23. Trigonometric Ratios and Trigonometric Identities

Let us Work Out 23.1

1. Question

I have drawn a right angled triangle ABC whose hypotenuse AB = 10 cm., base BC = 8 cm. and perpendicular AC = 6 cm. Let us determine the values of sine and tangent ∠ABC.

Answer

Given, a right angled triangle with AB = 10cm(hypothesis), BC = 8cm (base) and AC = 6cm (perpendicular)

Need to find out the value of sinθ and tanθ

⇒ now, we know that $\sin\theta = \frac{\text{perpendicular}}{\text{hypothesis}}$

And $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

⇒ $\sin\theta = \frac{AC}{AB}$

$= \frac{6}{10}$

= 0.6 cm

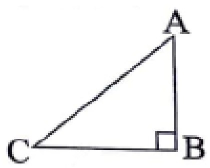
⇒ $\tan\theta = \frac{AC}{BC}$

$= \frac{6}{8}$

= 0.75cm

2. Question

Soma has drawn a right angled triangle ABC whose ABC = 90, AB = 24 cm. and BC = 7 cm. By calculating, let us write the value of sinA, cosA, tanA and cosec A.



Answer

Given, AB = 24, BC = 7

Need to find the values of sinA, cosA, tanA and cosecA

⇒ [from the figure, AC is hypothesis, AB is base and CB is perpendicular]

⇒ $AC^2 = AB^2 + BC^2$

⇒ $AC^2 = (24)^2 + (7)^2$

= 576 + 49

= 625

⇒ $AC^2 = 625$

⇒ AC = 25

⇒ $\sin A = \frac{\text{perpendicular}}{\text{hypothesis}}$

$= \frac{CB}{AC}$

$= \frac{7}{25}$

= 0.28

⇒ sinA = 0.28

⇒ $\cos A = \frac{\text{base}}{\text{hypothesis}}$

$= \frac{AB}{AC}$

$= \frac{24}{25}$

= 0.96

⇒ cosA = 0.96

⇒ $\tan A = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{CB}{AB}$

$= \frac{7}{24}$

= 0.29

⇒ tanA = 0.29

⇒ $\operatorname{cosec} A = \frac{\text{hypothesis}}{\text{perpendicular}}$

$= \frac{AC}{CB}$

$= \frac{25}{7}$

= 3.57

⇒ cosecA = 3.57

3. Question

If in a right angled triangle ABC, ∠C=90°, DC = 21 unit and AB = 29 units, then let us find the values of sinA, cosA, sinB and cosB.

Answer

Given, BC = 21 and AB = 29

Need to find the values of sinA, cosA,sinB,cosB

⇒ $\sin\theta = \frac{\text{perpendicular}}{\text{hypothesis}}$ and $\cos\theta = \frac{\text{base}}{\text{hypothesis}}$

⇒ $AB^2 = AC^2 + BC^2$

⇒ $(29)^2 = AC^2 + (21)^2$

⇒ $841 = AC^2 + 441$

⇒ $400 = AC^2$

⇒ AC = 20

⇒ $\sin A = \frac{BC}{AB}$

$= \frac{21}{29} = 0.72$

⇒ $\cos A = \frac{AC}{AB}$

$= \frac{20}{29} = 0.68$

⇒ $\sin B = \frac{AC}{AB}$

$= \frac{20}{29} = 0.68$

⇒ $\cos B = \frac{BC}{AB}$

$= \frac{21}{29} = 0.72$

4. Question

If $\cos\theta = \frac{7}{25}$, cos then let us determine the values of all trigonometric ratio of the angle θ.

Answer

Given, $\cos\theta = \frac{7}{25}$

Need to find the trigonometric ratios

$\Rightarrow \cos\theta = \frac{\text{base}}{\text{hypothesis}}$

[we know that hypothesis^2 = perpindicular^2 + base^2]

$\Rightarrow (25)^2 = P^2 + (7)^2$

$\Rightarrow 625 = P^2 + 49$

$\Rightarrow 576 = P^2$

$\Rightarrow P = 24$

$\Rightarrow \sin\theta = \frac{\text{perpendicular}}{\text{hypothesis}}$

$= \frac{24}{25} = 0.96$

$\Rightarrow \cos\theta = \frac{\text{base}}{\text{hypothesis}}$

$= \frac{7}{25} = 0.28$

$\Rightarrow \tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{24}{7} = 3.42$

$\Rightarrow \cot\theta = \frac{\text{base}}{\text{perpendicular}}$

$= \frac{7}{24} = 0.291$

$\Rightarrow \text{cosec}\theta = \frac{\text{hypothesis}}{\text{perpendicular}}$

$= \frac{25}{24} = 1.04$

$\Rightarrow \sec\theta = \frac{\text{hypothesis}}{\text{base}}$

$= \frac{25}{7} = 3.57$

5. Question

If cot θ = 2, then let us determine the values of tan θ and sec θ and show that $1+\tan^2\theta = \sec^2\theta$.

Answer

Given, cotθ = 2

Need to show $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \cot\theta = \frac{\text{base}}{\text{perpendicular}} = 2$

$\Rightarrow \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{1}{2}$

$\Rightarrow \text{hypothesis}^2 = \text{perpindicular}^2 + \text{base}^2$

$= 1 + 2^2$

$= 5$

$\Rightarrow \text{hypothesis} = \sqrt{5}$

$\Rightarrow \sec\theta = \frac{\text{hypothesis}}{\text{base}} = \frac{\sqrt{5}}{2}$

$\Rightarrow 1 + \tan^2\theta = 1 + \left(\frac{1}{2}\right)^2$

$= 1 + \frac{1}{4}$

$= \frac{5}{4}$

$\Rightarrow \sec^2\theta = \left(\frac{\sqrt{5}}{2}\right)^2$

$= \frac{5}{4}$

∴ LHS = RHS

6. Question

If cos θ = 0.6, then let us show that, (5 sin θ – 3 tan θ) = 0.

Answer

Given, cosθ = 0.6

$\Rightarrow \cos\theta = \frac{\text{base}}{\text{hypothesis}}$

$= 0.6 = \frac{6}{10} = \frac{3}{5}$

[we know that hypothesis^2 = perpindicular^2 + base^2]

$\Rightarrow (5)^2 = P^2 + 3^2$

$\Rightarrow 25 = P^2 + 9$

$\Rightarrow P = 4$

$\Rightarrow \sin\theta = \frac{\text{perpendicular}}{\text{hypothesis}}$

$= \frac{4}{5}$

$\Rightarrow \tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{4}{3}$

$\Rightarrow 5\sin\theta - 3\tan\theta$

$= 5\left(\frac{4}{5}\right) - 3\left(\frac{4}{3}\right)$

$= 4-4$

$= 0$

Hence, proved

7. Question

If $\cot A = \frac{4}{7.5}$, cot then let us determine the values of cos A and cosec A and show that $1 + \cot^2 A = \operatorname{cosec}^2 A$.

Answer

Given, $\cot A = \frac{4}{7.5}$

$\Rightarrow \cot A = \frac{\text{base}}{\text{perpendicular}}$

$= \frac{4}{7.5}$

[we know that hypothesis^2 = perpindicular^2 + base^2]

$\Rightarrow h^2 = 7.5^2 + 4^2$

$= 72.25$

$\Rightarrow h = 8.5$

$\Rightarrow \cos A = \frac{\text{base}}{\text{hypothesis}}$

$= \frac{4}{8.5}$

$\Rightarrow \operatorname{cosec} A = \frac{\text{hypothesis}}{\text{perpendicular}}$

$= \frac{8.5}{7.5}$

$\Rightarrow 1 + \cot^2 A = 1 + \left(\frac{4}{7.5}\right)^2$

$= 1 + \frac{16}{56.25}$

$= \frac{72.25}{56.25}$

$= 1.28$

$\Rightarrow \operatorname{cosec}^2 A = \left(\frac{8.5}{7.5}\right)^2$

$= \frac{72.25}{56.25}$

$= 1.28$

$\therefore 1 + \cot^2 A = \operatorname{cosec}^2 A$

8. Question

If $\sin C = \frac{2}{3}$, then let us write by calculating, the value of cos C × cosec C.

Answer

Given, $\sin C = \frac{2}{3}$

$\Rightarrow \sin C = \frac{\text{perpendicular}}{\text{hypothesis}}$

$= \frac{2}{3}$

[we know that hypothesis^2 = perpindicular^2 + base^2]

$\Rightarrow 3^2 = 2^2 + b^2$

$\Rightarrow 9 = 4 + b^2$

$\Rightarrow b = \sqrt{5}$

$\Rightarrow \cos C = \frac{\text{base}}{\text{hypothesis}}$

$= \frac{\sqrt{5}}{3}$

$\Rightarrow \text{cosecC} = \frac{\text{hypothesis}}{\text{perpendicular}}$

$= \frac{3}{2}$

$\Rightarrow \text{cosC} \times \text{cosecC} = \frac{\sqrt{5}}{3} \times \frac{3}{2}$

$= \frac{\sqrt{5}}{2}$

9. Question

Let us write with reason whether the following statements are true or false :

i. The value of tan A is always greater than 1.

ii. The value of cot A is always less than 1.

iii. For an angle θ, it may be possible that $\sin\theta = \frac{4}{3}$.

iv. For an angle α, it may be possible that, $\sec\alpha = \frac{12}{5}$.

v. For an angle β (Beta) it may be possible that, $\operatorname{cosec}\beta = \frac{5}{13}$.

vi. For an angle θ, it may be possible that, $\cos\theta = \frac{3}{5}$.

Answer

(i) TRUE

Let us consider ABC as a right angled triangle where ∠B = 90°

$\Rightarrow \text{tanA} = \frac{\text{opposite side}}{\text{adjacent side}}$

$= \frac{BC}{AB}$

Let us assume that BC is greater than AB

Ex: BC = 6 and AB = 2

$\Rightarrow \text{tanA} = \frac{6}{2}$

= 3

∴ $\tan A > 1$

(ii) FALSE

Let us consider ABC as a right angled triangle where $\angle B = 90°$

$\Rightarrow \cot A = \frac{\text{adjaent side}}{\text{opposite side}}$

$= \frac{AB}{BC}$

Let us assume that AB is greater than BC

Ex: BC = 2 and AB = 8

$\Rightarrow \cot A = \frac{8}{2}$

= 4

∴ $\cot A > 1$

(iii) FALSE

Given, $\sin\theta = \frac{4}{3}$

$\Rightarrow \sin\theta = 1.33$

⇒ From the trigonometric ratios we know that $\sin\theta$ value must lie between 0° to 90°

And sin90 = 1

(iv) FALSE

Given, $\sec\alpha = \frac{12}{5}$

$\Rightarrow \sec\alpha = 2.4$

⇒ 2.4 does not lie between 0 and 2

(v) TRUE

Given, $\text{cose}\beta = \frac{5}{13}$

$\Rightarrow \operatorname{cosec}\beta = 0.38$

⇒ The value of cosec lie between 2 and 1

(vi) TRUE

Given, $\cos\theta = \frac{3}{5}$

$\Rightarrow \cos\theta = 0.6$

⇒ The value of cos lie between 0 and 1, i.e 0° to 90°

Let us Work Out 23.2

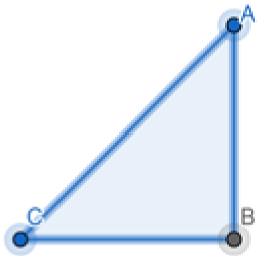
1. Question

In the window of our house, there is a ladder at an angle of 60^{0} with the ground. If the ladder is $2\sqrt{3}$m. long, then let us write by calculating with figure, the height of our window above the ground.

Answer

Given, length of ladder is $2\sqrt{3}$ m

Angle is 60°



⇒ ∠C = 60° and AC = $2\sqrt{3}$

Need to find out AB

⇒ we know, $\sin 60 = \frac{\sqrt{3}}{2}$

$$\Rightarrow \sin\theta = \frac{\text{perpendicular}}{\text{hypothesis}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{p}{2\sqrt{3}}$$

$$\Rightarrow P = 2\sqrt{3} \times \frac{\sqrt{3}}{2}$$

= 3

∴ the height of the window = 3m

2. Question

ABC is a right angled triangle with its ∠B is 1 right angle. If AB = $8\sqrt{3}$ cm. andBC = 8 cm., then let us write by calculating, the values of ∠ACB and ∠BAC.

Answer

Given, ∠ B = 90°

Need to find ∠ A and ∠ C

⇒ from the figure ∠ ACB, $\tan\theta = \frac{8\sqrt{3}}{8}$

$\Rightarrow \tan\theta = \sqrt{3}$

3. Question

In a right angled triangle ABC, ∠B=90°, ∠A=30° and AC = 20 cm. Let us determine the lengths of two sides BC and AB.

Answer

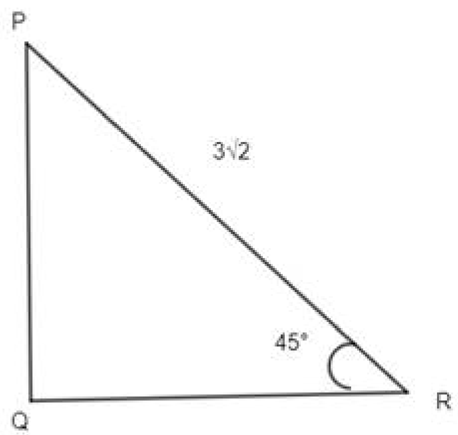
Given, ∠ B = 90° and ∠ A = 30°

AC = 20

4. Question

In a right angled triangle PQR, ∠Q=90°, ∠R=45°; if PR = 3√2, then let us find out the lengths of two sides PQ and QR.

Answer



To Find: PQ and QR

Given: PR = 3√2 and ∠Q=90°, ∠R=45°

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin 45° = \frac{PQ}{3\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{PQ}{3\sqrt{2}}$$

PQ = 3

Now, we also know that

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos 45° = \frac{QR}{3\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{QR}{3\sqrt{2}}$$

QR = 3

Hence PQ = 3 units and QR = 3 units

5 A. Question

Let us determine the values of :

$\sin^2 45° - \text{cosec}^2 60° + \sec^2 30°$

Answer

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \text{cosec} 60 = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec 30 = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin^2 45 - \text{cosec}^2 60 + \sec^2 30$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{1}{2} - \frac{4}{3} + \frac{4}{3}$

$= \frac{1}{2}$

5 B. Question

Let us determine the values of :

$\sec^2 45° - \cot^2 45° + \sin^2 30° - \sin^2 60°$

Answer

$\Rightarrow \sec 45 = \sqrt{2}$

$\Rightarrow \cot 45 = 1$

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \sin 60 = \frac{\sqrt{3}}{2}$

$\Rightarrow \sec^2 45 - \cot^2 45 - \sin^2 30 - \sin^2 60$

$= \sqrt{2}^2 - 1^2 - \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 - 1 - \frac{1}{4} - \frac{3}{4}$

$= \frac{4-1-3}{4}$

= 0

5 C. Question

Let us determine the values of :

$$3\tan^2 45^\circ - \sin^2 60^\circ - \frac{1}{3}\cot^2 30^\circ - \frac{1}{8}\sec^2 45^\circ$$

Answer

⇒ tan45 = 1

⇒ $\sin 60 = \frac{\sqrt{3}}{2}$

⇒ $\cot 30 = \sqrt{3}$

⇒ $\sec 45 = \sqrt{2}$

⇒ $3\tan^2 45 - \sin^2 60 - \frac{1}{3}\cot^2 30 - \frac{1}{8}\sec^2 45$

$= 3(1) - \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{3}(\sqrt{3})^2 - \frac{1}{8}(\sqrt{2})^2$

$= 3 - \frac{3}{4} - 1 - \frac{2}{8}$

$= 2 - \frac{3}{4} - \frac{1}{4}$

= 1

5 D. Question

Let us determine the values of :

$$\frac{4}{3}\cot^2 30^\circ + 3\sin^2 60^\circ - 2\operatorname{cosec}^2 60^\circ - \frac{3}{4}\tan^2 30^\circ$$

Answer

⇒ $\sin 60 = \frac{\sqrt{3}}{2}$

⇒ $\cot 30 = \sqrt{3}$

$\Rightarrow \text{cosec}60 = \frac{2}{\sqrt{3}}$

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{4}{3}\cot^2 30 + 3\sin^2 60 - 2\,\text{cosec}^2 60 - \frac{3}{4}\tan^2 30$

$= \frac{4}{3}\left(\sqrt{3}\right)^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$

$= 4 + 3\left(\frac{3}{4}\right) - 2\left(\frac{4}{3}\right) - \frac{1}{4}$

$= \frac{48 + 27 - 32 - 3}{12}$

$= \frac{40}{12}$

5 E. Question

Let us determine the values of :

$$\frac{\frac{1}{3}\cos 30^\circ}{\frac{1}{2}\sin 45^\circ} + \frac{\tan 60^\circ}{\cos 30^\circ}$$

Answer

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan 60 = \sqrt{3}$

$\Rightarrow \frac{\frac{1}{3}\cos 30}{\frac{1}{2}\sin 45} + \frac{\tan 60}{\cos 30}$

$= \frac{\frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)} + \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$= \left(\frac{\sqrt{3}}{6} \times 2\sqrt{2}\right) + 2$

$= \frac{\sqrt{6}}{3} + 2$

5 F. Question

Let us determine the values of :

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\ \sin 30^\circ$$

Answer

$\Rightarrow \cot 30 = \sqrt{3}$

$\Rightarrow \cos 60 = \frac{1}{2}$

$\Rightarrow \sec 45 = \sqrt{2}$

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \cot^2 30 - 2\cos^2 60 - \frac{3}{4}\sec^2 45 - 4\sin^2 30$

$= \sqrt{3}^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}\sqrt{2}^2 - 4\left(\frac{1}{2}\right)^2$

$= 3 - \frac{2}{4} - \frac{6}{4} - \frac{4}{4}$

$= \frac{3-2-6-4}{4}$

$= -\frac{9}{4}$

5 G. Question

Let us determine the values of :

$$\sec^2 60^\circ - \cot^2 30^\circ - \frac{2\tan 30^\circ \operatorname{cosec} 60^\circ}{1 + \tan^2 30^\circ}$$

Answer

$\Rightarrow \cot 30 = \sqrt{3}$

$\Rightarrow \operatorname{cosec} 60 = \frac{2}{\sqrt{3}}$

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \sec 60 = 2$

$\Rightarrow \frac{2\tan 30 \operatorname{cosec} 60}{1 + \tan^2 30}$

$= \frac{2 \times \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\frac{4}{3}}{1+\frac{1}{3}} = 1$

$\Rightarrow \sec^2 60 - \cot^2 30 - 1$

$= 2^2 - \sqrt{3}^2 - 1$

$= 0$

5 H. Question

Let us determine the values of :

$$\frac{\tan 60^\circ - \text{ta } 30^\circ}{1+\tan 60^\circ \tan 30^\circ} + \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

Answer

$\Rightarrow \cos 60 = \frac{1}{2}$

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan 60 = \sqrt{3}$

$\Rightarrow \sin 60 = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \frac{\tan 60 - \tan 30}{1 + \tan 60 \tan 30}$

$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$

$= \frac{\frac{2}{\sqrt{3}}}{2}$

$= \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\tan 60 - \tan 30}{1 + \tan 60 \tan 30} + \cos 60 \cos 30 + \sin 60 \sin 30$

$= \frac{1}{\sqrt{3}} + \frac{1}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\frac{1}{2}$

$= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

5 I. Question

Let us determine the values of :

$$\frac{1-\sin^2 30^\circ}{1+\sin^2 30^\circ} \times \frac{\cos^2 60^\circ + \cos^2 30^\circ}{\operatorname{cosec}^2 90^\circ - \cot^2 90^\circ} \div (\sin 60^\circ \tan 30^\circ)$$

Answer

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \cos 60 = \frac{1}{2}$

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow$ cosec90 = 1

$\Rightarrow$ cot90 = 0

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \sin 60 = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1-\sin^2 30}{1+\sin^2 30}$

$= \frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}$

$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}}$

$= \frac{\frac{3}{4}}{\frac{5}{4}}$

$= \frac{3}{5}$

6 A. Question

Let us show that,

$\sin^2 45° + \cos^2 45° = 1$

Answer

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin^2 45 + \cos^2 45$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{2}{2}$

= 1

6 B. Question

Let us show that,

$\cos 60° = \cos^2 30° – \sin^2 30°$

Answer

$\Rightarrow \cos 60 = \frac{1}{2}$

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \cos^2 30 - \sin^2 30$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$= \cos 60$

6 C. Question

Let us show that,

$$\frac{2\tan 30^{o}}{1-\tan^{2} 30^{o}} = \sqrt{3}$$

Answer

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{2\tan 30}{1-\tan^2 30}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \frac{3}{\sqrt{3}}$

6 D. Question

Let us show that,

$$\sqrt{\frac{1+\cos 30^\circ}{1-\cos 30^\circ}} = \sec 60^\circ + \tan 60^\circ$$

Answer

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow$ sec60 = 2

$\Rightarrow \tan 60 = \sqrt{3}$

$\Rightarrow \sqrt{\frac{1+\cos 30}{1-\cos 30}}$

$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}}$

$= \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$

6 E. Question

Let us show that,

$$\frac{2\tan^2 30^\circ}{1-\tan^2 30^\circ} + \sec^2 45^\circ - \cot^2 45^\circ = \sec 60^\circ$$

Answer

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow$ sec45 = $\sqrt{2}$

$\Rightarrow$ cot45 = 1

$\Rightarrow$ sec60 = 2

$\Rightarrow \frac{2\tan^2 30}{1-\tan^2 30}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{2}{\sqrt{3}-1}$

$\Rightarrow \frac{2\tan^2 30}{1-\tan^2 30} + \sec^2 45 - \cot^2 45$

$= \frac{2}{\sqrt{3}-1} + \sqrt{2}^2 - 1$

$= 1 + \frac{2}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

6 F. Question

Let us show that,

$$\tan^2\frac{\pi}{4}\sin\frac{\pi}{3}\tan\frac{\pi}{6}\tan^2\frac{\pi}{3} = 1\frac{1}{2}$$

Answer

$\Rightarrow \tan\frac{\pi}{4} = 1$

$\Rightarrow \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\frac{\pi}{3} = \sqrt{3}$

$\Rightarrow \tan^2\frac{\pi}{4}\sin\frac{\pi}{3}\tan\frac{\pi}{6}\tan^2\frac{\pi}{3}$

$= 1 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \times 3$

$= \frac{3}{2}$

6 G. Question

Let us show that,

$$\sin\frac{\pi}{3}\tan\frac{\pi}{6}+\sin\frac{\pi}{2}\cos\frac{\pi}{3}=2\sin^2\frac{\pi}{4}$$

Answer

$\Rightarrow \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\Rightarrow \sin\frac{\pi}{2} = 1$

$\Rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\frac{\pi}{3}\tan\frac{\pi}{6} + \sin\frac{\pi}{2}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}} + 1\times\frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2}$

= 1

$\Rightarrow 2\sin^2\frac{\pi}{4}$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2$

= 1

7 A. Question

If x sin 45° cos 45° tan 60°= $\tan^2$ 45° - cos 60°, then let us determine the value of x.

Answer

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \tan 60 = \sqrt{3}$

$\Rightarrow \cos 60 = \frac{1}{2}$

$\Rightarrow \tan 45 = 1$

$\Rightarrow x\sin 45 \cos 45 \tan 60 = \tan^2 45 - \cos 60$

$\Rightarrow x \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \sqrt{3} = 1 - \frac{1}{2}$

$\Rightarrow \frac{\sqrt{3}x}{2} = \frac{1}{2}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$

7 B. Question

If $x \sin 60^\circ \cos^2 30^\circ = \frac{\tan^2 45^\circ \sec 60^\circ}{\operatorname{cosec} 60^\circ}$, then let us determine the value of x.

Answer

$\Rightarrow \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 60 = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan 45 = 1$

$\Rightarrow \sec 60 = 2$

$\Rightarrow \operatorname{cosec} 60 = \frac{2}{\sqrt{3}}$

$\Rightarrow x \sin 60 \cos^2 30 = \frac{\tan^2 45 \sec 60}{\operatorname{cosec} 60}$

$\Rightarrow x \frac{\sqrt{3}}{2} \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{2}{\frac{2}{\sqrt{3}}}$

$\Rightarrow \frac{3x\sqrt{3}}{8} = \sqrt{3}$

$\Rightarrow x = \frac{8}{3}$

7 C. Question

If $x^2 = \sin^2 30° + 4 \cot^2 45° - \sec^2 60°$, then let us determine the value of x.

Answer

$\Rightarrow \sin 30 = \frac{1}{2}$

$\Rightarrow \cot 45 = 1$

$\Rightarrow \sec 60 = 2$

$\Rightarrow x^2 = \sin^2 30 + 4\cot^2 45 - \sec^2 60$

$\Rightarrow x^2 = \left(\frac{1}{2}\right)^2 + 4(1)^2 - (2)^2$

$\Rightarrow x^2 = \frac{1}{4} + 0$

$\Rightarrow x = \frac{1}{\sqrt{4}}$

$\Rightarrow x = \frac{1}{2}$

8. Question

If x tan 30° + y cot 60° = 0 and 2x – y tan 45° = 1, then let us write, by calculating the values of x and y.

Answer

⇒ tan45 = 1

$\Rightarrow \tan 30 = \frac{1}{\sqrt{3}}$

$\Rightarrow \cot 60 = \frac{1}{\sqrt{3}}$

⇒ xtan30 + ycot60 = 0

$\Rightarrow x\left(\frac{1}{\sqrt{3}}\right) + y\left(\frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow \frac{x+y}{\sqrt{3}} = 0$

⇒ x + y = 0

⇒ x = -y

⇒ 2x-ytan45 = 1

⇒ 2(-y)-y(1) = 1

⇒ -3y = 1

$\Rightarrow y = -\frac{1}{3}$

$\Rightarrow x = \frac{1}{3}$

9 A. Question

If A = B = 45°, then let us justify

sin (A+B) = sin A cos B + cos A sin B

Answer

Given, A = B = 45

⇒ A + B = 90

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 45 = \frac{1}{\sqrt{2}}$

⇒ sin90 = 1

⇒ sin90 = sin45 cos45 + cos45 sin45

$\Rightarrow 1 = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$

$\Rightarrow 1 = \frac{1}{2} + \frac{1}{2}$

⇒ 1 = 1

9 B. Question

If A = B = 45°, then let us justify

cos (A+B) = cos A cos B – sin A sin B

Answer

Given, A = B = 45

⇒ A + B = 90

$\Rightarrow \sin 45 = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 45 = \frac{1}{\sqrt{2}}$

⇒ cos90 = 0

⇒ cos(A + B) = cosA cosB-sinA sinB

⇒ cos90 = cos45 cos45-sin45 sin45

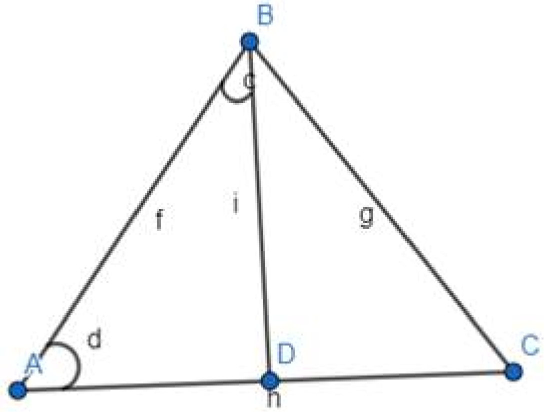
$\Rightarrow 0 = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$

⇒ 0 = 0

10 A. Question

In an equilateral triangle ABC, BD is a median. Let us prove that, ∠ ABD = cot ∠BAD

Answer



To Prove : cot ∠ ABD = cot ∠BAD

Proof:

In the figure shown above ABC is an equilateral triangle

Now BD is a median on side AC, Therefore, AD = DC

And ∠ ADB = 90°

(By property of equilateral triangle if a median is dropped from one vertex to opposite side, it is perpendicular to the side, and ∠ ABD = ∠BAD = 45°)

If the angles are equal, their values of cot will also be equal

So,

Cot ∠ ABD = cot ∠BAD

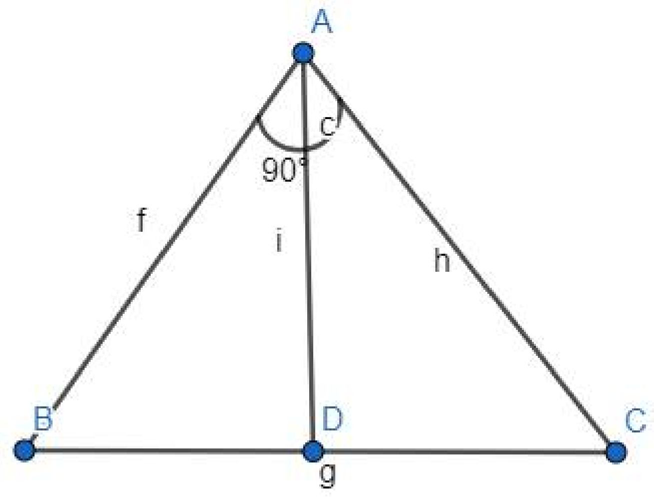
Hence, Proved

10 B. Question

In an isosceles triangle ABC, AB = AC and ∠BAC = 90°; the bisector of ∠BAC intersects the side BC at the point D.

Let us prove that, $\frac{\sec \angle ACD}{\sin \angle CAD} = \operatorname{cosec}^2 \angle CAD$

Answer



To Prove: $\frac{\sec \angle ACD}{\sin \angle CAD} = \operatorname{cosec}^2 \angle CAD$

Proof:

Since, ABC is an isosceles triangle with AB = AC then by the property of isosceles triangle that the angles opposite to equal sides are also equal, we have

$\angle ABC = \angle ACB$

From Triangle ABC,

Sum of angles of triangle = 180°

Therefore,

$\angle ABC + \angle BAC + \angle ACB = 180°$

$2\angle ABC + 90° = 180°$

$\angle ABC = 45°$

And $\angle ACB = 45°$

Sec $\angle ACD = \sec 45° = \sqrt{2}$

$\angle CAD = 45°$ ($\angle CAD = \angle ACB$)

Sin $\angle CAD = \sin 45° = \frac{1}{\sqrt{2}}$

$\operatorname{cosec}^2 \angle CAD = \operatorname{cosec}^2 45° = (\sqrt{2})^2 = 2$

$$\frac{\sec \angle ACD}{\sin \angle CAD} = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = 2$$

Hence,

$$\frac{\sec \angle ACD}{\sin \angle CAD} = \operatorname{cosec}^2 \angle CAD = 2$$

Hence, Proved.

11. Question

Let us determine the value/value of $\theta\left(0^\circ \le \theta \le 90^\circ\right)$, for which 2 $\cos^2\theta$ – 3 cos θ + 1 = 0 will be true.

Answer

Given, $2\cos^2\theta - 3\cos\theta + 1$

Need to check the equation is true for different values of theta

⇒ let θ = 0°

⇒ $2\cos^2 0° - 3\cos 0° + 1$

⇒ cos0° = 1

⇒ $2(1)^2 - 3(1) + 1$

= 2-3 + 1

= -1 + 1

= 0

Hence, if θ is 0 the given equation will be true

Let us Work Out 23.3

1 A. Question

If $\sin\theta = \frac{4}{5}$, then let us write the value of $\frac{\operatorname{cosec}\theta}{1+\cot\theta}$ by determining it.

Answer

Given, $\sin\theta = \frac{4}{5}$

⇒ $\sin^2\theta + \cos^2\theta = 1$

⇒ $\left(\frac{4}{5}\right)^2 + \cos^2\theta = 1$

⇒ $\cos^2\theta = 1 - \frac{16}{25}$

$\Rightarrow \cos^2\theta = \frac{9}{25}$

$\Rightarrow \cos\theta = \sqrt{\frac{9}{25}}$

$\Rightarrow \cos\theta = \frac{3}{5}$

$\Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \cot\theta = \frac{\frac{3}{5}}{\frac{4}{5}}$

$\Rightarrow \cot\theta = \frac{3}{4}$

$\Rightarrow \text{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\frac{4}{5}}$

$= \frac{5}{4}$

1 B. Question

If $\tan\theta = \frac{3}{4}$, then let us show that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1}{2}$

Answer

Given, $\tan\theta = \frac{3}{4}$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta = 1 + \left(\frac{3}{4}\right)^2$

$= 1 + \frac{9}{16}$

$= \frac{25}{16}$

$\Rightarrow \sec\theta = \frac{5}{4}$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$

$\Rightarrow \cos\theta = \frac{1}{\frac{5}{4}}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\frac{16}{25}}$

$= \frac{3}{5}$

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$\Rightarrow \sqrt{\frac{1-\frac{3}{5}}{1+\frac{3}{5}}}$

$= \sqrt{\frac{2}{8}}$

$= \sqrt{\frac{1}{4}}$

$= \frac{1}{2}$

1 C. Question

If tan θ=1, then let us determine the value of $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta-2\cos^3\theta+7\cos\theta}$.

Answer

Given, tan θ = 1

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow 1 = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta = 1 + 1^2$

$\Rightarrow \sec\theta = \sqrt{2}$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$

$= \frac{8\left(\frac{1}{\sqrt{2}}\right) + 5\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\left(\frac{1}{\sqrt{2}}\right)^3 + 7\left(\frac{1}{\sqrt{2}}\right)}$

$= \frac{\frac{13}{\sqrt{2}}}{-\left(\frac{1}{\sqrt{2}}\right)^3 + \frac{7}{\sqrt{2}}}$

$= \frac{\frac{13}{\sqrt{2}}}{\frac{1}{\sqrt{2}}\left(7 - \left(\frac{1}{\sqrt{2}}\right)^2\right)}$

$= \frac{13}{\left(7 - \frac{1}{2}\right)}$

$= \frac{13}{\frac{13}{2}}$

$= \frac{1}{2}$

2. Question

i. Let us express cosec θ and tan θ in term of sin θ.

ii. Let us write cosec θ and tan θ in term cos θ.

Answer

(i) $\Rightarrow \text{cosec}\theta\text{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta$

(ii) $\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \text{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\sqrt{1-\cos^2\theta}}$

$\Rightarrow \text{cosec}\theta = \frac{1}{\sqrt{1-\cos^2\theta}}$

3 A. Question

If sec θ + tan θ=2, then let us determine the value of (sec θ-tan θ).

Answer

Given, secθ + tanθ = 2

⇒ $\sec^2\theta = 1 + \tan^2\theta$

⇒ $\sec^2\theta - \tan^2\theta = 1$

⇒ (secθ + tanθ)(secθ- tanθ) = 1

⇒ 2(secθ- tanθ) = 1

⇒ $\sec\theta - \tan\theta = \frac{1}{2}$

3 B. Question

If cosec θ – cot θ = √2 – 1, then let us write by calculating, the value of (cosec θ + cot θ).

Answer

Given, cosecθ –cotθ = √2 -1

⇒ $\text{cosec}^2\theta = 1 + \cot^2\theta$

⇒ $\text{cosec}^2\theta - \cot^2\theta = 1$

⇒(cosecθ + cotθ)(cosecθ – cotθ) = 1

⇒(cosecθ + cotθ)(√2-1) = 1

⇒ $\text{cosec}\theta + \cot\theta = \frac{1}{\sqrt{2}-1}$

3 C. Question

If sin θ + cos θ = 1, then let us determine the value of sin θ × cos θ.

Answer

Given, sinθ + cosθ = 1

⇒ squaring on both sides

⇒ $(\sin\theta + \cos\theta)^2 = 1^2$

⇒ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

⇒ 1 + 2sinθ cosθ = 1

⇒ 2sinθ cosθ = 0

⇒ sinθ × cosθ = 0

3 D. Question

If tan θ × cot θ = 2, then let us determine the value of (tan θ - cot θ).

Answer

Given, tanθ + cotθ = 2

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)(\tan\theta - 1) = 0$

$\therefore \tan\theta = 1$

$\Rightarrow \cot\theta = \frac{1}{\tan\theta} = 1$

$\Rightarrow \tan\theta - \cot\theta = 0$

3 E. Question

If $\sin\theta - \cos\theta = \frac{7}{13}$, then let us determine the value of sin θ + cos θ.

Answer

Given, $\sin\theta - \cos\theta = \frac{7}{13}$

⇒ squaring on both sides

$\Rightarrow (\sin\theta - \cos\theta)^2 = \left(\frac{7}{13}\right)^2$

$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{49}{169}$

$\Rightarrow 1 - 2\sin\theta\cos\theta = \frac{49}{169}$

$\Rightarrow 1 - \frac{49}{169} = 2\sin\theta\cos\theta$

$\Rightarrow \frac{120}{169} = 2\sin\theta\cos\theta$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$, Can be written as

$\Rightarrow (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 - \frac{120}{169} = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 1 - \frac{120}{169}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = \frac{49}{169}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{\frac{49}{169}}$

$\Rightarrow \sin\theta + \cos\theta = \frac{7}{13}$

3 F. Question

If $\sin\theta\cos\theta = \frac{1}{2}$, then let us write by calculating, the value of (sin θ + cos θ).

Answer

$\Rightarrow \sin\theta\cos\theta = \frac{1}{2}$

$\Rightarrow 2\sin\theta\cos\theta = 1$

⇒ we know that, $\sin^2\theta + \cos^2\theta = 1$

Can be written as

$\Rightarrow (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 - 1 = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 2$

$\Rightarrow (\sin\theta + \cos\theta) = \sqrt{2}$

3 G. Question

If $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$, then let us determine the values of both sec θ and tan θ.

Answer

Given, $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$-----eq(1)

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow (\sec\theta + \tan\theta)\left(\frac{1}{\sqrt{3}}\right) = 1$

$\Rightarrow \sec\theta + \tan\theta = \sqrt{3}$………eq(2)

From adding both the equations we get

$\Rightarrow 2\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow 2\sec\theta = \frac{4}{\sqrt{3}}$

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{2}{\sqrt{3}} - \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \tan\theta$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

3 H. Question

If cosec θ + cot θ = √3, then let us determine the values of both cosec θ and cot θ.

Answer

Given, cosecθ + cotθ = √3

⇒ cosecθ + cotθ = √3 ……..eq(1)

⇒ $\text{cosec}^2\theta - \cot^2\theta = 1$

⇒ (cosecθ + cotθ)(cosecθ −cotθ) = 1

⇒ (√3)(cosecθ −cotθ) = 1

$\Rightarrow \text{cosec}\theta - \cot\theta = \frac{1}{\sqrt{3}}$ ………eq(2)

From adding both the equations we get

$\Rightarrow 2\text{cosec}\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow 2\text{cosec}\theta = \frac{4}{\sqrt{3}}$

$\Rightarrow \text{cosec}\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \text{cosec}\theta + \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2}{\sqrt{3}} + \cot\theta = \sqrt{3}$

$\Rightarrow \cot\theta = \sqrt{3} - \frac{2}{\sqrt{3}}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$

3 I. Question

If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 7$, then let us write by calculating, the value of tan θ.

Answer

Given, $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 7$

$\Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{7}{1}$ can be written as

$\Rightarrow \frac{\sin\theta + \cos\theta + \sin\theta - \cos\theta}{\sin\theta + \cos\theta - \sin\theta + \cos\theta} = \frac{7+1}{7-1}$ [by compendo and dividend rule]

$\Rightarrow \frac{2\sin\theta}{2\cos\theta} = \frac{8}{6}$

$\Rightarrow \tan\theta = \frac{4}{3}$

3 J. Question

If $\frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3}$, then let us write by calculating, the value of sin θ.

Answer

Given, $\frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3}$

$\Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \frac{\operatorname{cosec}\theta + \sin\theta + \operatorname{cosec}\theta - \sin\theta}{\operatorname{cosec}\theta + \sin\theta - \operatorname{cosec}\theta + \sin\theta} = \frac{5+3}{5-3}$

$\Rightarrow \frac{2\operatorname{cosec}\theta}{2\sin\theta} = \frac{8}{2}$

$\Rightarrow \frac{\operatorname{cosec}\theta}{\sin\theta} = 4$

$\Rightarrow \frac{\frac{1}{\sin\theta}}{\sin\theta} = 4$

$\Rightarrow \frac{1}{\sin^2\theta} = 4$

$\Rightarrow \frac{1}{4} = \sin^2\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

3 K. Question

If $\sec\theta + \cos\theta = \frac{5}{3}$, then let us write by calculating, the value of (sec θ-cos θ).

Answer

Given, $\sec\theta + \cos\theta = \frac{5}{3}$

$\Rightarrow \sec\theta + \frac{1}{\sec\theta} = \frac{5}{3}$

$\Rightarrow 3(\sec^2\theta + 1) = 5\sec\theta$

$\Rightarrow 3\sec^2\theta - 5\sec\theta + 3 = 0$

3 L. Question

Let us determine the value of from the relation $5\sin^2\theta + 4\cos^2\theta = \frac{9}{2}$.

Answer

Given, $5\sin^2\theta + 4\cos^2\theta = \frac{9}{2}$

Need to find tantθ value

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 5(1 - \cos^2\theta) + 4\cos^2\theta = \frac{9}{2}$

$\Rightarrow 5 - 5\cos^2\theta + 4\cos^2\theta = \frac{9}{2}$

$\Rightarrow 5 - \frac{9}{2} = \cos^2\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin^2\theta + \frac{1}{2} = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{1}{2}$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

⇒ $\tan\theta = 1$

3 M. Question

If $\tan^2\theta + \cot^2\theta = \frac{10}{3}$, then let us determine the values of $\tan\theta + \cot\theta$ and $\tan\theta - \cot\theta$ and from these let us write the value of $\tan\theta$.

Answer

Given, $\tan^2\theta + \cot^2\theta = \frac{10}{3}$

⇒ $\tan^2\theta + \frac{1}{\tan^2\theta} = \frac{10}{3}$

⇒ $\frac{\tan^4\theta + 1}{\tan^2\theta} = \frac{10}{3}$

⇒ $3\tan^4\theta + 3 - 10\tan^2\theta = 0$

⇒ $(3\tan^2\theta - 1)(\tan^2\theta - 3) = 0$

⇒ $\tan\theta = \sqrt{3}$ and $\cot\theta = \frac{1}{\sqrt{3}}$

⇒ $\tan\theta = \frac{1}{\sqrt{3}}$ and $\cot\theta = \sqrt{3}$

⇒ $\tan\theta + \cot\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

⇒ $\tan\theta - \cot\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

3 N. Question

If $\sec^2\theta + \tan^2\theta = \frac{13}{12}$, then let us write by calculating, the value of $(\sec^4\theta - \tan^4\theta)$.

Answer

Given, $\sec^2\theta + \tan^2\theta = \frac{13}{12}$

⇒ $\sec^2\theta = 1 + \tan^2\theta$

⇒ $1 + \tan^2\theta + \tan^2\theta = \frac{13}{12}$

⇒ $1 + 2\tan^2\theta = \frac{13}{12}$

$\Rightarrow 2\tan^2\theta = \frac{13}{12} - 1$

$\Rightarrow \tan^2\theta = \frac{1}{24}$

$\Rightarrow \sec^2\theta = 1 + \frac{1}{24}$

$\Rightarrow \sec^2\theta = \frac{25}{24}$

$\Rightarrow (\sec^4\theta - \tan^4\theta)$

$= (\sec^2\theta)^2 - (\tan^2\theta)^2$

$= \left(\frac{25}{24}\right)^2 - \left(\frac{1}{24}\right)^2$

$= \frac{625}{576} - \frac{1}{576}$

$= \frac{624}{576}$

4 A. Question

In ΔPQR, ∠Q is right angle. If PR = √5 units and PQ – RQ = 1 unit, then let us determine the value of cosP – cosR.

Answer

Given, ∠ Q = 90°

PR = √5 and PQ –QR = 1

$\Rightarrow$ cosP –cosR

$= \frac{PQ}{PR} - \frac{QR}{PR}$

$= \frac{PQ - QR}{PR}$

$= \frac{1}{\sqrt{5}}$

4 B. Question

In ΔXYZ, ∠Y is right angle. If XY = 2√3 units and XZ – YZ = 2 units then let us determine the values of (secX - tanX).

Answer

Given, ∠ Y = 90°

XY = 2√3 and XZ-YZ = 2

$\Rightarrow$ secX-tanX

$= \frac{XZ}{XY} - \frac{YZ}{XY}$

$= \frac{XZ - YZ}{XY}$

$= \frac{2}{2\sqrt{3}}$

5 A. Question

Let us eliminate ‘θ’ from the relations :

x = 2 sin θ, y = 3 cos θ

Answer

Given, x = 2sinθ and y = 3cosθ

$\Rightarrow \sin\theta = \frac{x}{2}$

$\Rightarrow \cos\theta = \frac{y}{3}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$

5 B. Question

Let us eliminate ‘θ’ from the relations :

5x = 3 sec θ, y = 3 tan θ

Answer

Given, 5x = 3secθ and y = 3tanθ

$\Rightarrow \sec\theta = \frac{5x}{3}$ and $\tan\theta = \frac{y}{3}$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{5x}{3}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$

$\Rightarrow \frac{25x^2}{9} - \frac{y^2}{9} = 1$

6 A. Question

If $\sin\alpha = \frac{5}{13}$, then let us show that, tan α + sec α = 1.5.

Answer

Given, $\sin\alpha = \frac{5}{13}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \left(\frac{5}{13}\right)^2 + \cos^2\alpha = 1$

$\Rightarrow \frac{25}{169} + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 1 - \frac{25}{169}$

$\Rightarrow \cos^2\alpha = \frac{144}{169}$

$\Rightarrow \cos\alpha = \frac{12}{13}$

$\Rightarrow \tan\alpha + \sec\alpha = 1.5$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{1}{\cos\alpha} = 1.5$

$\Rightarrow \frac{\frac{5}{13}}{\frac{12}{13}} + \frac{1}{\frac{12}{13}} = 1.5$

$\Rightarrow \frac{5}{12} + \frac{13}{12} = 1.5$

$\Rightarrow \frac{18}{12} = 1.5$

$\Rightarrow 1.5 = 1.5$

6 B. Question

If $\tan A = \frac{n}{m}$, then let us determine the values of both sinA and secA.

Answer

Given, $\tan A = \frac{n}{m}$

$\Rightarrow \frac{\sin A}{\cos A} = \frac{n}{m}$

$\Rightarrow \sec^2 A = 1 + \tan^2 A$

$= 1 + \left(\frac{n}{m}\right)^2$

$= \frac{m^2 + n^2}{m^2}$

$\Rightarrow \sec A = \sqrt{\frac{m^2 + n^2}{m^2}}$

$\Rightarrow \cos A = \frac{1}{\sec A}$

$= \frac{1}{\sqrt{\frac{m^2+n^2}{m^2}}}$

$= \frac{m}{\sqrt{m^2 + n^2}}$

$\Rightarrow \sin A = \frac{n}{m} \frac{m}{\sqrt{m^2 + n^2}}$

6 C. Question

If $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$, then let us show that, x sin θ = y cos θ.

Answer

Given, $\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{x}{\sqrt{x^2 + y^2}}\right)^2$

$= \frac{x^2 + y^2 - x^2}{x^2 + y^2}$

$\Rightarrow \sin^2\theta =$

$\Rightarrow \sin\theta = \sqrt{\frac{y^2}{x^2 + y^2}}$

$\Rightarrow \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta = \frac{\frac{y}{\sqrt{x^2+y^2}}}{\frac{x}{\sqrt{x^2+y^2}}}$

$\Rightarrow \tan\theta = \frac{y}{x}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

$\Rightarrow$ xsinθ = ycosθ

6 D. Question

If $\sin\alpha = \dfrac{a^2 - b^2}{a^2 + b^2}$, then let us show that, $\cot\alpha = \dfrac{2ab}{a^2 - b^2}$.

Answer

Given, $\sin\alpha = \dfrac{a^2 - b^2}{a^2 + b^2}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - \left(\dfrac{a^2 - b^2}{a^2 + b^2}\right)^2$

$= 1 - \left(\dfrac{{a^2}^2 + {b^2}^2 - 2a^2b^2}{{a^2}^2 + {b^2}^2 + 2a^2b^2}\right)$

$= \dfrac{4a^2b^2}{(a^2 + b^2)^2}$

$\Rightarrow \cos\alpha = \dfrac{2ab}{a^2 + b^2}$

$\Rightarrow \cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

$= \dfrac{\frac{2ab}{a^2 + b^2}}{\frac{a^2 - b^2}{a^2 + b^2}}$

$\Rightarrow \cot\alpha = \dfrac{2ab}{a^2 - b^2}$

6 E. Question

If $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y}$, then let us show that, $\sin\theta - \cos\theta = \dfrac{x - y}{\sqrt{x^2 + y^2}}$

Answer

Given, $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y}$

$\Rightarrow \sin\theta = \dfrac{x\cos\theta}{y}$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\dfrac{x\cos\theta}{y}\right)^2 + \cos^2\theta = 1$

$$\Rightarrow \frac{x^2}{y^2}\cos^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta\left(\frac{x^2}{y^2} + 1\right) = 1$$

$$\Rightarrow \cos\theta = \sqrt{\frac{y^2}{x^2+y^2}}$$

$$\Rightarrow \sin\theta = \frac{x\cos\theta}{y}$$

$$\Rightarrow \sin\theta = \frac{x}{y}\cos\theta$$

$$\Rightarrow \sin\theta = \frac{x}{y}\sqrt{\frac{y^2}{x^2+y^2}}$$

$$\Rightarrow \sin\theta - \cos\theta = \frac{x}{y}\sqrt{\frac{y^2}{x^2+y^2}} - \sqrt{\frac{y^2}{x^2+y^2}}$$

$$= \sqrt{\frac{y^2}{x^2+y^2}}\left(\frac{x}{y} - 1\right)$$

$$= \frac{x-y}{y}\frac{y}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \sin\theta - \cos\theta = \frac{(x-y)}{\sqrt{x^2+y^2}}$$

6 F. Question

If $(1+4x^2)$ cos A = 4x, then let us show that, $\operatorname{cosec} A + \cot A = \frac{1+2x}{1-2x}$.

Answer

Given, $\cos A = \frac{4x}{1+4x^2}$

$\Rightarrow \sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 A$

$$= 1 - \left(\frac{4x}{1+4x^2}\right)^2$$

$$= \frac{\left(1+4x^2\right)^2 - (4x)^2}{(1+4x^2)^2}$$

$$= \frac{\left(1+4x^2-4x\right)\left(1+4x^2+4x\right)}{(1+4x^2)^2}$$

$$= \frac{(1-2x)^2(1+2x)^2}{(1+4x^2)^2}$$

$\Rightarrow \sin A = \sqrt{\frac{(1-2x)^2(1+2x)^2}{(1+4x^2)^2}}$

$\Rightarrow \sin A = \frac{(1-2x)(1+2x)}{1+4x^2}$

$\Rightarrow \operatorname{cosec}A + \cot A$

$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$

$= \frac{1+\cos A}{\sin A}$

$= \frac{1+\frac{4x}{1+4x^2}}{\frac{(1-2x)(1+2x)}{1+4x^2}}$

$= \frac{1+4x^2+4x}{(1-2x)(1+2x)}$

$= \frac{(1+2x)^2}{(1-2x)(1+2x)}$

$= \frac{1+2x}{(1-2x)}$

7. Question

If x = a sin θ and y = b tan θ, then let us prove that, $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1.$

Answer

Given, x = asinθ and y = btanθ

$\Rightarrow \sin\theta = \frac{x}{a}$

$\Rightarrow \tan\theta = \frac{y}{b}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{y}{b}$

$\Rightarrow b\sin\theta = y\cos\theta$

$\Rightarrow \frac{\frac{x}{a}}{\frac{y}{b}} = \cos\theta$

$\Rightarrow \cos\theta = \frac{bx}{ay}$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec\theta = \frac{ay}{bx}$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{ay}{bx}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$

$\Rightarrow \left(\frac{y}{b}\right)^2 \left(\left(\frac{a}{x}\right)^2 - 1\right) = 1$

$\Rightarrow \frac{a^2}{x^2} - 1 = \frac{b^2}{y^2}$

$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

8. Question

If $\sin\theta + \sin^2\theta = 1$, then let us prove that, $\cos^2\theta + \cos^4\theta = 1$.

Answer

Given, $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta + \sin^2\theta = \sin^2\theta + \cos^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

⇒ squaring on both sides

$\Rightarrow (\sin\theta)^2 = (\cos^2\theta)^2$

$\Rightarrow \sin^2\theta = \cos^4\theta$

$\Rightarrow \sin\theta + \sin^2\theta = 1$

$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

9 A1. Question

If $3x = \operatorname{cosec}\alpha$ and $\frac{3}{x} = \cot\alpha$, then the value of $3\left(x^2 - \frac{1}{x^2}\right)$ is

A. $\frac{1}{27}$

B. $\frac{1}{81}$

C. $\frac{1}{3}$

D. $\frac{1}{9}$

Answer

Given, 3x = cosecα and $\frac{3}{x} = \cot\alpha$

$\Rightarrow x = \frac{\text{cosec}\alpha}{3}$ and $\frac{1}{x} = \frac{\cot\alpha}{3}$

$\Rightarrow 3\left(x^2 - \frac{1}{x^2}\right)$

$= 3\left(\frac{\text{cosec}^2\alpha}{9} - \frac{\cot^2\alpha}{9}\right)$

$= \frac{3\left(\text{cosec}^2\alpha - \cot^2\alpha\right)}{9}$

$= \frac{1}{3}$

⇒ option A is incorrect as it does not satisfy the given equation

⇒ option B is incorrect as it does not satisfy the given equation

⇒ option D is incorrect as it does not satisfy the given equation

9 A2. Question

If 2x = sec A and $\frac{2}{x} = \tan A$, then the value of $2\left(x^2 - \frac{1}{x^2}\right)$ is

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

Answer

⇒ Given, 2x = secA and $\frac{2}{x} = \tan A$

$\Rightarrow x = \frac{\sec A}{2}$ and $\frac{1}{x} = \frac{\tan A}{2}$

$\Rightarrow 2\left(x^2 - \frac{1}{x^2}\right)$

$= 2\left(\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4}\right)$

$= \frac{2(\sec^2 A - \tan^2 A)}{4}$

$= \frac{1}{2}$

⇒ Option B is incorrect as it does not satisfy the value

⇒ Option C is incorrect as it does not satisfy the value

⇒ Option D is incorrect as it does not satisfy the value

9 A3. Question

If tan α + cot α = 2, then the value of $(\tan^{13}\alpha + \cot^{13}\alpha)$ is

A. 1

B. 0

C. 2

D. none of these

Answer

⇒ tanα + cotα = 2

$\Rightarrow \tan\alpha + \frac{1}{\tan\alpha} = 2$

$\Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$

$\Rightarrow 1 + \tan^2\alpha - 2\tan\alpha = 0$

$\Rightarrow (\tan\alpha - 1)^2 = 0$

⇒ tanα = 1

⇒ cotα = 1

$\Rightarrow \tan^{13}\alpha + \cot^{13}\alpha = 1 + 1 = 2$

⇒ Option A is incorrect as it does not satisfy the value

⇒ Option B is incorrect as it does not satisfy the value

⇒ Option D is incorrect as it does not satisfy the value

9 A4. Question

If $\sin\theta - \cos\theta = 0\left(0^\circ \le \theta \le 90^\circ\right)$ and sec θ + cosec θ + x, then the value of x is

A. 1

B. 2

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer

Given, $\sin\theta - \cos\theta = 0$

⇒ squaring on both sides

⇒ $(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) = 0$

⇒ $(1 - 2\sin\theta\cos\theta) = 0$

⇒ $\sin\theta\cos\theta = \frac{1}{2}$

⇒ $\sec\theta + \text{cosec}\theta = x$

⇒ $\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = x$

⇒ $\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = x$

⇒ $\sin\theta + \cos\theta = x\sin\theta\cos\theta$

⇒ $\sin\theta + \cos\theta = \frac{x}{2}$

Squaring on both sides

⇒ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{x^2}{4}$

⇒ $1 + 2\left(\frac{1}{2}\right) = \frac{x^2}{4}$

⇒ $x^2 = 8$

⇒ $x = 2\sqrt{2}$

⇒ Option A is incorrect as it does not satisfy the value

⇒ Option B is incorrect as it does not satisfy the value

⇒ Option C is incorrect as it does not satisfy the value

9 A5. Question

If 2 cos 3 θ = 1, then the value of θ is

A. 10^o

B. 15^o

C. 20^o

D. 30^o

Answer

Given, $\cos 3\theta = \frac{1}{2}$

⇒ we know $\cos 60 = \frac{1}{2}$

∴ θ = 20

⇒ cos3(20) = cos60

⇒ Option A is incorrect as it does not satisfy the value

⇒ Option B is incorrect as it does not satisfy the value

⇒ Option D is incorrect as it does not satisfy the value

9 B. Question

Let us write whether the following statements are true or false :

i. If $0^o \le \alpha < 90^o$, then the least value of ($\sec^2 \alpha + \cos^2 \alpha$) is 2.

ii. The value of $\left(\begin{array}{l}\cos 0^o \times \cos 1^o \times \cos 2^o \times \\ \cos 3^o \times \ldots\ldots\ldots\cos 90^o\end{array}\right)$ is 1.

Answer

(i) TRUE

⇒ let us consider α as 0°

⇒ $\sec^2 0 + \cos^2 0 = 1 + 1 = 2$

(ii) FALSE

⇒ cos90 = 0

9 C. Question

Let us fill in the blanks :

i. The value of $\left(\frac{4}{\sec^2\theta}+\frac{1}{1+\cot^2\theta}+3\sin^2\theta\right)$ is ______.

ii. If $\sin\left(\theta-30^\circ\right)=\frac{1}{2}$, then the value of cos θ is ________

iii. If $\cos^2\theta-\sin^2\theta=\frac{1}{2}$, then the value of $\cos^4\theta - \sin^4\theta$ is ________

Answer

(i) Given, $\frac{4}{\sec^2\theta} + \frac{1}{1+\cot^2\theta} + 3\sin^2\theta$

$\Rightarrow 4\cos^2\theta + \frac{1}{\text{cosec}^2\theta} + 3\sin^2\theta$

$= 4\cos^2\theta + \sin^2\theta + 3\sin^2\theta$

= 4

(ii) Given, $\sin(\theta - 30) = \frac{1}{2}$

$\Rightarrow \theta = 0$

$\Rightarrow \cos 0^\circ = 1$

(iii) Given, $\cos^2\theta - \sin^2\theta = \frac{1}{2}$

$\Rightarrow \cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= \left(\frac{1}{2}\right)(1)$

$= \frac{1}{2}$

10 A. Question

If and then let us determine the values of both r and θ.

Answer

Given, $r\cos\theta = 2\sqrt{3}$, $r\sin\theta = 2$

$\Rightarrow \cos\theta = \frac{2\sqrt{3}}{r}$ and $\sin\theta = \frac{2}{r}$

$\Rightarrow \tan\theta = \frac{\frac{2}{r}}{\frac{2\sqrt{3}}{r}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30$

$\Rightarrow r\sin\theta = 2$

$\Rightarrow r\sin 30 = 2$

$\Rightarrow \frac{r}{2} = 2$

$\Rightarrow r = 4$

10 B. Question

If sin A + sin B = 2 where $0^\circ \leq A \leq 90^\circ$ and $0^\circ \leq B \leq 90^\circ$, then let us find out the value of (cos A + cos B).

Answer

Given, sinA + sinB = 2

$\Rightarrow$ let A = B = 90°

$\Rightarrow$ sin90 + sin90 = 2

$\Rightarrow$ cosA + cosB = cos90° + cos90° = 0

10 C. Question

If 0° < θ < 90°, then let us calculate the least value of ($9\tan^2\theta + 4\cot^2\theta$).

Answer

Let θ = 45

$\Rightarrow (9\tan^2\theta + 4\cot^2\theta)$ [tan45 = 1, cot45 = 1]

$\Rightarrow 9 + 4$

$= 13$

10 D. Question

Let us calculate the value of ($\sin^2\alpha + \cos^6\alpha + 3\sin^2\alpha\cos^2\alpha$).

Answer

Given, $\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha\cos^2\alpha$

$\Rightarrow \sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha\cos^2\alpha$

$= (\sin^2\alpha)^3 + (\cos^2\alpha)^3 + 3\sin^2\alpha\cos^2\alpha$

$= (\sin^2\alpha + \cos^2\alpha)(\sin^4\alpha - \sin^2\alpha\cos^2\alpha + \cos^4\alpha) + 3\sin^2\alpha\cos^2\alpha$

$= \sin^4\alpha + \cos^4\alpha - \sin^2\alpha\cos^2\alpha + 3\sin^2\alpha \cos^2\alpha$

$= (\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha + 3\sin^2\alpha \cos^2\alpha$

$= 1$

10 E. Question

If $\operatorname{cosec}^2 \theta = 2 \cot \theta$ and $0° < \theta < 90°$, then let us determine the value of θ.

Answer

Given, $\operatorname{cosec}^2\theta = 2\cot\theta$

Let $\theta = 45$

$\Rightarrow \operatorname{cosec}^2\theta = (\sqrt{2})^2$

$= 2$

$\Rightarrow 2\cot\theta = 2(1) = 2$