 - 27}{x - 3}$

Let x = 3 – h

Then $\lim_{h \to 0} \frac{(3 - h)^3 - 27}{(3 - h - 3)} = \lim_{h \to 0} \frac{-27h^2 + 9h^2 - h^3}{-h} = 27$

Also, $\lim_{x \to 3} \frac{x^3 - 27}{x - 3} = \lim_{x \to 3} \frac{(x - 3)(x^2 + 3x + 9)}{x - 3} = 9 + 9 + 9 = 27$

Question-50

Find the positive integer n such that $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$.

Solution:

$\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$

$n\, 3^{n - 1} = 108$

Put n = 4, then $4.3^3 = 4 \times 27 = 108$

$\therefore n = 4$

Question-51

Evaluate $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$ [Hint: Take e^x or $e^{\sin x}$ as common factor in numerator]

Solution:

$$\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x} = \lim_{x \to 0} \frac{e^x\left(1 - \frac{e^{\sin x}}{e^x}\right)}{(x - \sin x)}$$

$$= \frac{e^x(1 - e^{\sin x - x})}{(x - \sin x)} = \frac{e^x(1 - e^{-(x - \sin x)})}{(x - \sin x)}$$

$$= e^x \left[\frac{1 - (1 - (x - \sin x) + x - \sin x)^2 + \ldots\ldots}{(x - \sin x)}\right]$$

$$= e^x \left[\frac{(x - \sin x) - (x - \sin x)^2 + \ldots\ldots}{(x - \sin x)}\right]$$

$$= \lim_{x \to 0} e^x [1 - (x - \sin x) \ldots\ldots] = e^0 = 1$$

Question-52

If $f(x) = \frac{ax^2 + b}{x^2 - 1}$, $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to \infty} f(x) = 1$

Solution:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{ax^2 + b}{x^2 - 1} = b/-1 = -b = 1$$

$\therefore b = -1$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{a + \frac{b}{x^2}}{1 - \frac{1}{x^2}} = a = 1$$

$\therefore a = 1$

$$\therefore f(x) = \frac{x^2 - 1}{x^2 - 1} = 1$$

$f(2) = 1$ and $f(-2) = 1$

Question-53

Evaluate $\lim_{x \to 0} \frac{|x|}{x}$ and $\lim_{x \to 0} \frac{|x|}{x}$. What can you say about $\lim_{x \to 0} \frac{|x|}{x}$?

Solution:

Let f(x) = $\frac{x}{|x|}$ where $|x| = x \quad x \geq 0$

$= -x \quad x < 0$

Then,

$\underset{x \to 0}{\text{Lt}} f(x) = \underset{x \to 0}{\text{Lt}} \frac{x}{|x|} = \underset{x \to 0}{\text{Lt}} \frac{x}{x} = 1$

$\underset{x \to 0}{\text{Lt}} f(x) = \underset{x \to 0}{\text{Lt}} \frac{x}{|x|} = \underset{x \to 0}{\text{Lt}} \frac{x}{-x} = -1$

$\therefore \underset{x \to 0}{\text{Lt}} f(x) \neq \underset{x \to 0}{\text{Lt}} f(x)$

$\therefore \underset{x \to 0}{\text{Lt}} \frac{x}{|x|}$ does not exist.

Question-54

Compute $\lim_{x \to 0} \frac{a^x - b^x}{x}, a, b > 0$. Hence evaluate $\lim_{x \to 0} \frac{5^x - 6^x}{x}$.

Solution:

$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{(a^x - 1) - (b^x - 1)}{x} = \lim_{x \to 0} \left(\frac{a^x - 1}{x}\right) - \lim_{x \to 0} \left(\frac{b^x - 1}{x}\right) = \log a - \log b = \log \left(\frac{a}{b}\right)$

$\therefore \lim_{x \to 0} \frac{5^x - 6^x}{x} = \log\left(\frac{5}{6}\right)$

Question-55

Without using the series expansion of log(1 + x), prove that $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

Solution:

$\lim_{x \to 0} \frac{\log(1+x)}{x}$

Let y = log(1 + x) Then as $x \to 0$, $y \to 0$

$\lim_{y \to 0} \frac{y}{e^y - 1} = \lim_{y \to 0} \frac{y}{1 + y + \frac{y^2}{2} + \ldots\ldots - 1} = \lim_{y \to 0} \frac{y}{y\left(1 + \frac{y}{2} + \frac{y^2}{3} + \ldots\infty\right)} = \frac{1}{1} = 1$

Question-56

Differentiate the following with respect to x:

(i) $x^{7} + e^{x}$

(ii) $\log_7 x + 200$

(iii) $3 \sin x + 4 \cos x - e^{x}$

(iv) $e^{x} + 3 \tan x + \log x^{6}$

(v) $\sin 5 + \log_{10} x + 2 \sec x$

(vi) $x^{-3/2} + 8e + 7\tan x$

(vii) $\left(x+\frac{1}{x}\right)^{3}$

(viii) $\frac{(x-3)(2x^{2}-4)}{x}$

Solution:

(i) $y = x^{7} + e^{x}$

$$\frac{dy}{dx} = 7x^{6} + e^{x}$$

(ii) $y = \log_7 x + 200 = \log_e x \,.\, \log_{10} e + 200$

$$\frac{dy}{dx} = \log_{10} e\left(\frac{1}{x}\right)$$

(iii) $y = 3 \sin x + 4 \cos x - e^{x}$

$\frac{dy}{dx}$ = 3 cos x - 4 sin x – e^x

(iv) y = e^x + 3 tanx + 6 log x

$\frac{dy}{dx} = e^x + 3\sec^2 x + \frac{6}{x}$

(v) y = sin 5 + $\log_{10}x$ + 2 sec x

= sin 5 + $\log_e x \log_{10} e$ + 2 sec x

$\frac{dy}{dx} = 0 + \frac{\log_{10} e}{x} + 2\sec x \tan x$

(vi) y = $x^{-3/2}$ + 8e + 7tanx

$\frac{dy}{dx} = -\frac{3}{2}x^{-5/2} + 7\sec^2 x$

(vii) y = $\left(x + \frac{1}{x}\right)^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = x^3 + 3x + 3x^{-1} + x^{-3}$

$\frac{dy}{dx} = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$

(viii) $\frac{(x-3)(2x^2-4)}{x} = \frac{2x^3 - 6x^2 - 4x + 12}{x}$

y = $2x^2 - 6x - 4 + \frac{12}{x}$

$\frac{dy}{dx} = 4x - 6 - \frac{12}{x^2}$

Question-57

Differentiate the following functions with respect to x.

(i) $e^x \cos x$

(ii) $\sqrt[n]{x} \log \sqrt{x}, x > 0$

(iii) $6 \sin x \log_{10} x + e$

(iv) $(x^4 - 6x^3 + 7x^2 + 4x + 2)(x^3 - 1)$

(v) (a – b six (1 – 2 cos x)

(vi) cosec x . cotx

(vii) $\sin^2 x$

(viii) $\cos^2 x$

(ix) $(3x^2 + 1)^2$

(x) $(4x^2 - 1)(2x + 3)$

(xi) $(3 \sec x - 4 \operatorname{cosec} x)(2 \sin x + 5 \cos x)$

(xii) $x^2 e^x \sin x$

(xiii) $\sqrt{x} e^x \cos x$

Solution:

(i) $y = e^x \cos x$

$\frac{dy}{dx}$ = - e^x sin x + cos x e^x

(ii) y = $x^{1/n}\log(\sqrt{x})$ = $\frac{1}{2}x^{1/n}\log x$

$$\frac{dy}{dx} = \frac{1}{2}\left[x^{1/n}\frac{1}{x} + \log x\left(\frac{1}{n}x^{1/n-1}\right)\right] = \frac{1}{2}\left[x^{1/n-1} + \frac{1}{n}\log x . x^{\frac{1}{n}-1}\right]$$

(iii) y = 6 sin x $\log_{10}$ x + e = 6 sin x $\log_e x$. $\log_{10}$ e + e

$\frac{dy}{dx}$ = 6 $\log_{10}$ e$\left(\sin x \frac{1}{x} + \log x \cos x\right)$

(iv) y = $(x^4 - 6x^3 + 7x^2 + 4x + 2)(x^3 - 1)$

$\frac{dy}{dx}$ = $(x^4 - 6x^3 + 7x^2 + 4x + 2)(3x^2) + (x^3 - 1)(4x^3 - 18x^2 + 14x + 4)$

(v) y = (a – b six (1 – 2 cos x)

$\frac{dy}{dx}$ = (a – b sinx) (2 sin x) + (1 – 2 cos x) (-b cosx)

$\frac{dy}{dx}$ = 2a sinx – 2b $\sin^2$ x – b cos x + 2b $\cos^2$x

(vi) y = cosec x. cot x

$\frac{dy}{dx}$ = -cosec x cosec^2x(-cosec x cotx) = -cosec^3 x – $\cot^2$x cosec x

Question-58

Differentiate the following function using quotient rule.

$\frac{5}{x^2}$

Solution:

Let y = $\frac{5}{x^2}$

$$\frac{dy}{dx} = \frac{(x^2).\frac{d}{dx}(5).-5\frac{d}{dx}(x^2)}{(x^2)^2} = \frac{x^2.0 - 5(2x)}{x^4} = \frac{-10x}{x^4} = \frac{-10}{x^3}$$

Question-59

Differentiate the following function using quotient rule.

$\frac{2x-3}{4x+5}$

Solution:

Let y = $\frac{2x-3}{4x+5}$

$$\frac{dy}{dx} = \frac{(4x+5)\frac{d}{dx}(2x-3^2)-(2x-3)\frac{d}{dx}(4x+5)}{(4x+5)^2}$$

$$= \frac{(4x+5)(2)-(2x-3)(4)}{(4x+5)^2}$$

$$= \frac{8x+10-8x+12}{(4x+5)^2}$$

$$= \frac{22}{(4x+5)^2}$$

Question-60

Differentiate the following function using quotient rule.

$\frac{x^7-4^7}{x-4}$

Solution:

Let y = $\frac{x^7-4^7}{x-4}$

$$\frac{dy}{dx} = \frac{(x-4)7x^6-(x^7-4^7).1}{(x-4)^2} = \frac{6x^7-28x^6+4^7}{(x-4)^2}$$

Question-61

Differentiate the following function using quotient rule.

$$\frac{\cos x + \log x}{x^2 + e^x}$$

Solution:

Let $y = \frac{\cos x + \log x}{x^2 + e^x}$

$$\frac{dy}{dx} = \frac{(x^2 + e^x)\left(-\sin x + \frac{1}{x}\right) - (\cos x + \log x)(2x + e^x)}{(x^2 + e^x)^2}$$

$$\frac{dy}{dx} = \frac{e^x\left(\frac{1}{x} - \sin x - \cos x - \log x\right) - 2x(\cos x + \log x) + x - x^2 \sin x}{(x^2 + e^x)^2}$$

Question-62

Differentiate the following function using quotient rule.

$$\frac{\log x - 2x^2}{\log x + 2x^2}$$

Solution:

Let $y = \frac{\log x - 2x^2}{\log x + 2x^2}$

$$\frac{dy}{dx} = \frac{(\log x + 2x^2)\left(\frac{1}{x} - 4x\right) - (\log x - 2x^2)\left(\frac{1}{x} + 4x\right)}{(\log x + 2x^2)^2} = \frac{4x(1 - 2\log x)}{(\log x + 2x^2)^2}$$

Question-63

Differentiate the following function using quotient rule.

$$\frac{\log x}{\sin x}$$

Solution:

Let $y = \frac{\log x}{\sin x}$

$$\frac{dy}{dx} = \frac{\sin x\left(\frac{1}{x}\right) - \log x \cos x}{\sin^2 x}$$

Question-64

Differentiate the following function using quotient rule.

$$\frac{1}{ax^2 + bx + c}$$

Solution:

Let $y = \frac{1}{ax^2 + bx + c}$

$$\frac{dy}{dx} = \frac{(ax^2 + bx + c).0 - 1(2ax + b)}{(ax^2 + bx + c)^2} = -\frac{(2ax + b)}{(ax^2 + bx + c)^2}$$

Question-65

Differentiate the following function using quotient rule.

$$\frac{\tan x + 1}{\tan x - 1}$$

Solution:

Let $y = \frac{\tan x + 1}{\tan x - 1}$

$$\frac{dy}{dx} = \frac{(\tan x - 1)(\sec^2 x) - (\tan x + 1)\sec^2 x}{(\tan x - 1)^2} = \frac{-2\sec^2 x}{(\tan x - 1)^2}$$

Question-66

Differentiate the following function using quotient rule.

$\frac{\sin x + x\cos x}{x\sin x - \cos x}$

Solution:

Let y = $\frac{\sin x + x\cos x}{x\sin x - \cos x}$

$$\frac{dy}{dx} = \frac{(x\sin x - \cos x)(\cos x - x\sin x + \cos x) - (\sin x + x\cos x)(x\cos x + \sin x + \sin x)}{(x\sin x - \cos x)^2}$$

$$= \frac{x\sin x\cos x - \cos^2 x - x^2\sin^2 x + x\sin x\cos x + x\sin\cos x - \cos^2 x - x\sin x\cos x - x^2\cos^2 x - \sin^2 x + x\sin x\cos x - \sin^2 x - x\sin x\cos x)}{(x\sin x - \cos x)^2}$$

$$= \frac{-2\cos^2 x - 2\sin^2 x - x^2\sin^2 x + x^2\cos^2 x}{(x\sin x - \cos x)^2}$$

$$= \frac{-(2 + x^2)}{(x\sin x - \cos x)^2}$$

Question-67

Differentiate the following function using quotient rule.

Solution:

Let y = $\frac{\log x^2}{e^x} = \frac{2\log x}{e^x}$

$$\frac{dy}{dx} = \frac{e^x\frac{2}{x} - 2\log x.e^x}{(e^x)^2} = \frac{2e^x\left(\frac{1}{x} - \log x\right)}{(e^x)^2} = \frac{2\left(\frac{1}{x} - \log x\right)}{e^x} = e^{-x}\left(\frac{2}{x} - 2\log x\right)$$

Question-68

Differentiate the following function with respect to x.

Solution:

$y = \log(\sin x)$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$y = \log u$

$\frac{dy}{du} = \frac{1}{u} = \frac{1}{\sin x}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u}.\cos x = \frac{1}{\sin x}\cos x = \cot x$

Question-69

Differentiate the following function with respect to x.

$e^{\sin x}$

Solution:

$y = e^{\sin x}$

Put $u = \sin x$

$\frac{du}{dx} = \cos x$

$y = e^{u}$

$\frac{dy}{du} = e^{u} = e^{\sin x}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx} = e^{\sin x} . \cos x$

Question-70

Differentiate the following function with respect to x. $\sqrt{1+\cot x}$

Solution:

y = $\sqrt{1+\cot x}$

Put u = 1 + cotx

$\frac{du}{dx} = -\text{cosec}^2 x$

y = $u^{1/2}$

$\frac{dy}{du} = \frac{1}{2}u^{-1/2} = \frac{1}{2}(1+\cot x)^{-1/2}$

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{1}{2}(1+\cot x)^{1/2}(-\text{cosec}^2 x)$

Question-71

Differentiate the following function with respect to x. tan(logx)

Solution:

y = tan(logx)

Put u = logx

$\frac{du}{dx} = \frac{1}{x}$

y = tan u

$\frac{dy}{du} = \sec^2 u$

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \sec^2 u.\frac{1}{x} = \frac{\sec^2(\log x)}{x}$

Question-72

Differentiate the following function with respect to x.

$$\frac{e^{bx}}{\cos(a+b)}$$

Solution:

$$y = \frac{e^{bx}}{\cos(a+b)}$$

$$\frac{dy}{dx} = \frac{\cos(ax+b).\frac{d}{dx}(e^{bx}) - e^{bx}\frac{d}{dx}(\cos(ax+b))}{\cos^2(ax+b)}$$

$$= \frac{\cos(ax+b).e^{bx}.b + e^{bx}.\sin(ax+b)a}{\cos^2(ax+b)}$$

$$= \frac{e^{bx}(b\cos(ax+b) + a\sin(ax+b)}{\cos^2(ax+b)}$$

Question-73

Differentiate the following function with respect to x.

$\log\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Solution:

$y = \log\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Put $u = \frac{\pi}{4}+\frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$y = \log\sec u$

$y = \log v$

Put $v = \sec u$

$\frac{dv}{du} = \sec u\tan u$

$\frac{dy}{dv} = \frac{1}{v}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{v} \cdot \sec\left(\frac{\pi}{4} + \frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{\sec\left(\frac{\pi}{4} + \frac{x}{2}\right)} \sec\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2} \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Question-74

Differentiate the following function with respect to x.
log sin(e^x + 4x + 5)

Solution:

y = log sin(e^x + 4x + 5)

$$\frac{dy}{dx} = \frac{1}{\sin(e^x + 4x + 5)} \cos(e^x + 4x + 5)(e^x + 4)$$

$$= (e^x + 4) \frac{\cos(e^x + 4x + 5)}{\sin(e^x + 4x + 5)}$$

$$= (e^x + 4)\cot(e^x + 4x + 5)$$

Question-75

Differentiate the following function with respect to x.
$\sin(x^{3/2})$

Solution:
$y = \sin(x^{3/2})$

Put $u = x^{3/2}$

$y = \sin u$

$\frac{du}{dx} = \frac{3}{2} x^{1/2}$

$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u. \frac{3}{2} x^{1/2} = \cos x^{3/2} \left(\frac{3}{2} x^{1/2}\right)$

Question-76

Differentiate the following function with respect to x.
$\cos(\sqrt{x})$

Solution:
$y = \cos u$

Put $u = \sqrt{x}$

$\frac{dy}{du} = -\sin u$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\sin u \cdot \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

Question-77

Differentiate the following function with respect to x.
$e^{\sin(\log x)}$

Solution:
$y = e^{\sin(\log x)}$

Put u = logx

$\frac{du}{dx} = \frac{1}{x}$

$y = e^{\sin u}$

Put v = sin u

$\frac{dv}{du} = \cos u = \cos(\log x)$

Put $y = e^v$

$\frac{dy}{dv} = e^v$
Hence $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx} = e^v \cdot \cos(\log x) \cdot \frac{1}{x} = e^{\sin(\log x)} \cos^{(\log s)} \cdot \frac{1}{x}$