9. Direct Proportion and Inverse Proportion

- Two quantities, x and y, are said to be in **direct proportion**, if they increase (or decrease) together in such a manner that the ratio of their corresponding values remains constant. That is, $\frac{x}{y} = k$ where k is a positive number. ,

  For example, price of wheat per kg and the weight of wheat that can be brought are in direct proportion as more the weight of wheat, more will be the cost.

- If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ is a case of direct proportion.
- Two variables x and y will be in direct proportion if $\frac{x}{y} = k$ or $x = ky$, where the constant k is known as constant of proportionality of the direct proportion. Thus, to check whether the variables x and y are in direct proportion, we need to find the ratio $\frac{x}{y}$ for their corresponding values. If this ratio remains constant, then the variables are in direct proportion, otherwise they are not.

- Two quantities, x and y, are said to be in **inverse proportion**, if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, $xy = k$, where k is a positive number.
- Two variables x and y will be in inverse proportion if $xy = k$, where the constant k is known as constant of proportionality of the inverse proportion. Thus, to check whether the two variables x and y of a given situation are in inverse proportion or not, we have to calculate the product of the value of variable x with its corresponding value of the variable y. If all these products are equal, then we can say that the variables x and y are in inverse proportion, otherwise not.

  For example, $x = 1$, $y = 20$ and $x = 5$, $y = 4$ are in inverse proportion.

  Here, $1 \times 20 = 20$

  $5 \times 4 = 20$

  It can be seen that $x \times y = 20$, which is constant for both observations.

  Therefore, x and y are in inverse proportion.

When two or more persons start a business enterprise jointly, they are called partners and the deal is known as partnership. The partners start a business by investing money. They share the profit earnedor the loss incurred in the business in proportion to the money invested by each partner.

Distribution of Profit:

i) When investments are for the same time:
The profit or loss is distributed among the partners in the ratio of their investments. Suppose A and B invest ₹x and ₹y respectively for one year in a business, then at the end of the year:
A’s share of profit : B's share of profit = $x : y$

ii) When investments are for different time periods:
In this case, equivalent capitals are calculated for a unit of time by taking the product of the capital and the number of units of time. Now, profit or loss is divided in the ratio of these equivalent capitals. Suppose A invests ₹x for p months and B invests ₹y for q months, then
A’s share of profit : B's share of profit = $xp : yq$

Let us consider an example to understand this concept.
Two persons A and B invested ₹12,000 and ₹15,000 respectively and started a business. We need to find the share of each, out of an annual profit of ₹1,800.
First we find the ratio of profit.
A’s share of profit : B's share of profit
= A’s investment : B's investment
= ₹12,000 : ₹15,000
= 4 : 5
Now, we can easily find the share of each A and B.
A's share of profit=₹1,800×44+5=₹800B's share of profit=₹1,800×54+5=₹1,000

Now, if A invested ₹12,000 for 10 months and B invested ₹15,000 for one year, then let us find the share of each, out of an annual profit of ₹1,800.
In that case, A and B share the profit in the ratio of their equivalent capitals.
A’s share of profit : B's share of profit
= A’s investment × time period : B's investment × time period
= 12000 × 10 : 15000 × 12
= 2 : 3
Now,
A's share of profit=₹1,800×22+3=₹720B's share of profit=₹1,800×32+3=₹1,080