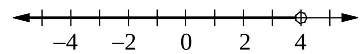
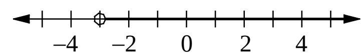
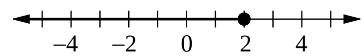
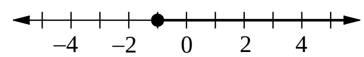
CHAPTER 4
Linear Inequalities and Graphs

4-1. Solving Inequalities

An inequality is a mathematical sentence that contains inequality symbols such as $>$, $<$, $\geq$, $\leq$. between numerical or variable expressions. Four types of simple inequalities and their graphs are shown below.

Verbal Expressions	**Inequality**	**Graph**
All real numbers **less than** 4.	$x < 4$	
All real numbers **greater than** -3.	$x > -3$	
All real numbers **less than or equal to** 2. All real numbers **at most** 2. All real numbers **no greater than** 2.	$x \leq 2$	
All real numbers **greater than or equal to** -1. All real numbers **at least** -1. All real numbers **no less than** -1.	$x \geq -1$	

Notice on the graphs that we use an open dot for $>$ or $<$ and a solid dot for $\geq$ or $\leq$.

Properties of Inequalities

For all real numbers a, b, and c, the following are true.

Transitive Property — If $a < b$ and $b < c$, then $a < c$.

Addition and Subtraction Properties — If $a < b$, then $a + b < a + c$ and $a - c < b - c$.

Multiplication and Division Properties — If $a < b$ and c is positive, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

If $a < b$ and c is negative, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Example 1 □ Write each statement as an inequality.

a. The number x is no greater than -2.

b. The amount of calories n meet or exceed 1,200.

Solution □ a. $x \leq -2$

b. $n \geq 1,200$

Example 2 □ Solve $4n - 9 \geq 12 - 3n$.

Solution □

$4n - 9 \geq 12 - 3n$	
$4n - 9 + 3n \geq 12 - 3n + 3n$	Add $3n$ to each side.
$7n - 9 \geq 12$	Simplify.
$7n - 9 + 9 \geq 12 + 9$	Add 9 to each side.
$7n \geq 21$	Simplify.
$\frac{7n}{7} \geq \frac{21}{7}$	Divide each side by 7.
$n \geq 3$	Simplify.

Exercises - Translating Words into Inequalities

1

If $-3+n \le 25$, which inequality represents the possible range of values for $4n-12$?

A) $4n-12 \le -100$

B) $4n-12 \le 100$

C) $4n-12 \ge -100$

D) $4n-12 \ge 100$

2

Which of the following numbers is NOT a solution to the inequality $\frac{1}{2}x-\frac{1}{3}>\frac{7}{9}+\frac{5}{2}x$?

A) $-\frac{7}{2}$

B) $-\frac{5}{2}$

C) $-\frac{3}{2}$

D) $-\frac{1}{2}$

3

If $-3a+7 \ge 5a-17$, what is the greatest possible value of $3a+7$?

A) 16

B) 14

C) 12

D) 10

4

Nine is not more than the sum of a number and 17.

Which of the following inequalities represents the statement above?

A) $9 \ge 17n$

B) $9 \ge n+17$

C) $9 \le 17n$

D) $9 \le n+17$

5

The product of 7 and number n is no less than 91.

Which of the following inequalities represents the statement above?

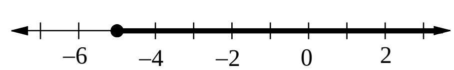
A) $7n \le 91$

B) $7n < 91$

C) $7n \ge 91$

D) $7n > 91$

6



Which of the following inequalities represents the graph above?

A) $n \le -5$

B) $n < -5$

C) $n \ge -5$

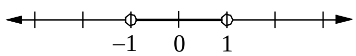
D) $n > -5$

4-2. Compound and Absolute Value Inequalities

Two or more inequalities that are connected by the words *and* or *or* are called a **compound inequality**. A compound inequality containing *and* is true only if both inequalities are true. Its graph is the **intersection** of the graphs of the two inequalities. A compound inequality containing *or* is true if one or more of the inequalities is true. Its graph is the **union** of the graphs of the two inequalities.

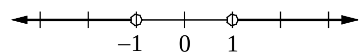
Since the absolute value of any number x is its distance from zero on a number line, $|x| < 1$ means that the distance from zero to x is less than 1 unit and $|x| > 1$ means that the distance from zero to x is greater than 1 unit. Therefore the inequality $|x| < 1$ is equivalent to $-1 < x < 1$ and $|x| > 1$ is equivalent to $x < -1$ or $x > 1$.

Graph of $|x| < 1$



The distance between x and 0 is less than1.

Graph of $|x| > 1$



The distance between x and 0 is greater than1.

To translate more general absolute value inequalities into compound inequalities, use the following properties.

1. The inequality $|ax + b| < c$ is equivalent to $-c < ax + b < c$, in which $0 \le c$.
2. The inequality $|ax + b| > c$ is equivalent to $ax + b < -c$ or $ax + b > c$, in which $0 \le c$.

In the statements above, $<$ could be replaced by $\le$ and $>$ could be replaced by $\ge$.

Example 1 □ Solve $7 - 2x > 15$ and $10 + 3x > -11$

Solution □

$7 - 2x > 15$	and	$10 + 3x > -11$
$7 - 2x - 7 > 15 - 7$	and	$10 + 3x - 10 > -11 - 10$
$-2x > 8$	and	$3x > -21$
$\frac{-2x}{-2} < \frac{8}{-2}$	and	$\frac{3x}{3} > \frac{-21}{3}$
$x < -4$	and	$x > -7$

Example 2 □ Solve each inequality.

a. $|2x - 3| \le 7$ b. $|n - 4| > 3$

Solution □ a. $|2x - 3| \le 7$

$-7 \le 2x - 3 \le 7$	$\lvert ax + b \rvert < c$ is equivalent to $-c < ax + b < c$.
$-7 + 3 \le 2x - 3 + 3 \le 7 + 3$	Add 3 to each expression.
$-4 \le 2x \le 10$	Simplify.
$\frac{-4}{2} \le \frac{2x}{2} \le \frac{10}{2}$	Divide each expression by 2.
$-2 \le x \le 5$	Simplify.

b. $|n - 4| > 3$

$n - 4 < -3$	or	$n - 4 > 3$	$\lvert ax + b \rvert > c$ is equivalent to $ax + b < -c$ or $ax + b > c$.
$n - 4 + 4 < -3 + 4$	or	$n - 4 > 3 + 4$	Add 4 to each side.
$n < 1$	or	$n > 7$	Simplify.

Exercises - Compound and Absolute Value Inequalities

1

Which of the following numbers is NOT a solution to the inequality $3-n<-2$ or $2n+3\le -1$?

A) -6

B) -2

C) 2

D) 6

2

Which of the following numbers is a solution to the inequality $5w+7>2$ and $6w-15\le 3(-1+w)$?

A) -1

B) 2

C) 5

D) 8

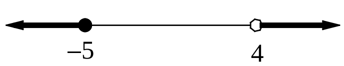
3

Which of the following is the graph of

$-x\le 5$ and $7-\frac{1}{2}x>x+1$?

A) 

B) 

C) 

D) 

4

If $-2<n<-1$, what is the value of $7+\frac{1}{2}n$ rounded to the nearest whole number?

5

Which of the following numbers is NOT a solution to the inequality $\left|\frac{1}{2}x-1\right|\le 1$?

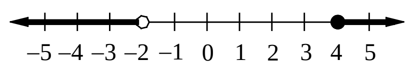
A) 0

B) 2

C) 4

D) 6

6



Which of the following is the compound inequality for the graph above?

A) $x<-2$ or $4\le x$

B) $x\le -2$ or $4<x$

C) $-2<x\le 4$

D) $-2\le x<4$

7

If $\frac{1}{4}x-1\le -x+5$, what is the greatest possible value of x?

8

If $\left|\frac{3}{4}n-2\right|<1$ and n is an integer, what is one possible value of n?

4-3. Graphing Inequalities in Two Variables

A **linear inequality** in x and y is an inequality that can be written in one of the following forms.

$ax+by<c$, $ax+by\le c$, $ax+by>c$, $ax+by\ge c$

An ordered pair (a,b) is a **solution** to a linear inequality in x and y if the inequality is true when a and b are substituted for x and y , respectively.

Sketching the Graph of a Linear Inequality

1. Sketch the graph of the corresponding linear equation. Use a dashed line for inequalities with $<$ or $>$ and a solid line for inequalities with $\le$ or $\ge$. This line divides the coordinate plane into two **half planes**.
2. Test a point in one of the half planes to find whether it is a solution of the inequality.
3. If the test point is a solution, shade its half plane. If not, shade the other half plane.

Example 1 □ Graph each inequality.

a. $2y+x\le 4$

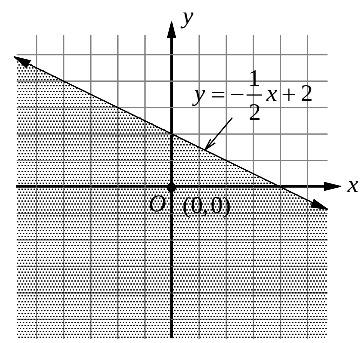
b. $3x-2y>2$

Solution □ a. The corresponding equation is $2y+x=4$.

Write the equation in slope-intercept form.

$y=-\frac{1}{2}x+2$ Slope-intercept form of the corresponding equation.

Graph the line that has a slope of $-\frac{1}{2}$ and a y- intercept of 2. The boundary should be drawn as a solid line.



Select a point in one of the half planes and test it. Let's use $(0,0)$.

$2y+x\le 4$	Original inequality
$2(0)+0\le 4$	$x=0$, $y=0$
$0\le 4$	True

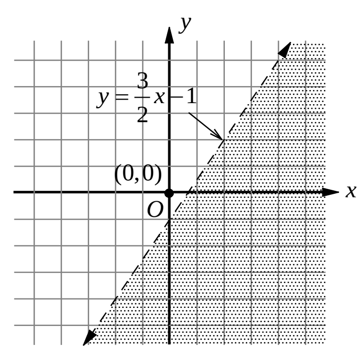
Since the statement is true, shade the half plane containing the origin.

b. The corresponding equation is $3x-2y=2$.

Write the equation in slope-intercept form.

$y=\frac{3}{2}x-1$ Slope-intercept form of the corresponding equation.

Graph the line that has a slope of $\frac{3}{2}$ and a y- intercept of -1 . The boundary should be drawn as a dashed line.



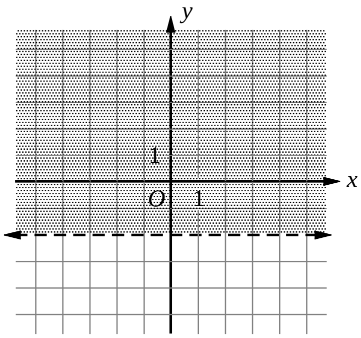
Select a point in one of the half planes and test it. Let's use $(0,0)$.

$3x-2y>2$	Original inequality
$3(0)-2(0)>2$	$x=0$, $y=0$
$0>2$	False

Since the statement is false, shade the other half plane not containing the origin.

Exercises - Graphing Inequalities in Two Variables

1



Which of the following inequalities represents the graph above?

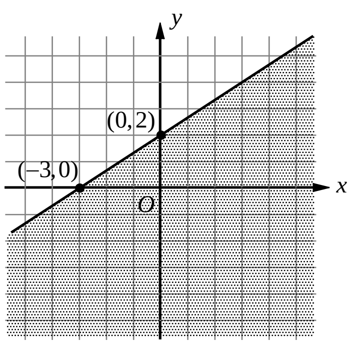
A) $x > -2$

B) $x < -2$

C) $y > -2$

D) $y < -2$

2



Which of the following inequalities represents the graph above?

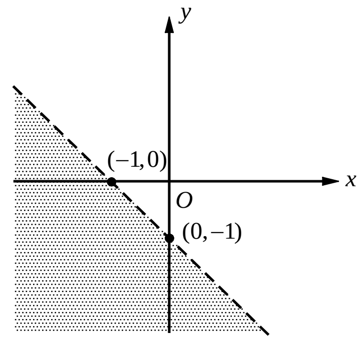
A) $2y - 3x \geq 6$

B) $2y - 3x \leq 6$

C) $3y - 2x \geq 6$

D) $3y - 2x \leq 6$

3



Which of the following inequalities represents the graph above?

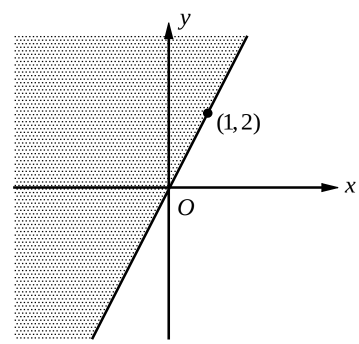
A) $x + y < -1$

B) $x + y > -1$

C) $x + y \leq -1$

D) $x + y \geq -1$

4



Which of the following inequalities represents the graph above?

A) $2x - y \geq 0$

B) $2x - y \leq 0$

C) $x - 2y \geq 0$

D) $x - 2y \leq 0$

4-4. Graphing Systems of Inequalities

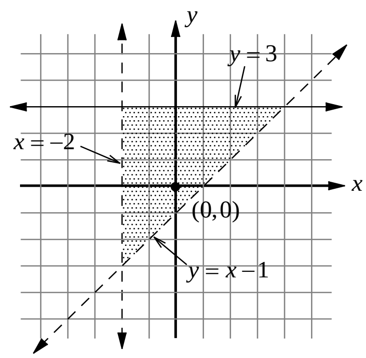
A **system of inequalities** is a set of two or more inequalities with the same variables. The ordered pairs that satisfy all inequalities is a solution to the system. The solution set is represented by the intersection, or overlap, of the graphs.

Example 1 □ Solve each system of inequalities by graphing.

a. $x > -2$
$y \le 3$
$y > x - 1$

b. $y \le 3x - 1$
$y > -2x$

Solution □ a. The corresponding equations are $x = -2$, $y = 3$, and $y = x - 1$.



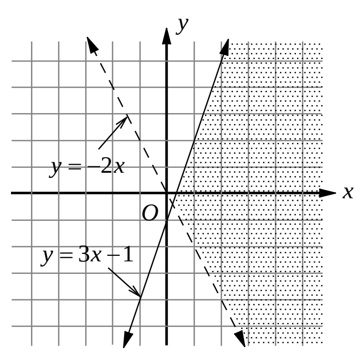
Use a dashed line for inequalities with $<$ or $>$ and a solid line for inequalities with $\le$ or $\ge$.

Select $(0, 0)$ as a test point for each inequality.

$0 > -2$	True
$0 \le 3$	True
$0 > 0 - 1$	True

The graph of the first inequality is the half plane right of the vertical line. The graph of the second inequality is the half plane on and below the horizontal line. The graph of the third inequality is the half plane above the line $y = x - 1$. The graph of the system is the shaded region shown above.

b. The corresponding equations are $y = 3x - 1$ and $y = -2x$.



Use a dashed line for inequalities with $<$ or $>$ and a solid line for inequalities with $\le$ or $\ge$.

Select $(0, 0)$ as a test point for the inequality $y \le 3x - 1$.

$0 \le 3(0) - 1$ False

The point $(0, 0)$ is not part of the solution. Shade the half plane on or below the line $y = 3x - 1$.

We cannot select $(0, 0)$ as a test point for the inequality $y > -2x$, because $(0, 0)$ is on the boundary line. Let's use $(0,1)$ as a test point.

$y > -2x$.

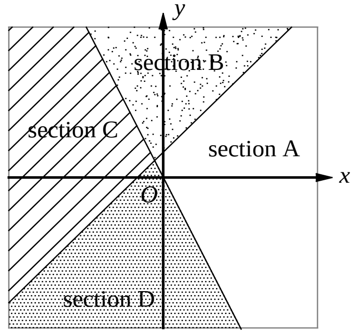
$1 > -2(0)$ True

The point $(0,1)$ is part of the solution. Shade the half plane above the line $y = -2x$. The graph of the system is the shaded region shown above.

Exercises - Graphing Systems of Inequalities

1

$$\begin{cases} y - x \geq 1 \\ y \leq -2x \end{cases}$$



A system of inequalities and a graph are shown above. Which section or sections of the graph could represent all of the solutions to the system?

A) Section A

B) Section B

C) Section C

D) Section D

2

Which of the following ordered pairs (x, y) is a solution to the system of inequalities $y > x - 4$ and $x + y < 5$?

A) $(4, -2)$

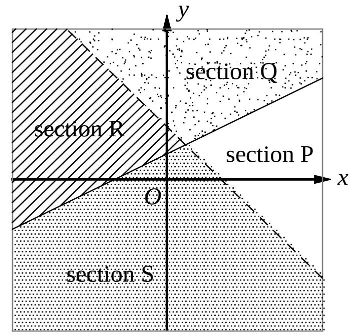
B) $(0, 2)$

C) $(5, 3)$

D) $(0, -5)$

3

$$\begin{cases} x - 2y \leq -2 \\ y < -x + 2 \end{cases}$$



A system of inequalities and a graph are shown above. Which section or sections of the graph could represent all of the solutions to the system?

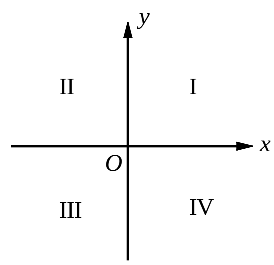
A) Section P

B) Section Q

C) Section R

D) Section S

4



If the system of inequalities $2 - y < 2x$ and $-x \leq 4 - y$ is graphed on the xy- plane above, which quadrant contains no solutions to the system?

A) Quadrant II

B) Quadrant III

C) Quadrant IV

D) There are solutions in all four quadrants.

Chapter 4 Practice Test

1

The sum of $120k$ and $215j$ does not exceed 2,500.

Which of the following inequalities represents the statement above?

A) $120k + 215j < 2,500$

B) $120k + 215j > 2,500$

C) $120k + 215j \le 2,500$

D) $120k + 215j \ge 2,500$

2

One half of a number decreased by 3 is at most -5.

Which of the following inequalities represents the statement above?

A) $\frac{1}{2}n - 3 \le -5$

B) $3 - \frac{1}{2}n \le -5$

C) $\frac{1}{2}n - 3 < -5$

D) $3 - \frac{1}{2}n < -5$

3

Which of the following numbers is NOT a solution to the inequality $\frac{3b+5}{-2} \ge b - 8$?

A) 0

B) 1

C) 2

D) 3

4

Which of the following inequalities is equivalent to $0.6(k-7) - 0.3k > 1.8 + 0.9k$?

A) $k < 10$

B) $k < -10$

C) $k > 10$

D) $k > -10$

5

$4m - 3 \le 2(m+1)$ or $7m + 23 < 15 + 9m$

Which of the following numbers is a solution to the compound inequality above?

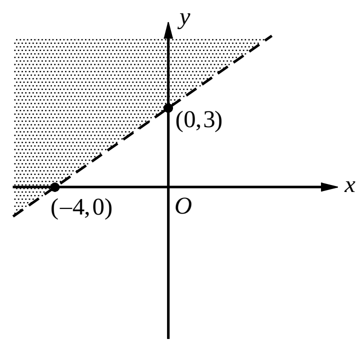
A) 2

B) 3

C) 4

D) 5

6



Which of the following inequalities represents the graph above?

A) $4y - 3x > 12$

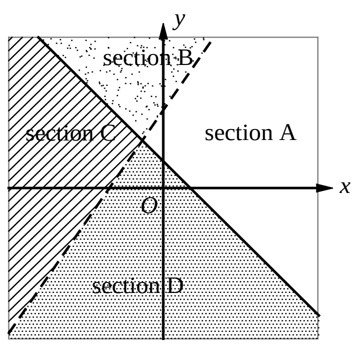
B) $4y - 3x < 12$

C) $3y - 4x > 12$

D) $3y - 4x < 12$

7

$$\begin{cases} 2y - 3x \le 6 \\ y > 1 - x \end{cases}$$



A system of inequalities and a graph are shown above. Which section or sections of the graph could represent all of the solutions to the system?

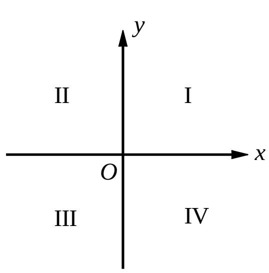
A) Section A

B) Section B

C) Section C

D) Section D

8



If the system of inequalities $3 \ge x$ and $-1 \le y$ is graphed in the xy- plane above, which quadrant contains no solutions to the system?

A) Quadrant II

B) Quadrant III

C) Quadrant IV

D) All four quadrants contain solutions.

9

$$\begin{cases} y < ax + 1 \\ y > bx - 1 \end{cases}$$

In the xy- plane, if $(1, 0)$ is a solution to the system of inequalities above, which of the following must be true?

I. $a > -1$

II. $a + b = 0$

III. $b < 1$

A) I only

B) I and II only

C) I and III only

D) I, II, and III

10

$$\begin{cases} y \ge 12x + 600 \\ y \ge -6x + 330 \end{cases}$$

In the xy- plane, if (x, y) lies in the solution set of the system of inequalities above, what is the minimum possible value of y ?

11

If $-6 \le 3 - 2x \le 9$, what is the greatest possible value of $x - 1$?

12

For what integer value of x is $4x - 2 > 17$ and $3x + 5 < 24$?

Answer Key

Section 4-1

1. B	2. D	3. A	4. D	5. C
6. C				

Section 4-2

1. B	2. B	3. D	4. 6	5. D
6. A	7. $\frac{24}{5}$	8. 2 or 3		

Section 4-3

1. C	2. D	3. A	4. B

Section 4-4

1. C	2. B	3. C	4. B

Chapter 4 Practice Test

1. C	2. A	3.D	4. B	5. A
6. A	7. A	8. D	9. C	10. 420
11. $\frac{7}{2}$ or 3.5	12. 5 or 6			

Answers and Explanations

Section 4-1

1. B

$-3+n \le 25$

$-3+n+3 \le 25+3$ — Add 3 to each side.

$n \le 28$ — Simplify.

$4n \le 4 \cdot 28$ — Multiply each side by 4.

$4n \le 112$ — Simplify.

$4n-12 \le 112-12$ — Subtract 12 from each side.

$4n-12 \le 100$ — Simplify.

2. D

$\frac{1}{2}x-\frac{1}{3} > \frac{7}{9}+\frac{5}{2}x$

$\frac{1}{2}x-\frac{1}{3}-\frac{5}{2}x > \frac{7}{9}+\frac{5}{2}x-\frac{5}{2}x$ — Subtract $\frac{5}{2}x$ from each side.

$-2x-\frac{1}{3} > \frac{7}{9}$ — Simplify.

$-2x-\frac{1}{3}+\frac{1}{3} > \frac{7}{9}+\frac{1}{3}$ — Add $\frac{1}{3}$ to each side.

$-2x > \frac{10}{9}$ — Simplify.

$-\frac{1}{2}(-2x) < -\frac{1}{2}(\frac{10}{9})$ — Multiply each side by $-\frac{1}{2}$ and change $>$ to $<$.

$x < -\frac{5}{9}$ — Simplify.

Therefore, $-\frac{1}{2}$ is not a solution to the inequality.

3. A

$-3a+7 \ge 5a-17$

Add $-5a-7$ to each side of the inequality.

$-3a+7+(-5a-7) \ge 5a-17+(-5a-7)$

$-8a \ge -24$ — Simplify.

$(-8a)\div(-8) \le (-24)\div(-8)$ — Divide each side by -8 and change $\ge$ to $\le$.

$a \le 3$ — Simplify.

$3a \le 3(3)$ — Multiply each side by 3.

$3a \le 9$ — Simplify.

$3a+7 \le 9+7$ — Add 7 to each side.

$3a+7 \le 16$ — Simplify.

Therefore, the greatest possible value of $3a+7$ is 16.

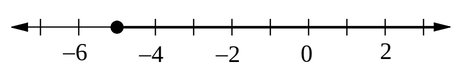
4. D

$$\underbrace{9}_{\text{nine}} \quad \underbrace{\le}_{\text{is not more than}} \quad \underbrace{n+17}_{\text{the sum of a number and 17}}$$

5. C

$$\underbrace{7n}_{\text{the product of 7 and a number } n} \quad \underbrace{\ge}_{\text{is no less than}} \quad \underbrace{91}_{91}$$

6. C



The solution set is $n \ge -5$.

Section 4-2

1. B

$3-n < -2$	or	$2n+3 \le -1$
$3-n-3 < -2-3$	or	$2n+3-3 \le -1-3$
$-n < -5$	or	$2n \le -4$
$n > 5$	or	$n \le -2$

$-6 \le -2$, $-2 \le -2$, and $6 > 5$ are true. 2 is not a solution to the given compound inequality.

2. B

$5w+7>2$	and	$6w-15\le 3(-1+w)$
$5w+7-7>2-7$	and	$6w-15\le -3+3w$
$5w>-5$	and	$3w\le 12$
$w>-1$	and	$w\le 4$

Thus, 2 is a solution to the inequality.

3. D

$-x\le 5$	First inequality
$(-1)(-x)\ge(-1)(5)$	Multiply each side by -1 and change $\ge$ to $\le$.
$x\ge -5$	First inequality simplified.
$7-\frac{1}{2}x>x+1$	Second inequality
$7-\frac{1}{2}x-7>x+1-7$	Subtract 7 from each side.
$-\frac{1}{2}x>x-6$	Simplify.
$-\frac{1}{2}x-x>x-6-x$	Subtract x from each side.
$-\frac{3}{2}x>-6$	Simplify.
$(-\frac{2}{3})(-\frac{3}{2}x)<(-\frac{2}{3})(-6)$	Multiply each side by $-\frac{2}{3}$ and change $>$ to $<$.
$x<4$	Simplify.

The inequality can be written as $-5\le x<4$, so answer choice D is correct.

4. 6

$-2<n<-1$	
$(\frac{1}{2})(-2)<(\frac{1}{2})n<(\frac{1}{2})(-1)$	Multiply each side by $\frac{1}{2}$.
$-1<\frac{1}{2}n<-\frac{1}{2}$	Simplify.
$7-1<7+\frac{1}{2}n<7-\frac{1}{2}$	Add 7 to each side.
$6<7+\frac{1}{2}n<6.5$	Simplify

Thus, $7+\frac{1}{2}n$ rounded to the nearest whole number is 6.

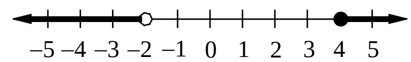
5. D

$\left|\frac{1}{2}x-1\right|\le 1$ is equivalent to $-1\le\frac{1}{2}x-1\le 1$.

$-1+1\le\frac{1}{2}x-1+1\le 1+1$	Add 1 to each side.
$0\le\frac{1}{2}x\le 2$	Simplify.
$2\cdot 0\le 2\cdot\frac{1}{2}x\le 2\cdot 2$	Multiply each side by 2.
$0\le x\le 4$	Simplify.

Thus, 6 is not a solution of the given inequality.

6. A



The compound inequality $x<-2$ or $4\le x$ represents the graph above.

7. $\frac{24}{5}$

$\frac{1}{4}x-1\le -x+5$

Add $x+1$ to each side of the inequality.

$\frac{1}{4}x-1+(x+1)\le -x+5+(x+1)$

$\frac{5}{4}x\le 6$	Simplify.
$\frac{4}{5}(\frac{5}{4}x)\le\frac{4}{5}(6)$	Multiply each side by $\frac{4}{5}$.
$x\le\frac{24}{5}$	Simplify.

The greatest possible value of x is $\frac{24}{5}$.

8. 2 or 3

$\left|\frac{3}{4}n-2\right|<1$ is equivalent to $-1<\frac{3}{4}n-2<1$.

$-1+2<\frac{3}{4}n-2+2<1+2$	Add 2 to each side.
$1<\frac{3}{4}n<3$	Simplify.
$\frac{4}{3}\cdot 1<\frac{4}{3}\cdot\frac{3}{4}n<\frac{4}{3}\cdot 3$	Multiply each side by $\frac{4}{3}$.
$\frac{4}{3}<n<4$	Simplify.

Since n is an integer, the possible values of n are 2 and 3.

Section 4-3

1. C

The equation of the boundary line is $y = -2$. Any point above that horizontal has a y- coordinate that satisfies $y > -2$. Since the boundary line is drawn as a dashed line, the inequality should not include an equal sign.

2. D

The slope-intercept form of the boundary line is $y = \frac{2}{3}x + 2$. The standard form of the line is $3y - 2x = 6$. Since the boundary line is drawn as a solid line, the inequality should include an equal sign. Select a point in the plane which is not on the boundary line and test the inequalities in the answer choices. Let's use $(0,0)$.

C) $3y - 2x \geq 6$

$3(0) - 2(0) \geq 6$	$x = 0,\ y = 0$
$0 \geq 6$	false

D) $3y - 2x \leq 6$

$3(0) - 2(0) \leq 6$	$x = 0,\ y = 0$
$0 \leq 6$	true

Since the half-plane containing the origin is shaded, the test point $(0,0)$ should give a true statement. Answer choice D is correct. Choices A and B are incorrect because the equations of the boundary lines are not correct.

3. A

The slope-intercept form of the boundary line is $y = -x - 1$. The standard form of the line is $x + y = -1$. Since the boundary line is drawn as a dashed line, the inequality should not include an equal sign. Select a point in the plane which is is not on the boundary line and test the inequalities in the answer choices. Let's use $(0,0)$.

A) $x + y < -1$

$0 + 0 < -1$	$x = 0,\ y = 0$
$0 < -1$	false

Since the half-plane containing the origin is not shaded, the test point $(0,0)$ should give a false statement. Answer choice A is correct. Choices C and D are incorrect because the inequalities include equal signs.

4. B

The slope-intercept form of the boundary line is $y = 2x$. The standard form of the line is $2x - y = 0$. Since the boundary line is drawn as a solid line, the inequality should include an equal sign. Select a point in the plane which is not on the boundary line and test the inequalities in the answer choices. We cannot use $(0,0)$ for this question because $(0,0)$ is on the boundary line. Let's use $(0,1)$.

A) $2x - y \geq 0$

$2(0) - (1) \geq 0$	$x = 0,\ y = 1$
$-1 \geq 0$	false

B) $2x - y \leq 0$

$2(0) - (1) \leq 0$	$x = 0,\ y = 1$
$-1 \leq 0$	true

Since the half-plane containing the $(0,1)$ is shaded, the test point $(0,1)$ should give a true statement. Answer choice B is correct. Choices C and D are incorrect because the equations of the boundary lines are not correct.

Section 4-4

1. C

$y - x \geq 1$
$y \leq -2x$

Select a point from each section, then test them on the inequalities. Let's use $(3,0)$, $(0,3)$, $(-3,0)$, and $(0,-3)$, from each section as test points.

$0 - 3 \geq 1$	$x = 3,\ y = 0$ is false.
$0 \leq -2(3)$	$x = 3,\ y = 0$ is false.
$3 - 0 \geq 1$	$x = 0,\ y = 3$ is true.
$3 \leq -2(0)$	$x = 0,\ y = 3$ is false.
$0 - (-3) \geq 1$	$x = -3,\ y = 0$ is true.
$0 \leq -2(-3)$	$x = -3,\ y = 0$ is true.

Since $x = -3$ and $y = 0$ are true for both inequalities, section C represents all of the solutions to the system.

2. B

$y > x - 4$
$x + y < 5$

Check each answer choice, to determine which ordered pair (x, y) is a solution to the system of inequalities.

A) $(4, -2)$

$-2 > 4 - 4$ $\quad$ $x = 4,\ y = -2$ is false.
$4 + (-2) < 5$ $\quad$ $x = 4,\ y = -2$ is true.

B) $(0, 2)$

$2 > 0 - 4$ $\quad$ $x = 0,\ y = 2$ is true.
$0 + 2 < 5$ $\quad$ $x = 0,\ y = 2$ is true.

$(0, 2)$ is a solution to the system of inequalities because the ordered pair gives a true statement for both pairs of inequalities.

3. C

$x - 2y \le -2$
$y < -x + 2$

Select a point from each section, then test them on the inequalities. Let's use $(3, 0)$, $(0, 3)$, $(-3, 0)$, and $(0, -3)$, from each section as test points.

$3 - 2(0) \le -2$ $\quad$ $x = 3,\ y = 0$ is false.

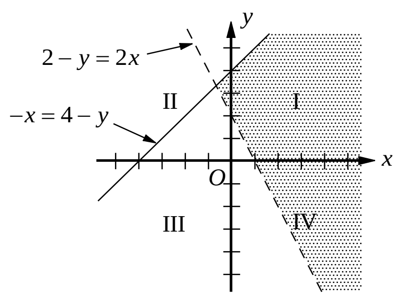
If the first statement is false, we don't need to check the second statement because the ordered pair must give a true statement for both pairs of the inequalities.

$0 - 2(3) \le -2$ $\quad$ $x = 0,\ y = 3$ is true.
$3 < -(0) + 2$ $\quad$ $x = 0,\ y = 3$ is false.

$-3 - 2(0) \le -2$ $\quad$ $x = -3,\ y = 0$ is true.
$0 < -(-3) + 2$ $\quad$ $x = -3,\ y = 0$ is true.

Since $x = -3$ and $y = 0$ are true for both inequalities, section R represents all of the solutions to the system.

4. B



To determine which quadrant does not contain any solution to the system of inequalities, graph the inequalities. It is easier to use x-intercept and y-intercept to graph the boundary line. Graph the inequality $2 - y < 2x$ by drawing a dashed line through the x-intercept $(1, 0)$ and y-intercept $(0, 2)$. Graph the inequality $-x \ge 4 - y$ by drawing a solid line through the x-intercept $(-4, 0)$ and y-intercept $(0, 4)$. The solution to the system of inequalities is the shaded region as shown in the graph above. It can be seen that the solutions only include points in quadrants I, II, and IV and do not include any points in quadrant III.

Chapter 4 Practice Test

1. C

$$\underbrace{120k + 215j}_{\text{the sum of } 120k \text{ and } 215j} \quad \underbrace{\le}_{\text{does not exceed}} \quad \underbrace{2{,}500}_{2{,}500}$$

2. A

$$\underbrace{\frac{1}{2}n}_{\text{one half of a number}} \quad \underbrace{-3}_{\text{decreased by 3}} \quad \underbrace{\le}_{\text{is at most}} \quad \underbrace{-5}_{-5}$$

3. D

$\dfrac{3b+5}{-2} \ge b - 8$

$-2\left(\dfrac{3b+5}{-2}\right) \le -2(b-8)$ $\quad$ Multiply each side by -2 and change $\ge$ to $\le$.

$3b + 5 \le -2b + 16$ $\quad$ Simplify.

$3b + 5 + 2b \le -2b + 16 + 2b$ $\quad$ Add $2b$ to each side.

$5b + 5 \le 16$ $\quad$ Simplify.

$5b + 5 - 5 \le 16 - 5$ $\quad$ Subtract 5.

$5b \le 11$ $\quad$ Simplify.

$\dfrac{5b}{5} \le \dfrac{11}{5}$ $\quad$ Divide each side by 5.

$b \le \dfrac{11}{5}$ $\quad$ Simplify.

So, 3 is not a solution to the inequality.

4. B

$0.6(k - 7) - 0.3k > 1.8 + 0.9k$
$\Rightarrow\ 0.6k - 4.2 - 0.3k > 1.8 + 0.9k$
$\Rightarrow\ 0.3k - 4.2 > 1.8 + 0.9k$

$\Rightarrow \ 0.3k - 4.2 - 0.9k > 1.8 + 0.9k - 0.9k$

$\Rightarrow \ -0.6k - 4.2 > 1.8 \ \Rightarrow \ -0.6k > 6$

$\Rightarrow \ \dfrac{-0.6k}{-0.6} < \dfrac{6}{-0.6} \ \Rightarrow \ k < -10$

5. A

$4m - 3 \le 2(m+1)$	or	$7m + 25 < 15 + 9m$
$4m - 3 \le 2m + 2$	or	$-2m + 25 < 15$
$2m \le 5$	or	$-2m < -10$
$m \le \dfrac{5}{2}$	or	$m > 5$

Thus, among the answer choices, 2 is the only solution to the compound inequality.

6. A

Slope m of the boundary line is

$m = \dfrac{3-0}{0-(-4)} = \dfrac{3}{4}$. The y- intercept is 3. So, the slope-intercept form of the line is $y = \dfrac{3}{4}x + 3$.

The standard form of the line is $4y - 3x = 12$.

Select a point in the shaded region and test each inequality.

Let's use $(0,4)$, as a test point.

A) $4y - 3x > 12$

$4(4) - 3(0) > 12$	$x = 0, \ y = 4$
$16 > 12$	true

Since the half-plane containing $(0,4)$ is shaded, the test point $(0,4)$ should give a true statement. Answer choice A is correct.

Choices C and D are incorrect because the equations of the boundary lines are not correct.

7. A

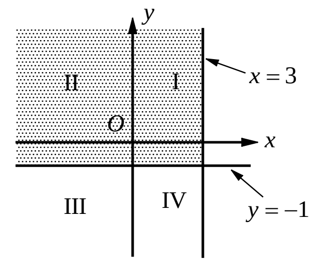
Let's check $(3,0)$, which is in section A.

$2(0) - 3(3) \le 6$	$x = 3, \ y = 0$ is true.
$0 > 1 - 3$	$x = 3, \ y = 0$ is true.

Since $x = 3$ and $y = 0$ are true for both inequalities, section A represents all of the solutions to the system.

8. D

To determine which quadrant does not contain any solution to the system of inequalities, graph the inequalities.



The solution to the system of inequalities is the shaded region shown in the graph above. Its solutions include points in all four quadrants. D is correct answer.

9. C

$y < ax + 1$ and $y > bx - 1$

Since $(1,0)$ is a solution to the system of inequalities, substitute $x = 1$ and $y = 0$ in the given inequalities.

$0 < a(1) + 1$	and	$0 > b(1) - 1$	$x = 1, \ y = 0$
$-1 < a$	and	$1 > b$	Simplify.

Statements I and III are true. But we do not know the exact value of a or b, so statement II is not true.

10. 420

$y \ge 12x + 600$	First inequality
$y \ge -6x + 330$	Second inequality

Multiply each side of the second inequality by 2 and then add it to the first inequality.

$2y \ge -12x + 660$	2nd inequality multiplied by2.
$+ \ y \ge 12x + 600$	First inequality
$3y \ge 1260$	Sum of two inequalities
$\dfrac{3y}{3} \ge \dfrac{1260}{3}$	Divide each side by 3.
$y \ge 420$	Simplify.

Therefore, the minimum possible value of y is 420.

11. $\dfrac{7}{2}$ or 3.5

$-6 \le 3 - 2x \le 9$	
$-6 - 3 \le 3 - 2x - 3 \le 9 - 3$	Subtract 3 from each side.
$-9 \le -2x \le 6$	Simplify.
$\dfrac{-9}{-2} \ge \dfrac{-2x}{-2} \ge \dfrac{6}{-2}$	Divide each side by -2 and change $\le$ to $\ge$.
$\dfrac{9}{2} \ge x \ge -3$	Simplify.

$\frac{9}{2} - 1 \geq x - 1 \geq -3 - 1$ Subtract 1 from each side.

$\frac{7}{2} \geq x - 1 \geq -4$ Simplify.

The greatest possible value of $x - 1$ is $\frac{7}{2}$.

12. 5 or 6

$4x - 2 > 17$ and $3x + 5 < 24$

$4x > 19$ and $3x < 19$

$x > \frac{19}{4}$ and $x < \frac{19}{3}$

Since x is between $\frac{19}{4}(= 4.75)$ and $\frac{19}{3}(\approx 6.33)$, the integer value of x is 5 or 6.