

Ex 5.1

Algebra of Matrices Ex 5.1 Q1

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 8 elements, we have to find all the ordered pairs of natural numbers whose products is 8.

The ordered pairs are: (1×8) , (8×1) , (2×4) , (4×2)

$(1,5)$ and $(5,1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1

Algebra of Matrices Ex 5.1 Q2

$$\text{If } A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix} \text{ and } B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$(i) \quad a_{22} + b_{21} = 4 + (-3) = 1$$

$$\text{Hence, } a_{22} + b_{21} = 1$$

$$(ii) \quad a_{11} b_{11} + a_{22} b_{22} = (2)(2) + (4)(4) = 4 + 16 = 20$$

Hence,

$$a_{11} b_{11} + a_{22} b_{22} = 20$$

Algebra of Matrices Ex 5.1 Q3

Here, $A = [a_{ij}]_{3 \times 4}$

R_1 = first row of $A = [a_{11} a_{12} a_{13} a_{14}]_{1 \times 4}$

So, order of $R_1 = 1 \times 4$

C_2 = Second column of A

$$= \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}_{3 \times 1}$$

Order of $C_2 = 3 \times 1$

Let $A = (a_{ij})_{2 \times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad \dots (i)$$

$$(i) \quad a_{ij} = i \cdot j$$

$$a_{11} = 1 \cdot 1 = 1, \quad a_{12} = 1 \cdot 2 = 2, \quad a_{13} = 1 \cdot 3 = 3$$

$$a_{21} = 2 \cdot 1 = 2, \quad a_{22} = 2 \cdot 2 = 4, \quad a_{23} = 2 \cdot 3 = 6$$

So, using equation (i)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

$$(ii) \quad a_{ij} = 2i - j$$

$$a_{11} = 2(1) - 1 = 1, \quad a_{12} = 2(1) - 2 = 0, \quad a_{13} = 2(1) - 3 = -1$$

$$a_{21} = 2(2) - 1 = 3, \quad a_{22} = 2(2) - 2 = 2, \quad a_{23} = 2(2) - 3 = 1$$

Using equation (i)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(iii) \quad a_{ij} = i + j$$

$$a_{11} = 1 + 1 = 2, \quad a_{12} = 1 + 2 = 3, \quad a_{13} = 1 + 3 = 4$$

$$a_{21} = 2 + 1 = 3, \quad a_{22} = 2 + 2 = 4, \quad a_{23} = 2 + 3 = 5$$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$(iv) \quad a_{ij} = \frac{(i + j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2, \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}, \quad a_{13} = \frac{(1+3)^2}{2} = 8$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \quad a_{22} = \frac{(2+2)^2}{2} = 8, \quad a_{23} = \frac{(2+3)^2}{2} = \frac{25}{2}$$

Using equation (i),

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q5

Here,

$$A = [a_{ij}]_{2 \times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{---(i)}$$

$$(i) \quad a_{ij} = \frac{(i+j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2, \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \quad a_{22} = \frac{(2+2)^2}{2} = 8,$$

Using equation (i)

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

$$(ii) \quad a_{ij} = \frac{(i-j)^2}{2}$$

$$a_{11} = \frac{(1-1)^2}{2} = 0, \quad a_{12} = \frac{(1-2)^2}{2} = \frac{1}{2},$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1}{2}, \quad a_{22} = \frac{(2-2)^2}{2} = 0,$$

Using equation (i)

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$(iii) \quad a_{ij} = \frac{(i - 2j)^2}{2}$$

$$a_{11} = \frac{(1 - 2(1))^2}{2} = \frac{1}{2}, \quad a_{12} = \frac{(1 - 2(2))^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2 - 2(1))^2}{2} = 0, \quad a_{22} = \frac{(2 - 2(2))^2}{2} = 2,$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

$$(iv) \quad a_{ij} = \frac{(2i + j)^2}{2}$$

$$a_{11} = \frac{(2(1) + 1)^2}{2} = \frac{9}{2}, \quad a_{12} = \frac{(1(1) + 2)^2}{2} = 8,$$

$$a_{21} = \frac{(2(2) + 2)^2}{2} = \frac{25}{2}, \quad a_{22} = \frac{(2(2) + 2)^2}{2} = 18$$

Using equation (i)

$$A = \begin{bmatrix} \frac{9}{2} & 8 \\ \frac{25}{2} & 18 \end{bmatrix}$$

$$(v) \quad a_{ij} = \frac{(|2i - 3j|)^2}{2}$$

$$a_{11} = \frac{|2(1) - 3(1)|}{2} = \frac{1}{2}, \quad a_{12} = \frac{|2(1) - 3(2)|}{2} = 2$$

$$a_{21} = \frac{|2(2) - 3(1)|}{2} = \frac{1}{2}, \quad a_{22} = \frac{|2(2) - 3(2)|}{2} = 1$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q6

$$\text{Here, } A = (a_{ij})_{3 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \quad \dots (i)$$

$$(i) \quad a_{ij} = i + j$$

$$a_{11} = 1 + 1 = 2, \quad a_{12} = 1 + 2 = 3, \quad a_{13} = 1 + 3 = 4, \quad a_{14} = 1 + 4 = 5$$

$$a_{21} = 2 + 1 = 3, \quad a_{22} = 2 + 2 = 4, \quad a_{23} = 2 + 3 = 5, \quad a_{24} = 2 + 4 = 6$$

$$a_{31} = 3 + 1 = 4, \quad a_{32} = 3 + 2 = 5, \quad a_{33} = 3 + 3 = 6, \quad a_{34} = 3 + 4 = 7$$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

$$(ii) \quad a_{ij} = i - j$$

$$a_{11} = 1 - 1 = 0, \quad a_{12} = 1 - 2 = -1, \quad a_{13} = 1 - 3 = -2, \quad a_{14} = 1 - 4 = -3$$

$$a_{21} = 2 - 1 = 1, \quad a_{22} = 2 - 2 = 0, \quad a_{23} = 2 - 3 = -1, \quad a_{24} = 2 - 4 = -2$$

$$a_{31} = 3 - 1 = 2, \quad a_{32} = 3 - 2 = 1, \quad a_{33} = 3 - 3 = 0, \quad a_{34} = 3 - 4 = -1$$

Using equation (i)

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

$$(iii) \quad a_{ij} = 2i$$

$$a_{11} = 2(1) = 2, \quad a_{12} = 2(1) = 2, \quad a_{13} = 2(1) = 2, \quad a_{14} = 2(1) = 2$$

$$a_{21} = 2(2) = 4, \quad a_{22} = 2(2) = 4, \quad a_{23} = 2(2) = 4, \quad a_{24} = 2(2) = 4$$

$$a_{31} = 2(3) = 6, \quad a_{32} = 2(3) = 6, \quad a_{33} = 2(3) = 6, \quad a_{34} = 2(3) = 6$$

Using Equation (i) ,

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

$$(iv) \quad a_{ij} = j$$

$$a_{11} = 1, \quad a_{12} = 2, \quad a_{13} = 3, \quad a_{14} = 4$$

$$a_{21} = 1, \quad a_{22} = 2, \quad a_{23} = 3, \quad a_{24} = 4$$

$$a_{31} = 1, \quad a_{32} = 2, \quad a_{33} = 3, \quad a_{34} = 4$$

Using Equation (i) ,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Here,

$$A = [a_{ij}]_{4 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

$$(a) \quad a_{ij} = 2i + \frac{i}{j}$$

$$a_{11} = 2(1) + \frac{1}{1} = 3, \quad a_{12} = 2(1) + \frac{1}{2} = \frac{5}{2}, \quad a_{13} = 2(1) + \frac{1}{3} = \frac{7}{3}$$

$$a_{21} = 2(2) + \frac{2}{1} = 6, \quad a_{22} = 2(2) + \frac{2}{2} = 5, \quad a_{23} = 2(2) + \frac{2}{3} = \frac{14}{3}$$

$$a_{31} = 2(3) + \frac{3}{1} = 9, \quad a_{32} = 2(3) + \frac{3}{2} = \frac{15}{2}, \quad a_{33} = 2(3) + \frac{3}{3} = 7$$

$$a_{41} = 2(4) + \frac{4}{1} = 12, \quad a_{42} = 2(4) + \frac{4}{2} = 10, \quad a_{43} = 2(4) + \frac{4}{3} = \frac{28}{3}$$

Using equation (i),

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

$$(c) \quad a_{ij} = i$$

$$a_{11} = 1, \quad a_{12} = 1, \quad a_{13} = 1,$$

$$a_{21} = 2, \quad a_{22} = 2, \quad a_{23} = 2$$

$$a_{31} = 3, \quad a_{32} = 3, \quad a_{33} = 3$$

$$a_{41} = 4, \quad a_{42} = 4, \quad a_{43} = 4$$

Using equation (i)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

Given,

$$\begin{bmatrix} 3x + 4y & 2 & x - 2y \\ a + b & 2a - b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.

So,

$$3x + 4y = 2 \quad \text{--- (i)}$$

$$x - 2y = 4 \quad \text{--- (ii)}$$

$$a + b = 5 \quad \text{--- (iii)}$$

$$2a - b = -5 \quad \text{--- (iv)}$$

Solving equation (i) and (ii)

$$3x - 4y = 2$$

$$\begin{array}{r} 3x - 4y = 2 \\ (-) \quad (+) \quad (-) \end{array}$$

$$10y = -10$$

$$y = \frac{-10}{10} = -1$$

Put $y = -1$ in equation (ii)

$$x - 2y = 4$$

$$x - 2(-1) = 4$$

$$x = 4 - 2$$

$$x = 2$$

Now, solving equation (iii) and (iv),

$$2a + 2b = 10$$

$$\begin{array}{r} 2a + 2b = 10 \\ (-) \quad (+) \quad (+) \end{array}$$

$$3b = 15$$

$$b = \frac{15}{3}$$

$$b = 5$$

Put the value of b in equation of (iii)

$$a + b = 5$$

$$a + 5 = 5$$

$$a = 5 - 5$$

$$a = 0$$

Hence,

$$x = 2, y = -1, a = 0, b = 5$$

Given,

$$\begin{bmatrix} 2x - 3y & a - b & 3 \\ 1 & x + 4y & 3a + 4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.

So,

$$2x - 3y = 1 \quad \text{--- (i)}$$

$$x - b = -2 \quad \text{--- (ii)}$$

$$x - 4y = 6 \quad \text{--- (iii)}$$

$$3a + 4b = 29 \quad \text{--- (iv)}$$

Solving equation (i) and (iii)

$$2x - 3y = 1$$

$$\begin{array}{r} 2x - 8y = 12 \\ (-) \quad (-) \quad (-) \\ \hline \end{array}$$

$$-11y = -11$$

$$y = \frac{-11}{-11}$$

$$y = 1$$

Put the value of y in equation (i),

$$2x - 3y = 1$$

$$2x - 3(1) = 1$$

$$2x - 3 = 1$$

$$2x = 1 + 3$$

$$2x = 4$$

$$x = 2$$

Solving equation (ii) and (iv)

$$4a - 4b = -8$$

$$\begin{array}{r} 3a - 4b = 29 \\ \hline \end{array}$$

$$7a = 21$$

$$a = \frac{21}{7}$$

$$a = 3$$

Put $a = 3$ in equation (ii),

$$3 - b = -2$$

$$b = 3 + 2$$

$$b = 5$$

Hence,

$$x = 2, y = 1, a = 3, b = 5$$

Algebra of Matrices Ex 5.1 Q10

As the given matrices are equal, therefore their corresponding elements must be equal.

Comparing the corresponding elements, we get

$$2a + b = 4 \quad \text{----(i)}$$

$$a - 2b = -3 \quad \text{----(ii)}$$

$$5c - d = 11 \quad \text{----(iii)}$$

$$4c + 3d = 24 \quad \text{----(iv)}$$

Multiplying (i) by 2 and adding to (ii)

$$5a = 5 \Rightarrow a = 1$$

$$(i) \Rightarrow b = 4 - 2.1 = 2$$

Multiplying (iii) by 3 and adding to (iv)

$$19c = 57 \Rightarrow c = 3$$

$$(iii) \Rightarrow d = 5.3 - 11 = 4$$

Hence, $a = 1, b = 2, c = 3, d = 4$

Algebra of Matrices Ex 5.1 Q11

Given,

$$A = B$$

$$\begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x - 2 = y \quad \text{---(i)}$$

$$3 = z \quad \text{---(ii)}$$

$$2z = 6 \quad \text{---(iii)}$$

$$18z = 6 \quad \text{---(iv)}$$

$$y + 2 = x \quad \text{---(v)}$$

$$6z = 2y \quad \text{---(vi)}$$

Equation (ii) gives, $z = 3$

Put the value of z in equal (iv),

$$18z = 6y$$

$$18(3) = 6y$$

$$54 = 6y$$

$$y = \frac{54}{6}$$

$$y = 9$$

Put $y = 9$ in equation (v)

$$y + 2 = x$$

$$9 + 2 = x$$

$$11 = x$$

Hence,

$$x = 11, y = 9, z = 3$$

Algebra of Matrices Ex 5.1 Q12

Given,

$$\begin{bmatrix} x & 3x - y \\ 2x + z & 3y - w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x = 3 \quad \text{---(i)}$$

$$3x - y = 2 \quad \text{---(ii)}$$

$$2x + z = 4 \quad \text{---(iii)}$$

$$3y - w = 7 \quad \text{---(iv)}$$

Put the value of $x = 3$ from equation on (i) in equation(ii),

$$3x - y = 2$$

$$3(3) - y = 2$$

$$9 - y = 2$$

$$y = 9 - 2$$

$$y = 7$$

Put the value of $y = 7$ in equation (iv),

$$3y - w = 7$$

$$3(7) - w = 7$$

$$w = 21 - 7$$

$$w = 14$$

Put the value of $x = 3$ in equation(iii),

$$2x + z = 4$$

$$2(3) + z = 4$$

$$6 + z = 4$$

$$z = 4 - 6$$

$$z = -2$$

Hence,

$$x = 3, y = 7, z = -2, w = 14$$

Algebra of Matrices Ex 5.1 Q13

Given,

$$\begin{bmatrix} x - y & z \\ 2x - y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$\begin{aligned} x - y &= -1 & \text{---(i)} \\ z &= 4 & \text{---(ii)} \\ 2x - y &= 0 & \text{---(iii)} \\ w &= 5 & \text{---(iv)} \end{aligned}$$

Solving equation (i) and (iii)

$$\begin{array}{r} x - y = -1 \\ 2x - y = 0 \\ \hline (-) \quad (+) \quad (-) \\ -x = -1 \\ x = 1 \end{array}$$

Put $x = 1$ in equation (i),

$$\begin{aligned} x - y &= -1 \\ 1 - y &= -1 \\ -y &= -1 - 1 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

equation (ii) and (iv) give the values of z and w respectively, so

$$z = 4, \quad w = 5$$

Hence,

$$x = 1, y = 2, z = 4, w = 5$$

By the definition of equality of matrices we know that if two matrices

$$A = [a_{ij}]_{m \times n} \text{ and } B = [b_{ij}]_{m \times n}$$

are equal then $a_{ij} = b_{ij}$ for $i = 1, 2, 3, \dots, m$ and $j = 1, 2, 3, \dots, n$.

$$\text{Given that } \begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

$\therefore$ Equating the entries gives:

$$\begin{aligned} x+3 &= 0, \quad z+4 = 6 \text{ and } 2y-7 = 3y-2 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } 2y-3y = -2+7 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } -y = 5 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } y = -5 \end{aligned}$$

Similarly, $a-1 = -3$ and $2c+2 = 0$

$$\begin{aligned} \Rightarrow a &= -3+1 \text{ and } 2c = -2 \\ \Rightarrow a &= -2 \text{ and } c = -1 \end{aligned}$$

Lastly, $b-3 = 2b+4$

$$\begin{aligned} \Rightarrow b-2b &= 4+3 \\ \Rightarrow -b &= 7 \\ \Rightarrow b &= -7 \end{aligned}$$

The values of x, y, z, a, b, c are $-3, -5, 2, -2, -7, -1$ respectively.

Algebra of Matrices Ex 5.1 Q15

$$\text{Given that } \begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$$

The corresponding entries of the equal matrices are equal.

$$\begin{aligned} \Rightarrow 2x+1 &= x+3, \quad y^2+1 = 26, \\ \Rightarrow 2x-x &= 2, \quad y^2 = 25 \\ \Rightarrow x &= 2, \quad y = \pm 5 \\ \Rightarrow x &= 2, \quad y = 5 \text{ or } x = 2, \quad y = -5 \\ \therefore x+y &= 7 \text{ or } -3 \end{aligned}$$

Algebra of Matrices Ex 5.1 Q16

$$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$$

The corresponding entries of the two equal matrices are equal.

$$\Rightarrow xy = 8 \dots\dots(1),$$

$$w = 4 \dots\dots\dots(2),$$

$$z+6 = 0 \dots\dots\dots(3),$$

$$\text{and } x+y = 6 \dots\dots\dots(4)$$

from equation (2) and equation(3) we get $z = -6$ and $w = 4$.

from equation(4) we have,

$$x+y = 6,$$

$$\Rightarrow x = 6-y,$$

substituting value of x in equation (1) we get,

$$\Rightarrow (6-y)y = 8,$$

$$\Rightarrow y^2-6y+8 = 0,$$

$$\Rightarrow (y-2)(y-4) = 0,$$

$$\Rightarrow y = 2, 4$$

substituting the value of y in equation(1) we get ,

$$\Rightarrow x = 4, 2$$

Therefore, value of x, y, z, w are $2, 4, -6, 4$ or $4, 2, -6, 4$.

Algebra of Matrices Ex 5.1 Q17

(i) We know that,

Order of a row matrix = $1 \times n$

order of a column matrix = $m \times 1$

So, order of a row as well as column matrix = 1×1

Therefore,

$$\text{Required matrix} = [a]_{1 \times 1}$$

(ii) A diagonal matrix has only a_{11}, a_{22}, a_{33} for a 3×3 matrix such that a_{11}, a_{22}, a_{33} are equal or different and all other entries zero while scalar matrix has

$a_{11} = a_{22} = a_{33} = m$ (say) So, A diagonal matrix which is not scalar, must have,

$a_{11} \neq a_{22} \neq a_{33}$ and $a_{ij} = 0$ for $i \neq j$, So

$$\text{Required Matrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) A triangular matrix is a square matrix $A = [a_{ij}]$ such that $a_{ij} = 0$ for all $i > j$, so

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q18

Given data is,

For January 2013:

Dealer A	DeluxePremium	Standard Cars
	5	3
Dealer B	7	2

For January-February :

Dealer A	DeluxePremium	Standard Cars
	8	7
Dealer B	10	5

Hence,

$$A = \begin{matrix} & \text{Deluxe} & \text{Premium} & \text{Standard} \\ \text{Dealer A} & 5 & 3 & 4 \\ \text{Dealer B} & 7 & 2 & 3 \end{matrix}$$

$$B = \begin{matrix} & \text{Deluxe} & \text{Premium} & \text{Standard} \\ \text{Dealer A} & 8 & 7 & 6 \\ \text{Dealer B} & 10 & 5 & 7 \end{matrix}$$

Algebra of Matrices Ex 5.1 Q19

Given,

$$A = B$$

$$\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Since equal matrices has all corresponding entries equal,

So,

$$2x+1 = x+3 \quad \text{---(i)}$$

$$2y = y^2+2 \quad \text{---(ii)}$$

$$y^2-5y = -6 \quad \text{---(iii)}$$

Solving equation (i)

$$2x+1 = x+3$$

$$2x-x = 3-1$$

$$x = 2$$

Solving equation (ii)

$$2y = y^2+2$$

$$y^2-2y+2 = 0$$

$$D = b^2-4ac$$

$$= (-2)^2 - 4(1)(2)$$

$$= 4-8$$

$$= -4$$

So, There is no real value of y from equation (ii).

Solving equation (iii)

$$y^2-5y = -6$$

$$y^2-5y+6 = 0$$

$$y^2-3y-2y+6 = 0$$

$$y(y-3)-2(y-3) = 0$$

$$(y-3)(y-2) = 0$$

$$y = 3 \quad \text{or} \quad y = 2$$

From solution of equation (i), (ii) and (iii), We can say that A and B can not be equal for any value of y .

Given,

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x+10 = 3x+4 \quad \text{---(i)}$$

$$y^2+2y = 3 \quad \text{---(ii)}$$

$$-4 = y^2-5y \quad \text{---(iii)}$$

Solving equation (i),

$$x+10 = 3x+4$$

$$x-3x = 4-10$$

$$-2x = -6$$

$$x = \frac{6}{2}$$

Solving equation (ii),

$$y^2+2y = 3$$

$$y^2+2y-3 = 0$$

$$y^2+3y-y-3 = 0$$

$$y(y+3)(y-1) = 0$$

$$\Rightarrow y = -3 \text{ and } y = 1$$

Solving equation (iii)

$$-4 = y^2-5y$$

$$y^2-5y+4 = 0$$

$$y^2-4y-y+(y-4) = 0$$

$$y(y-4)-1(y-4) = 0$$

$$(y-4)(y-1) = 0$$

$$\Rightarrow y = 4 \text{ and } y = 1$$

From equation (ii) and (iii),

The common value of $y = 1$

So, $x = 3, y = 1$

Algebra of Matrices Ex 5.1 Q21

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Given that $A = B$

Corresponding element of two equal matrices are equal

$$\Rightarrow a+4 = 2a+2, 3b = b^2+2 \text{ and } -6 = b^2-5b$$

$$\Rightarrow a-2a = 2-4, b^2-3b+2 = 0 \text{ and } b^2-5b+6 = 0$$

$$\Rightarrow -a = -2, (b-1)(b-2) = 0 \text{ and } (b-2)(b-3) = 0$$

$$\Rightarrow a = 2, b = 1, 2 \text{ and } b = 2, 3$$

So value of $a = 2, b = 2$ respectively.

Ex 5.2

Algebra of Matrices Ex 5.2 Q1

$$\begin{aligned} \text{(i)} \quad & \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

$$\begin{aligned} \text{(ii)} \quad & \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$2A - 3B$$

$$\begin{aligned} &= 2 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix} \\ &= \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(ii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$B - 4C$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix} \\ &= \begin{bmatrix} 1+8 & 3-20 \\ -2-12 & 5-16 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$B - 4C = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$3A - C$$

$$\begin{aligned} &= 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix} \end{aligned}$$

Hence,

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iv)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$3A - 2B + 3C$$

$$\begin{aligned} &= 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix} \\ &= \begin{bmatrix} 6-2-6 & 12-6+15 \\ 9+4+9 & 6-10+12 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix} \end{aligned}$$

Hence,

$$3A - 2B + 3C = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q3

Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \Rightarrow$

(i) $A + B$

$A + B$ is not possible as order of A is 2×2 and order of B is 2×3 .

And we know that sum of matrix is possible only when their order is same.

Hence,

$A + B$ is not possible

$B + C$

$$\begin{aligned} &= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1-1 & 0+2 & 2+3 \\ 3+2 & 4+1 & 1+0 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix} \end{aligned}$$

So,

$$B + C = \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

We need to find $2B + 3A$ and $3C - 4B$

Thus, $2B + 3A$ does not exist as the order of A and B are different.

Let us find $3C - 4B = 3 \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 4 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$\begin{aligned} &= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 8 \\ 12 & 16 & 4 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -8 \\ -12 & -16 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 6 & 1 \\ -6 & -13 & -4 \end{bmatrix} \end{aligned}$$

Algebra of Matrices Ex 5.2 Q4

Given, $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$

$2A - 3B + 4C$

$$\begin{aligned} &= 2 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} - 3 \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} + 4 \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{bmatrix} \\ &= \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15+8 \\ 6-3+24 & 2+9+0 & 8-3-16 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q5

Given, $A = \text{diag} (2 \quad -5 \quad 9), B = \text{diag} (1 \quad 1 \quad -4)$
and $C = \text{diag} (-6 \quad 3 \quad 4)$

$$\begin{aligned} (i) \quad A - 2B &= \text{diag} (2 \quad -5 \quad 9) - 2\text{diag} (1 \quad 1 \quad -4) \\ &= \text{diag} (2 \quad -5 \quad 9) - \text{diag} (2 \quad 2 \quad -8) \\ &= \text{diag} (2 - 2 \quad -5 - 2 \quad 9 + 8) \\ &= \text{diag} (0 \quad -7 \quad 17) \end{aligned}$$

$$\text{So, } A - 2B = \text{diag} (0 \quad -7 \quad 17)$$

$$\begin{aligned} (ii) \quad B + C - 2A &= \text{diag} (1 \quad 1 \quad -4) + \text{diag} (-6 \quad 3 \quad 4) - 2\text{diag} (2 \quad -5 \quad 9) \\ &= \text{diag} (1 \quad 1 \quad -4) + \text{diag} (-6 \quad 3 \quad 4) - \text{diag} (4 \quad -10 \quad 18) \\ &= \text{diag} (1 - 6 - 4 \quad 1 + 3 + 10 \quad -4 + 4 - 18) \\ &= \text{diag} (-9 \quad 14 \quad -18) \end{aligned}$$

$$\text{So, } B + C - 2A = \text{diag} (-9 \quad 14 \quad -18)$$

$$\begin{aligned} (iii) \quad 2A + 3B - 5C &= 2 \text{diag} (2 \quad -5 \quad 9) + 3 \text{diag} (1 \quad 1 \quad -4) - 5 \text{diag} (-6 \quad 3 \quad 4) \\ &= \text{diag} (4 \quad -10 \quad 18) + \text{diag} (3 \quad 3 \quad -12) - \text{diag} (-30 \quad 15 \quad 20) \\ &= \text{diag} (4 + 3 + 30 \quad -10 + 3 - 15 \quad 18 - 12 - 20) \\ &= \text{diag} (37 \quad -22 \quad -14) \end{aligned}$$

$$\text{So, } 2A + 3B - 5C = \text{diag} (37 \quad -22 \quad -14)$$

Algebra of Matrices Ex 5.2 Q6

Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}, C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$\text{LHS} = (A+B)+C$$

$$= \left\{ \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \right\} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 1+7 & 1-1 \\ 3+2 & -1+5 & 0+4 \\ 0+2 & 2+1 & 4+6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$\text{LHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \quad \text{---(i)}$$

$$\text{RHS} = A+(B+C)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left\{ \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and , we get

$$(A + B) + C = A + (B + C)$$

Algebra of Matrices Ex 5.2 Q7

We have

$$(X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\text{Also, } (X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 5 - 3 & 2 - 6 \\ 0 - 0 & 9 + 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q8

$$\begin{aligned}
2X + Y &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \\
\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \\
\Rightarrow 2X &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix} \\
\Rightarrow 2X &= \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} \\
\therefore X &= \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}
\end{aligned}$$

Algebra of Matrices Ex 5.2 Q9

Given,

$$2x - y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \quad \text{---(i)}$$

$$x + 2y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \text{---(ii)}$$

Now find

$$2(2x - y) + (x + 2y) = 2 \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow 4x - 2y + x + 2y = \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$\Rightarrow 5x = \begin{bmatrix} 12+3 & -12+2 & 0+5 \\ -8-2 & 4+1 & 2-7 \end{bmatrix}$$

$$\Rightarrow 5x = \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$\Rightarrow 5x = 5 \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

Now find,

$$(2x - y) - 2(x + 2y) = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow 2x - y - 2x - 4y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 10 \\ -4 & 2 & -14 \end{bmatrix}$$

$$\Rightarrow -y - 4y = \begin{bmatrix} 6-6 & -6-4 & 0-10 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$$

$$\Rightarrow -5y = \begin{bmatrix} 0 & -10 & -10 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\Rightarrow -5y = -5 \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q10

Given,

$$x - y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x + y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

Now find,

$$(x - y) + (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2x = 2 \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now find,

$$(x - y) - (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow x - y - x - y = \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1+1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow -2y = \begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow -2y = -2 \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}, y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Given,

$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 9-1 & -1-2 & 1-4 \\ 4+1 & -2-0 & 3-9 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

$$\text{Given, } A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$$

$$\text{Let, } C = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

Since, $5A + 3B + 2C$ is a null matrix, so

$$5A + 3B + 2C = 0$$

$$\Rightarrow 5 \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3 \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2x & 2y \\ 2z & 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45 + 3 + 2x & 5 + 15 + 2y \\ 35 + 21 + 2z & 40 + 36 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48 + 2x & 20 + 2y \\ 56 + 2z & 76 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal.

$$48 + 2x = 0$$

$$x = -\frac{48}{2}$$

$$x = -24$$

$$20 + 2y = 0$$

$$y = -\frac{20}{2}$$

$$y = -10$$

$$56 + 2z = 0$$

$$z = -\frac{56}{2}$$

$$z = -28$$

$$76 + 2w = 0$$

$$w = -\frac{76}{2}$$

$$w = -38$$

$$\text{Hence, } C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

Given,

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

And

$$2A + 3X = 5B$$

$$\Rightarrow 3X = 5B - 2A$$

$$\Rightarrow 3X = 5 \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} - 2 \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 40 - 4 & 0 + 4 \\ 20 - 8 & -10 - 4 \\ 15 + 10 & 30 - 2 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} \frac{36}{3} & \frac{4}{3} \\ \frac{12}{3} & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

$$x = \begin{bmatrix} 4 & \frac{-1}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q14

Given,

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

And

$$A + B + C = 0$$

$$\Rightarrow C = -A - B + 0$$

$$\Rightarrow C = -A - B$$

$$\Rightarrow C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -1-2 & 3+1 & -2+1 \\ -2-1 & 0-0 & -2+1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Hence,

$$C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q15(i)

$$\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 2-2 & -2+2 \\ 4+1 & x+0 & 6-1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 0 & 0 \\ 5 & x & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

Use know that, corresponding entries of equal matrices are equal. So,

$$x - y + 3 = 6$$

$$\Rightarrow x - y = 3 \quad \text{---(i)}$$

$$\text{and } x = 2x + y$$

$$\Rightarrow 2x - x + y = 0$$

$$\Rightarrow x + y = 0 \quad \text{---(ii)}$$

Adding equation (i), (ii),

$$x - y + x + y = 3 + 0$$

$$\Rightarrow 2x = 3$$

$$\Rightarrow x = \frac{3}{2}$$

Put in equation (i),

$$x - y = 3$$

$$\Rightarrow \frac{3}{2} - y = 3$$

$$\Rightarrow -y = \frac{3-3}{2}$$

$$\Rightarrow y = \frac{-3}{2}$$

Hence,

$$x = \frac{3}{2}, y = \frac{-3}{2}$$

Algebra of Matrices Ex 5.2 Q15(ii)

$$\begin{aligned}
& [x \ y + 2 \ z - 3] + [y \ 4 \ 5] = [4 \ 9 \ 12] \\
\Rightarrow & [x + y \ y + 2 + 4 \ z - 3 + 5] = [4 \ 9 \ 12] \\
\Rightarrow & [x + y \ y + 6 \ z + 2] = [4 \ 9 \ 12]
\end{aligned}$$

We know that, corresponding entries, of equal matrices are equal, So

$$\begin{aligned}
x + y &= 4 & \text{--- (i)} \\
y + 6 &= 9 & \text{--- (ii)} \\
z + 2 &= 12 & \text{--- (iii)}
\end{aligned}$$

From equation (ii), We get

$$\begin{aligned}
y &= 9 - 6 \\
y &= 3
\end{aligned}$$

Put the value of y in equation (i),

$$\begin{aligned}
& x + y = 4 \\
\Rightarrow & x + 3 = 4 \\
\Rightarrow & x = 4 - 3 \\
\Rightarrow & x = 1
\end{aligned}$$

From equation (iii)

$$\begin{aligned}
z + 2 &= 12 \\
z &= 12 - 2 \\
z &= 10
\end{aligned}$$

Hence,

$$x = 1, y = 3, z = 10$$

Algebra of Matrices Ex 5.2 Q16

Given,

$$\begin{aligned}
& 2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\
\Rightarrow & \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\
\Rightarrow & \begin{bmatrix} 6 + 1 & 8 + y \\ 10 + 0 & 2x + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\
\Rightarrow & \begin{bmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}
\end{aligned}$$

Since corresponding entries of equal matrices are equal, So

$$\begin{aligned}
8 + y &= 0 \\
y &= -8
\end{aligned}$$

And

$$\begin{aligned}
2x + 1 &= 5 \\
2x &= 5 - 1 \\
x &= \frac{4}{2} \\
x &= 2
\end{aligned}$$

Hence,

$$x = 2, y = -8$$

Algebra of Matrices Ex 5.2 Q17

Given,

$$\lambda \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} + 2 \begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \lambda & 0 & 2\lambda \\ 3\lambda & 4\lambda & 5\lambda \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ -2 & -6 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \lambda + 2 & 4 & 2\lambda + 6 \\ 3\lambda - 2 & 4\lambda - 6 & 5\lambda + 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$\lambda + 2 = 4$$

$$\Rightarrow \lambda = 2$$

and

$$3\lambda - 2 = 4$$

$$3\lambda = 6$$

$$\Rightarrow \lambda = 2$$

Hence,

$$\lambda = 2$$

Algebra of Matrices Ex 5.2 Q18(i)

Given,

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

and

$$2A + B + X = 0$$

$$\Rightarrow X = -2A - B$$

$$\Rightarrow X = -2 \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 - 3 & -4 + 2 \\ -6 - 1 & -8 - 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Hence,

$$X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q18(ii)

$$\text{Given, } A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

Also we have $2A + 3X = 5B$

Thus, we have, $3X = 5B - 2A$

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow 3X = 5 \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2 \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 10 - 16 & -10 - 0 \\ 20 - 8 & 10 - (-4) \\ -25 - 6 & 5 - 12 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & \frac{-10}{3} \\ 4 & \frac{14}{3} \\ \frac{-31}{3} & \frac{-7}{3} \end{bmatrix}$$

$$3 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3t = 2t + 3$$

$$\Rightarrow t = 3$$

$$3z = -1 + z + t$$

$$\Rightarrow 2z = -1 + t = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } t = 3$$

Algebra of Matrices Ex 5.2 Q19(ii)

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Comparing the corresponding elements from both sides,

$$2x + 3 = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$$

$$2y - 4 = 14 \Rightarrow 2y = 18 \Rightarrow y = 9$$

Hence, $x = 2, y = 9$

Algebra of Matrices Ex 5.2 Q20

Let us solve this problem using simultaneous linear equation and algebra of matrices.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (2)$$

multiplying the first equation by 3 and second equation by 2 we get,

$$6X + 9Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (3),$$

$$6X + 4Y = 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (4)$$

Subtracting equation (4) from equation (3) we have,

$$5Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{5}\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

Similarly, multiplying the equation (1) by 2 and equation (2) by 3 we get,

$$4X + 6Y = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (5),$$

$$9X + 6Y = 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (6)$$

Subtracting equation (6) from equation (5) we have,

$$-5X = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = -\frac{1}{5}\begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Hence the value of $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$.

Let A represent the post allocation matrix for a college, So

$$A = \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix} \begin{matrix} \text{Peons} \\ \text{Clerks} \\ \text{Typist} \\ \text{Section officer} \end{matrix}$$

The total number of posts of each kind in 30 colleges is given by:

$$= 30A$$

$$= 30 \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$

$$30A = \begin{bmatrix} 450 \\ 90 \\ 30 \\ 30 \end{bmatrix} \begin{matrix} \text{Peons} \\ \text{Clerks} \\ \text{Typists} \\ \text{Section Officers} \end{matrix}$$

Ex 5.3

Algebra of Matrices Ex 5.3 Q1

$$\begin{aligned}
 \text{(i)} \quad & \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \\
 &= \begin{bmatrix} (a)(a) + (b)(b) & (a)(-b) + (b)(a) \\ (-b)(a) + (a)(b) & (-b)(-b) + (a)(a) \end{bmatrix} \\
 &= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix} \\
 &= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

$$\begin{aligned}
 \text{(ii)} \quad & \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} (1)(1) + (-2)(-3) & (1)(2) + (-2)(2) & (1)(3) + (-2)(-1) \\ (2)(1) + (3)(-3) & (2)(2) + (3)(2) & (2)(3) + (3)(-1) \end{bmatrix} \\
 &= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

$$\begin{aligned}
 \text{(iii)} \quad & \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} (2)(1) + (3)(0) + (4)(3) & (2)(-3) + (3)(2) + (4)(0) & (2)(5) + (3)(4) + (4)(5) \\ (3)(1) + (4)(0) + (5)(3) & (3)(-3) + (4)(2) + (5)(0) & (3)(5) + (4)(4) + (5)(5) \\ (4)(1) + (5)(0) + (6)(3) & (4)(-3) + (5)(2) + (6)(0) & (4)(5) + (5)(4) + (6)(5) \end{bmatrix} \\
 &= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} \\
 &= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}
 \end{aligned}$$

Algebra of Matrices Ex 5.3 Q2(i)

$$\text{Given, } A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q2(ii)

$$\text{Given, } A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii), $AB \neq BA$

Algebra of Matrices Ex 5.3 Q2(iii)

$$\text{Given, } A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \quad \text{---(ii)}$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q3(i)

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

Since order of A is 2×2 and order of B is 2×3 ,

So AB is possible but BA is not possible order of AB is 2×3 .

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (1)(1) + (-2)(2) & (1)(2) + (-2)(3) & (1)(3) + (-2)(1) \\ (2)(1) + (3)(2) & (2)(2) + (3)(3) & (2)(3) + (3)(1) \end{bmatrix} \\ &= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exist

Algebra of Matrices Ex 5.3 Q3(ii)

Here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Order of $A = 3 \times 2$ and order of $B = 2 \times 3$ So,

AB and BA Both exists and order of $AB = 3 \times 3$ and order of $BA = 2 \times 2$

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} (3)(4) + (2)(0) & (3)(5) + (2)(1) & (3)(6) + (2)(2) \\ (-1)(4) + (0)(0) & (-1)(5) + (0)(1) & (-1)(6) + (0)(2) \\ (-1)(4) + (1)(0) & (-1)(5) + (1)(1) & (-1)(6) + (1)(2) \end{bmatrix} \\ &= \begin{bmatrix} 12 + 0 & 15 + 2 & 18 + 4 \\ -4 + 0 & -5 + 0 & -6 + 0 \\ -4 + 0 & -5 + 0 & -6 + 2 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix} \\ BA &= \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (4)(3) + (5)(-1) + (6)(-1) & (4)(2) + (5)(0) + (6)(1) \\ (0)(3) + (1)(-1) + (2)(-1) & (0)(2) + (1)(0) + (2)(1) \end{bmatrix} \\ &= \begin{bmatrix} 12 - 5 - 6 & 8 + 0 + 6 \\ 0 - 1 - 2 & 0 + 0 + 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iii)

Here,

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

Order of $A = 1 \times 4$ and order of $B = 4 \times 1$ So,

AB and BA both exist and order of $AB = 1 \times 1$ and order of $BA = 4 \times 4$, So

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \\ &= [(1)(0) + (-1)(1) + (2)(3) + (3)(2)] \\ &= [0 - 1 + 6 + 6] \\ AB &= [11] \end{aligned}$$

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\begin{aligned} BA &= \begin{bmatrix} (0)(1) & (0)(-1) & (0)(2) & (0)(3) \\ (1)(1) & (1)(-1) & (1)(2) & (1)(3) \\ (3)(1) & (3)(-1) & (3)(2) & (3)(3) \\ (2)(1) & (2)(-1) & (2)(2) & (2)(3) \end{bmatrix} \\ BA &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{aligned} AB &= [11] \\ BA &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix} \end{aligned}$$

Algebra of Matrices Ex 5.3 Q3(iv)

$$\begin{aligned}
& \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\
&= \begin{bmatrix} ac + bd \end{bmatrix} + \begin{bmatrix} a^2 + b^2 + c^2 + d^2 \end{bmatrix} \\
&= \begin{bmatrix} ac + bd + a^2 + b^2 + c^2 + d^2 \end{bmatrix}
\end{aligned}$$

Hence,

$$\begin{aligned}
& \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\
&= \begin{bmatrix} ac + bd + a^2 + b^2 + c^2 + d^2 \end{bmatrix}
\end{aligned}$$

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 3 + 6 & 3 + 6 - 9 & -1 - 3 + 4 \\ -4 + 1 + 6 & 6 - 2 - 9 & -2 + 1 + 4 \\ -6 + 0 + 6 & 9 + 0 - 9 & -3 + 0 + 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (i)}$$

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 - 3 & -6 - 3 + 0 & 2 - 3 + 1 \\ -1 + 4 - 3 & -3 - 2 + 0 & 1 - 2 + 1 \\ -6 + 18 - 12 & -18 - 9 + 0 & 6 - 9 + 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$AB \neq BA$$

$$A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \quad \text{--- (i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-22+9 & -4+10-5 & -9+0+1 \\ 30-44+10 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii)

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q5(i)

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(ii)

$$\begin{aligned}
& \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\
&= \begin{bmatrix} 1+4+0 & 0+0+3 & 2+0+6 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 3 & 10 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\
&= \begin{bmatrix} 10+12+60 \end{bmatrix} \\
&= \begin{bmatrix} 82 \end{bmatrix}
\end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 82 \end{bmatrix}$$

$$\begin{aligned}
& \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}
\end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right) = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

$$\text{Given, } A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I_2 \quad \text{--- (i)}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = I_2 \quad \text{--- (ii)}$$

$$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^2 = I_2 \quad \text{--- (iii)}$$

Hence,

From equation (i), (ii) and (iii),

$$A^2 = B^2 = C^2 = I_2$$

Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$$\begin{aligned}
 & 3A^2 - 2B + I \\
 &= 3 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= 3 \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= 3 \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 3-0+1 & -12+8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}
 \end{aligned}$$

Hence,

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Given, $A = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$

$$(A - 2I)(A - 3I)$$

$$= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4-2 & 2-0 \\ -1-0 & -1-2 \end{bmatrix} \begin{bmatrix} 4-3 & 2-0 \\ -1-0 & -1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$(A - 2I)(A - 3I) = 0$$

$$\text{Given, } A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Hence,

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\text{Given, } A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \\ &= \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

Hence,

$$A^2 = 0$$

Algebra of Matrices Ex 5.3 Q11

$$\text{Given, } A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\begin{aligned} A^2 &= A.A \\ &= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin^2 2\theta + \cos 2\theta \sin^2 2\theta \\ -\cos 2\theta \sin^2 2\theta - \sin^2 2\theta \cos 2\theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos 4\theta & 2 \sin^2 2\theta \cos 2\theta \\ -2 \sin^2 2\theta \cos 2\theta & \cos 4\theta \end{bmatrix} \\ &\quad \{\text{since } \cos^2 \theta - \sin^2 \theta = \cos 2\theta\} \\ &= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} \\ &\quad \{\text{since } \sin^2 \theta = 2 \sin \theta \cos \theta\} \end{aligned}$$

Hence,

$$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q12

$$\text{Given, } A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} -2 - 3 + 5 & 6 + 9 - 15 & 10 + 15 + 25 \\ 1 + 4 - 5 & -3 - 12 + 15 & -5 - 20 + 25 \\ -1 - 3 + 4 & 3 + 9 - 12 & 5 + 15 - 20 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$AB = O_{3 \times 3} \quad \text{---(i)}$$

$$\begin{aligned} BA &= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -2 - 3 + 5 & 3 + 12 - 15 & 5 + 15 - 20 \\ 2 + 3 - 5 & -3 - 12 + 15 & -5 - 15 + 20 \\ -2 - 3 + 5 & 3 + 12 - 15 & 5 + 15 - 20 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$BA = O_{3 \times 3} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$AB = BA = O_{3 \times 3}$$

$$\text{Given, } A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}, B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + abc - abc & 0 + b^2c - b^2c & 0 + bc^2 - bc^2 \\ -a^2c + 0 + a^2c & -abc + 0 + abc & -ac^2 + 0 + ac^2 \\ a^2b - a^2b + 0 & ab^2 - ab^2 + 0 & abc - abc + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3 \times 3} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$AB = BA = O_{3 \times 3}$$

$$\text{Given, } A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4 + 3 - 5 & -4 - 9 + 10 & -8 - 12 + 15 \\ -2 - 4 + 5 & 2 + 12 - 10 & 4 + 16 - 15 \\ 2 + 3 - 4 & -2 - 9 + 18 & -4 - 12 + 12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \end{aligned}$$

$$AB = A$$

$$\begin{aligned} BA &= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 4 + 2 - 4 & -6 - 8 + 12 & -10 - 10 + 16 \\ -2 - 3 + 4 & 3 + 12 - 12 & 5 + 15 - 16 \\ 2 + 2 - 3 & -3 - 8 + 9 & -5 - 10 + 12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \end{aligned}$$

$$BA = B$$

$$\text{Given, } A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix} \\ A^2 &= \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} B^2 &= \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix} \\ B^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{---(ii)} \end{aligned}$$

Subtracting equation(ii) from equation(i),

$$\begin{aligned} A^2 - B^2 &= \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix} \\ &= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix} \end{aligned}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

$$\text{Given, } A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & +0+0+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \text{----(i)}$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \text{---(ii)}$$

$$\sqrt{-4} \quad \sqrt{-4}$$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+6 & -4+2-3 & 4+4+3 \\ 1+0+4 & -1+1-2 & 1+2+2 \\ 3+0+2 & -3+0-1 & 3+0+1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-15+0 & 20+0+0 & -10+5+11 \\ 5-6+0 & 10+0+0 & -5-2+5 \\ 5-12+0 & 10+0+0 & -5-4+4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{---(i)}$$

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1-3+0 & 2+0+0 & -1-1+1 \\ 0+3+0 & 0+0+0 & 0+1+2 \\ 2-3+0 & 4+0+0 & -2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & -1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8+6-3 & 8+0+12 & -4+6-6 \\ -2+3-2 & 2+0+8 & -1+3-4 \\ -6+0-1 & 6+0+4 & -3+0-2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{--(ii)}$$

From equation (i) and (ii),

$$(AB)C = A(BC)$$

Given, $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B + C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$AB + AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

Using equation (i) and (ii),

$$A(B + C) = AB + AC$$

Given, $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$A(B + C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{--- (i)}$$

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$A(B + C) = AB + AC$$

Algebra of Matrices Ex 5.3 Q18

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \left[\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \quad \text{---(i)}$$

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} - \begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6+0+0 \\ 0-2-1 & -10+1+1 & -4+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -14-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$A(B - C) = AB - AC$$

Algebra of Matrices Ex 5.3 Q19

Given,

$$\begin{aligned}
 A &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix} \\
 A &= \begin{bmatrix} 0-3+0 & 0+2+0 \\ 4+0+8 & -2+0+6 \\ 0-9+8 & 0+6+6 \\ 8+0+16 & -4+0+12 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix} \\
 A &= \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix} \\
 A &= \begin{bmatrix} 0+6 & -3-6 & 3+8 & -6-8 & 6+0 \\ 0+12 & 12-12 & -12+16 & 24-16 & -24+0 \\ 0+36 & -1-36 & 1+48 & -2-48 & 2+0 \\ 0+24 & 24-24 & -24+34 & 48-32 & -48+0 \end{bmatrix} \\
 A &= \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}
 \end{aligned}$$

Here, $a_{43} = 8, a_{22} = 0$

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$\begin{aligned} A^2 &= A \times A \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \\ &= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^3 &= A^2 \times A \\ &= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \\ &= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix} \end{aligned}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned} pI + qA + rA^2 &= p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix} \\ &= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} pI + qA + rA^2 &= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \end{aligned}$$

Algebra of Matrices Ex 5.3 Q21

Given, W is a complex cube root of unity,

$$\begin{aligned}
 & \left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \\
 &= \begin{bmatrix} 1+w & w+w^2 & w^2+1 \\ w+w^2 & w^2+1 & 1+w \\ w^2+w & 1+w^2 & w+1 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \\
 &= \begin{bmatrix} -w^2 & -1 & -w \\ -1 & -w & -w^2 \\ -1 & -w & -w^2 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \quad \left\{ \begin{array}{l} \text{Since } 1+w+w^2=0 \\ \text{and } w^3=1 \end{array} \right\} \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} -w^2-w-w^3 \\ -1-w^2-w^4 \\ -1-w^2-w^2 \end{bmatrix} \\
 &= \begin{bmatrix} -w(1+w+w^2) \\ -1-w^2-w^3w \\ -1-w^2-w^3w \end{bmatrix} \\
 &= \begin{bmatrix} -w.0 \\ -1-w^2-w \\ -1-w^2-w \end{bmatrix} \quad \{\text{using reason (i)}\} \\
 &= \begin{bmatrix} 0 \\ -(1+w+w^2) \\ -(1+w+w^2) \end{bmatrix} \\
 &= \begin{bmatrix} 0 \\ -(0) \\ -(0) \end{bmatrix} \\
 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
 \end{aligned}$$

Algebra of Matrices Ex 5.3 Q22

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$\begin{aligned}
 A^2 &= A, A \\
 &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 4+3-5 & -6-12+15 & -10-15+20 \\ -2-4+5 & 3+16-15 & 5+20-20 \\ 2+3-4 & -3-12+12 & -5-15+16 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\
 &= A
 \end{aligned}$$

Hence,

$$A^2 = A$$

Given, $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 3 - 12 & -4 + 0 + 4 & -16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 + 0 + 12 \\ 12 - 3 - 9 & -3 + 0 + 3 & -12 + 4 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

Hence,

$$A^2 = I_3$$

Given,

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0+2x & 0+2+x & 2+1+0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+1 & 2+x & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [2x+1+2+x+3] = 0$$

$$\Rightarrow 3x+6=0$$

$$\Rightarrow x = -\frac{6}{3}$$

$$\Rightarrow x = -2$$

Algebra of Matrices Ex 5.3 Q24(ii)

$$\text{Given that } \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

By multiplication of matrices, we have,

$$\begin{bmatrix} 2 \times 1 + 3 \times (-2) & 2 \times (-3) + 3 \times 4 \\ 5 \times 1 + 7 \times (-2) & 5 \times (-3) + 7 \times 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow x = 13$$

Algebra of Matrices Ex 5.3 Q25

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x + 4 + 0 & x + 0 + 2 & 2x + 8 - 4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x + 4 & x + 2 & 2x + 4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [(2x + 4)x + 4(x + 2) - 1(2x + 4)] = 0$$

$$\Rightarrow 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow 2x^2 + 6x + 4 = 0$$

$$\Rightarrow 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow 2x(x + 1) + 4(x + 1) = 0$$

$$\Rightarrow (x + 1)(2x + 4) = 0$$

$$\Rightarrow x + 1 = 0 \text{ or } 2x + 4 = 0$$

$$\Rightarrow x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Given,

$$\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 - 2 + x & 1 - 1 + x & -1 - 3 + x \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x - 2 & x & x - 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [0(x - 2) + x \cdot 1 + 1 \cdot (x - 4)] = 0$$

$$\Rightarrow 0 + x + x - 4 = 0$$

$$\Rightarrow 2x - 4 = 0$$

$$\Rightarrow x = 2$$

Hence,

$$x = 2$$

Algebra of Matrices Ex 5.3 Q27

$$\text{Given, } A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} & A^2 - A + 2I \\ &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 - 3 + 2 & -2 + 2 + 0 \\ 4 - 4 + 0 & -4 + 2 + 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

Hence,

$$A^2 - A + 2I = 0$$

Algebra of Matrices Ex 5.3 Q28

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And

$$A^2 = 5A + \lambda I$$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3+2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+\lambda & 5 \\ -5 & 10+\lambda \end{bmatrix}$$

Since, Corresponding entries of equal matrices are equal, So

$$8 = 15 + \lambda$$

$$\lambda = 8 - 15$$

$$\lambda = -7$$

Algebra of Matrices Ex 5.3 Q29

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^2 - 5A + 7I_2$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence, $A^2 - 5A + 7I_2 = 0$

Algebra of Matrices Ex 5.3 Q30

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$A^2 - 2A + 3I_2$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 2A + 3I_2 = 0$$

Algebra of Matrices Ex 5.3 Q31

$$\text{Given, } A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

$$\text{Hence, } A^3 - 4A^2 + A$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 - 28 + 2 & 45 - 48 + 3 \\ 15 - 16 + 1 & 26 - 28 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$\text{So, } A^3 - 4A^2 + A = 0$$

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$\begin{aligned}
 & A^2 - 12A - I \\
 &= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12 \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= 0
 \end{aligned}$$

Since $A^2 - 12A - I = 0$

So,

$$A \text{ is a root of the equation } A^2 - 12A - I = 0$$

Algebra of Matrices Ex 5.3 Q33

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$\begin{aligned}
 & A^2 = 5A - 14I \\
 &= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} \\
 &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} \\
 &= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= 0
 \end{aligned}$$

So,

$$A^2 - 5A - 14I = 0$$

Algebra of Matrices Ex 5.3 Q34

$$\text{Given, } A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

$$\text{Hence, } A^3 - 4A^2 + A$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$\text{So, } A^3 - 4A^2 + A = 0$$

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{aligned} \therefore A^2 &= A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$\begin{aligned} &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= O = \text{R.H.S.} \end{aligned}$$

$$\therefore A^2 - 5A + 7I = O$$

Since $A^2 - 5A + 7I = O$, we have

$$A^2 = 5A - 7I$$

Therefore, $A^4 = A^2 \times A^2 = (5A - 7I)(5A - 7I)$

$$\Rightarrow A^4 = 25A^2 - 35AI - 35IA + 49I$$

$$\Rightarrow A^4 = 25A^2 - 70A + 49I$$

$$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I$$

$$\Rightarrow A^4 = 125A - 175I - 70A + 49I$$

$$\Rightarrow A^4 = 55A - 126I$$

$$\Rightarrow A^4 = 55 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 - 126 & 55 - 0 \\ -55 - 0 & 110 - 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

$$\begin{aligned}
 A^2 = A \cdot A &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}
 \end{aligned}$$

Now $A^2 = kA - 2I$

$$\begin{aligned}
 \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}
 \end{aligned}$$

Comparing the corresponding elements, we have:

$$\begin{aligned}
 3k - 2 &= 1 \\
 \Rightarrow 3k &= 3 \\
 \Rightarrow k &= 1
 \end{aligned}$$

Thus, the value of k is 1.

Algebra of Matrices Ex 5.3 Q36

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$\begin{aligned}
 A^2 - 8A + kI &= 0 \\
 \Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= 0 \\
 \Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

Since,

corresponding entries of equal matrices are equal, so

$$\begin{aligned}
 -7 + k &= 0 \\
 k &= 7
 \end{aligned}$$

Algebra of Matrices Ex 5.3 Q37

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$$f(A) = A^2 - 2A - 3I$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$f(A) = 0$$

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$$2\lambda + \mu = 7 \quad \text{--- (i)}$$

$$\lambda = 4 \quad \text{--- (ii)}$$

Put λ from equation (ii) in equation (i),

$$2(4) + \mu = 7$$

$$\mu = 7 - 8$$

$$\mu = -1$$

Hence, $\lambda = 4, \mu = -1$

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^3 - 4A^2 + A$$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-7+2 & 45-12+3 \\ 15-4+1 & 26-7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A = \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$

Given,

$$\begin{aligned} & \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Since, corresponding entries of equal matrices are equal, so

$$5x = 1 \quad \text{and} \quad 10x - 2 = 0$$

$$\Rightarrow x = \frac{1}{5} \quad \text{and} \quad x = \frac{1}{5}$$

$$\text{Hence, } x = \frac{1}{5}$$

Here,

$$\begin{aligned} & \begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0 \\ \Rightarrow & \begin{bmatrix} x-2 & 0-3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0 \\ \Rightarrow & [(x-2)x - 15] = 0 \\ \Rightarrow & x^2 - 2x - 15 = 0 \\ \Rightarrow & x^2 - 5x + 3x - 15 = 0 \\ \Rightarrow & x(x-5) + 3(x-5) = 0 \\ \Rightarrow & (x-5)(x+3) = 0 \\ \Rightarrow & x-5 = 0 \quad \text{or} \quad x+3 = 0 \\ \Rightarrow & x = 5 \quad \text{or} \quad x = -3 \end{aligned}$$

So,

$$x = 5 \text{ or } -3$$

We have:

$$\begin{aligned}
 \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} &= O \\
 \Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} &= O \\
 \Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} &= O \\
 \Rightarrow \begin{bmatrix} x(x-2)-40+2x-8 \end{bmatrix} &= O \\
 \Rightarrow \begin{bmatrix} x^2-2x-40+2x-8 \end{bmatrix} &= \begin{bmatrix} 0 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} x^2-48 \end{bmatrix} &= \begin{bmatrix} 0 \end{bmatrix} \\
 \therefore x^2-48 &= 0 \\
 \Rightarrow x^2 &= 48 \\
 \Rightarrow x &= \pm 4\sqrt{3}
 \end{aligned}$$

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$

$$\begin{aligned}
 & A^2 - 4A + 3I_3 \\
 &= \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1+6+0 & 2-8+0 & 0+10+0 \\ 3-12+0 & 6+16-5 & 0-20+15 \\ 0-3+0 & 0+4-3 & 0-5+9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 7-4+3 & -6-8+0 & 10-0+0 \\ -9-12+0 & 17+16+3 & -5-20+0 \\ -3-0+0 & 1+4+0 & 4-12+3 \end{bmatrix} \\
 &= \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}
 \end{aligned}$$

Hence,

$$A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q42

Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

And $f(x) = x^2 - 2x$

$\Rightarrow f(A) = A^2 - 2A$

$\Rightarrow f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 0+4+0 & 0+5+4 & 0+0+6 \\ 0+20+0 & 4+25+0 & 8+0+0 \\ 0+8+0 & 0+10+6 & 0+0+9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4-0 & 9-2 & 6-4 \\ 20-8 & 29-10 & 8-0 \\ 8-0 & 16-4 & 9-6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\text{And } f(x) = x^3 + 4x^2 - x$$

$$\Rightarrow f(A) = A^3 + 4A^2 - A \quad \text{---(i)}$$

$$A^2 = A \times A$$

$$\begin{aligned} &= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix} \end{aligned}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$\begin{aligned} &= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix} \end{aligned}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$$f(A) = A^3 + 4A^2 - A$$

$$\begin{aligned} &= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4 \begin{bmatrix} 4 & -15 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix} \end{aligned}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Given that, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $f(x) = x^3 - 6x^2 + 7x + 2$

Therefore, $f(A) = A^3 - 6A^2 + 7A + 2I_3$

First find A^2 :

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, Let us find A^3 :

$$A^3 = A^2 \times A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Thus,

$$f(A) = A^3 - 6A^2 + 7A + 2I_3$$

$$\begin{aligned} &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 21-30+7+2 & 0 & 34-48+14+0 \\ 12-12+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0 & 55-78+21+2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \end{aligned}$$

Thus, A is a root of the polynomial.

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 - 4A - 5I$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

Hence,

$$A^2 - 4A - 5I = 0$$

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 7A + 10I_3$$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7 \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \quad \text{---(i)}$$

$$-2x + 3z = 7 \quad \text{---(ii)}$$

$$5y - 7u = -6 \quad \text{---(iii)}$$

$$-2y + 3u = 2 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$

$$\underline{-10x + 15z = 35}$$

$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$

$$\Rightarrow 5x = 16 + 21$$

$$\Rightarrow 5x = 37$$

$$\Rightarrow x = 7.4$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$

$$\underline{-10y + 15u = 10}$$

$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$

$$\Rightarrow 5y - 7(-2) = -6$$

$$\Rightarrow 5y + 14 = -6$$

$$\Rightarrow 5y = -20$$

$$\Rightarrow y = -4$$

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{Since, } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2 \times 2} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2 \times 3}$$

$\Rightarrow$ A is a matrix of order 2×3

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+d & b+e & c+f \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$d = 1, e = 0, f = 1$$

$$\text{And } a + d = 3$$

$$a + 1 = 3$$

$$a = 3 - 1$$

$$a = 2$$

$$b + e = 3$$

$$b + 0 = 3$$

$$b = 3$$

$$\text{And } c + f = 5$$

$$c + 1 = 5$$

$$c = 4$$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

It is given that:

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

$$\text{Now, let } X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c=-7, \quad 2a+5c=-8, \quad 3a+6c=-9$$

$$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$$

$$\text{Now, } a+4c=-7 \Rightarrow a=-7-4c$$

$$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$$

$$\Rightarrow -3c=6$$

$$\Rightarrow c=-2$$

$$\therefore a=-7-4(-2)=-7+8=1$$

$$\text{Now, } b+4d=2 \Rightarrow b=2-4d$$

$$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$$

$$\Rightarrow -3d=0$$

$$\Rightarrow d=0$$

$$\therefore b=2-4(0)=2$$

$$\text{Thus, } a=1, b=2, c=-2, d=0$$

$$\text{Hence, the required matrix } X \text{ is } \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.$$

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

Therefore,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3},$$

$$\Rightarrow \begin{bmatrix} 4 \times (x_1) & 4 \times (x_2) & 4 \times (x_3) \\ 1 \times (x_1) & 1 \times (x_2) & 1 \times (x_3) \\ 3 \times (x_1) & 3 \times (x_2) & 3 \times (x_3) \end{bmatrix}_{3 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\Rightarrow \begin{bmatrix} 4x_1 & 4x_2 & 4x_3 \\ x_1 & x_2 & x_3 \\ 3x_1 & 3x_2 & 3x_3 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\Rightarrow 4x_1 = -4, \quad 4x_2 = 8, \quad 4x_3 = 4$$

$$\text{Solving } x_1 = -1, \quad x_2 = 2, \quad x_3 = 1$$

$$\text{So, matrix } A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}.$$

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

$$\text{Let, } A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } A_1.A_2 &= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \\ &= \begin{bmatrix} (2 \times -1) + (1 \times -1) + (3 \times 0) & (2 \times 0) + (1 \times 1) + (3 \times 1) & (2 \times -1) + (1 \times 0) + (3 \times 1) \end{bmatrix} \\ &= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } (A_1.A_2)A_3 &= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} (-3 \times 1) + (4 \times 0) + (1 \times -1) \end{bmatrix} \\ (A_1.A_2)A_3 &= \begin{bmatrix} -4 \end{bmatrix} = A \end{aligned}$$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating $(A_2.A_3)$ first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Algebra of Matrices Ex 5.3 Q49

$$\text{Let, } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Given,

$$\begin{aligned} A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} &= 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} &= \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} &= \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \end{aligned}$$

Since, corresponding entries of equal matrices are equal, so

$$a + b = 6 \quad \text{---(i)}$$

$$-2a + 4b = 0 \quad \text{---(ii)}$$

$$c + d = 0 \quad \text{---(iii)}$$

$$-2c + 4d = 6 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$\begin{aligned} 4a + 4b &= 24 \\ -2a + 4b &= 0 \\ \begin{array}{r} (+) \quad (-) \\ \hline 6a \end{array} &= 24 \\ \Rightarrow a &= \frac{24}{6} \\ a &= 4 \end{aligned}$$

Put $a = 4$ in equation (i)

$$a + b = 6$$

$$4 + b = 6$$

$$b = 6 - 4$$

$$b = 2$$

Solving equation (iii) and (iv)

$$2c + 2d = 0$$

$$\underline{-2c + 4d = 6}$$

$$6d = 6$$

$$d = \frac{6}{6}$$

$$d = 1$$

Put $d = 1$ in equation (iii)

$$c + d = 0$$

$$c = -1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q50

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$\begin{aligned}
 A^2 &= A \times A \\
 &= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 A^4 &= A^2 \times A^2 \\
 &= 0 \times 0 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 A^{16} &= A^4 \times A^4 \\
 &= 0 \times 0 \\
 &= 0
 \end{aligned}$$

So,

A^{16} is a null matrix

Solving the LHS of the given equation we have ,

$$\Rightarrow A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} (0 \times 0) + ((-x+1) \times (x+1)) & (0 \times (-x+1)) + ((-x+1) \times 0) \\ ((x+1) \times 0) + (0 \times (x+1)) & ((x+1) \times (-x+1)) + (0 \times 0) \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}.$$

Solving the RHS we get,

$$\Rightarrow A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0 \times 0) + ((-x) \times (x)) & (0 \times (-x)) + ((-x) \times 0) \\ ((x) \times 0) + (0 \times (x)) & ((x) \times (-x)) + (0 \times 0) \end{bmatrix} + \begin{bmatrix} (0 \times 0) + (1 \times 1) & (0 \times 1) + (1 \times 0) \\ (1 \times 0) + (0 \times 1) & (1 \times 1) + (0 \times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

Substituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow (A+B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow (A+B)^2 = A^2 + B^2.$$

Algebra of Matrices Ex 5.3 Q52

Solving the LHS i.e.

$$\begin{aligned} A^2 + A &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix} \end{aligned}$$

Solving the RHS i.e.

$$\begin{aligned} A(A+I) &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix} \end{aligned}$$

So, LHS = RHS verified.

Algebra of Matrices Ex 5.3 Q53

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } A^2 = AA &= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3 \times 3) + (-5 \times -4) & (3 \times -5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + (2 \times 2) \end{bmatrix} \\ &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}, \end{aligned}$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \quad \text{and} \quad -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 - 5A - 14I &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} \\ &= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 + 0 \\ -20 + 20 + 0 & 24 - 10 + -14 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Now,

$$A^2 - 5A - 14I = 0$$

$$\Rightarrow A^2 = 5A + 14I$$

$$\Rightarrow A^3 = A^2 \cdot A = (5A + 14I) A$$

$$\Rightarrow A^3 = A^2 \cdot A = 5A^2 + 14A \quad \left[\begin{array}{l} \text{By using dist. of matrices over} \\ \text{matrix addition} \end{array} \right]$$

$$\Rightarrow A^3 = 5 \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q54

We have,

$$P(x). P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$\Rightarrow P(x). P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$\Rightarrow P(x). P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

Now,

$$P(y). P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow P(y). P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$$

$$\Rightarrow P(y). P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

$$\therefore P(x). P(y) = P(x+y) = P(y). P(x)$$

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\begin{aligned} \text{So, } PQ &= \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \\ &= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix} \\ &= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } QP &= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \\ &= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix} \\ &= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & cz \end{bmatrix} \end{aligned}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$$\therefore PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then,

$$\begin{aligned} A^2 &= A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + 0 \times 2 + 1 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 0 \times 3 + 1 \times 0 \\ 2 \times 2 + 1 \times 2 + 3 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 1 \times 3 + 3 \times 0 \\ 1 \times 2 + -1 \times 2 + 0 \times 1 & 1 \times 0 + -1 \times 1 + 0 \times -1 & 1 \times 1 + -1 \times 3 + 0 \times 0 \end{bmatrix} \\ -5A &= \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{aligned} A^2 - 5A + 4I &= \begin{bmatrix} 5-10+4 & -1+0+0 & 5-5+0 \\ 9-10+0 & -2-5+4 & 5-15+0 \\ 0-5-0 & -1+5+0 & -2+0+4 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} \end{aligned}$$

Now, given is $A^2 - 5A + 4I + X = 0$

$$\Rightarrow X = -(A^2 - 5A + 4I)$$

$$X = - \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \quad \text{---(i)}$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\} \\ &= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix} \end{aligned}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that A^n is true for $n = k + 1$ whenever it is true for $n = k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q52

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

$$\text{To prove } A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \quad \text{we will use the principle of mathematical induction.}$$

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \frac{b(a^1 - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

$$A^n \text{ is true for } n = 1$$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \quad \text{---(i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1} - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} a^k & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} && \text{\{using equation (i) and given\}} \\ &= \begin{bmatrix} a^{k+1} + 0 & a^k b + \frac{b(a^k - 1)}{a - 1} \\ 0 + 0 & 0 + 1 \end{bmatrix} \\ &= \begin{bmatrix} a^{k+1} & \frac{a^{k+1}b - a^k b + a^k b - b}{a - 1} \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Algebra of Matrices Ex 5.3 Q57 pending

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1} - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

So,

$$A^n \text{ is true for } n = k + 1 \text{ whenever it is true } n = k.$$

Hence, by principle of mathematical induction A^n is true for all positive integer n .

Algebra of Matrices Ex 5.3 Q58

Given,

$$A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

To show that,

$$A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Put $n = 1$

$$A^1 = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

So,

$$A^n \text{ is true for } n = 1$$

Let, A^n is true for $n = k$, so

$$A^k = \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \quad \text{---(i)}$$

Now, we have to show that,

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now, $A^{k+1} = A^k \times A$

$$\begin{aligned} &= \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos k\theta \cos \theta + i^2 \sin k\theta \sin \theta & i^2 \cos k\theta \sin \theta + i \sin k\theta \cos \theta \\ i \sin k\theta \cos \theta + i \cos k\theta \sin \theta & i^2 \sin k\theta \sin \theta + \cos \theta \cos k\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos k\theta \cos \theta - \sin k\theta \sin \theta & i (\cos k\theta \sin \theta + \sin k\theta \cos \theta) \\ i (\sin k\theta \cos k\theta \sin \theta) & \cos k\theta \cos \theta - \sin k\theta \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix} \end{aligned}$$

So, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q59

Given,

$$A = \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix}$$

To prove $P(n)$: $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2} \sin n\alpha \\ -\sqrt{2} \sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show $P(1)$ is true.

A^n is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos \alpha + \sin \alpha) - 2 \sin \alpha \sin k\alpha & (\cos k\alpha + \sin k\alpha)\sqrt{2} \sin \alpha \\ (\cos \alpha + \sin \alpha)(-\sqrt{2} \sin k\alpha) - \sqrt{2} \sin \alpha (\cos k\alpha - \sin k\alpha) & +\sqrt{2} \sin k\alpha (\cos \alpha - \sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos k\alpha \cos \alpha + \sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha & \sqrt{2} \cos k\alpha \sin \alpha + \sqrt{2} \sin \alpha \sin k\alpha + \\ + \sin \alpha \sin k\alpha - 2 \sin \alpha \sin k\alpha & \sqrt{2} \sin k\alpha \cos \alpha - \sqrt{2} \sin k\alpha \sin \alpha \\ -\sqrt{2} \cos \alpha \sin \alpha - \sqrt{2} \sin \alpha \sin k\alpha - \sqrt{2} \sin \alpha & -2 \sin k\alpha \sin \alpha + \cos k\alpha \cos \alpha - \cos \alpha \\ \cos k\alpha + \sqrt{2} \sin \alpha \sin k\alpha & \sin k\alpha - \sin \alpha \cos k\alpha \sin \alpha \sin k\alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha \cos k\alpha + \sin \alpha \sin k\alpha & \sqrt{2} (\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha) \\ \sin \alpha \cos k\alpha + \sin k\alpha \cos \alpha & \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha - \\ -\sqrt{2} (\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha) & (\sin k\alpha \cos \alpha + \sin \alpha \cos k\alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix} \end{aligned}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Algebra of Matrices Ex 5.3 Q60

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k + 1$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} && \text{(using equation (i) and given)} \\ &= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Hence, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n .

We will prove $P(n): A^{n+1} = B^n [B + (n+1)C]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C, \quad BC = CB, \quad C^2 = 0 \\ A = B + C$$

Squaring both the sides, so

$$\begin{aligned} A^2 &= (B + C)^2 \\ \Rightarrow A^2 &= (B + C)(B + C) \\ \Rightarrow A^2 &= B \times B + BC + CB + C \times C && \text{(using distributive property)} \\ \Rightarrow A^2 &= B^2 + BC + BC + C^2 && \text{(using } BC = CB \text{ given)} \\ \Rightarrow A^2 &= B^2 + 2BC + 0 && \text{(since, given } C^2 = 0\text{)} \\ \Rightarrow A^2 &= B^2 + 2BC && \text{---(1)} \\ A^2 &= B(B + 2C) \end{aligned}$$

Now, consider

$$P(n): A^{n+1} = B^n [B + (n+1)C]$$

Step 1: To prove $P(1)$ is true, put $n = 1$

$$\begin{aligned} A^{1+1} &= B^1 [B + (1+1)C] \\ A^2 &= B [B + 2C] \\ A^2 &= B^2 + 2BC \end{aligned}$$

From equation (i), $P(1)$ is true.

Step 2: Suppose $P(k)$ is true.

$$\therefore A^{k+1} = B^k [B + (k+1)C] \quad \text{---(2)}$$

Step 3: Now, we have to show that $P(k+1)$ is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1} [B + (k+2)C]$$

Now,

$$\begin{aligned} A^{k+2} &= A^k \times A^2 \\ &= B^{(k-1)} [B + kC] \times [B + 2C] \\ &= B^k [B + kC] \times [B + 2C] \\ &= B^k [B \times B + B \times 2C + kC \times B + 2kC^2] \\ &= B^k [B^2 + 2BC + kBC + 2k \times 0] && \text{(since } BC = CB, \quad C^2 = 0\text{)} \\ &= B^k [B^2 + BC(2+k)] \\ &= B^k \times B [B + (k+2)C] \\ &= B^{k+1} [B + (k+2)C] \end{aligned}$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}(a^n, b^n, c^n)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}(a^1, b^1, c^1)$$

$$A = \text{diag}(a, b, c)$$

So,

$$A^n \text{ is true for } n = 1$$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}(a^k, b^k, c^k) \quad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \text{diag}(a^k, b^k, c^k) \times \text{diag}(a, b, c) \quad \text{(using equation (i) and given)} \end{aligned}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Given,

$$\text{order of matrix } X = (a+b) \times (a+2)$$

$$\text{order of matrix } Y = (b+1) \times (a+3)$$

Given, $X_{(a+b) \times (a+2)} \cdot Y_{(b+1) \times (a+3)}$ exist.

$$\Rightarrow a+2 = b+1$$

$$\Rightarrow a - b = -1 \quad \text{---(i)}$$

And

$$Y_{(b+1) \times (a+3)} \cdot X_{(a+b) \times (a+2)} \text{ exists.}$$

$$\Rightarrow a+3 = a+b$$

$$\Rightarrow b = 3$$

Put $b = 3$ in equation (i),

$$a - b = -1$$

$$a - 3 = -1$$

$$a = 3 - 1$$

$$a = 2$$

$$\text{So, } a = 2, b = 3$$

So,

$$\text{Order of } X = (a+b) \times (a+2)$$

$$= (2+3) \times (2+2)$$

$$= 5 \times 4$$

$$\text{Order of } Y = (b+1) \times (a+3)$$

$$= (3+1) \times (2+3)$$

$$= 4 \times 5$$

$$\text{Order of } X_{5 \times 4} \cdot Y_{4 \times 5} = 5 \times 5$$

$$\text{Order of } X_{4 \times 5} \cdot Y_{5 \times 4} = 4 \times 4$$

So, order of XY and YX are not same and they are not equal but both are square matrices.

Algebra of Matrices Ex 5.3 Q65(i)

$$\text{Let, } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned} BA &= \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \end{aligned}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation (i) and (ii)

$$AB \neq BA$$

$$\text{when } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$\text{Let, } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$\begin{aligned} AB &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = 0$$

When,

$$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$\text{Let, } A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$AB = 0$$

$$\begin{aligned} BA &= \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Let, } A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Here,

$$A \neq 0, B \neq C$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

So,

$$\text{for } A \neq 0, BC \neq 0 \text{ but } AB = AC$$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matrices of same order

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= A(A+B) + B(A+B) && \text{(using distributive property)} \\ &= A \times A + AB + BA + B^2 \\ &= A^2 + AB + BA + B^2 \end{aligned}$$

But,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ is possible only when } AB = BA$$

Here, we can not say that $AB = BA$

So,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ does not hold.}$$

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

$$\begin{aligned}
 \text{(i) } (A+B)^2 &= (A+B)(A+B) \\
 &= A(A+B) + B(A+B) && \{\text{using distributive property}\} \\
 &= A \times A + AB + BA + B \times B \\
 &= A^2 + AB + BA + B^2 \\
 &\neq A^2 + 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$)

$$\text{So, } (A+B)^2 \neq A^2 + 2AB + B^2$$

$$\begin{aligned}
 \text{(ii) } (A-B)^2 &= (A-B)(A-B) \\
 &= A(A-B) - B(A-B) && \{\text{using distributive property}\} \\
 &= A \times A - AB - BA + B \times B \\
 &= A^2 - AB - BA + B^2 \\
 &\neq A^2 - 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$), so

$$\text{So, } (A-B)^2 \neq A^2 - 2AB + B^2$$

$$\begin{aligned}
 \text{(iii) } (A+B)(A-B) &= A(A-B) + B(A-B) && \{\text{using distributive property}\} \\
 &= A \times A - AB + BA - B \times B \\
 &= A^2 - AB + BA - B^2 \\
 &= A^2 - B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$),

$$\text{So, } (A+B)(A-B) \neq A^2 - B^2$$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that $AB = BA$ else the result is not true in general.

$$\text{Example: Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
 \text{Here } AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 0 + 0 \times 2 + 0 \times 0 & 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 0 + 1 \times 2 + 0 \times 0 & 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{and } BA &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$AB \neq BA$$

$$\begin{aligned}
\text{Now, } (AB)^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 \times 0 + 1 \times 1 + 0 \times 0 & 0 \times 1 + 1 \times 2 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 1 \times 0 + 2 \times 1 + 0 \times 0 & 1 \times 1 + 1 \times 2 + 0 \times 0 & 1 \times 0 + 2 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
A^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
B^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 \times 0 + 1 \times 2 + 0 \times 0 & 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 0 + 1 \times 2 + 0 \times 0 & 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
A^2 B^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 \times 2 + 0 \times 2 + 0 \times 0 & 1 \times 1 + 0 \times 2 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 2 \times 2 + 1 \times 2 + 0 \times 0 & 2 \times 1 + 1 \times 2 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 2 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 1 & 0 \\ 6 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2 B^2$

Algebra of Matrices Ex 5.3 Q69

Given,

A and B two square matrices of same order such that

$AB = BA$.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$\begin{aligned}
(A+B)^2 &= (A+B)(A+B) \\
&= A(A+B) + B(A+B) && \left[\begin{array}{l} \text{by dist. of matrix multiplication} \\ \text{over addition} \end{array} \right] \\
&= A^2 + AB + BA + B^2 && \left[\begin{array}{l} \text{by dist. of matrix multiplication} \\ \text{over addition} \end{array} \right] \\
&= A^2 + 2AB + B^2 && [As, AB = BA] \\
&= \text{RHS}
\end{aligned}$$

Hence proved.

Algebra of Matrices Ex 5.3 Q70

$$\text{Given, } A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12 \end{bmatrix} \\ AB &= \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} AC &= \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0 \end{bmatrix} \\ AC &= \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \quad \text{---(ii)} \end{aligned}$$

From equation (i) and (ii)

$$AB = AC$$

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{matrix} & \begin{matrix} \text{Notebook} & \text{Pens} & \text{Pencils} \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{matrix}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{matrix} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{matrix}$$

Individual bill can be calculated by

$$\begin{aligned} XY &= \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \\ XY &= \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix} \\ XY &= \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix} \end{aligned}$$

So,

Bill of A = Rs 157.80

Bill of B = Rs 167.40

Bill of C = Rs 281.40

Algebra of Matrices Ex 5.3 Q72

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{matrix} & \begin{matrix} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{matrix} \\ \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \end{matrix}$$

Matrix representation of sellin price (Rs.) of each book is given by

$$Y = \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \begin{matrix} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{matrix}$$

So, totaol amount recieved by the store from sellin all the items is given by,

$$\begin{aligned} XY &= \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \\ &= [(120)(8.30) + (96)(3.45) + (60)(4.50)] \\ &= [996 + 331.20 + 270] \\ &= [1597.20] \end{aligned}$$

Required amount = Rs 1597.20

Algebra of Matrices Ex 5.3 Q73

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and Y is given by.

$$B = \begin{bmatrix} & \text{Telephone} & \text{Housecall} & \text{Letter} \\ 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Total amount spent by the group in the two cities X and Y can be given by

$$\begin{aligned} BA &= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \\ &= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix} \\ &= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix} \end{aligned}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Algebra of Matrices Ex 5.3 Q74

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \quad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

Algebra of Matrices Ex 5.3 Q75

The cost for each mode per attempt is represented by 3×1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3×3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the organization for the three villages separately is given by matrix multiplication

$$\begin{aligned} BA &= \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} \\ BA &= \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix} \\ &= \begin{bmatrix} 30,000 \\ 23,000 \\ 39,000 \end{bmatrix} \end{aligned}$$

Note: The answer given in the book is incorrect.

Algebra of Matrices Ex 5.3 Q76

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{array}{c} \text{Men} \quad \text{Women} \quad \text{Children} \\ \text{Family A} \begin{bmatrix} 4 & 6 & 2 \end{bmatrix} \\ \text{Family B} \begin{bmatrix} 2 & 2 & 4 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \text{Calories} \quad \text{Protein} \\ \text{Men} \begin{bmatrix} 2400 & 45 \end{bmatrix} \\ \text{Women} \begin{bmatrix} 1900 & 55 \end{bmatrix} \\ \text{Children} \begin{bmatrix} 1800 & 33 \end{bmatrix} \end{array}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R thus ,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{array}{c} \text{Calories} \quad \text{Protein} \\ \text{Family A} \begin{bmatrix} 24600 & 576 \end{bmatrix} \\ \text{Family B} \begin{bmatrix} 15800 & 332 \end{bmatrix} \end{array}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Algebra of Matrices Ex 5.3 Q77

The cost per contact (in paisa) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{array}{c} \text{Telephone} \quad \text{House calls} \quad \text{Letters} \\ \text{City X} \begin{bmatrix} 1000 & 500 & 5000 \end{bmatrix} \\ \text{City Y} \begin{bmatrix} 3000 & 1000 & 10000 \end{bmatrix} \end{array}$$

The total amount of money spent by party in each of the city for the election is given by the matrix multiplication :

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix}$$

$$= \begin{array}{c} \text{City X} \begin{bmatrix} 990000 \end{bmatrix} \\ \text{City Y} \begin{bmatrix} 2120000 \end{bmatrix} \end{array}$$

The total amount of money spent by party in each of the city for the election in rupees is given by

$$= \left(\frac{1}{100} \right) \begin{array}{c} \text{City X} \begin{bmatrix} 990000 \end{bmatrix} \\ \text{City Y} \begin{bmatrix} 2120000 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \text{City X} \begin{bmatrix} 9900 \end{bmatrix} \\ \text{City Y} \begin{bmatrix} 21200 \end{bmatrix} \end{array}$$

One could consider social activities before casting his/her vote to the party.

Ex 5.4

Algebra of Matrices Ex 5.4 Q1(i)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(2A)^T = 2 \times A^T$$

$$\Rightarrow \left(2 \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2A^T$$

Algebra of Matrices Ex 5.4 Q1(ii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A + B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 2+1 & -7+2 \\ -3+0 & 5-4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A + B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q1(iii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A - B)^T = A^T - B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 2-1 & -7-2 \\ -3-0 & 5+4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A - B)^T = A^T - B^T$$

Algebra of Matrices Ex 5.4 Q1(iv)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2 - 6 & 0 + 12 \\ -7 + 10 & 0 - 20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2 - 6 & -7 + 10 \\ 0 + 12 & 0 - 20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

$$\Rightarrow \text{HS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Given,

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}^T \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q3(ii)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q3(iii)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(2A)^T = 2A^T$$

$$\Rightarrow \quad \left(2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}^T = 2 \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Given,

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & -6 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Given,

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$(AB)^T$$

$$\begin{aligned} &= \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix} \right) \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} 6 - 4 - 2 & 8 + 8 - 1 \\ -3 + 0 + 4 & -4 + 0 + 2 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix} \end{aligned}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

Given,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+0+15 & -2+20 \\ 4+0+0 & -4+2+0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}^T = \begin{bmatrix} 2+0+15 & 4+0+0 \\ -2+2+0 & -4+2+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Given,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Given that $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

We need to find $A^T - B^T$.

Given that, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Let us find $A^T - B^T$:

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

(i)

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Algebra of Matrices Ex 5.4 Q9

(ii)

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Algebra of Matrices Ex 5.4 Q10

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \begin{cases} l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \\ l_2 l_3 + m_2 m_3 + n_2 n_3 = 0 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 = 0 \end{cases} \quad \text{---(A)}$$

And,

$$\begin{cases} l_1^2 + m_1^2 + n_1^2 = 1 \\ l_2^2 + m_2^2 + n_2^2 = 1 \\ l_3^2 + m_3^2 + n_3^2 = 1 \end{cases} \quad \text{---(B)}$$

Given,

$$\begin{aligned} A &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \\ AA^T &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T \\ &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_1 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \\ &= \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & l_2^2 + m_2^2 + n_2^2 & l_2 l_3 + m_2 m_3 + n_2 n_3 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 & l_2 l_3 + m_2 m_3 + n_2 n_3 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(Using (A) and (B))} \\ &= I \end{aligned}$$

Hence,

$$AA^T = I$$

Ex 5.5

Algebra of Matrices Ex 5.5 Q1

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{aligned} (A - A^T) &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T \right) \\ &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \right) \\ &= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix} \\ (A - A^T) &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \\ -(A - A^T)^T &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)} \end{aligned}$$

From (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $(A - A^T)$ is skew symmetric.

Algebra of Matrices Ex 5.5 Q2

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$\begin{aligned} A - A^T &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} \end{aligned}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} \end{aligned}$$

$$-(A - A^T)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that, x is skewsymmetric matrix if $x = -x^T$

So, $(A - A^T)$ is skewsymmetric matrix.

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

$$\text{So, } x = a_{13} = a_{31} = 4$$

$$y = a_{21} = a_{12} = 2$$

$$z = a_{22} = a_{22} = z$$

$$t = a_{32} = a_{23} = -3$$

Hence,

$$x = 4, y = 2, t = -3 \text{ and } z \text{ can have any value.}$$

Algebra of Matrices Ex 5.5 Q4

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$

$$\begin{aligned} \therefore X &= \frac{1}{2}(A + A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix} \\ \Rightarrow X &= \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, } Y &= \frac{1}{2}(A - A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix} \\
 Y &= \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}
 \end{aligned}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = X$$

$\Rightarrow$ X is a symmetric matrix

Now,

$$\begin{aligned}
 -Y^T &= - \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix} \\
 &= - \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T
 \end{aligned}$$

$$\Rightarrow -Y' = Y$$

$\therefore Y$ is skew symmetric matrix.

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2} + \frac{1}{2} & \frac{5}{2} + \frac{9}{2} \\ \frac{3}{2} - \frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2} - \frac{9}{2} & 4+1 & 8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

$$= A$$

Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Given,

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

$$\begin{aligned} \text{Let } X &= \frac{1}{2}(A + A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} \\ X^T &= \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} = X \end{aligned}$$

$\therefore X$ is symmetric matrix

Now,

$$\begin{aligned} Y &= \frac{1}{2}(A - A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix} \\ \therefore Y &= \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} \\ \Rightarrow -Y^T &= - \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} = Y \end{aligned}$$

$\Rightarrow Y$ is a skew symmetric matrix.

Now,

$$\begin{aligned} X + Y &= \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} \\ &= \begin{bmatrix} 4+0 & \frac{5}{2}-\frac{1}{2} & 0-1 \\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2} \\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} = A \end{aligned}$$

A square matrix A is called a symmetric matrix, if $A^T = A$

Here,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\begin{aligned} A + A^T &= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}^T \\ &= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 4+5 \\ 5+4 & 6+6 \end{bmatrix} \end{aligned}$$

$$A + A^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \quad \text{---(i)}$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}^T$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$(A + A^T)^T = (A + A^T)$$

So,

$(A + A^T)$ is a symmetric matrix.

Algebra of Matrices Ex 5.5 Q7

Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\text{Let, } X = \frac{1}{2}(A + A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}$$

$$\text{Now, } X^T = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} = X$$

$\Rightarrow X$ is symmetric matrix

$$\text{Let } Y = \frac{1}{2}(A - A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\text{Now, } -Y^T = - \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = - \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow Y$ is skew symmetric

$$\text{Now, } X + Y = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & -\frac{3}{2}-\frac{5}{2} \\ -\frac{3}{2}+\frac{5}{2} & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Algebra of Matrices Ex 5.5 Q8

Let,

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Let, } X &= \frac{1}{2}(A + A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\text{Now, } X^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix

$$\begin{aligned} \text{Let, } Y &= \frac{1}{2}(A - A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} \end{aligned}$$

$$-Y^T = - \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric matrix

$$X + Y = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{1}{2}-\frac{5}{2} & \frac{-5}{2}-\frac{3}{2} \\ \frac{1}{2}+\frac{5}{2} & -2+0 & -2-3 \\ \frac{-5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

$$\text{Hence, Symmetric matrix } X = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$