TALENT & OLYMPIAD

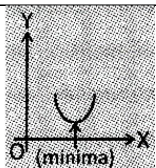
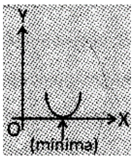
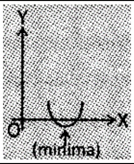
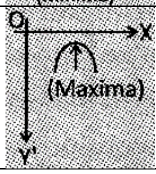
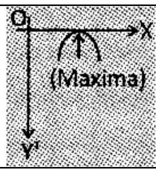
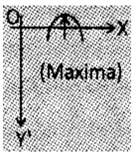
Introduction

Polynomials are an algebraic expression having many terms. We have studied about the polynomials in one variable in previous classes. There are different types of polynomials. The highest power of the polynomials is called degree of the polynomials. The polynomials of degree one is called linear polynomial. The polynomials of degree two are called quadratic polynomials. The polynomials of degree three are called cubic polynomials and the polynomials of degree four are called biquadratic polynomials. A real number which satisfies the given polynomials is called zeroes of the polynomials.

Geometrical Meaning of Zeroes of Polynomial

If we represent linear polynomials on the graph we get a straight line. The straight line intersects the x axis at only one point. Thus number of zeroes of the linear polynomials is one. Hence we can say that the number of zeroes of the polynomials is the number of times the graph of the polynomials intersect x axis. The quadratic polynomials will have two zeroes and the cubic polynomials will have three zeroes.

Graphical Representation of Different Forms of Quadratic Equation

Characteristic of the function	$b^2-4ac<0$	$b^2-4ac\ 0$	$b^2-4ac>0$
When 'a' is positive			
When 'a' negative i.e. a < 0			

Relationship between the Zeroes of the Polynomials and Coefficient of Polynomials

If $ax^2+bx+c=0$ is a quadratic equation whose roots are a and p, then the relation between the roots of the equation is given by

Sum of the roots = $\alpha+\beta-\frac{b}{a}$,

Product of the roots = $\alpha\beta-\frac{c}{a}$.

Fora cubic equation $ax^3+bx^2+cx+d=0$, the relation between the roots whose roots are $\alpha,\beta\, and\, \gamma$, is given by

Sum of roots = $\alpha+\beta+\gamma=-\frac{b}{a}$,

Sum of the product of roots =

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$

Product of the roots = $\alpha\beta\gamma=-\frac{d}{a}$.

Commonly Asked QUESTIONS

The graphical representation of the equation $f(x) = x^2 + 2x + 10$ is:

(a) Straight line
(b) Circle
(c) Parabola
(d) Ellipse
(e) None of these

Answer: (c)

Explanation

The above given equation is a quadratic equation which traces a parabola on the graph.

The zeroes of the polynomials $f(x) = abx^2 + (b^2 + ac)x + bc$ is ____.

(a) $\left(\frac{b}{a} \& -\frac{c}{a}\right)$
(b) $\left(-\frac{b}{a} \& -\frac{c}{a}\right)$
(c) $\left(-\frac{b}{a} \& \frac{c}{a}\right)$
(d) $\left(\frac{b}{a} \& \frac{c}{a}\right)$
(e) None of these

Answer: (b)

Explanation

We have,

$$abx^2 + (b^2 + ac)x + bc$$
$$= abx^2 + b^2x + acx + bc$$
$$= bx(ax + b) + c(ax + b)$$
$$= (ax + b)(bx + c)$$

Therefore, $x = \left(-\frac{b}{a} \& -\frac{c}{a}\right)$

If a and (3 are the roots of the given equation $2\sqrt{3}x^2 + 4x - 3\sqrt{3}$, then the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is ____.

(a) 0
(b) $-\frac{280\sqrt{3}}{243}$
(c) $+\frac{280\sqrt{3}}{243}$
(d) $\frac{280}{243}$
(e) None of these

Answer: (c)

Explanation

The sum of the roots is $\alpha+\beta=-\frac{b}{a}=-\frac{2}{\sqrt{3}}$

Product of the roots $\alpha\beta=\frac{c}{a}=-\frac{3}{2}$

Now, $=\frac{1}{\alpha^3}\frac{1}{\beta^3}=\frac{\left(\alpha+\beta^3\right)-3\alpha\beta\left(\alpha+\beta\right)}{\left(\alpha\beta\right)^3}$

If α *and* β are the roots of the polynomials $ky^2+6y-18\, such\, that\, \alpha^2+\beta^2=36$ then find the value of k.

(a) $\frac{1\pm\sqrt{2}}{2}$ (b) $\frac{1\pm\sqrt{3}}{2}$

(c) $\frac{1+\sqrt{3}}{2}$ (d) $\frac{1\pm\sqrt{5}}{2}$

(e) None of these

Answer: (d)
Explanation

We have, $\alpha^2+\beta^2=\left(\alpha+\beta\right)^2-2\alpha\beta$

$\frac{36}{k^2}+\frac{36}{k}=36$

$k^2-k-1=0$

$k=\frac{1\pm\sqrt{5}}{2}$

If a and b are the roots of the equation m^2+5m-8, find a polynomial whose roots are $2a+1 and\ 2b+1$.

(a) $h(m)=3m^2+4m+1$ (b) $k(m^2+9m+41)$

(c) $(m^2+8m-)41$ (d) $k(m^2-9m-41)$

(e) None of these

Answer: (a)

Explanation

We have sum of the roots = a + b = - 5
Product of the roots = ab = - 8
Now for the required equation,
Sum of the roots = 2 (a + b) + 2 = - 10 + 2 = - 8
Product of the roots = 4ab + 2(a + b) + 1 = - 41
Therefore, required equation is k
$(m^2+8m-)41$.

Commonly Asked Questions

Based on Cubic Polynomials

The zeroes of the polynomials $f(y) = y^3 - 8y^2 + 24 + 64$, if two zeroes are equal in magnitude but opposite in sign.

(a) (5, 8, & 9) (b) $(2\sqrt{2}, -2\sqrt{2} \,\&\, 8)$

(c) $(-2\sqrt{2}, 5, \&8)$ (d) $(2\sqrt{2}, 5\sqrt{2} \,\&\, 8)$

(e) None of these

Answer: (b)

Explanation

If $\alpha, \beta \, and \, \gamma$ are the roots of the equation, then sum of the roots $= \alpha + \beta + \gamma = 8$ s

Product of the roots $= \alpha\beta \; \gamma = -64$

Putting the value we have, $\alpha\beta = -8$

Therefore, $\alpha, 2\sqrt{2} = a \, and \; \beta = -2\sqrt{2} \, and \; \gamma = 8$

Find the values of k for which the zeroes of the polynomial $f(n) = n^3 + 12n^2 + 39n + k$ are in A.P.

(a) 60 (b) 25

(c) - 25 (d) 28

(e) None of these

Answer: (a)

The cubic polynomial whose three roots are 3, -1 and - 3 is:

(a) $n^3 + n^2 - 9n - 9$ (b) $n^3 - n^2 - 9n - 9$

(c) $n^3 + n^2 + 9n + 9$ (d) $n^3 - n^2 + 9n + 9$ s

(e) None of these

Answer: (a)

If the zeroes of the polynomial $f(y) y^3 - 3y^2 + 9y + 8 \, are \, (p - q), p \, and \, (p + q)$. The value of p and q is:

(a) (1, 3) (b) $(1, \pm 3)$

(c) $(-1, \pm 3)$ (d) $(-1, -3)$

(e) None of these

Answer: (b)

If $2 \pm \sqrt{3}$ are the two zeroes of the polynomial $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$, then the other two zeroes of the polynomials f(x) are:

(a) (- 5, 3) (b) (1, 3)

(c) (- 1, 3) (d) (- 5, 7)

(e) None of these

Answer: (d)

Division of Polynomial

Previously we have studied about the division of the real numbers, in which we obtained quotient and remainders which satisfies the relation,

Dividend = Quotient $\times$ Divisor + Remainder

This is also known as Euclid's division lemma. In this section we will discuss about the division of the polynomials which is known as the division algorithm for polynomials. The concept of division of the polynomials can be used for finding the zeroes of the cubic or biquadratic polynomials.

Commonly Asked QUESTIONS

Divide the polynomial $g(x) = x^3 - 3x^2 + 3x^2 + 3x - 5$ by the polynomials $h(x) = x^2 + x + 1$ and find the quotient and remainder.

(a) $(x-4, 6x-1)$ (b) $(x+4, 6x+1)$

(c) $(x^2+1, 3x+2)$ (d) $(x^2+1, x-2)$

(e) None of these

Answer: (a)

Explanation

When we divide $g(x) by\, h(x)$ we have,

$$x^2 + x + 1 \overline{\smash{)}\, x^3 - 3x^2 + 3x - 5}$$

$$\underline{\pm x^3 \pm x^2 x}$$

$$-4x^2 \mp 2x - 5$$

$$\underline{\mp 4x^2 \mp 4x \mp 4}$$

$$6x - 1$$

Find the remaining two zeroes of the polynomial $h(y) = 3y^4 + 6y^3 - 2y - 10y5$ if the two zeroes of the polynomial is $\pm\sqrt{\frac{5}{3}}$.

(a) (- 1, 1) (b) (-1, - 1)

(c) (+ 1, 2) (d) (- 2 , 2)

(e) None of these

Answer: (b)

Explanation

$$
\begin{array}{r|l}
 & y^2+2y+1 \\ \hline
3y^2-5 & 3y^4-6y^3-2y^2-10y-5 \\
 & \underline{\pm 3y^4 \pm 0y^3k \mp 5y^2} \\
 & 6y^3+3y^2-10y-5 \\
 & \underline{6y^3 \pm 0y^2 \mp 10y} \\
 & 3y^2 \mp 5 \\
 & \underline{\pm 3y^2 \pm 5} \\
 & \quad 0
\end{array}
$$

Find the value of m and n such that z^2+1 is the factor of $g(z)=z^4+z^3+8z^2+mz+n.$.

(a) (- 1, - 7) (b) (- 1, - 1)
(c) (1, 2) (d) (1, 7)
(e) None of these

Answer: (d)

Find the value of k and p in the polynomial $m(z)=z^4-6z^3+16z^2-25z+10$ is divisible by $n(z)=z^2-2z+k$, gives the remainder z + p.

(a) $(k=-5, p=-7)$ (b) $(k=-5, p=-1)$
(c) $(k=5, p=-5)$ (d) $(k=1, p=7)$
(e) None of these

Answer: (c)

If the polynomial $g(m)=6m^4+8m^3+17m^2+21m+7$ is divisible by another polynomial $h(m)=3m^2+4m+1$ gives the remainder qm + a, then find the value of q and a.

(a) (q = 5, a = -7) (b) (q = 1, a = 2)
(c) (q = 2, a = -5) (d) (q = 1, a = -2)
(e) None of these

Answer: (b)

You Must KNOW

- The number of trees with 10 is the number 106.
- The smallest possible value of the longest edge in a Heronian Tetrahedron is the number 117.
- The smallest number to appear 6 times in Pascal's triangle is 120.
- The smallest number that is not of the form 12x - 3y | is 41.
- 50 is the smallest number that can be written as the sum of 2 squares in 2 ways.

SUMMARY

- For any polynomials $p(x) = a_0 + a_1x + a_2x^2 + ----+a_nx^n, x$ is a variable and $a_0, a_1, a_2, ---a_n$ are constants.
- The highest power of the variable in the polynomial is called degree of the polynomial.
- A polynomial of degree zero is called constant polynomial.
- A real number 'a' is called zeroes of the polynomial if F (a) = 0.
- A polynomial of degree n will have n zeroes.

Self Evaluation TEST

Duration 10 Minutes

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1. **If one of the zeroes of the polynomial $f(z)=(p^2+4)z^2+13z+40$ is reciprocal of the another root of the polynomial, then the value of p is:**

(a) ± 2
(b) - 2
(c) 1
(d) - 1
(e) None of these

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2. **If the zeroes of the polynomial $p(n)=2n^3-6n^2+5n-7$ are m - n, m and m + n, then find the value of m is:**

(a) 4
(b) 3
(c) 1
(d) 0
(e) None of these

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3. **If a, b, c are the zeroes of the polynomial $g(m)=m^3+\ell m^2+rm-q$, then 111 find the value of $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}$ is:**

(a) $-\frac{\ell}{q}$
(b) $-\frac{\ell}{qm}$
(c) $\frac{q}{\ell}$
(d) 0
(e) None of these

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4. **If l, m, n are the roots of the polynomial $z(y)=3y^3-8y^2+9y-48$, then find the value of $l^2+m^2+n^2$**

(a) $-\frac{1}{9}$
(b) $-\frac{10}{9}$
(c) $\frac{10}{9}$
(d) $\frac{1}{9}$
(e) None of these

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5. **If α,β,γ are the roots of the polynomials $h(n)=2n^3-6zn^2+9n-24$, then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$.**

(a) $-\frac{4}{81}$
(b) $-\frac{10}{9}$
(c) $-\frac{3}{8}$
(d) $\frac{3}{8}$
(e) None of these

6. **If a and b are the roots of the polynomial $f(x)=8x^2-62x+96$, then find 1 the value of $\frac{1}{a^4}+\frac{1}{b^4}$.**

(a) $\frac{259201}{5308416}$ (b) 0

(c) $\frac{1}{5308416}$ (d) $\frac{25481}{5308416}$

(e) None of these

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7. **If the zeroes of the polynomials $h(z)-z^3-3mz^2+nz-p$, are in A.P., then the relation between m, n, and p is given by:**

(a) $2m^3=mn+p$ (b) $2m^3=mn-p$

(c) $2n^3=mn+p$ (d) $2n^3=mn-p$

(e) None of these

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8. **Find the remaining zeroes of the polynomial $p(z)=2z^2+z^2-6z-3$, if two of its zeroes are $\sqrt{3}\, and -\sqrt{3}$.**

(a) $-\frac{1}{2}$ (b) 0

(c) $-\frac{1}{4}$ (d) $\frac{3}{5}$

(e) None of these

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9. **The cubic equation whose three roots are -3, 5, 8 is given by:**

(a) z^3+10z^2z+120 (b) $z^3=10z^2-z+120$

(c) $z^3-10z^2+z+120$ (d) $z^3-10z^2-z-120$

(e) None of these

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10. **Which one of the following is not the zeroes of the polynomial given by**
$f(y)=2m^4+7m^3-19m^2-14m+30$.

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) - 5 (d) $-2\sqrt{5}$

(e) None of these

Answers – Self Evaluation Test																			
1.	A	**2.**	C	**3.**	A	**4.**	C	**5.**	D	**6.**	A	**7.**	B	**8.**	A	**9.**	C	**10.**	D