CHAPTER 17

Quadratic Equations and Inequations

Quadratic Equation

Equation of order (degree) two are called quadratic equations.

The general quadratic equation is given by

$$ax^2 + bx + c = 0$$

where a, b, c are real numbers and $a \neq 0$.

- A quadratic equation containing both the second power and the first power of the variable is called a complete quadratic equation. *i.e.*, $ax^2 + bx + c = 0$
- A quadratic equation in which first power term is missing is called a pure quadratic equation *i.e.*, $ax^2 + c = 0$

Roots of a Quadratic Equation

A value of a variable which satisfies the particular quadratic equation is called root of that equation or solution of the equation.

e.g., The equation is $x^2 - 6x + 8 = 0$.

Here, if $x = 2$, then $2^2 - 6(2) + 8 = 0$

So, $x = 2$ is root of the quadratic equation.

Solutions of Quadratic Equations

1. By Factorization

If we are able to factorize $ax^2 + bx + c = 0$

As $(dx + e)(fx + g), d \neq 0, f \neq 0$

Then, $(dx + e)(fx + g) = 0$

or $(dx + e) = 0$ or $(fx + g) = 0$

$dx = -e$ or $fx = -g$

$x = -\frac{e}{d}$ or $x = -\frac{g}{f}$

So, $x = -e/d$ and $-g/f$ are roots of equation.

Example 1. Solve the equation $x^2 - 7x + 12 = 0$, then the value of x is

(a) $-3, -4$ (b) $-3, 4$ (c) $3, -4$ (d) $3, 4$

Sol. (*d*) $x^2 - 7x + 12 = 0$

Splitting the middle term

$x^2 - 3x - 4x + 12 = 0$

$\Rightarrow x(x - 3) - 4(x - 3) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow (x - 3) = 0$ or $(x - 4) = 0 \Rightarrow x = 3$ or $x = 4$

So, $x = 3, 4$ are roots of equation.

Example 2. Solve the equation $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$, then the value of x is

(a) $\frac{2b}{3a}, \frac{2b}{a}$ (b) $\frac{-2b}{3a}, \frac{2b}{a}$ (c) $\frac{-2b}{3a}, \frac{-2b}{a}$ (d) $\frac{2b}{3a}, \frac{-2b}{a}$

Sol. (*c*) The equation is

$3a^2x^2 + 8abx + 4b^2 = 0$

or $3a^2x^2 + 2abx + 6abx + 4b^2 = 0$

or $ax(3ax + 2b) + 2b(3ax + 2b) = 0$

$\therefore (3ax + 2b)(ax + 2b) = 0$

$\Rightarrow 3ax + 2b = 0$ or $ax + 2b = 0 \Rightarrow x = \frac{-2b}{3a}$ or $\frac{-2b}{a}$

2. By Using the Quadratic Formula

The roots of the equation $ax^2 + bx + c = 0$, where $a, b, c \in R$, $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If α and β be considered as roots of the quadratic equation, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- The quantity $b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$ and is denoted by D.

So, $D = b^2 - 4ac$

Example 3. Solve the equation $x^2-9x+18=0$, then the value of x is

(a) 3, 6 (b) −3, −6 (c) −3, 6 (d) 3, −6

Sol. (a) We have, $x^2-9x+18=0$ Here, $a=1, b=-9$ and $c=18$

$$\therefore \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-9)\pm\sqrt{(-9)^2-4(1)(18)}}{2(1)}$$

$$=\frac{9\pm\sqrt{81-72}}{2}=\frac{9\pm\sqrt{9}}{2}=\frac{9\pm3}{2}$$

$$\therefore \quad x=\frac{9+3}{2} \text{ or } x=\frac{9-3}{2}\Rightarrow x=6 \text{ or } 3$$

Nature of Roots of Equation

Let $D=b^2-4ac$ be the discriminant of the given equation, then $ax^2+bx+c=0$

(i) If $D>0$, then the two roots are real and unequal.
(ii) If $D=0$, then the two roots are real and equal.
(iii) If $D<0$, then there are no real root i.e., imaginary roots.
(iv) The roots are real when $D\geq 0$.

- If $D>0$ and D is perfect square, then roots are rational.
- If $D>0$ and D is not a perfect square, then roots are irrational.
- If one of the root of the quadratic equation is $a+\sqrt{b}$, then its other root is $a-\sqrt{b}$.

Sum and Product of the Roots

Let α, β be the roots of the equation $ax^2+bx+c=0$

(i) The sum of the roots $=\alpha+\beta=-\frac{b}{a}=\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

(ii) The product of the roots $=\alpha\beta=\frac{c}{a}=\frac{\text{constant term}}{\text{coefficient of } x^2}$

Some Important Conditions for the Roots of Quadratic Equation

1. Condition for having both the roots positive
$$\frac{-b}{a}>0 \text{ and } \frac{c}{a}>0.$$
2. Condition for having both the roots negative
$$\frac{-b}{a}<0 \text{ and } \frac{c}{a}<0$$
3. Condition for having both roots equal in magnitude but opposite in sign $\frac{-b}{a}=0$ or $b=0$.
4. Condition for having both the roots common of two quadratic equations $ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$
$$\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$$
5. Condition for having only one root common of two quadratic equations $ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$
$$\frac{x^2}{b_1c_2-c_1b_2}=\frac{-x}{a_2c_1-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$
6. If the roots of the equation $ax^2+bx+c=0$ are α and β, the equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ will be $cx^2+bx+a=0$
i.e., for this condition $a=c$
7. If the roots of the equation $ax^2+bx+c=0$ are reciprocals of each other i.e., roots are α and $\frac{1}{\alpha}$, then $a=c$

Formation of a Quadratic Equation

If the roots of equation are ogiven. Let roots be α and β, then

$$S=\text{sum f roots}=\alpha+\beta$$
$$P=\text{product of roots}=\alpha\beta$$

Then, quadratic equation is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

or
$$x^2-Sx+P=0$$

Equations Reducible to Quadratic Equations

The equations which at the out set are not quadratic equations, but can be reduced to quadratic equations by suitable substitutions are called equations reducible to quadratic equations.

We will explain the method of solution by the example.

Type I $ax^{2n}+bx^n+c=0$

Here, put $x^n=y\Rightarrow x^{2n}=y^2$ and equation reduces to

$$ay^2+by+c=0$$

Example 4. Solve the quadratic equation
$$x^4-26x^2+25=0,$$
then the value of x is

(a) ±1, ±25 (b) ±1, ±5 (c) 1, 5 (d) 1, 25

Sol. (b) Put $x^2=z\Rightarrow x^4=z^2$

$x^4-26x^2+25=0$ can be written as

$$z^2-26z+25=0$$
$$z^2-25z-z+25=0$$
$$z(z-25)-1(z-25)=0$$
$$(z-1)(z-25)=0 \Rightarrow z=1 \text{ or } z=25$$

when $z=1\Rightarrow x^2=1\Rightarrow x=\pm1$

when $z=25\Rightarrow x^2=25\Rightarrow x=\pm5$

Hence, $x=\pm1, \pm5$ is solution.

Type II $Px+\frac{Q}{x}=R$

Here, reduce the given equation to a quadratic equation by multiplying both sides by 'x'. So, $Px^2+Q=Rx$

or $$Px^2-Rx+Q=0$$

Example 5. Solve for x, $2x-\frac{3}{x}=5$, then the value of x is

(a) 1/3, −3 (b) 1/2, −3 (c) −1/2, 3 (d) 1/2, 4

Sol. (c) Here, $2x-\frac{3}{x}=5$

$$2x^2-3=5x$$
$$2x^2-5x-3=0$$
$$2x^2-6x+x-3=0$$
$$(2x+1)(x-3)=0 \Rightarrow x=-1/2 \text{ or } x=3$$

Hence, $x=-1/2, 3$ is solution.

Type III $\sqrt{x+b}=c$ or $\sqrt{a-x^2}=bx+c$

- Squaring both sides gives a quadratic equation without the radical.
- Solve the factorization or by quadratic formula.

Example 6. Solve $\sqrt{2x+9}+x=13$, then the value of x is

(a) 4, 10 (b) 8, 20 (c) 3, 15 (d) −4, −10

Sol. (b) $$\sqrt{2x+9}+x=13$$
$$\sqrt{2x+9}=13-x$$

Squaring both sides

$$(\sqrt{2x+9})^2=(13-x)^2$$
$$2x+9=169+x^2-26x$$
$$x^2-28x+160=0$$
$$x^2-20x-8x+160=0$$
$$(x-8)(x-20)=0 \Rightarrow x=8 \text{ or } x=20$$

Hence, $x=8, 20$ is solution.

Example 7. Solve $\sqrt{2x^2-2x+1}-2x+3=0$, then the value of x is

(a) 1, 4 (b) −1, −4
(c) 2, 3 (d) None of these

Sol. (a) $$\sqrt{2x^2-2x+1}-2x+3=0$$
$$\sqrt{2x^2-2x+1}=2x-3$$

Squaring both sides

$$(\sqrt{2x^2-2x+1})^2=(2x-3)^2$$
$$2x^2-2x+1=4x^2+9-12x$$
$$x^2-5x+4=0$$
$$x^2-4x-x+4=0$$
$$(x-4)(x-1)=0 \Rightarrow x=4 \text{ or } 1$$

Hence, $x=4, 1$ is solution.

Type IV Equation of the form $\sqrt{ax+b}\pm\sqrt{cx+d}=e$

or $$\sqrt{ax+b}\pm\sqrt{cx+d}\pm\sqrt{ex+f}=0$$

- Squaring both sides once, so that only one term containing radical is obtained.
- Keep only the term containing radical on one side and all other terms on the other side.
- Squaring again and solve the quadratic equation so obtained.

Example 8. Solve the following equation
$$\sqrt{4-x}+\sqrt{x+9}=5,$$
then the value of x is

(a) 1, 5 (b) −1, 3 (c) 0, −5 (d) 1, 4

Sol. (c) $$\sqrt{4-x}+\sqrt{x+9}=5$$

Squaring both sides

$$(\sqrt{4-x}+\sqrt{x+9})^2=25$$
$$(4-x)+(x+9)+2(\sqrt{4-x}\sqrt{x+9})=25$$
$$2(\sqrt{4-x})(\sqrt{x+9})=12$$
$$(\sqrt{4-x})(\sqrt{x+9})=6$$

Again, squaring both sides

$$(4-x)(x+9)=36$$
$$-x^2+36-5x=36$$
$$x^2+5x=0$$
$$x(x+5)=0 \Rightarrow x=0 \text{ or } x=-5$$

Hence, $x=0, -5$ is solution.

Example 9. Solve $\sqrt{11y-6}+\sqrt{y-1}-\sqrt{4y+5}=0$, then the value of y is

(a) {5, 6/5} (b) {1, 2}
(c) {3, 5/2} (d) None of these

Sol. (a) $$\sqrt{11y-6}+\sqrt{y-1}-\sqrt{4y+5}=0$$
$$\sqrt{11y-6}+\sqrt{y-1}=\sqrt{4y+5}$$

Squaring both sides

$$[\sqrt{11y-6}+\sqrt{y-1}]^2=[\sqrt{4y+5}]^2$$
$$(11y-6)+(y-1)+2\sqrt{(11y-6)(y-1)}=(4y+5)$$
$$12y-7+2\sqrt{11y^2-17y+6}=4y+5$$
$$2\sqrt{11y^2-17y+6}=12-8y$$
$$\sqrt{11y^2-17y+6}=6-4y$$

Again, squaring both sides

$$11y^2-17y+6=(6-4y)^2$$
$$11y^2-17y+6=36+16y^2-48y$$
$$\Rightarrow 5y^2-31y+30=0$$

Here, $a=5, b=-31, c=30$

$$\Rightarrow y=\frac{+31\pm\sqrt{(-31)^2-4(5)(30)}}{2\times5}$$
$$\Rightarrow y=\frac{31\pm\sqrt{361}}{10} \Rightarrow y=\frac{31\pm19}{10}$$
$$\Rightarrow y=\frac{31+19}{10} \text{ and } y=\frac{31-19}{10}$$
$$\Rightarrow y=\left\{5,\frac{6}{5}\right\} \text{ is solution of equation.}$$

Type V Equation of the type

$$(x+a)(x+b)(x+c)(x+d)+k=0$$

Here, k may or may not be zero.

- Sum of any two constants a, b, c, d is equal to the sum of the other.
- Multiply the products which satisfies first condition.
- Put first two term containing x^2 and x and solve it as follows.

Example 10. Solve the equation

$$(x+1)(x+2)(x+3)(x+4)-8=0,$$

then the number of real roots of this equation is

(a) 1 (b) 2
(c) 4 (d) No real roots

Sol. (b) $(x+1)(x+2)(x+3)(x+4)-8=0$

Here, $1+4=2+3$, so,

$$[(x+1)(x+4)][(x+2)(x+3)]-8=0$$
$$[x^2+5x+4][x^2+5x+6]-8=0$$

Put $x^2+5x=T$

$$(T+4)(T+6)-8=0$$
$$T^2+10T+24-8=0$$
$$T^2+10T+16=0$$
$$T^2+8T+2T+16=0$$
$$T(T+8)+2(T+8)=0$$
$$(T+8)(T+2)=0 \Rightarrow T=-8 \text{ or } -2$$

Now,

$x^2+5x=-8$	$x^2+5x=-2$
$x^2+5x+8=0$	$x^2+5x+2=0$
$x=\dfrac{-5\pm\sqrt{25-32}}{2}$	$x=\dfrac{-5\pm\sqrt{25-8}}{2}$
$x=\dfrac{-5\pm\sqrt{-7}}{2}$	$x=\dfrac{-5\pm\sqrt{17}}{2}$
No real roots.	

Hence, $x=\dfrac{-5\pm\sqrt{17}}{2}$ are roots.

Type VI $A^{x+a}+A^{-x+b}=C$

Here, $A=2,3,4,\ldots\ldots$

Example 11. Solve the equation $2^{x+2}+2^{-x}=5$, then the roots of the equation is

(a) {0, 2} (b) {−2, 0} (c) {0, 1} (d) {−1, 0}

Sol. (b) The given equation is $2^{x+2}+2^{-x}=5$

$$2^x2^2+2^{-x}=5$$
$$4\cdot2^x+\frac{1}{2^x}=5$$

Put $2^x=y$

$$\Rightarrow \quad 4y+\frac{1}{y}=5$$
$$\Rightarrow \quad 4y^2+1=5y \Rightarrow 4y^2-5y+1=0$$

Solving equation, we have

$$(y-1)(4y-1)=0$$
$$\Rightarrow \quad y-1=0 \text{ or } 4y-1=0$$
$$\Rightarrow \quad y=1 \text{ or } 4y=1 \Rightarrow y=1, \frac{1}{4}$$

By condition,

$2^x=y$	$2^x=y$
$2^x=1$	$2^x=\frac{1}{4}$
$2^x=2^0$	$2^x=\frac{1}{2^2}=2^{-2}$
$x=0$	$x=-2$

Here, $x=\{0,-2\}$ are roots of equation.

Example 12. Solve the equation

$$2^{2y+3}=65(2^y-1)+57,$$

then the value of y is

(a) {−3, 3} (b) {0, 3} (c) {−3, 0} (d) {−2, 2}

Sol. (a) $2^{2y+3}=65(2^y-1)+57$

$$\Rightarrow \quad 2^{2y}\cdot2^3=65\cdot2^y-65+57 \Rightarrow 8\cdot2^{2y}=65\cdot2^y-8$$
$$\Rightarrow \quad 8\cdot2^{2y}-65\cdot2^y+8=0$$

Put $2^y=x \Rightarrow 2^{2y}=x^2 \Rightarrow 8x^2-65x+8=0$

$$\Rightarrow \quad (8x-1)(x-8)=0 \Rightarrow x=1/8, 8$$

By condition,

$2^y=x$	$2^y=x$
$2^y=\frac{1}{8}$	$2^y=8$
$2^y=\frac{1}{2^3}$	$2^y=2^3$
$2^y=2^{-3}$	$y=3$
$y=-3$	

So, $y=\{3,-3\}$ is the solution.

Type VII Equation of the form

(i) $a\left(x^2+\dfrac{1}{x^2}\right)+b\left(x+\dfrac{1}{x}\right)+c=0$

(ii) $a\left(x^2+\dfrac{1}{x^2}\right)+b\left(x-\dfrac{1}{x}\right)+c=0$

Put $\left(x+\dfrac{1}{x}\right)=y$ in case (i) and find $x^2+\dfrac{1}{x^2}$ and $\left(x-\dfrac{1}{x}\right)=y$ in case (ii) and find $x^2+\dfrac{1}{x^2}$ and evaluate as follows.

Example 13. Solve for value of x

$$4\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-7=0; \; x\neq0, \text{ which is}$$

(a) {−2, −1/2} (b) {0, 2}
(c) {2, 1/2} (d) None of these

Sol. (c) $4\left(x^2+\dfrac{1}{x^2}\right)-4\left(x+\dfrac{1}{x}\right)-7=0$ ...(i)

Put $x+\dfrac{1}{x}=y$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=y^2 \Rightarrow x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

$\Rightarrow \quad 4(y^2-2)-4y-7=0,$ [from Eq. (i)]

$\Rightarrow \quad 4y^2-8-4y-7=0 \Rightarrow 4y^2-4y-15=0$

$\Rightarrow \quad (2y-5)(2y+3)=0 \Rightarrow y=\frac{5}{2}$ or $-\frac{3}{2}$

But $x+\frac{1}{x}=y$, so

For $y=\frac{5}{2}$	For $y=-\frac{3}{2}$
$x+\frac{1}{x}=\frac{5}{2}$	$x+\frac{1}{x}=-\frac{3}{2}$
$\frac{x^2+1}{x}=\frac{5}{2}$	$\frac{x^2+1}{x}=-\frac{3}{2}$
$2x^2+2=5x$	$2x^2+2=-3x$
$2x^2-5x+2=0$	$2x^2+3x+2=0$
$(x-2)(2x-1)=0$	Here, $D=9-16$
$\Rightarrow x=2$ or $x=\frac{1}{2}$	$(\because D=b^2-4ac)$
	$D=-7$
	So, no real root.

Here, $x=\left\{2,\frac{1}{2}\right\}$ is solution.

Type VIII Equation of the type

$$ax^4+bx^3+cx^2+bx+a=0$$

- Coefficient of x^4 and constant value should be same.
- Coefficient of x^3 and coefficient of x should be same.
- Divide the equation by x^2.
- Collect the like term.
- Now, follow steps of type VII

Example 14. Solve $x^4-3x^3-2x^2+3x+1=0$, $x\neq 0$, then the roots of the equation is

(a) $\left\{\pm 2, \frac{-3\pm\sqrt{13}}{2}\right\}$ (b) $\left\{\pm 1, \frac{3\pm\sqrt{13}}{2}\right\}$

(c) $\left\{\pm 3, \frac{3\pm\sqrt{11}}{2}\right\}$ (d) None of these

Sol. (b) $x^4-3x^3-2x^2+3x+1=0$

On dividing throughout by x^2 and rearranging the term

$$\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0 \quad \text{...(i)}$$

Put $\quad x-\frac{1}{x}=y$

Squaring both sides

$$x^2+\frac{1}{x^2}-2=y^2$$

$\Rightarrow \quad x^2+\frac{1}{x^2}=y^2+2$

$\Rightarrow \quad (y^2+2)-3y-2=0,$ [from Eq. (i)]

$\Rightarrow \quad y^2-3y=0$

$\Rightarrow \quad y(y-3)=0$

$\Rightarrow \quad y=0$ or 3

For $y=0$	For $y=3$
$x-\frac{1}{x}=0$	$x-\frac{1}{x}=3$
$\frac{x^2-1}{x}=0$	$\frac{x^2-1}{x}=3$
$x^2=1$	$x^2-3x-1=0$
$x=\pm 1$	$x=\frac{3\pm\sqrt{9+4}}{2}$
	$x=\frac{3\pm\sqrt{13}}{2}$

So, $x=\left\{\pm 1, \frac{3\pm\sqrt{13}}{2}\right\}$ is solution of equation.

Symmetric Functions of the Roots

An expression in α, β is called symmetrical if by interchanging α and β, the expression is not changed.

e.g. $\alpha^2+\beta^2, \alpha^3+\beta^3, \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$; etc.

Formulae to be Remember

- $(\alpha^2+\beta^2)=[(\alpha+\beta)^2-2\alpha\beta]$
- $(\alpha^3+\beta^3)=[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)]$
- $(\alpha-\beta)^2=[(\alpha+\beta)^2-4\alpha\beta]$
- $(\alpha^3-\beta^3)=[(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)]$
- $(\alpha^2-\beta^2)=(\alpha+\beta)(\alpha-\beta)=(\alpha+\beta)[\sqrt{(\alpha+\beta)^2-4\alpha\beta}]$
- $(\alpha^4+\beta^4)=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2=[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2$
- $\alpha^4-\beta^4=(\alpha+\beta)(\alpha^2+\beta^2)(\alpha-\beta)$
  $=(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta]\cdot\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

Inequation

An inequation is a statement involving a sign of inequality. An inequation may contain one or more variables just like an equation. *e.g.*, $px+q\geq 0$, $ax+by+c\leq 0$, $ax+by+c>0$ etc. are inequations.

- The symbol $>, <, \geq, \leq$ are called the signs of inequations.
- The values of the variables for which an inequations holds true are called the solutions of the inequation or solution space or solution set of inequation.

General Rules for Solving an Inequation Algebraically

- A quantity or constant can be added to or subtracted from both sides of an inequation. *e.g.*, If $p\leq q$, then $p+x\leq q+x$
- We can multiply (or divide) both sides of an inequation by a positive number. *e.g.*, $a\leq b \Rightarrow 4a\leq 4b$

- When an inequation is multiplied both sides by a negative number, the signs of inequality are reversed.

e.g., $x \le y \Rightarrow -2x \ge -2y$
$a \ge b \Rightarrow -3a \le -3b$

- The square root on both the sides of an inequation cannot be taken in the new take the square root on both sides of an equation.

e.g., $x^2 = 16 \Rightarrow x = \pm 4$

But $x^2 \le 16 \Rightarrow x \le \pm 4$

But, it is taken as

$x^2 \le 16 \Rightarrow |x| \le 4 \Rightarrow -4 \le x \le 4$

or $x^2 \ge 16 \Rightarrow |x| \ge 4 \Rightarrow x \le -4$ or $x \ge 4$

Linear Inequations

An inequation is said to be linear if each term of the algebraic expressions of the inequation contains variable of first degree.

e.g., $px + qy \le 0$

Graphical Solution of Linear Inequations in Two Variables

Let inequation be $ax + by + c \ge 0$.

Step 1 Consider it as $ax + by + c = 0$. Draw the graph of this equation.

Step 2 Choose any point [if possible (0, 0)]

If it satisfied the given inequation, then the shaded part of the plane contains the point, otherwise shaded the other part as solution.

Example 15. The solution set of inequation $2x + 3y \ge 6$, $x \ge 0$ and $y \ge 0$ is

(a) $x = 1, y = 1$
(b) $x = -1, y = -1$
(c) $x = 3, y = 1$
(d) None of these

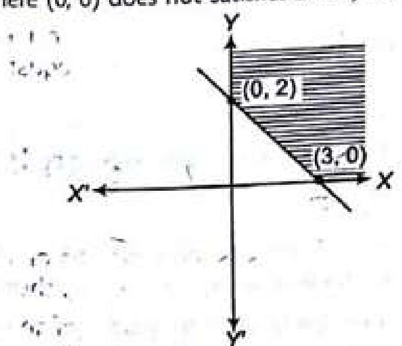
Sol. (c) Here, draw the graph of $2x + 3y \ge 6$.

Let $2x + 3y = 6$

The values of (x, y) satisfying $2x + 3y = 6$ are

x	0	3
y	2	0

Clearly, here (0, 0) does not satisfies $2x + 3y \ge 6$.



Also, $x \ge 0$ consists of y-axis and plane on the right hand side of y-axis.

While $y \ge 0$ consists of x-axis and the plane above x-axis.

Hence, the shaded region represents the solution set of the given system of inequations.

Example 16. The solution set of the following simultaneous linear inequations $x + 2y \le 10$, $x + y \le 6$, $x \le 4$, $x \ge 0$ and $y \ge 0$ is

(a) $x = 6, y = 3$
(b) $x = 4, y = 5$
(c) $x = 2, y = 2$
(d) None of these

Sol. (c) Consider $x + y \le 6$ ...(i)

Draw graph for $x + y = 6$

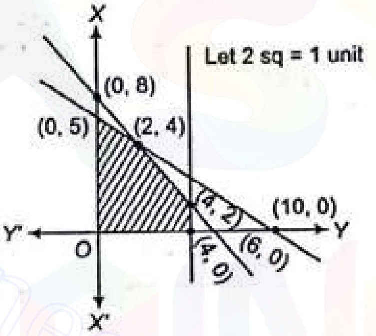
The values of x, y satisfying $x + y = 6$ are

x	0	6
y	6	0

Also, value of (x, y) satisfying $x + 2y = 10$ are

x	0	4
y	5	3

Clearly, here (0, 0) does not lies on both equations but satisfies the inequations. The equation $x = 4$ is a straight line parallel to y-axis and $x \le 4$ represent the region on left hand side of $x = 4$.



Also, $x \ge 0$ consists of y-axis and plane on the right hand side of y-axis, also $y \ge 0$ consists of x-axis and the plane above x-axis. So, the shaded region represent solution set.

Quadratic Inequations

An equation of the form

$$ax^2 + bx + c \ge 0$$

or $$ax^2 + bx + c \le 0$$

or $$ax^2 + bx + c > 0$$

or $$ax^2 + bx + c < 0$$

where $a \ne 0$ is called a quadratic inequation in one variable x.

- Can be solved graphically or algebraically.

Solution of Quadratic Inequation

- Factorize the quadratic inequation.
- If discriminant of $b^2 - 4ac$ of the corresponding equation $ax^2 + bx + c = 0$ is positive, then $ax^2 + bx + c$ will always have distinct linear factor.
- When the product of the two factors is positive or > 0, then either both the factor are positive or both are negative.

- When the product of the two factors is negative, then two factors will be of opposite signs.
- If $b^2 - 4ac = 0$, then $ax^2 + bx + c$ will be a perfect square.
- If $b^2 - 4ac < 0$, then $ax^2 + bx + c$ will be not have any real factor *i.e.*, imaginary factors.

Example 17. The inequation $x^2 + 4x + 3 \geq 0$ have the solution is

(a) $x < -3$ and $x > -1$ (b) $x \leq -3$ and $x \geq -1$
(c) $x < -3$ and $x \geq -1$ (d) $x \leq -3$ or $x > -1$

Sol. (*b*) We have, $x^2 + 4x + 3 \geq 0$

$\Rightarrow \quad x^2 + 3x + x + 3 \geq 0$

$\Rightarrow \quad (x+1)(x+3) \geq 0$

Either $(x+1) \geq 0$ and $x + 3 \geq 0$ or $x + 1 \leq 0$ and $(x+3) \leq 0$

$\Rightarrow \quad x \geq -1$ and $x \geq -3$ or $x \leq -1$ and $x \leq -3$

$\Rightarrow \quad x \geq -1$ or $x \leq -3$

Let a and b be real numbers, such that $a<b$

(i) $x < b \Leftrightarrow (x-a) > 0$ and $(x-b) > 0 \Leftrightarrow (x-a)(x-b) > 0$

(ii) $x < a \Leftrightarrow (x-a) < 0$ and $(x-b) < 0 \Leftrightarrow (x-a)(x-b) > 0$

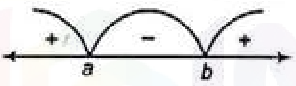
(iii) $a < x < b \Leftrightarrow (x-b) < 0$ and $(x-a) > 0 \Leftrightarrow (x-a)(x-b) < 0$

How to be Remember?

Let a and b be real numbers, such that $a < b$, then

(i) $(x-a)(x-b) > 0$ when $x > b$ also when $x < a$.
If > 0, then choose positive loops.

(ii) $(x-a)(x-b) < 0$ when $a < x < b$.



If < 0 choose the negative loops.

Example 18. The solution set of the inequation $2x^2 - 5x + 2 \leq 0$ is

(a) (–1/2, 2) (b) (–1/2, 2) (c) [–1/2, 2) (d) [–1/2, 2]

Sol. (*d*) $2x^2 - 5x + 2 \leq 0 \Leftrightarrow (2x+1)(x-2) \leq 0$

$\Leftrightarrow \quad 2\left(x + \frac{1}{2}\right)(x-2) \leq 0$

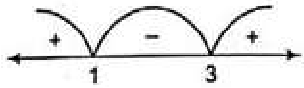
$\Leftrightarrow \quad \left(x + \frac{1}{2}\right)(x-2) \leq 0$ as $2 \neq 0 \Leftrightarrow -\frac{1}{2} \leq x \leq 2$

$\therefore$ Solution set is $\left[-\frac{1}{2}, 2\right]$.

Example 19. The real values of x which satisfy $x^2 - 4x + 3 \geq 0$ and $x^2 - 3x - 4 \leq 0$ is

(a) $(-1, 1) \cup (3, 4)$ (b) $[-1, 1] \cap [3, 4]$
(c) $[-1, 1] \cup [3, 4]$ (d) $(-1, 1) \cup [3, 4]$

Sol. (*c*) Here, $x^2 - 4x + 3 \geq 0$

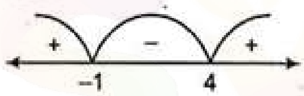


$\Rightarrow \quad (x-1)(x-3) \geq 0 \Leftrightarrow (x \leq 1$ or $x \geq 3)$

$\Leftrightarrow \quad x \in]-\infty, 1]$ or $x \in [3, \infty[$

$\Leftrightarrow \quad x \in]-\infty, 1] \cup [3, \infty[$

Again, $x^2 - 3x - 4 \leq 0$



$\Leftrightarrow \quad (x+1)(x-4) \leq 0 \Leftrightarrow x \in [-1, 4]$

$\therefore$ Solution set for x

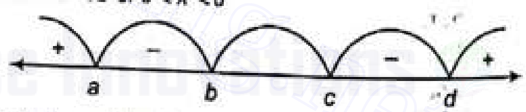
$=]-\infty, 1] \cup [3, \infty[\cap [-1, 4] = [-1, 1] \cup [3, 4]$

Formulae to be Remember

If a, b, c, d are real numbers such that $a < b < c < d$, then

(i) $(x-a)(x-b)(x-c)(x-d) > 0$
when $x > d$ or $d < x < c$ or $x < a$

(ii) $(x-a)(x-b)(x-c)(x-d) < 0$
when $a < x < b$ or $c < x < d$



i.e., for > 0 take positive loops
and for < 0 take negative loops.

Exercise

1. The solution set for equation $4x^2 - 6x = 0$ when $x \in N$ is

(a) {0, 1} (b) {1, 2} (c) {0} (d) ϕ

2. The quadratic equation has the maximum

(a) one root (b) two roots (c) four roots (d) three roots

3. Which of the following are quadratic equations?

I. $x^2 + \frac{1}{x^2} = 2$

II. $x + \frac{3}{x} = x^2$

III. $2x^2 - x + 2 = x^2 + 4x - 4$

IV. $x^3 + 6x^2 + 2x - 1 = 0$

(a) I, III are quadratic (b) II, III and IV are quadratic
(c) III is only quadratic (d) None of the above

4. In each of the following, determine which of the given values are solutions of the equation.

I. $3x^2 - 2x - 1 = 0;\ x = 1$

II. $x^2 + \sqrt{2}x - 4 = 0;\ x = \sqrt{2}, x = -2\sqrt{2}$.

III. $x^2 + x + 1 = 0;\ x = 1, x = -1$.

IV. $9x^2 - 3x - 2 = 0;\ x = -\frac{1}{3}, x = \frac{2}{3}$.

(a) I and II only (b) I, II and IV
(c) III, IV only (d) IV only

5. The values of x in the equation $a^2b^2x^2-(a^2+b^2)x+1=0,\ a\neq 0,\ b\neq 0$ is
   (a) $1/a^2$ (b) $1/b^2$
   (c) $1/a^2, 1/b^2$ (d) None of these

6. If $6\leq x\leq 8$, then which one of the following is correct? **(CDS 2011 II)**
   (a) $(x-6)(x-8)\geq 0$ (b) $(x-6)(x-8)>0$
   (c) $(x-6)(x-8)\leq 0$ (d) $(x-6)(x-8)<0$

7. The product of the roots of $x^2-3kx+2k^2-1=0$ is 7 for a fixed k. What is the nature of roots? **(CDS 2007 I)**
   (a) Integral and positive (b) Integral and negative
   (c) Irrational (d) Rational but not integral

8. The value of 'a' for which the equation $ax^2-2\sqrt{5}\,x+4=0$ has equal roots is
   (a) 5/4 (b) 4/5 (c) $-5/4$ (d) $-5/3$

9. Match list I with list II. List I contains quadratic polynomials and list II contains the conditions for these polynomials to be factorizable into a product of real linear factors.

	List I		List II
A.	$4x^2+kx+1$	1.	$k\leq 1/2$
B.	kx^2-4x+k	2.	$k\geq 4$ or $k\leq -4$
C.	kx^2-2x+2	3.	$k\geq 8$ or $k\leq 0$
D.	$2x^2-kx+k$	4.	$-2\leq k\leq 2$

Codes

	A	B	C	D		A	B	C	D
(a)	4	1	3	2	(b)	3	2	1	4
(c)	1	3	4	2	(d)	2	4	1	3

10. The equation $(1+m^2)x^2+2mcx+c^2-a^2=0$ has equal roots, if
    (a) $a^2=c^2(1-m^2)$ (b) $c^2=a^2(1-m^2)$
    (c) $a^2=c^2(1+m^2)$ (d) $c^2=a^2(1+m^2)$

11. If one root of $3x^2=8x+(2k+1)$ is seven times the other, then the value of k is
    (a) 5/3 (b) $-5/3$ (c) 2/3 (d) $-3/2$

12. Match list I with list II

	List I		List II
A.	Roots of $2x^2-13x+21=0$	1.	7/2 and 1
B.	Roots of $2x^2-9x+7=0$	2.	3 and 7/2
C.	Roots of $x^2-6x+9=0$	3.	3 and 3
D.	Roots of $2x^2-21x+49=0$	4.	7 and 7/2

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	4	1	3	2
(c)	2	1	3	4	(d)	1	4	3	2

13. The quadratic equation whose roots are $\frac{4+\sqrt{7}}{2}$ and $\frac{4-\sqrt{7}}{2}$ is
    (a) $4x^2+16x+9=0$ (b) $4x^2-16x-9=0$
    (c) $4x^2-16x+9=0$ (d) $4x^2+16x-9=0$

14. If one of the roots of the equation $ax^2+x-3=0$ is -1.5, then what is the value of a? **(CDS 2010 I)**
    (a) 4 (b) 3 (c) 2 (d) -2

15. The solution set of inequation $2x+1\geq 7$ is
    (a) $x\geq 8$ (b) $x\geq 3$
    (c) $x\geq 6$ (d) None of these

16. The values of x satisfying inequation $\frac{x-1}{3}\geq 4$ is
    (a) $x\leq 13$ (b) $x\geq 12$ (c) $x\geq 13$ (d) $x=13$

17. The values of x satisfying $-2x+3\geq 11$ is
    (a) $x\leq -4$ (b) $x\leq 4$ (c) $x\geq 4$ (d) $x\leq 14$

18. The values of x satisfying $3x+2\leq 5x-(4-x)$ is
    (a) $x\leq 2$ (b) $x\geq 2$
    (c) $x=2$ (d) None of these

19. If one root of the equation $ax^2+x-3=0$ is -1, then what is the other root? **(CDS 2010 II)**
    (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

20. How many real values of x satisfy the equation $x^{2/3}+x^{1/3}-2=0$? **(CDS 2007 I)**
    (a) Only 1 value (b) 2 values
    (c) 3 values (d) No value

21. If α and β are the roots of the equation $x^2-8x+P=0$ and $\alpha^2+\beta^2=40$, then P is equal to
    (a) 12 (b) 10 (c) 9 (d) 11

22. With respect to the roots of $x^2-x-2=0$, we can say that
    (a) both of them are integers
    (b) both of them are natural numbers
    (c) the latter of the two is negative
    (d) None of the above

23. Match list I with list II.
    Let $f(x)=ax^2+bx+c$ be a quadratic equation

	List I		List II
A.	Roots of $f(x)=0$ are real and equal	1.	$b^2-4ac<0$
B.	Roots of $f(x)=0$ are real and distinct	2.	$b^2-4ac>0$
C.	Product of roots of $f(x)=0$	3.	$b^2-4ac=0$
D.	Sum of roots of $f(x)=0$	4.	c/a
		5.	$-b/a$

Codes

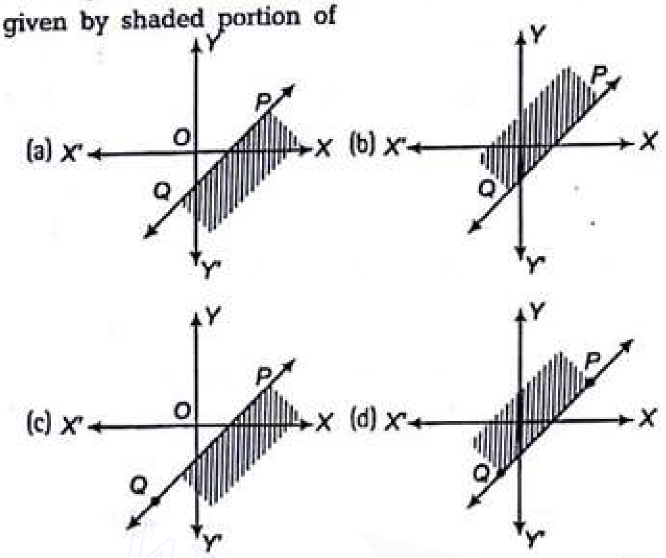
	A	B	C	D		A	B	C	D
(a)	4	2	1	5	(b)	3	2	4	5
(c)	1	3	4	5	(d)	1	2	4	5

24. If α and β are the roots of the equation $x^2-5x+6=0$, then the value of $\alpha^2-\beta^2$
    (a) 5 (b) -5 (c) ± 5 (d) ± 4

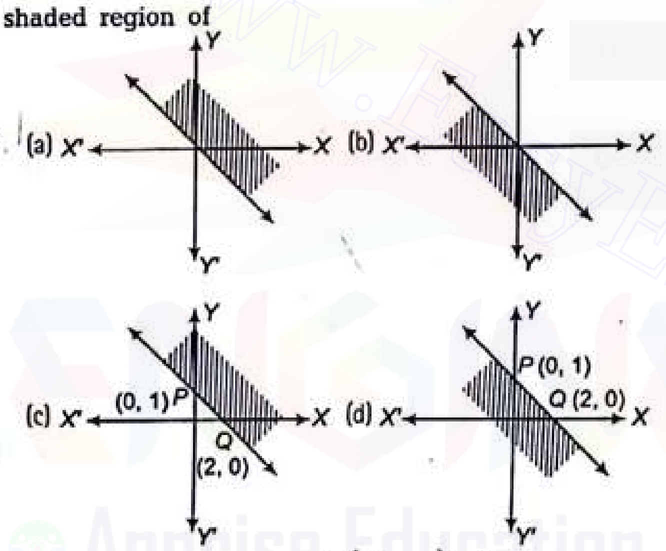
25. What is the magnitude of difference of the roots of $x^2-ax+b=0$? **(CDS 2009 I)**
    (a) $\sqrt{a^2-4b}$ (b) $\sqrt{b^2-4a}$ (c) $2\sqrt{a^2-4b}$ (d) $\sqrt{b^2-4ab}$

26. The solution set of x for the inequations $2x+3\geq 8$ and $3x+1\leq 12$ is
    (a) $\frac{5}{2}<x\leq\frac{11}{3}$ (b) $\frac{5}{2}<x<\frac{11}{3}$ (c) $\frac{5}{2}\leq x\leq\frac{11}{3}$ (d) $\frac{5}{2}\geq x\geq\frac{11}{3}$

27. The region represented by the inequation $2x - y \geq 1$ is given by shaded portion of



28. The region represented by $x + 2y > 2$ is given by shaded region of



29. The values of x satisfying $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$ are

(a) $x \geq 120$ (b) $x \leq 120$ (c) $x \leq 12$ (d) $x \geq 12$

30. $|x| > 3$, then the values of x are

(a) $x \leq 3$ or $x < -3$ (b) $x > -3$ or $x < 3$
(c) $x < -3$ or $x < 3$ (d) $x < -3$ or $x > 3$

31. What are the roots of the equation $4^x - 3 \cdot 2^{x+2} + 32 = 0$? **(CDS 2010 II)**

(a) 1, 2 (b) 3, 4 (c) 2, 3 (d) 1, 3

32. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

(a) $bx^2 + ax + c = 0$ (b) $ax^2 - bx + c = 0$
(c) $cx^2 + ax + b = 0$ (d) $cx^2 + bx + a = 0$

33. If α, β are the roots of a quadratic equation such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, then the equation is

(a) $x^2 - 24x - 128 = 0$ (b) $x^2 + 24x + 128 = 0$
(c) $x^2 + 24x - 128 = 0$ (d) None of these

34. What are the roots of the quadratic equation $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$? **(CDS 2011 II)**

(a) $\frac{1}{a^2}, \frac{1}{b^2}$ (b) $-\frac{1}{a^2}, -\frac{1}{b^2}$ (c) $\frac{1}{a^2}, \frac{1}{b^2}$ (d) $-\frac{1}{a^2}, \frac{1}{b^2}$

35. The side (in cm) of a right triangle are $x - 1$, x and $x + 1$. Then, the area of triangle is

(a) $x(x + 1)\,\text{cm}^2$ (b) $7\,\text{cm}^2$
(c) $6\,\text{cm}^2$ (d) $(x^2 - 1^2)\,\text{cm}^2$

36. Divide 16 into two parts such that the twice of the square of the greater part exceeds, the square of the smaller part by 164. Then, the greater part is

(a) 58 (b) 10 (c) 6 (d) 15

37. The number of straight lines that can connect 'x' points is given by the equation $y = \frac{x(x-1)}{2}$. How many points does a figure have if only 15 lines can be drawn connecting them?

(a) 15 (b) 10 (c) 6 (d) 5

38. If α, β are the roots of the quadratic equation $2x^2 - 4x + 1 = 0$. Then, the value of $\frac{1}{\alpha + 2\beta} + \frac{1}{\beta + 2\alpha}$ is equal to

(a) $\frac{12}{17}$ (b) $\frac{17}{12}$ (c) $\frac{11}{17}$ (d) $\frac{13}{17}$

39. An equation equivalent to the quadratic equation $x^2 - 6x + 5 = 0$ is

(a) $x^2 - 5x + 6 = 0$ (b) $5x^2 - 6x + 1 = 0$
(c) $|x - 3| = 2$ (d) $6x^2 - 5x + 1 = 0$

40. Consider the following statements

I. A quadratic equation can have maximum two roots.
II. If α, β are the roots of the equation $2x^2 - 3x + 1 = 0$, then roots of equation $x^2 - 3x + 2 = 0$ are $\frac{1}{\alpha}, \frac{1}{\beta}$.
III. If the roots of the equation $ax^2 + bx + c = 0$ are negative reciprocal of each other, then $a + c = 0$.
IV. If α, β are the roots of the equation $2x^2 - 4x + 1 = 0$, the value of $\frac{1}{\alpha + 2\beta}, \frac{1}{\beta + 2\alpha}$ is $\frac{12}{17}$.

Of these statements

(a) I, II are correct (b) I, II and IV are correct
(c) All are correct (d) None is correct

41. The roots of the equation $x^2 + px + q = 0$ are 1 and 2. The roots of the equation $qx^2 - px + 1 = 0$ must be

(a) $\frac{-1}{2}$ and 1 (b) $\frac{1}{2}$ and 1
(c) $\frac{-1}{2}$ and -1 (d) None of these

42. Which one of the following is the quadratic equation whose roots are reciprocal to the roots of the quadratic equation $2x^2 - 3x - 4 = 0$? **(CDS 2008 II)**

(a) $3x^2 - 2x - 4 = 0$ (b) $4x^2 + 3x - 2 = 0$
(c) $3x^2 - 4x - 2 = 0$ (d) $4x^2 - 2x - 3 = 0$

43. The sum of the roots of the equation $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$ is zero. What is the product of the roots of the equation? **(CDS 2010 I)**

(a) $-\frac{(a+b)}{2}$ (b) $\frac{(a+b)}{2}$ (c) $-\frac{(a^2+b^2)}{2}$ (d) $\frac{(a^2+b^2)}{2}$

44. $4x^2-1\le 0$, then the solution set is

(a) $\frac{-1}{2}\le x\le\frac{1}{2}$ (b) $\frac{-1}{2}<x<\frac{1}{2}$

(c) $\frac{-1}{2}\ge x\ge\frac{1}{2}$ (d) None of these

45. $4+x^2-4x\ge 0$, then x belongs to

(a) I^+ (b) I^- (c) I (d) R

46. $x^2-10x<-25$, then x belongs to

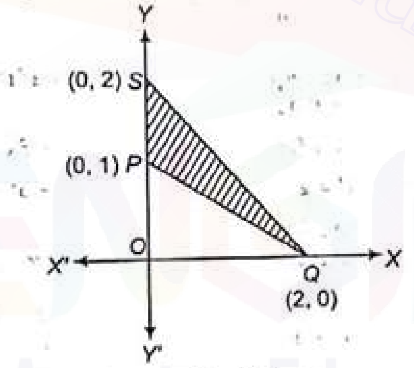
(a) I^+ (b) I^- (c) I (d) ϕ

47. The region specified by $x\ge 0, x+y\ge 0$ includes which of the following as a whole

(a) Ist quadrant (b) IInd quadrant

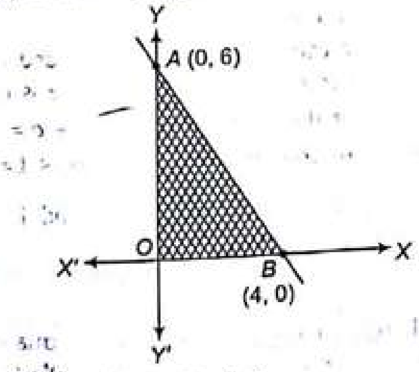
(c) IIIrd quadrant (d) IVth quadrant

48. The shaded region, including the boundary in the given figure is exactly represented by



(a) $x+2y\ge 2, x+y<2, x\ge 0$
(b) $x+2y\ge 2, x+y\le 2, x>0$
(c) $x+2y>2, x+y\le 2, x>0$
(d) $x+2y\ge 2, x+y\le 2, x\ge 0$

49. The shaded region, including the boundary in the give graph, is exactly represented by



(a) $3x+2y\le 12, x<0, y\ge 0$
(b) $3x+2y\le 12, x\ge 0, y\ge 0$
(c) $3x+2y<12, x\ge 0, y\ge 0$
(d) $3x+2y>12, x\ge 0, y\ge 0$

50. The value of x satisfying the equation $\sqrt{x+4}=x-2$ is

(a) 0, 5 (b) 0, 4

(c) 5 (d) None of these

51. The two successive natural number whose squares have sum 221 are

(a) 10 and 11 (b) 11 and 12

(c) -10 and -11 (d) None of these

52. The two consecutive positive odd integers the sum of whose squares is 130.

(a) -7 and -9 (b) 7 and 9

(c) 7 and 5 (d) 3 and -5

53. If α and β are the roots of the equation $x^2+px+q=0$, then $-\alpha^{-1}, -\beta^{-1}$ are the roots of which one of the following equations? **(CDS 2010 II)**

(a) $qx^2-px+1=0$ (b) $q^2+px+1=0$

(c) $x^2+px-q=0$ (d) $x^2-px+q=0$

54. If the roots of the quadratic equation $px^2+qx+r=0$ are reciprocal to each other, then

(a) $q=r$ (b) $p=r$

(c) q divides r (d) p divides q

55. Sum of roots is -1 and sum of their reciprocals is 1/6, then equation is

(a) $x^2-6x+1=0$ (b) $x^2-x+6=0$

(c) $6x^2+x+1=0$ (d) $x^2+x-6=0$

56. If sum of the roots of the equation $ax^2+bx+c=0$ is equal to the sum of their squares, then which one of the following is correct? **(CDS 2007 I)**

(a) $a^2+b^2=c^2$ (b) $a^2+b^2=a+b$

(c) $2ac=ab+b^2$ (d) $2c+b=0$

57. If the roots of $x^2-lx+m=0$ differ by 1, then

(a) $l^2=4m-1$ (b) $l^2=4m+2$

(c) $l=4m^2+1$ (d) $l^2=4m+1$

58. If α, β are the roots of the equation $x^2-(1+a^2)x+\frac{1}{2}(1+a^2+a^4)=0$, then $\alpha^2+\beta^2$ is equal to

(a) a^4+a^2 (b) a^2

(c) $(a^2+a^4)^2$ (d) None of these

59. If the roots of $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude and opposite in sign, then product of roots is

(a) $-\frac{1}{2}(p^2+q^2)$ (b) $\frac{p^2+q^2}{2}$

(c) $\frac{p+q}{2}$ (d) $\frac{1}{2}(p+q)^2$

60. If α, β are the roots of $3x^2+2x+1=0$, then the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$ is

(a) $x^2+2x+3=0$ (b) $x^2-2x+3=0$

(c) $x^2+2x-3=0$ (d) $x^2-2x-3=0$

61. If x is real, then the value of the polynomial $\dfrac{x^2+2x+1}{x^2+2x+7}$ lies between

(a) 1 and 2 (b) -1 and 1
(c) 0 and 1 (d) 1/2 and 1

62. If one root of $px^2+qx+r=0$ is double of the other root, then which one of the following is correct? **(CDS 2007 II)**

(a) $2q^2=9pr$ (b) $2q^2=9p$
(c) $4q^2=9r$ (d) $9q^2=2pr$

63. If the roots of the equation $x^2+x+1=0$ are in the ratio $m:n$, then

(a) $\sqrt{\dfrac{m}{n}}+\sqrt{\dfrac{n}{m}}+1=0$ (b) $\sqrt{m}+\sqrt{n}+1=0$
(c) $\dfrac{m}{n}+\dfrac{n}{m}+1=0$ (d) $m+n+1=0$

64. A group of students decided to buy a tape recorder from ₹ 170 to ₹ 195. But at the last moment two students backed out of the decision so that the remaining students had to pay ₹ 1 more than they had planned. What was the price of the tape recorder if the students paid equal shares?

(a) ₹ 175 (b) ₹ 180 (c) ₹ 185 (d) ₹ 190

65. A factory kept increasing its output by the same percentage every year. Find the percentage, if it is known that the output doubled in the last two years

(a) $100(\sqrt{2}+1)\%$ (b) $100(\sqrt{2}-1)\%$
(c) $\left(\dfrac{100-1}{\sqrt{2}}\right)\%$ (d) 25%

66. What is one of the value of x in the equation $\sqrt{\dfrac{x}{1-x}}+\sqrt{\dfrac{1-x}{x}}=\dfrac{13}{6}$? **(CDS 2007 II)**

(a) $\dfrac{5}{13}$ (b) $\dfrac{7}{13}$ (c) $\dfrac{9}{13}$ (d) $\dfrac{11}{3}$

67. The solution set for the quadratic inequation $x^2-5x+6>0$ is

(a) $]-\infty,2[\cup]3,\infty[$ (b) $]-\infty,2]\cup]3,\infty[$
(c) $]-\infty,2]\cup[3,\infty[$ (d) $[2,3]$

68. The values of x satisfying the quadratic inequation $x^2-8x+16\le 0$ is/are

(a) $]-\infty,4]$ (b) $[-4,4]$ (c) 4 (d) R

69. All real values of x for which $\dfrac{x-2}{3x+1}<\dfrac{x-3}{3x-2}$ are

(a) $\left]\dfrac{-1}{3},\dfrac{2}{3}\right]$ (b) $\left[\dfrac{-1}{3},\dfrac{2}{3}\right[$ (c) $\left[\dfrac{-1}{3},\dfrac{2}{3}\right]$ (d) R

70. Area of the rectangular region $2\le x\le 5$, $-1\le y\le 3$ is

(a) 9 sq units (b) 10 sq units
(c) 12 sq units (d) 15 sq units

71. What are the roots of the equation $(a+b+x)^{-1}=a^{-1}+b^{-1}+x^{-1}$? **(CDS 2007 II)**

(a) a, b (b) $-a, b$ (c) $a, -b$ (d) $-a, -b$

72. If a and b are the roots of the equation $x^2+ax+b=0$, then

(a) $a=1$ (b) $a=-2$
(c) $a=1$ or 0 (d) $a=-2$ or 0

73. For the equation $|y|^2+|y|-6=0$

(a) there are four distinct roots
(b) there are only three distinct roots
(c) there are only two distinct roots
(d) there is only one root

74. The number of roots of the quadratic equation $8\sec^2\phi-6\sec\phi+1=0$ is

(a) n (b) 2 (c) 0 (d) infinite

75. If a, b are the roots of the equation $x^2+x+1=0$, then a^2+b^2 is equal to

(a) 1 (b) 2 (c) -1 (d) 3

76. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2-bx+c=0$, then which one of the following is correct? **(CDS 2011 II)**

(a) $a^2+b^2+2ac=0$ (b) $a^2-b^2+2ac=0$
(c) $a^2+c^2+2ab=0$ (d) $a^2-b^2-2ac=0$

77. The equation $(1+n^2)x^2+2ncx+(c^2-a^2)=0$ will have equal roots, if **(CDS 2011 I)**

(a) $c^2=1+a^2$ (b) $c^2=1-a^2$
(c) $c^2=1+n^2+a^2$ (d) $c^2=(1+n^2)a^2$

78. If $e^{\cos x}-e^{-\cos x}=4$, then the value of $\cos x$ is

(a) $\log(2+\sqrt{5})$ (b) $-\log(3+\sqrt{5})$
(c) $\log(-2+\sqrt{5})$ (d) None of these

79. The number of real roots of $3^{2x^2-7x+7}=9$ is

(a) 3 (b) 1 (c) 2 (d) 4

80. The value of x for which $2^{x+4}-2^{x+2}=3$ is

(a) 2 (b) 1 (c) -2 (d) 3

81. If $x=\sqrt{2}+2$, then

(a) $x^2+4x+2=0$ (b) $x^2-2x-2=0$
(c) $x^2-4x+2=0$ (d) $x^2-4x-2=0$

82. For what value of k will the equation $\dfrac{x^2-bx}{ax-c}=\dfrac{k-1}{k+1}$ have roots reciprocal to each other?

(a) $\dfrac{1+c}{1-c}$ (b) $\dfrac{c+1}{c-1}$ (c) $a-c$ (d) $\dfrac{b+c}{c-1}$

83. If 'α' and 'β' be the roots of $ax^2-bx+b=0$, the value of $\sqrt{\dfrac{\alpha}{\beta}}+\sqrt{\dfrac{\beta}{\alpha}}$ is

(a) a/b (b) $\sqrt{b/a}$ (c) $\sqrt{a/b}$ (d) $-a/b$

84. If the equations $2x^2-7x+3=0$ and $4x^2+ax-3=0$ have a common root, then what is the value of a? **(CDS 2007 II)**

(a) -11 or 4 (b) -11 or -4
(c) 11 or -4 (d) 11 or 4

85. If α and β are the roots of the equation $x^2 - 6x + 6 = 0$, what is $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$ equal to? **(CDS 2011 II)**

(a) 150 (b) 138 (c) 128 (d) 124

86. In the first four examination Rahul got 94, 73, 84, 72 marks. If a final average greater than or equal to 80 and less than 90 is needed to obtain a final grade '*B*' in a score. What range of marks on the fifth examination will result in Rahul receiving '*B*' in the courtse?

(a) $77 \le x < 127$ (b) $87 \le x \le 137$
(c) $67 \le x \le 97$ (d) $x = 100$

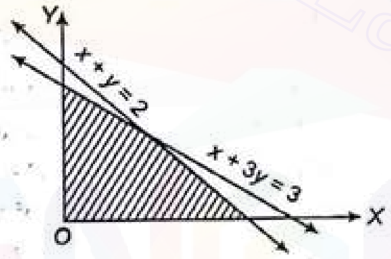
87. The system of linear inequations $x + y \le 0, x \ge 0$ and $y \ge 0$ has

(a) three solutions
(b) exactly one solution
(c) no solution
(d) an infinite number of solutions

88. If $x + 2y \le 3, x > 0$ and $y > 0$, then one of the solution is

(a) $x = -1, y = 2$ (b) $x = 2, y = 1$
(c) $x = 1, y = 1$ (d) $x = 0, y = 0$

89. The shaded region in the given figure is the solution set of the inequations



(a) $x + y \le 2, x + 3y \ge 3, x \ge 0, y \ge 0$
(b) $x + y \ge 2, x + 3y \ge 3, x \ge 0, y \ge 0$
(c) $x + y \ge 2, x + 3y \le 3, x \ge 0, y \ge 0$
(d) $x + y \le 2, x + 3y \le 3, x \ge 0, y \ge 0$

90. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ has equal roots, then which one of the following is correct? **(CDS 2010 II)**

(a) $ab = cd$ (b) $ad = bc$
(c) $a^2 + c^2 = b^2 + d^2$ (d) $ac = bd$

91. If the roots of $x^2 + bx + c = 0$, be α and β, also those of $x^2 + px + q = 0$ be $k\alpha$ and $k\beta$, then

(a) $cb^2 = qp^2$ (b) $qc^2 = b^2p^2$
(c) $qb^2 = cp^2$ (d) None of these

92. If $\left(\frac{14}{5}\right)^{2x-3} = \left(\frac{5}{14}\right)^{x-3}$, then the value of x is

(a) 2 (b) 1 (c) 3 (d) 0

93. Find the condition that one root of $px^2 + qx + r = 0$ may be double of the other.

(a) $2p^2 = 9qr$ (b) $2q^2 = pr$
(c) $2q^2 = 9pr$ (d) $2r^2 = 9pq$

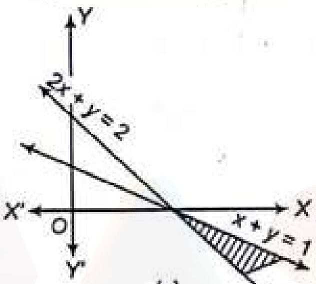
94. If -4 is a root of the equation $x^2 + px - 4 = 0$ and the equation $x^2 + px + q = 0$ has equal roots, then the values of p and q are, respectively

(a) -3 and $\frac{9}{4}$ (b) 3 and $\frac{9}{4}$ (c) $\frac{9}{4}$ and 3 (d) 4 and 3

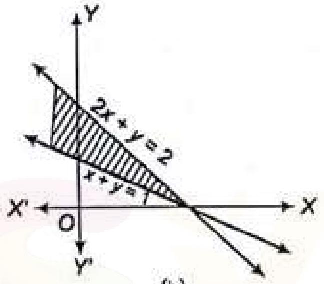
95. What is the condition that the equation $ax^2 + bx + c = 0$, where $a \ne 0$ has both the roots positive? **(CDS 2011 II)**

(a) a, b and c are of same sign.
(b) a and b are of same sign.
(c) b and c have the same sign opposite to that of a.
(d) a and c have the same sign opposite to that of b.

96. The region represented by $x + y \ge 1, 2x + y \le 2$ is given by the shaded portion of

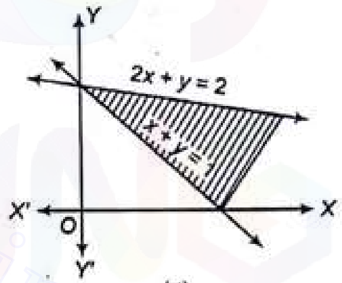


(a)



(b)

(c)



(d)

97. The water acidity in a pool is considered when the average pH reading of three daily measurement is between 7.2 and 7.8. If the first two pH readings are 7.48 and 7.85, the range of pH value for the third reading x that will result in the acidity level being normal is

(a) $6.2 < x < 8.09$ (b) $6.27 < x < 8.07$
(c) $6.7 < x < 8.7$ (d) None of these

98. A company manufactures cassettes and its cost equation for a week is $C = 300 + 1.5x$ and its revenue equation is $R = 2x$, where x is the number of cassettes sold in a week. How many cassettes must be sold for the company to realise a profit?

(a) $x > 700$ (b) $x > 650$
(c) $x > 600$ (d) $500 < x$

99. What is the least integral value of k for which the equation $x^2 - 2(k-1)x + (2k+1) = 0$ has both the roots positive? **(CDS 2010 I)**

(a) 1 (b) $-\frac{1}{2}$ (c) 4 (d) 0

100. If α, β are the roots of $2x^2 - 6x + 3 = 0$, then the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 3\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta$ is

(a) 12 (b) 23 (c) 13 (d) -13

101. The solution set of the equation

$$(x + 2)(x - 5)(x - 6)(x + 1) = 144 \text{ is}$$

(a) $\{7, -3\}$ (b) $\{7, -3, 2\}$
(c) $\{7, -3, 2, 1\}$ (d) $\{7, 2\}$

102. The solution set of the equation $\sqrt{x^2 - 16} - (x - 4) = \sqrt{x^2 - 5x + 4}$ is

(a) $\left\{4, 5, -\frac{13}{3}\right\}$ (b) $\{4, 5\}$

(c) $\{4\}$ (d) $\left\{5, -\frac{13}{3}\right\}$

103. If $(2x - 3y < 7)$ and $(x + 6y < 11)$, then which one of the following is correct? **(CDS 2008 I)**

(a) $x + y < 5$ (b) $x + y < 6$ (c) $x + y \le 5$ (d) $x + y \le 6$

104. In a group of children, each child gives a gift to every other child. If the number of gifts is 132, then the number of children

(a) 13 (b) 10 (c) 11 (d) 12

105. In a flight of 1600 km, an aircraft was slowed down by bad weather. Its average speed for the trip was reduced by 400 km/h and the time of flight increased by 40 min. Then the actual time of flight

(a) $1\frac{1}{2}$ h (b) $1\frac{1}{3}$ h (c) $1\frac{1}{4}$ h (d) 2 h

106. A person on tour has ₹ 360 for his daily expenses. If he exceeds his tour programme by 4 days, he must cut down his daily expenses by ₹ 3 per day. Then the number of days of his tour programme is

(a) 20 days (b) 24 days (c) 25 days (d) 23 days

107. If $a + b = 2m^2$, $b + c = 6m$, $a + c = 2$, where m is a real number and $a \le b \le c$, then which one of the following is correct? **(CDS 2009 II)**

(a) $0 \le m \le 1/2$ (b) $-1 \le m \le 0$
(c) $1/3 \le m \le 1$ (d) $1 < m \le 2$

Answers

1. (d)	2. (b)	3. (c)	4. (b)	5. (c)	6. (c)	7. (c)	8. (a)	9. (d)	10. (d)
11. (b)	12. (c)	13. (c)	14. (c)	15. (b)	16. (c)	17. (a)	18. (b)	19. (c)	20. (b)
21. (a)	22. (a)	23. (b)	24. (c)	25. (a)	26. (c)	27. (a)	28. (c)	29. (b)	30. (d)
31. (c)	32. (d)	33. (d)	34. (a)	35. (c)	36. (b)	37. (c)	38. (a)	39. (c)	40. (c)
41. (c)	42. (b)	43. (c)	44. (a)	45. (d)	46. (d)	47. (a)	48. (d)	49. (b)	50. (c)
51. (a)	52. (b)	53. (a)	54. (b)	55. (d)	56. (c)	57. (d)	58. (b)	59. (a)	60. (b)
61. (c)	62. (a)	63. (a)	64. (b)	65. (b)	66. (c)	67. (a)	68. (c)	69. (a)	70. (c)
71. (d)	72. (c)	73. (c)	74. (c)	75. (c)	76. (b)	77. (d)	78. (a)	79. (c)	80. (c)
81. (c)	82. (b)	83. (b)	84. (a)	85. (b)	86. (a)	87. (b)	88. (c)	89. (d)	90. (b)
91. (c)	92. (a)	93. (c)	94. (b)	95. (d)	96. (b)	97. (b)	98. (c)	99. (c)	100. (c)
101. (b)	102. (b)	103. (b)	104. (d)	105. (b)	106. (a)	107. (c)			

Hints and Solutions

1. $4x^2 - 6x = 0$ or $2x(2x - 3) = 0$

Now, either $2x = 0$ or $2x - 3 = 0$
$x = 0$ or $x = 3/2$

when $x \in N$, the equation has no solution. So, solution set is ϕ or empty set.

2. The degree of equation is equal to maximum number of roots it has roots, may be real or imaginary.

3. I. $x^2 + \frac{1}{x^2} = 2$ or $x^4 - 2x^2 + 1 = 0$ is not a quadratic polynomial.

II. $x + \frac{3}{x} = x^2 \Rightarrow \frac{x^2 + 3}{x} = x^2$

or $x^2 + 3 = x^3 \Rightarrow x^3 - x^2 - 3 = 0$

is not a quadratic polynomial.

III. $2x^2 - x + 2 = x^2 + 4x - 4$

or $2x^2 - x + 2 - x^2 - 4x + 4 = 0$

or $x^2 - 5x + 6 = 0$ is a quadratic polynomial.

IV. $x^3 + 6x^2 + 2x - 1 = 0$ is not a quadratic polynomial.

4. $x^2 + x + 1 = 0; x = 1, x = -1$

Put $x = 1$ in $x^2 + x + 1 = 0$, we get

$$1^2 + 1 + 1 = 3 \ne 0$$

$\therefore$ LHS $\ne$ RHS

$\therefore x = 1$ is not a solution.

Substituting $x = -1$ in the LHS, we get

$$(-1)^2 + (-1) + 1 = 1 - 1 + 1 = 1 \ne 0$$

$\therefore x = -1$ is not a solution.

5. $a^2b^2x^2 - a^2x - b^2x + 1 = 0$

$\Rightarrow a^2x(b^2x - 1) - 1(bx^2 - 1) = 0$

$\Rightarrow (b^2x - 1)(a^2x - 1) = 0$

$\Rightarrow a^2x - 1 = 0$ or $b^2x - 1 = 0$

$\Rightarrow a^2x = 1$ or $b^2x = 1$

$x = 1/a^2$ or $x = 1/b^2$

6. $6 \le x \le 8$

or $\quad x \in [6, 8]$

$\Rightarrow \quad (x-6)(x-8) \le 0$

7. Let α and β be the roots of the equation

$$x^2 - 3kx + 2k^2 - 1 = 0$$

$\therefore \quad \alpha\beta = 2k^2 - 1$

But $\quad \alpha\beta = 7 \quad$ (given)

$\therefore \quad 2k^2 - 1 = 7 \Rightarrow 2k^2 = 8 \Rightarrow k^2 = 4$

$\Rightarrow \quad k = \pm 2$

On putting $k = \pm 2$ in given equation, we get $x^2 \mp 6x + 7 = 0$.

Now, $\sqrt{b^2 - 4ac} = \sqrt{(6)^2 - 4 \times 7} = \sqrt{36 - 28} = 2\sqrt{2}$

Hence, roots of given equation are irrational.

8. As $ax^2 - 2\sqrt{5}x + 4 = 0$ has equal roots.

$\therefore \quad$ Discriminant $= (-2\sqrt{5})^2 - 4(a)4 = 0 \quad (\because D = B^2 - 4AC)$

$\Rightarrow \quad 20 - 16a = 0 \Rightarrow a = 5/4$

9. **A** $4x^2 + kx + 1$ is factorizable when $D \ge 0$, *i.e.*, $k^2 - 16 \ge 0$

i.e., $\quad k^2 \ge 16$ or $k \ge 4$ or $k \le -4$

B $kx^2 - 4x + k$ is factorizable when $16 - 4k^2 \ge 0$

i.e., when $\quad k^2 \le 4$ or $-2 \le k \le 2$

C $kx^2 - 2x + 2$ is factorizable when $4 - 8k \ge 0$ *i.e.*, when $k \le \frac{1}{2}$

D $2x^2 - kx + k$ is factorizable when $k^2 - 8k \ge 0$

But $\quad k(k-8) \ge 0$

$\Rightarrow \quad (k \ge 0)$ and $(k - 8 \ge 0)$ or $(k \le 0$ and $k - 8 \le 0)$

$\Rightarrow \quad k \ge 8$ or $k \le 0$

10. Equal roots, if $D = 0 \Rightarrow B^2 - 4AC = 0$

$$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4[c^2 + m^2c^2 - m^2a^2 - a^2] = 0$$

as $\quad 4 \ne 0$

$$c^2 - m^2a^2 - a^2 = 0 \Rightarrow c^2 = a^2(1+m^2)$$

11. Here, $3x^2 = 8x + (2k+1)$

or $\quad 3x^2 - 8x - (2k+1) = 0$

Let first root be α and second be 7α.

$\therefore \quad$ Sum of roots $= \alpha + 7\alpha = \frac{8}{3} \Rightarrow 8\alpha = \frac{8}{3} \Rightarrow \alpha = \frac{1}{3}$

So, roots are $\frac{1}{3}$ and $\frac{7}{3}$.

$\therefore \quad$ Product of roots $= \frac{1}{3} \times \frac{7}{3} = \frac{7}{9}$

Also, product of roots $= \frac{-(2k+1)}{3} \Rightarrow \frac{7}{9} = \frac{-(2k+1)}{3}$

$\Rightarrow \quad \frac{7}{3} = -(2k+1) \Rightarrow 7 = -6k - 3 \Rightarrow 10 = -6k \Rightarrow k = \frac{-5}{3}$

12. **A** $\quad 2x^2 - 13x + 21 = 0 \Rightarrow 2x^2 - 7x - 6x + 21 = 0$

$\Rightarrow \quad x(2x-7) - 3(2x-7) = 0 \Rightarrow (2x-7)(x-3) = 0$

$\Rightarrow \quad x = 7/2$ or $x = 3$

B $\quad 2x^2 - 9x + 7 = 0 \Rightarrow (2x-7)(x-1) = 0$

$\Rightarrow \quad x = \frac{7}{2}$ or $x = 1$

C $\quad x^2 - 6x + 9 = 0$

$\Rightarrow \quad (x-3)(x-3) = 0$

$\Rightarrow \quad x = 3$ or $x = 3$

D $\quad 2x^2 - 21x + 49 = 0$

$\Rightarrow \quad (2x-7)(x-7) = 0$

$\Rightarrow \quad x = \frac{7}{2}$ or $x = 7$

13. Here, sum of roots, $S = \frac{4+\sqrt{7}}{2} + \frac{4-\sqrt{7}}{2} = 4$

Product of roots, $P = \left(\frac{4+\sqrt{7}}{2}\right)\left(\frac{4-\sqrt{7}}{2}\right)$

$$= \frac{16-7}{4} = \frac{9}{4}$$

The required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - 4x + \frac{9}{4} = 0 \Rightarrow 4x^2 - 16x + 9 = 0$$

14. Since, -1.5 is a root of $ax^2 + x - 3 = 0$

$\therefore \quad a(-1.5)^2 + (-1.5) - 3 = 0$

$\Rightarrow \quad 2.25a - 4.5 = 0 \Rightarrow a = \frac{4.5}{2.25} \Rightarrow a = 2$

15. Here, $2x + 1 \ge 7 \Rightarrow 2x \ge 7 - 1 \Rightarrow 2x \ge 6 \Rightarrow x \ge 3$

16. Here, $\frac{x-1}{3} \ge 4 \Rightarrow x - 1 \ge 12 \Rightarrow x \ge 13$

17. Here, $-2x + 3 \ge 11 \Rightarrow -2x \ge 8 \Rightarrow 2x \le -8 \Rightarrow x \le -4$

18. Here, $\quad 3x + 2 \le 5x - (4 - x)$

$\Rightarrow \quad 3x + 2 \le 5x - 4 + x$

$\Rightarrow \quad 3x + 2 \le 6x - 4$

$\Rightarrow \quad 3x - 6x \le -4 - 2$

$\Rightarrow \quad -3x \le -6$

$\Rightarrow \quad -x \le -2 \Rightarrow x \ge 2$

19. Since, one root of the equation $ax^2 + x - 3 = 0$ is -1.

$\therefore \quad a(-1)^2 + (-1) - 3 = 0 \Rightarrow a = 4$

$\therefore \quad 4x^2 + x - 3 = 0$

Let other root of this equation is α.

$\therefore \quad P = -1 \cdot \alpha = -\frac{3}{4} \Rightarrow \alpha = \frac{3}{4}$

20. Given equation is $x^{2/3} + x^{1/3} - 2 = 0$

$\Rightarrow \quad (x^{1/3})^2 + x^{1/3} - 2 = 0 \Rightarrow X^2 + X - 2 = 0$

where $(X = x^{1/3})$

$\Rightarrow$ It is a quadratic equation in X.

$\therefore$ Discriminant of $X^2 + X - 2 = 0$ is

$$B^2 - 4AC = 1^2 - 4(1)(-2) = 9 \ge 0$$

Hence, two real values of x satisfy the given equation.

21. $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ...(i)

$$40=(8)^2-2(P)$$
$$40-64=-2P$$
$$-24=-2P \Rightarrow P=12$$

22. $x^2-x-2=0$

$$(x-2)(x+1)=0$$

So, $x=2, x=-1$ both the roots are integers.

23. Here, $f(x)=0 \Rightarrow ax^2+bx+c=0$

Sum of roots $=-b/a$, product of roots $=c/a$

If roots are real and equal, then $D=0$

i.e., $b^2-4ac=0$

If roots are real and distinct, then $D>0$

i.e., $b^2-4ac>0$

24. $\alpha+\beta=5$ and $\alpha\beta=6$

$$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=(5)^2-4\times6=1$$
$$\alpha-\beta=\pm1 \text{ and } \alpha+\beta=5$$
$$\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)=5(\pm1)$$
$$\alpha^2-\beta^2=\pm5$$

25. Let the roots of the given equation

$x^2-ax+b=0$ be α and β.

$\therefore$ $\alpha+\beta=a$ and $\alpha\beta=b$

Now, $|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{a^2-4b}$

26. Here, $2x+3\geq8 \Rightarrow 2x\geq8-3 \Rightarrow 2x\geq5 \Rightarrow x\geq\frac{5}{2}$

Again, $3x+1\leq12 \Rightarrow 3x\leq11 \Rightarrow x\leq\frac{11}{3}$

Combining values $\Rightarrow \frac{5}{2}\leq x\leq\frac{11}{3}$

27. Here, consider $2x-y=1$

$\therefore$ Values of (x,y) satisfying $2x-y=1$ are

x	2	0	1/2
y	3	−1	0

So, points $P(2,3)$ and $Q(0,-1)$ divides the plane of the paper. Also, point $(0,0)$ does not lies on it. So, shaded part of the plane divided by line PQ does not contain $(0,0)$.

28. On replacing inequality sign by equality.

We get $x+2y=2$

Values of (x,y) satisfying $x+2y=2$ are

x	0	2
y	1	0

So, points $P(0,1)$ and $Q(2,0)$ divides the plane of paper. Also, the point $(0,0)$ does not lies on it. So, shaded part of plane divided by PQ does not contain $(0,0)$.

29. $\frac{1}{2}\left(\frac{3}{5}x+4\right)\geq\frac{1}{3}(x-6) \Rightarrow \frac{3}{10}x+2\geq\frac{1}{3}x-2$

$\Rightarrow$ $9x+60\geq10x-60$

$\Rightarrow$ $-x\geq-120$ (multiplying both sides by -1)

$\Rightarrow$ $x\leq120$

Thus, all real numbers x which are less than or equal to 120 satisfies the inequation.

30. As $|x|>3$

When $x\geq0$, then $|x|=x$

$\therefore$ $x>3$

When $x<0$, then $|x|=-x$

$\therefore$ $-x>3 \Rightarrow x<-3$

So, $x<-3$ or $x>3$

31. Given, $4^x-3\cdot2^{x+2}+32=0$

$\Rightarrow$ $(2^x)^2-12(2^x)+32=0$

$\Rightarrow$ $2^{2x}-8\cdot2^x-4\cdot2^x+32=0$

$\Rightarrow$ $(2^x-8)(2^x-4)=0$

Either $2^x=8 \Rightarrow x=3$

or $2^x=4 \Rightarrow x=2$

32. Here, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$

Now, roots of required equation are $\frac{1}{\alpha},\frac{1}{\beta}$.

$$S=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-b/a}{c/a}=\frac{-b}{c}$$
$$P=\frac{1}{\alpha}\cdot\frac{1}{\beta}=\frac{1}{c/a}=\frac{a}{c}$$

$\therefore$ Quadratic equation is

$$x^2-\left(\frac{-b}{c}\right)x+\frac{a}{c}=0$$
$$cx^2+bx+a=0$$

33. As $\alpha+\beta=24$ and $\alpha-\beta=8$

Solving, we get $\alpha=16$ and $\beta=8$

$\therefore$ Sum of roots $=\alpha+\beta=24$

Product of roots $=16\times8=128$

$\therefore$ Required equation is $x^2-24x+128=0$.

34. Let roots of equation $a^2b^2x^2-(a^2+b^2)x+1=0$ be α and β.

$\therefore$ $\alpha+\beta=\frac{a^2+b^2}{a^2b^2}$ and $\alpha\beta=\frac{1}{a^2b^2}$

Now, $\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\left(\frac{a^2+b^2}{a^2b^2}\right)^2-\frac{4}{a^2b^2}}$

$\Rightarrow$ $\alpha-\beta=\sqrt{\frac{(a^2-b^2)^2}{(a^2b^2)^2}}=\frac{a^2-b^2}{a^2b^2}$

On solving, we get $\alpha=\frac{1}{b^2}$ and $\beta=\frac{1}{a^2}$

35. Here, $(x+1)^2=(x-1)^2+x^2$ (by pythagorus theorem)

$$x^2+2x+1=x^2-2x+1+x^2$$
$$x^2-4x=0 \Rightarrow x(x-4)=0 \Rightarrow x=4 \text{ or } x=0$$

But $x\neq0$ $\therefore$ $x=4$ and other sides are 3, 5.

$\therefore$ Area $=\frac{1}{2}b\times h=\frac{1}{2}\times3\times4=6\text{ cm}^2$

36. Let the smaller part $= x$ and other part $= 16 - x$

By condition, $2(16-x)^2 - x^2 = 164$

$$2(256 + x^2 - 32x) - x^2 = 164$$
$$x^2 - 64x + 348 = 0$$
$$(x-58)(x-6) = 0$$
$\Rightarrow \quad x = 58, x = 6$, here $x \neq 58$

$\therefore \quad x = 6$

Here, larger part $= 16 - x = 16 - 6 = 10$

37. Here, $y = 15$, then $15 = \dfrac{x(x-1)}{2} \Rightarrow x^2 - x - 30 = 0$

$\Rightarrow \quad x^2 - 6x + 5x - 30 = 0$

$\Rightarrow \quad x(x-6) + 5(x-6) = 0$

$\Rightarrow \quad (x+5)(x-6) = 0$

$\Rightarrow \quad$ Other $x = 6$ or $x = -5$

But $x \neq -5$ so $x = 6$

Thus, figure has 6 points.

38. Here, $\alpha + \beta = \dfrac{4}{2} = 2$

$$\alpha\beta = \frac{1}{2}$$

Now, $\dfrac{1}{\alpha + 2\beta} + \dfrac{1}{\beta + 2\alpha} = \dfrac{\beta + 2\alpha + \alpha + 2\beta}{(\alpha + 2\beta)(\beta + 2\alpha)}$

$$= \frac{3\alpha + 3\beta}{\alpha\beta + 2\alpha^2 + 2\beta^2 + 4\alpha\beta}$$
$$= \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2 + \alpha\beta} = \frac{3(2)}{2(2)^2 + \frac{1}{2}} = \frac{12}{17}$$

39. $x^2 - 6x + 5 = 0$

$\Rightarrow \quad (x-5)(x-1) = 0$

$\Rightarrow \quad x = 5$ or 1

Also, $|x-3| = 2 \Leftrightarrow (x-3) = 2$ or $-(x-3) = 2$

$$x = 5 \text{ or } x = 1$$

$\therefore \quad x^2 - 6x + 5 = 0$ and $|x-3| = 2$ are equivalent.

40. I. A quadratic equation has maximum two roots.

II. $2x^2 - 3x + 1 = 0$

$\Rightarrow \quad (2x-1)(x-1) = 0$

$\Rightarrow \quad x = \dfrac{1}{2}$ and $x = 1$

i.e., $\quad \alpha = \dfrac{1}{2}$ and $\beta = 1$

and $\quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad (x-2)(x-1) = 0$

$\Rightarrow \quad x = 2, x = 1$

So, $\quad \dfrac{1}{\alpha} = 2$ and $\dfrac{1}{\beta} = 1$

III. Let roots be α and $-\dfrac{1}{\alpha}$, then product $= -1$

$\therefore \quad \dfrac{c}{a} = -1 \Rightarrow c = -a$ or $c + a = 0$

IV. Here, $\alpha + \beta = 2$ and $\alpha\beta = \dfrac{1}{2}$

$$\frac{1}{\alpha + 2\beta} + \frac{1}{\beta + 2\alpha} = \frac{(\beta + 2\alpha) + (\alpha + 2\beta)}{(\alpha + 2\beta)(\beta + 2\alpha)}$$
$$= \frac{3(\alpha+\beta)}{2(\alpha^2 + \beta^2) + 5\alpha\beta}$$
$$= \frac{3(\alpha+\beta)}{2[(\alpha+\beta)^2 - 2\alpha\beta] + 5\alpha\beta} = \frac{3(\alpha+\beta)}{2(\alpha+\beta)^2 + \alpha\beta}$$
$$= \frac{3 \times 2}{2 \times 4 + \frac{1}{2}} = \frac{12}{17}$$

So, all I, II, III and IV are correct.

41. The equation is $x^2 + px + q = 0$.

Sum of roots $= -p = 1 + 2 \Rightarrow p = -3$

Product of roots $= q = 1 \times 2 = 2$

$\therefore$ Equation $qx^2 - px + 1 = 0$ becomes

$$2x^2 - (-3)x + 1 = 0 \text{ or } 2x^2 + 3x + 1 = 0$$

or $\quad (2x+1)(x+1) = 0 \therefore \; x = -\dfrac{1}{2}$ or $x = -1$

42. Given equation is $2x^2 - 3x - 4 = 0$

For getting a reciprocal roots, we replace x by $\dfrac{1}{x}$, we get

$$2\left(\frac{1}{x}\right)^2 - 3\left(\frac{1}{x}\right) - 4 = 0$$

$\Rightarrow \quad -4x^2 - 3x + 2 = 0 \Rightarrow 4x^2 + 3x - 2 = 0$

43. Given, $\quad \dfrac{1}{x+a} + \dfrac{1}{x+b} = \dfrac{1}{c} \Rightarrow \dfrac{(x+b) + (x+a)}{(x+a)(x+b)} = \dfrac{1}{c}$

$\Rightarrow \quad 2cx + (a+b)c = x^2 + (a+b)x + ab$

$\Rightarrow \quad x^2 + (a+b-2c)x + ab - ac - bc = 0$

Let the roots of above equation be α and β.

Given, $\quad \alpha + \beta = 0$

$\Rightarrow \quad -(a+b-2c) = 0$

$\Rightarrow \quad a + b = 2c \quad$ ...(i)

Now, $\quad \alpha\beta = ab - ac - bc = ab - (a+b)c$

$$= ab - (a+b) \cdot \frac{(a+b)}{2} \quad \text{[from Eq. (i)]}$$
$$= \frac{2ab - (a^2 + b^2 + 2ab)}{2} = -\frac{(a^2 + b^2)}{2}$$

44. We have, $4x^2 - 1 \leq 0$

$\Rightarrow \quad (2x)^2 - 1 \leq 0$

$\Rightarrow \quad (2x-1)(2x+1) \leq 0$

So, either $\quad (2x-1) \geq 0$ and $(2x+1) \leq 0 \quad$ ...(i)

or $\quad (2x-1) \leq 0$ and $(2x+1) \geq 0 \quad$ ...(ii)

From Eq. (i), $\quad 2x \geq 1$ and $2x \leq -1$

$x \geq \dfrac{1}{2}$ and $x \leq \dfrac{-1}{2}$ which is not possible.

From Eq. (ii), $(2x-1) \leq 0$ and $(2x+1) \geq 0$

$$2x \leq 1 \text{ and } 2x \geq -1$$
$$x \leq \frac{1}{2} \text{ and } x \geq \frac{-1}{2}$$

$\therefore \dfrac{-1}{2} \leq x \leq \dfrac{1}{2}$ is the required solution set.

45. Here, $x^2-4x+4\geq 0 \Rightarrow (x-2)^2 \geq 0$

It is true for all real values of x as $(x-2)^2$ is a perfect square.

So, $x \in R$.

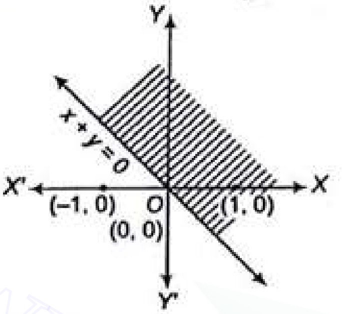
46. Here, $x^2-10x<-25 \Rightarrow x^2-10x+25<0$

$$\Rightarrow \quad (x-5)^2<0$$

As $(x-5)^2$ is a perfect square, it is always positive.

$\therefore (x-5)^2 \nless 0$, thus $(x-5)^2<0$ has no solution. Hence, $x \in \phi$.

47. Consider line $x=0$ and $x+y=0$.



Clearly, $x=0$ is y-axis and $(x+y)=0$ passes through origin *i.e.*, $(0,0)$ consider any point $(-1,0)$ inequality $x+y\geq 0$ does not satisfies it so shaded part does not contain $(-1,0)$.

Also, point $(1,0)$ satisfies the inequality so the shaded part contain $(1,0)$. Here, the first quadrant will be included as a whole in the region of x.

48. Here, $x\geq 0$ is the region of plane to the right hand side of y-axis.

Consider $x+2y=2 \Rightarrow \frac{x}{2}+y=1$. So, this straight line meet x-axis at $(2,0)$ and y-axis at $(0,1)$. So, this is the line joining $P(0,1)$ and $Q(2,0)$. Now, $(0,0)$ does not satisfy $(x+2y)\geq 2$ so the solution set is the portion of plane on and above the line PQ.

Again, consider $x+y\leq 2$ is the portion on line PQ and below QS. The three portion *i.e.*, $x\geq 0, x+2y\geq 2$ and $x+2y\leq 2$ intersect to give the shaded portion PQR.

49. Here, $3x+2y=12$ passes through the points $(0,6)$ and $(4,0)$. So, graph is line AB. Also, $3x+2y\leq 12$ as the shaded region is below AB.

$x=0$ and $y=0$ are the y-axis and x-axis, respectively. $x\geq 0$ and $y\geq 0 \Rightarrow$ the region on the right hand side of y-axis and region above x-axis, respectively.

The three portion $x\geq 0$, $y\geq 0$ and $3x+2y\leq 12$ intersects to give the shaded portion OAB.

50. As $\sqrt{x+4}=x-2$

Squaring both sides, we have

$$(x+4)=(x-2)^2 \Rightarrow x+4=x^2+4-4x$$

$$\Rightarrow \quad x^2-5x=0 \Rightarrow x=0, x=5$$

But for $x=0$, $\quad \sqrt{0+4}=0-2$

$$\sqrt{4}\neq -2$$

So, $x=5$ is the only solution.

51. Let the natural numbers be x and $x+1$.

$$x^2+(x+1)^2=221 \Rightarrow 2x^2+2x+1=221$$

$$\Rightarrow \quad 2x^2+2x-220=0 \Rightarrow x^2+x-110=0$$

$$\Rightarrow \quad (x+11)(x-10)=0$$

$$\Rightarrow \quad x=-11 \text{ and } x=10$$

But $x=10$, so next consecutive natural number

$$=x+1=10+1=11$$

52. Let the consecutive positive odd integers be $2x+1$ and $2x+3$,

so $(2x+1)^2+(2x+3)^2=130$

$$\Rightarrow \quad (4x^2+4x+1)+(4x^2+12x+9)=130$$

$$\Rightarrow \quad x^2+2x-15=0$$

$$\Rightarrow \quad (x+5)(x-3)=0$$

$$\Rightarrow \quad x=3, x=-5, \text{ but } x\neq -5$$

$\therefore$ Two consecutive integers are $2x+1=7$ and $2x+3=9$, *i.e.*, 7 and 9.

53. Since, α and β be the roots of the equation $x^2+px+q=0$

$$\therefore \quad \alpha+\beta=-p \text{ and } \alpha\beta=q$$

Now, $\quad -\alpha^{-1}-\beta^{-1}=-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=-\left(\frac{\alpha+\beta}{\alpha\beta}\right)=\frac{p}{q}$

and $\quad \left(-\frac{1}{\alpha}\right)\left(-\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}=\frac{1}{q}$

Hence, required equation is

$$x^2-(-\alpha^{-1}-\beta^{-1})x+(-\alpha^{-1})(-\beta^{-1})=0$$

$$\Rightarrow \quad x^2-\frac{p}{q}x+\frac{1}{q}=0 \Rightarrow qx^2-px+1=0$$

54. Let roots of equation be α and $\frac{1}{\alpha}$.

$\therefore$ Product of roots $=\alpha\times\frac{1}{\alpha}=\frac{r}{p}$ or $1=\frac{r}{p} \Rightarrow r=p$

55. Let roots be α and β, then $\alpha+\beta=-1$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{6} \Rightarrow \frac{\beta+\alpha}{\alpha\beta}=\frac{1}{6}$$

$$\frac{-1}{\alpha\beta}=\frac{1}{6} \Rightarrow \alpha\beta=-6$$

$\therefore$ Equation is $x^2+x-6=0$.

56. Let α, β be the roots of the equation

$$ax^2+bx+c=0$$

$\therefore \quad$ Sum of roots $(\alpha+\beta)=-\frac{b}{a}$

and $\quad$ product of roots $(\alpha\beta)=\frac{c}{a}$

By given condition,

$$\alpha+\beta=\alpha^2+\beta^2 \Rightarrow \alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$$

$$\Rightarrow \quad -\frac{b}{a}=\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)$$

$$\Rightarrow \quad -ba=b^2-2ca \Rightarrow 2ac=b^2+ab$$

57. Here, roots are α and $\alpha+1$.

$$\therefore \quad \alpha+(\alpha+1)=l \text{(sum of roots)}$$

$$2\alpha=l-1$$

$$\alpha=\frac{l-1}{2}$$

Also, $\quad \alpha(\alpha+1)=m$ or $\alpha^2+\alpha=m$

$$\Rightarrow \quad \left(\frac{l-1}{2}\right)^2+\left(\frac{l-1}{2}\right)=m$$
$$\Rightarrow \quad (l-1)^2+2(l-1)=4m$$
$$\Rightarrow \quad l^2-1=4m$$
$$\Rightarrow \quad l^2=4m+1$$

58. Here, $\alpha+\beta=(1+a^2)$

and $\alpha\beta=\frac{1}{2}(a^4+a^2+1)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ [formula]

$=(1+a^2)^2-(a^4+a^2+1)$

$=1+a^4+2a^2-a^4-a^2-1$

$\alpha^2+\beta^2=a^2$

59. Here, $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$\Rightarrow \quad r(x+p+x+q)=(x+p)(x+q)$

$\Rightarrow \quad x^2+(p+q-2r)x+pq-(p+q)r=0$

Let roots be α and $(-\alpha)$, then

$\alpha+(-\alpha)=0$

$\Rightarrow -(p+q-2r)=0 \Rightarrow r=\frac{p+q}{2}$

So, product of roots $=pq-(p+q)r=pq-(p+q)\cdot\frac{(p+q)}{2}$

$=pq-\frac{(p+q)^2}{2}=\frac{-1}{2}(p^2+q^2)$

60. Here, $\alpha+\beta=-\frac{2}{3}$ and $\alpha\beta=\frac{1}{3}$

$\Rightarrow \quad S=\frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}=\frac{(1-\alpha)(1+\beta)+(1+\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$

$=\frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}=\frac{2-2\times\frac{1}{3}}{1+(-2/3)+1/3}=\frac{4}{3}\times\frac{3}{2}=2$

$P=\frac{1-\alpha}{1+\alpha}\times\frac{1-\beta}{1+\beta}=\frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$

$=\frac{1-(-2/3)+1/3}{1+(-2/3)+1/3}=\frac{2}{2/3}=3$

$\therefore$ The required equation is

$x^2-Sx+P=0$ or $x^2-2x+3=0$

61. Let $y=\frac{x^2+2x+1}{x^2+2x+7}$

$\Rightarrow \quad y(x^2+2x+7)=x^2+2x+1$

$\Rightarrow \quad x^2(y-1)+2x(y-1)+(7y-1)=0$

Since, x is real.

$\Rightarrow$ Discriminant ≥ 0

$\therefore \quad [2(y-1)]^2-4(y-1)(7y-1)\geq 0$

$\Rightarrow \quad 4(y-1)(y-1-7y+1)\geq 0$

$\Rightarrow \quad -24y(y-1)\geq 0$

The above expression ≥ 0 and coefficient of $y^2<0$

$\therefore y$ lies between 0 and 1.

62. Given, $px^2+qx+r=0$

Let the roots be α and β.

By given condition, $\beta=2\alpha$

Product of roots $(\alpha\beta)=\frac{r}{p}=2\alpha^2$ ...(i)

Sum of roots $(\alpha+\beta)=-\frac{q}{p}=3\alpha$ ...(ii)

On squaring Eq. (ii), we get

$9\alpha^2=\frac{q^2}{p^2}$

$\Rightarrow \quad 9\left(\frac{r}{2p}\right)=\frac{q^2}{p^2}$ [from Eq. (i)]

$\Rightarrow \quad 9rp=2q^2$

63. Let roots be α,β, then $\frac{\alpha}{\beta}=\frac{m}{n}$ (given)

$\alpha+\beta=-1, \alpha\beta=1$

$\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}+1=\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+1=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+1$

$=\frac{-1}{\sqrt{1}}+1=-1+1=0$

64. Let price of tape recorder = ₹ x

And number of students be n.

Then, initially contribution per student = ₹ $\frac{x}{n}$

When two students backed out, number of students left to pay $=n-2$

$\therefore$ Contribution per student = ₹ $\frac{x}{n-2}$

By condition, $\frac{x}{n-2}-\frac{x}{n}=1 \Rightarrow \frac{xn-x(n-2)}{n(n-2)}=1$

$\Rightarrow \quad 2x=n^2-2n \Rightarrow x=\frac{n^2-2n}{2}$

But $170\leq x\leq 195$

$\Rightarrow \quad 170\leq\frac{n^2-2n}{2}\leq 195$

$\Rightarrow \quad 340\leq n^2-2n\leq 390$

$\Rightarrow \quad 341\leq n^2-2n+1\leq 391$

$\Rightarrow \quad 341\leq(n-1)^2\leq 391$

$\Rightarrow \quad \sqrt{341}\leq n-1\leq\sqrt{391}$

$\Rightarrow \quad 18.46\leq n-1\leq 19.77$

$\Rightarrow \quad 18.46+1\leq n\leq 19.77+1$

$\Rightarrow \quad 19.46\leq n\leq 20.77$

$\therefore \quad n=20$

and $x=\frac{n^2-2n}{2}=\frac{(20)^2-2(20)}{2}=\frac{360}{2}=$ ₹ 180

65. Let the increase in output be x% every year.

Let the output two years ago be P.

Then, last year's output

$=P+P\times\frac{x}{100}=P\left(1+\frac{x}{100}\right)$

Present output $= P\left(1+\frac{x}{100}\right)+P\left(1+\frac{x}{100}\right)\times\frac{x}{100}$

$$= P\left(1+\frac{x}{100}\right)\left(1+\frac{x}{100}\right)=P\left(1+\frac{x}{100}\right)^2$$

Since, output doubles in last two years.

$\therefore \quad P\left(1+\frac{x}{100}\right)^2=2P \Rightarrow \left(1+\frac{x}{100}\right)^2=2$

$\Rightarrow \quad 1+\frac{x}{100}=\sqrt{2} \Rightarrow x=100(\sqrt{2}-1)\%$

Shortcut method

Here, present output $=2P$

Output two years ago $=P$

Let rate be $x\%$.

$\Rightarrow \quad 2P=P\left(1+\frac{x}{100}\right)^2 \Rightarrow x=100(\sqrt{2}-1)\%$

66. Let $\sqrt{\frac{x}{1-x}}=y \Rightarrow \sqrt{\frac{1-x}{x}}=\frac{1}{y}$

$\therefore \quad y+\frac{1}{y}=\frac{13}{6} \Rightarrow (y^2+1)6=13y$

$\Rightarrow \quad 6y^2-13y+6=0$

$\Rightarrow \quad 6y^2-9y-4y+6=0$

$\Rightarrow \quad 3y(2y-3)-2(2y-3)=0 \Rightarrow (3y-2)(2y-3)=0$

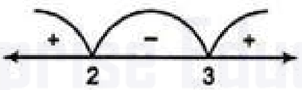
$\therefore \quad y=\frac{2}{3}$ and $\frac{3}{2}$

when $\quad y=\frac{2}{3} \Rightarrow \frac{x}{1-x}=\frac{4}{9} \Rightarrow 9x=4-4x \Rightarrow x=\frac{4}{13}$

when $\quad y=\frac{3}{2} \Rightarrow \frac{x}{1-x}=\frac{9}{4}$

$\Rightarrow 4x=9-9x \Rightarrow x=\frac{9}{13}$

67. $x^2-5x+6>0 \Leftrightarrow (x-2)(x-3)>0$



$\Leftrightarrow \quad x<2$ or $x>3$

$\Leftrightarrow \quad x\in]-\infty,2[$ or $x\in]3,\infty[$

$\Leftrightarrow \quad x\in]-\infty,2[\cup]3,\infty[$

68. $x^2-8x+16\le 0 \quad [\because (x-4)^2$ cannot be negative$]$

$\Leftrightarrow (x-4)^2\le 0 \Leftrightarrow x-4=0 \Leftrightarrow x=4$

69. $\frac{x-2}{3x+1}<\frac{x-3}{3x-2} \Leftrightarrow \frac{x-2}{3x+1}-\frac{x-3}{3x-2}<0$

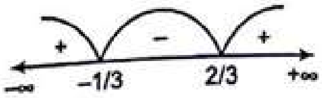
$\Leftrightarrow \quad \frac{(x-2)(3x-2)-(x-3)(3x+1)}{(3x+1)(3x-2)}<0$

$\Leftrightarrow \quad \frac{7}{(3x+1)(3x-2)}<0$

$\Leftrightarrow \quad (3x+1)(3x-2)<0$

$\Leftrightarrow \quad 3\left(x+\frac{1}{3}\right)3\left(x-\frac{2}{3}\right)<0$

$\Leftrightarrow \quad \left(x+\frac{1}{3}\right)\left(x-\frac{2}{3}\right)<0$



$\Leftrightarrow \quad \frac{-1}{3}<x<\frac{2}{3}$

So, $\quad x\in\left]\frac{-1}{3},\frac{2}{3}\right[$

70. The region is a rectangle bounded by the lines $x=2, y=5$, $y=-1$ and $y=3$.

$\therefore$ Length of rectangle $=5-2=3$ units

Breadth of rectangle $=3-(-1)=4$ units

$\therefore$ Area of the rectangle $=3\times 4$ sq units $=12$ sq units

71. Given, $\quad \frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\therefore \quad \frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$\Rightarrow \quad \frac{-(a+b)}{(a+b+x)x}=\frac{(a+b)}{ab} \Rightarrow x^2+(a+b)x+ab=0$

$\Rightarrow (x+a)(x+b)=0 \Rightarrow x=-a,-b$

72. Since, a and b are roots of the equation $x^2+ax+b=0$.

So, $\quad a^2+a^2+b=0$ and $b^2+ab+b=0$

$\Rightarrow \quad 2a^2+b=0$ and $b(b+a+1)=0$

$\Rightarrow \quad 2a^2=-b$ and $b=0$ or $b=-a-1$

Now, $\quad b=0$ and $2a^2+b=0 \Rightarrow a=0$

$\quad b=-a-1$ and $2a^2+b=0$

$\Rightarrow \quad 2a^2-a-1=0 \Rightarrow a=1$ or $-1/2$

So, $a=0$ or $=1$ or $=-1/2$

73. Here, we have $|y|^2+|y|-6=0$

$\Rightarrow \quad (|y|+3)(|y|-2)=0$

$\Rightarrow \quad |y|=2 \Rightarrow y=\pm 2 \quad (\because |y|+3\neq 0)$

So, equation has only two distinct roots.

74. $8\sec^2\phi-6\sec\phi+1=0 \Rightarrow 8\sec^2\phi-4\sec\phi-2\sec\phi+1=0$

$\Rightarrow \quad (4\sec\phi-1)(2\sin\phi-1)=0$

$\Rightarrow \quad \sec\phi=1/4$ or $\sec\phi=1/2$

But $\quad \sec\phi\ge 1$ or $\sec\phi\le -1$

Hence, the equation has no solution.

75. Here, $\quad a+b=-1$

$\quad ab=1$

$a^2+b^2=(a+b)^2-2ab=(-1)^2-2(+1)=1-2=-1$

76. Since, $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2-bx+c=0$.

$\therefore \quad \sin\theta+\cos\theta=\frac{b}{a}$ and $\sin\theta\cos\theta=\frac{c}{a}$

$\Rightarrow \quad (\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta=\frac{b^2}{a^2}$

$\Rightarrow \quad 1+2\left(\frac{c}{a}\right)=\frac{b^2}{a^2} \Rightarrow 2\left(\frac{c}{a}\right)=\frac{b^2-a^2}{a^2}$

$\Rightarrow 2ac=b^2-a^2 \Rightarrow a^2-b^2+2ac=0$

77. The equation will have equal roots, if

$$D = B^2 - 4AC = 0$$

$\therefore \quad (2nc)^2 - 4(1+n^2)(c^2 - a^2) = 0$

$\Rightarrow \quad 4n^2c^2 - 4(c^2 + n^2c^2 - a^2 - n^2a^2) = 0$

$\Rightarrow -4c^2 + 4a^2 + 4n^2a^2 = 0 \Rightarrow c^2 = a^2(1+n^2)$

78. Let $\quad e^{\cos x} - e^{-\cos x} = 4$

Put $\quad e^{\cos x} = z$

$$z - \frac{1}{z} = 4$$

$\Rightarrow \quad z^2 - 4z - 1 = 0$

$\Rightarrow \quad z = 2 \pm \sqrt{5}$, as $z > 0 \Rightarrow e^{\cos x} > 0$

So, $\quad z = 2 + \sqrt{5} \Rightarrow e^{\cos x} = 2 + \sqrt{5}$

$\Rightarrow \quad \cos x = \log(2 + \sqrt{5})$

79. Here, $3^{2x^2 - 7x + 7} = 9 = 3^2$

On comparing,

$$2x^2 - 7x + 7 = 2$$

$$2x^2 - 7x + 5 = 0$$

Here, $\quad D = b^2 - 4ac = 49 - 4(2)(5) = 9$

So, $D > 0$ so it has two real roots.

80. $2^{x+4} - 2^{x+2} = 3 \Rightarrow 2^x \cdot 2^4 - 2^x \cdot 2^2 = 3$

$\Rightarrow \quad 16 \cdot 2^x - 4 \cdot 2^x = 3 \Rightarrow 12 \cdot 2^x = 3$

$\Rightarrow \quad 2^x = \frac{3}{12} = \frac{1}{4} \Rightarrow 2^x = 2^{-2} \Rightarrow x = -2$

81. Here, $\quad x = \sqrt{2} + 2 \Rightarrow x - 2 = \sqrt{2}$

Squaring both sides

$$(x-2)^2 = 2 \Rightarrow x^2 - 4x + 4 = 2$$

$\Rightarrow \quad x^2 - 4x + 2 = 0$

82. $\dfrac{x^2 - bx}{ax - c} = \dfrac{k-1}{k+1}$

$\Rightarrow \quad (x^2 - bx)(k+1) = (k-1)(ax - c)$

$\Rightarrow \quad x^2k + x^2 - bxk - bx = kax - kc - ax + c$

$\Rightarrow \quad (k+1)x^2 - x(bk + b + ka - a) + kc - c = 0$

Since, roots are reciprocal to each other.

So, product $= 1$

i.e., $\quad \dfrac{kc - c}{(k+1)} = 1 \Rightarrow kc - c = k + 1$

$\Rightarrow \quad k(c-1) = c + 1 \Rightarrow k = \dfrac{c+1}{c-1}$

83. Here, $\quad \alpha + \beta = b/a, \alpha\beta = b/a$

So, $\quad \sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} = \dfrac{\alpha + \beta}{\sqrt{\alpha\beta}} = \dfrac{b/a}{\sqrt{b/a}} = \sqrt{\dfrac{b}{a}}$

84. Given, $\quad 2x^2 - 7x + 3 = 0$

$\therefore \quad 2x^2 - 6x - x + 3 = 0$

$\Rightarrow \quad 2x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (2x-1)(x-3) = 0$

when $\quad x = \dfrac{1}{2}$

$$4\left(\frac{1}{2}\right)^2 + a\left(\frac{1}{2}\right) - 3 = 0 \Rightarrow 1 + \frac{a}{2} - 3 = 0 \Rightarrow \frac{a}{2} = 2$$

$$a = 4$$

when $\quad x = 3$

$$4(3)^2 + a(3) - 3 = 0$$

$\Rightarrow \quad 36 + 3a - 3 = 0 \Rightarrow a = -11$

$\therefore \quad a = -11$ or 4

85. Here, $\quad \alpha + \beta = 6$ and $\alpha\beta = 6$

$\therefore \quad (\alpha+\beta)^2 = 6^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 36$

$\Rightarrow \quad \alpha^2 + \beta^2 = 36 - 2(6) = 24$

$\therefore \quad (\alpha^3 + \beta^3) + (\alpha^2 + \beta^2) + (\alpha + \beta)$

$= (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta) + (\alpha^2 + \beta^2) + (\alpha + \beta)$

$= 6(24 - 6) + (24) + (6)$

$= 6(18) + 30 = 108 + 30 = 138$

86. Let Rahul obtains x marks in his fifth examination. Then

$$80 \le \frac{x + 94 + 73 + 72 + 84}{5} < 90$$

$\Rightarrow \quad 80 \le \dfrac{323 + x}{5} < 90$

$\Rightarrow \quad 400 \le 323 + x < 450$

$\Rightarrow \quad 400 - 323 \le x < 450 - 323 \Rightarrow 77 \le x < 127$

Hence, Rahul must get 77 more marks on the fifth examination to receive grade 'B'.

87. Consider the equation $x + y = 0$, $x = 0$ and $y = 0$.

Clearly, $(x = 1, y = -1)$ and $(x = 2, y = -2)$ satisfies $x + y = 0$, plot the point A $(1, -1)$ and B $(2, -2)$ and join AB to obtain the group $x + y = 0$.

Also, $x = 0$ is y-axis and $y = 0$ is x-axis.

So, $x + y \le 0$ is the part below the line AB and line AW. While $x \ge 0$ and $y \ge 0$ is the point XOY.

So, $x = 0, y = 0$ is common point of the two parts. Here, system has exactly one solution $(0, 0)$.

88. Here, as $x > 0$ and $y > 0$ so both are positive and satisfies $x + 2y \le 3$ when $x = 3, y = 1$, we get $x + 2y = 5$.

Clearly, these does not satisfy $x + 2y \le 3$ when $x = 1, y = 1$, then $x + 2y \le 3$.

So, $x = 1, y = 1$ is one of the solutions.

89. As $(0, 0)$ satisfies $x + 3y \le 3$.

And $(0, 0)$ satisfies $x + y \le 2$.

Clearly, shaded region is the position common to the line $x + 3y = 3$ and below it and that on the line $x + y = 2$ and below it.

So, shaded region is the solution set

$$x + y \le 2,\ x + 3y \le 3,\ x \ge 0, y \ge 0$$

90. Since, the roots of equation

$(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal.

$\therefore \quad B^2 = 4AC \quad (\because D = 0)$

$\Rightarrow \quad 4(ac + bd)^2 = 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow \quad a^2c^2 + b^2d^2 + 2abcd = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$

$\Rightarrow \quad (ad - bc)^2 = 0 \Rightarrow ad = bc$

91. Here, for equation $x^2+bx+c=0$

$$\alpha+\beta=-b, \alpha\beta=c$$

and for $x^2+px+q=0$

$$k\alpha+k\beta=-p, k^2\alpha\beta=q$$

Now, $qb^2=k^2\alpha\beta(\alpha+\beta)^2=k^2(\alpha+\beta)^2\alpha\beta$

$$=[k(\alpha+\beta)]^2\alpha\beta$$

$$qb^2=p^2c$$

92. $\left(\frac{14}{5}\right)^{2x-3}=\left(\frac{5}{14}\right)^{x-3}$

$$\left(\frac{5}{14}\right)^{3-2x}=\left(\frac{5}{14}\right)^{x-3}$$

On comparing

$$3-2x=x-3 \Rightarrow 6=3x \Rightarrow x=2$$

93. Here, $px^2+qx+r=0$

Let roots be α and 2α.

$$\alpha+2\alpha=\frac{-q}{p}$$

$$\Rightarrow \quad 3\alpha=\frac{-q}{p} \quad \ldots(i)$$

$$2\alpha\cdot\alpha=\frac{r}{p} \Rightarrow 2\alpha^2=\frac{r}{p}$$

$$\frac{r}{p}=2\alpha^2=2\left(\frac{-q}{3p}\right)^2 \quad \left[\text{from Eq. (i)} \left(\because \alpha=\frac{-q}{3p}\right)\right]$$

$$\frac{r}{p}=\frac{2q^2}{9p^2} \Rightarrow 2q^2=9pr$$

94. Since, -4 is a root of $x^2+px-4=0$

$$(-4)^2+p(-4)-4=0 \Rightarrow p=3$$

As roots of $x^2+px+q=0$ being equal.

$$\therefore \quad D=B^2-4AC=0$$

$$\Rightarrow \quad p^2-4q=0 \text{ or } 3^2-4q=0 \Rightarrow q=\frac{9}{4}$$

So, $p=3, q=\frac{9}{4}$

95. a and c have the same sign opposite to that of b.

96. As point $O(0,0)$ does not satisfy $x+y\geq 1$

So, the part of plane on and above the line $x+y=1$ is the solution set of $x+y\geq 1$.
Also, $(0,0)$ satisfies $2x+y\leq 2$, so the portion of the plane on and below the line $2x+y=2$, is the solution of $2x+y\leq 2$.
$\therefore$ The region represented by $x+y\geq 1, 2x+y\leq 2$ is the portion common to both, which is given by the graph in (b).

97. As the third pH is x.

Then, $7.2<\frac{7.48+7.85+x}{3}<7.8$ (given condition)

$$\Rightarrow \quad 7.2<\frac{15.33+x}{3}<7.8$$

$$\Rightarrow \quad 21.6<15.33+x<23.4$$

$$\Rightarrow \quad 21.6-15.33<x<23.4-15.33 \Rightarrow 6.27<x<8.07$$

Hence, pH range is $6.27<x<8.07$

98. For profit

Revenue > Cost

$$2x>300+1.5x$$

$$\Rightarrow \quad 2x-1.5x>300+1.5x-1.5x$$

$$\Rightarrow \quad 0.5x>300$$

$$\Rightarrow$$

$$\Rightarrow \quad \frac{x}{2}>300 \Rightarrow x>300$$

$\therefore$ More than 600 cassettes must be sold for the company to realise profit.

99. $\because$ The condition for both the roots of the equation $ax^2+bx+c=0$ are positive, if

$$-\frac{b}{a}>0 \text{ and } \frac{c}{a}>0$$

Given equation is $x^2-2(k-1)x+(2k+1)=0$ whose roots are positive.

$$-\frac{b}{a}=\frac{2(k-1)}{1}>0 \Rightarrow k>1$$

and $\frac{c}{a}=\frac{2k+1}{1}>0 \Rightarrow k>-\frac{1}{2}$

$$\therefore \quad k>1$$

Hence, the least value k in given answer is 4.

100. Here, $\alpha+\beta=3, \alpha\beta=3/2$

$$\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+3\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+2\alpha\beta=\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{3(\alpha+\beta)}{\alpha\beta}+2\alpha\beta$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+\frac{3(\alpha+\beta)}{\alpha\beta}+2\alpha\beta$$

$$=\frac{3^2-2\cdot 3/2}{3/2}+\frac{3\cdot 3}{3/2}+2\left(\frac{3}{2}\right)$$

$$=\frac{6}{3/2}+\frac{9}{3/2}+3=4+6+3=13$$

101. $(x+2)(x-5)(x-6)(x+1)=144$

Here, $2+(-6)=-5+1=-4$

$$[(x+2)(x-6)][(x-5)(x+1)]=144$$

$$(x^2-4x-12)(x^2-4x-5)=144$$

Put $x^2-4x=y$

$$\Rightarrow \quad (y-12)(y-5)=144$$

$$\Rightarrow \quad y^2-17y-84=0, \text{ factorizing}$$

$$\Rightarrow \quad (y-21)(y+4)=0$$

$$\Rightarrow \quad y=21 \text{ or } -4$$

For $y=21$	For $y=-4$
$x^2-4x=21$	$x^2-4x=-4$
$x^2-4x-21=0$	$x^2-4x+4=0$
$(x-7)(x+3)=0$	$(x-2)^2=0$
$x=7$ or $x=-3$	$\Rightarrow$ $x=2$

So solution set is $\{7,-3,2\}$.

102. The equation is

$$\sqrt{x^2-16}-(x-4)=\sqrt{x^2-5x+4}$$

$$\sqrt{(x-4)(x+4)}-\sqrt{(x-4)(x-4)}=\sqrt{(x-1)(x-4)}$$

$$\sqrt{(x-4)}[\sqrt{x+4}-\sqrt{x-4}-\sqrt{x-1}]=0$$

So, either $\sqrt{x-4}=0 \Rightarrow x=4$

or $\sqrt{x+4}-\sqrt{x-4}-\sqrt{x-1}=0$

$$\sqrt{x+4}-\sqrt{x-4}=\sqrt{x-1}$$

Squaring both sides

$$(x+4)+(x-4)-2\sqrt{x^2-16}=x-1$$

$$x+1=2\sqrt{x^2-16}$$

Again squaring,

$$x^2+2x+1=4(x^2-16)$$

$\Rightarrow$ $3x^2-2x-65=0$

Here, $x=\dfrac{-(-2)\pm\sqrt{784}}{2\times 3}=\dfrac{2\pm 28}{6}$

$$x=\frac{30}{6} \text{ or } x=\frac{-26}{6}$$

$$x=5, \frac{-13}{2}$$

But $x=\dfrac{-13}{2}$ does not satisfies the equation, so solution set is {4,5}.

103. Given that,

$$2x-3y<7 \quad \text{...(i)}$$

and $x+6y<11$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2x-3y+x+6y<7+11$$

$\Rightarrow$ $3x+3y<18$

$\Rightarrow$ $x+y<6$

104. Let number of children be x.

$\because$ Each child gives a gift to every other child.

$\therefore$ One child gives $x-1$ gifts to others.

So, $x(x-1)=132$

$\Rightarrow$ $x^2-x-132=0$

$\Rightarrow$ $x^2-12x+11x-132=0$

$\Rightarrow$ $x(x-12)+11(x-12)=0$

$\Rightarrow$ $(x+11)(x-12)=0$

$\Rightarrow$ $x=-11$ or $x=12$

$x\neq -11$ so, $x=12$

Hence, the number of children is 12.

105. Let the average speed of aircraft be x km/h. Time taken to cover a distance of 1600 km/h

$$=\frac{1600}{x}\text{h} \quad \text{...(i)}$$

Speed of aircraft when reduced by 400 $=(x-400)$ km/h

So, time taken now $=\dfrac{1600}{x-400}$h ...(ii)

As by reducing the speed, the time is increased by 40 min

$$=\frac{40}{60}=\frac{2}{3}\text{h}$$

According to question,

$$\frac{1600}{x-400}-\frac{1600}{x}=\frac{2}{3}\Rightarrow 1600\left[\frac{1}{x-400}-\frac{1}{x}\right]=\frac{2}{3}$$

$\Rightarrow$ $2x(x-400)=1600\times 400\times 3$

$\Rightarrow$ $x^2-400x-960000=0$

$\Rightarrow$ $x=\dfrac{400\pm\sqrt{4000000}}{2}$

$$=\frac{400\pm 2000}{2}$$

$\Rightarrow$ $x=\dfrac{2400}{2}, \dfrac{-1600}{2}$

But $x\neq -800$

$\Rightarrow$ $x=1200$ km/h

$\therefore$ Actual time of flight $=\dfrac{1600}{x}=\dfrac{1600}{1200}=\dfrac{4}{3}$h

106. Let the number of days of tour $=x$

Daily expenditure = ₹ $\dfrac{360}{x}$

When number of days of tour are increased, then new number of days $=x+4$

Daily expenditure now = ₹ $\left(\dfrac{360}{x}-3\right)$

So, $(x+4)\left(\dfrac{360}{x}-3\right)=360$

$\Rightarrow$ $360+\dfrac{1440}{x}-12-3x=360$

$\Rightarrow$ $1440-12x-3x^2=0$

$\Rightarrow$ $x^2+4x-480=0$

$\Rightarrow$ $(x-20)(x+24)=0$

So, $x=2$ or $x=-24$

But $x\neq -24$

So, $x=20$

Number of days of his tour programme was 20.

107. Given, $a+b=2m^2$ ...(i)

$b+c=6m$ ...(ii)

and $a+c=2$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2(a+b+c)=2m^2+6m+2$$

$\Rightarrow$ $a+b+c=m^2+3m+1$ ...(iv)

Subtracting Eq. (ii) from Eq. (iv)

$$a=m^2-3m+1$$

Subtracting Eq. (iii) from Eq. (iv)

$$b=m^2+3m-1$$

Subtracting Eq. (i) from Eq. (iv)

$$c=-m^2+3m+1$$

As $a\le b$ and $b\le c$

$\Rightarrow$ $m^2-3m+1\le m^2+3m-1$

and $m^2+3m-1\le -m^2+3m+1$

$\Rightarrow$ $6m\ge 2$ and $2m^2\le 2$

$\Rightarrow$ $m\ge\dfrac{1}{3}$ and $-1\le m\le 1$

$\therefore$ $\dfrac{1}{3}\le m\le 1$