Sample Question Paper - 13
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

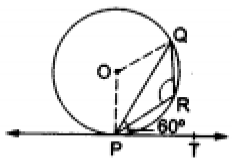
1. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 2 - 3n$. **[2]**

OR

Which term of an A.P. 150,147,144,..........is its first negative term?

2. Solve for x : **[2]**
$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$

3. In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT$ = 60°, find $\angle PRQ$. **[2]**



4. A sphere of diameter 5 cm is dropped into a cylindrical vessel partly filled with water. The diameter of the base of the vessel is 10 cm. If the sphere is completely submerged, by how much will the level of water rise? **[2]**

5. Find the modal class for the following distribution **[2]**

Marks	**Number of students**
Below 10	3
Below 20	12
Below 30	27
Below 40	57

Below 50	75
Below 60	80

6. Find the value of k for which the roots are real and equal of equation: **[2]**

$3x^2 - 5x + 2k = 0$

OR

Find the roots of the equation, if they exist, by applying the quadratic formula: $36x^2 - 12ax + (a^2 - b^2) = 0$.

Section B

7. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24. **[3]**

Age in years	0 - 10	10 -20	20 - 30	30 - 40	40 - 50
No. of persons	5	25	?	18	7

8. Divide a line segment of length 14 cm internally in the ratio 2: 5. Also, justify your construction. **[3]**

9. Calculate the arithmetic mean of the following frequency distribution, using the step-deviation method: **[3]**

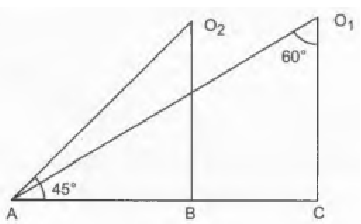
Class interval	**Frequency**
0 - 50	17
50 - 100	35
100 - 150	43
150 - 200	40
200 - 250	21
250 - 300	24

10. A ladder rests against a wall at an angle a to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle $\beta$with the horizontal. Show that **[3]**

$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

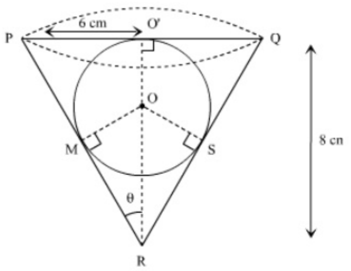
OR

What are the angles of depression from the observing positions O_1 and O_2 of the object at A?



Section C

11. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows? **[4]**

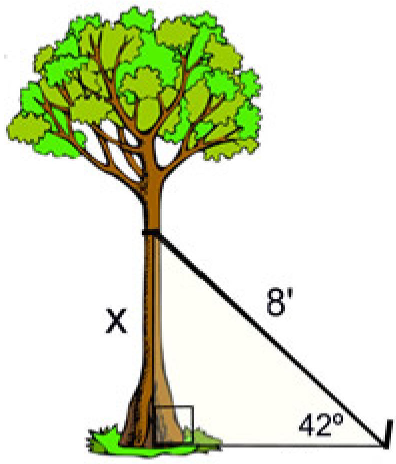


12. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC. **[4]**

OR

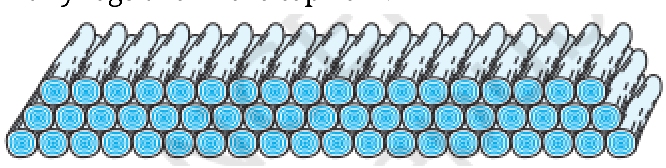
AB is a diameter and AC is chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Prove that BC = BD.

13. A nursery plants a new tree and attaches a guy wire to help support the tree while its roots take hold. An eight-foot wire is attached to the tree and to a stake in the ground. From the stake in the ground the angle of elevation of the connection with the tree is 42°. **[4]**



i. Find to the *nearest tenth of a foot*, the height of the connection point on the tree.
ii. If the angle of elevation changes to 30^{0}, keeping the height of the connection point on the tree same as calculated in (i), what should be the length of the wire?

14. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how many logs are in the top row? **[4]**



Solution

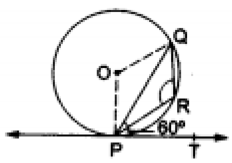
MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. According to the question,
   a_n = 2 - 3n
   Put n = 1,a_1 = 2 - 3 $\times$ 1 = 2 - 3 = -1
   Put n = 2, a_2 = 2 - 3 $\times$ 2 = 2 - 6 = -4
   Common difference(d) = (-4) - (-1) = -4 + 1 = -3
   S_n = $\frac{n}{2}[2a+(n-1)d]$
   S_{25} = $\frac{25}{2}[2(-1)+(25-1)(-3)]$
   = $\frac{25}{2}[-2-72]$
   $=\frac{25}{2}\times -74$
   = -925

   OR

   a = 150 and d = -3
   Let, $a_{n\ =}$ 0
   a + (n - 1)d = 0
   150 + (n - 1)(-3) = 0
   150 - 3n + 3 = 0
   -3n = -153
   3n =153
   n = 153/3
   ∴n > 51.
   Since, the first negative term is 52^{nd} term.
2. Given, $\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}$
   $\Rightarrow$ $\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2}$ = $\frac{4x-8-2x-3}{x-2}$
   $\Rightarrow$(2x^2 + 4) (x - 2) = (2x -11) (x^2 + x - 2)
   or, 5x^2 + 19x- 30 = 0
   $\Rightarrow$ 5x^2 + 25x - 6x - 30 = 0
   $\Rightarrow$(x+5) (5x-6) = 0
   $\Rightarrow$ x = -5, $\frac{6}{5}$
3. 

   In the given figure it is given that PQ is a chord of a circle with center O and ∠QPT = 60°.
   let X be the point on the tangent PT of the circle.
   Now from the given figure we have, ∠QPT + ∠QPX = 180° (Linear pair)
   ⇒ ∠QPX = 180° - ∠QPT = 180° - 60° = 120°
   and ∠QPX = ∠PRQ (Alternate segment theorem)
   ⇒ ∠PRQ = 120°
4. Diameter of sphere = 5 cm
   Radius of sphere = $\frac{5}{2}$ cm
   Diameter of cylinder = 10 cm
   Radius of Cylinder = $\frac{10}{2}$ = 5 cm
   Let level of water rise in cyinder = h cm

According to the question

Volume of sphere = Volume of water rise in cylinder

$\Rightarrow \quad \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 = \pi(5)^2 \times h$

$\Rightarrow \quad \frac{4}{3} \times \frac{125}{8} = 25 \times h$

$\Rightarrow \quad h = \frac{4\times125}{3\times8\times25}$ cm

$\Rightarrow \quad h = \frac{500}{600} = \frac{5}{6}$ cm

5.

Class Interval	**Frequency**
0 - 10	3
10 - 20	9
20 - 30	15
30 - 40	30
40 - 50	18
50 - 60	5

Here, modal class = 30 - 40 as it has maximum frequency,i.e; 30.

6. We have,

$3x^2 - 5x + 2k = 0$

Here, a = 3, b = -5 and c = 2k

$\therefore$ D = b^2 - 4ac

= $(-5)^2 - 4 \times 3 \times 2k$

= 25 - 24k

$\Rightarrow$ D = 25 - 24k

Now, it is given that given equation will has real and equal roots,

So, discriminant D = 0

$\Rightarrow$ 25 - 24k = 0

$\Rightarrow$ 24k = 25

$\Rightarrow k = \frac{25}{24}$

Hence, k $= \frac{25}{24}$ so that given equation has real and equal roots.

OR

The given equation is $36x^2 - 12ax + (a^2 - b^2) = 0$

Comparing it with $ax^2 + bx + c = 0$, we have

a = 36, b = -12a and c = $a^2 - b^2$

Now, discrimnant D = $b^2 - 4ac = (-12a)^2 - 4(36)(a^2 - b^2)$

= $144a^2 - 144a^2 + 144b^2 = 144b^2 > 0$

So, the given equation has real roots and are given by

$\alpha = \frac{-b+\sqrt{D}}{2A} = \frac{-(-12a)+\sqrt{144b^2}}{2\times36} = \frac{12a+12b}{72} = \frac{a+b}{6}$

$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-12a)-\sqrt{144b^2}}{2\times36} = \frac{12a-12b}{72} = \frac{a-b}{6}$

Hence, $\frac{a+b}{6}$ and $\frac{a-b}{6}$ are the roots of the given equation.

Section B

7.

Class interval	**Frequency**	**Cumulative frequency**
0-10	5	5
10-20	25	30
20-30	x	30 + x
30-40	18	48 + x
40-50	7	55 + x

	N = 55 + x	

Let the missing frequency be x

Given, Median = 24 ...(1)

From table, total frequency N = 55 + x Or, $\left(\frac{N}{2}\right)$ = 27.5 + $\left(\frac{x}{2}\right)$

Hence, c.f. just greater than $\left(\frac{N}{2}\right)$ is (30 + x), which corresponding class is 20 - 30.

Then, median class = 20 - 30

$\therefore$ l = 20, h = 30 - 20 =10, f =x, F = 30

$\therefore$ Median= l + $\frac{\frac{N}{2}-F}{f}\times$ h

$\Rightarrow$ 24 = 20 + $\frac{\frac{55+x}{2}-30}{x}\times 10$

$\Rightarrow$ 24 - 20 = $\frac{\frac{55+x}{2}-30}{x}\times$ 10

$\Rightarrow$ 4x = $\left(\frac{55+x}{2}-30\right)\times$ 10

$\Rightarrow$ 4x = 5 (55 + x) - 300

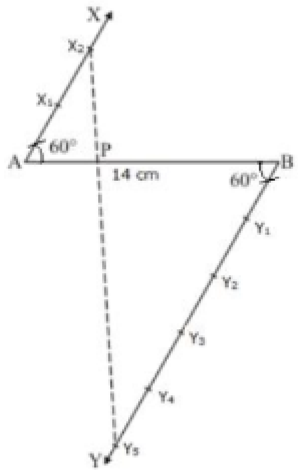
$\Rightarrow$ 4x - 5x = -25

$\Rightarrow$ -x = -25

$\Rightarrow$ x = 25

$\therefore$ Missing frequency = 25

8. Steps of construction



i. Draw a line segment AB of 14 cm.
ii. Through the points A and B, draw two parallel lines AX and BY on the opposite side of AB.
iii. Starting from A. Cut 2 equal parts on AX and starting from B cut 5 equal parts on BY such that $AX_1 = X_1X_2$ and $BY_1 = Y_1Y_2 = Y_2Y_3 = Y_3Y_4 = Y_4Y_5$
iv. Join X_2Y_5 which intersect AB at P

$\therefore \frac{AP}{PB} = \frac{2}{5}$

Justification

In $\triangle APX_2$ and $\triangle BPY_5$

$\angle APX_2 = \angle BPY_5$ (vertically opposite angles)

$\angle PAX_2 = \angle PBY_5$ (alternate interior angles)

then $\triangle APX_2 \sim \triangle BYY$ (by AA similarity)

$\therefore \frac{AP}{PB} = \frac{AX_2}{BY_5} = \frac{2}{5}$ (by c.p.c.t)

9.

Class Interval	Frequency(f_i)	Mid value x_i	$u_i = \frac{(x_i - A)}{h}$	$(f_i \times u_i)$
0 - 50	17	25	-2	-34
50 - 100	35	75	-1	-35
100 - 150	43	125 = A	0	0
150 - 200	40	175	1	40

200 - 250	21	225	2	42
250 - 300	24	275	3	72
	$\Sigma f_i = 180$			$\Sigma\left(f_i \times u_i\right) = 154 - 69 = 85$

let assumed mean is 125.

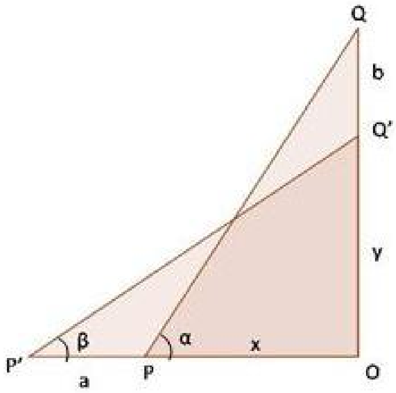
$A = 125, h = 50, \Sigma f_i = 180$ and $\Sigma\left(f_i u_i\right) = 85$

Mean, $\bar{x} = A + \left\{h \times \frac{\Sigma(f_i u_i)}{\Sigma f_i}\right\}$

$= 125 + \left\{50 \times \frac{85}{180}\right\}$

$= 125 + 23.61$

$= 148.61$

10. 

Let PQ be the ladder such that is top Q is on the wall OQ.

The ladder is pulled away from the wall through a distance a, so Q slides and takes position Q'.

Clearly, $PQ = P'Q'$.

In $\Delta's\ POQ\ and\ P'OQ'$,we have

$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}, \sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$

$\Rightarrow \sin\alpha = \frac{b+y}{PQ}, \cos\alpha\frac{x}{PQ}, \sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ}$ and

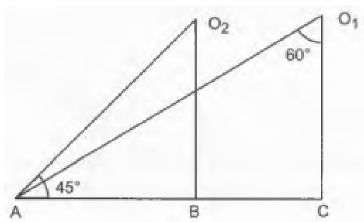
$\cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b}{PQ}$ and

$\cos\beta - \cos\alpha = \frac{a}{PQ}$

$\Rightarrow \frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

OR



In triangle O_1AC,

$\Rightarrow \angle A = 180^0 - (90^0 + 60^0)$

$\Rightarrow \angle A = 180^0 - 150^0$ We know that $[\angle A + \angle B + \angle C = 180^0]$

$\Rightarrow \angle A = 30^0$

Again,

In a triangle O_2AB,

$\Rightarrow \angle O_2 = 180^0 - (90^0 + 45^0)$

$\Rightarrow \angle O_2 = 180^0 - 135^0$

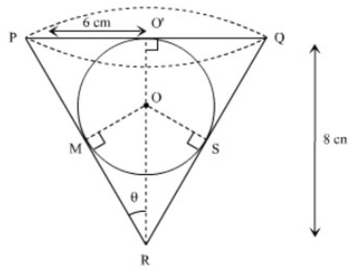
$\Rightarrow \angle O_2 = 45^o$

Hence the required angles are 30^o, 45^o.

Section C

11. Radius (R) of conical vessel = 6 cm

Height (H) of conical vessel = 8 cm



Volume of conical vessel (V_c) = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi \times 6 \times 6 \times 8$

= $96\pi cm^3$

Let the radius of the sphere be r cm

In right Δ PO'R by pythagoras theorem We have

$l^2 = 6^2 + 8^2$

l = $\sqrt{36+64}$ = 10 cm

In right triangle MRO

$\sin\theta = \frac{OM}{OR}$

$\Rightarrow \quad \frac{3}{5} = \frac{r}{8-r}$

$\Rightarrow$ 24 - 3r = 5r

$\Rightarrow$ 8r = 24

$\Rightarrow$ r = 3 cm

$\therefore V_1$ = Volume of the sphere ,= $\frac{4}{3}\pi \times 3^3 cm^3 = 36\pi cm^3$

V_2 = Volume of the water = Volume of the cone = $\frac{1}{3}\pi \times 6^2 \times 8cm^3 = 96\pi cm^3$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1.

$\therefore$ Fraction of the water that flows out =$V_1 : V_2 = 36\pi : 96\pi = 3 : 8$

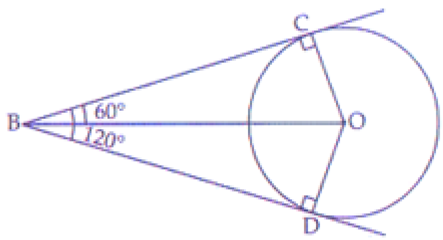
12. According to the question, we are a given that from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle$DBC = 120°, we have to prove that BC + BD = BO, i.e., BO = 2BC.

Given: A circle with centre O.

Tangents BC and BD are drawn from an external point B such that $\angle$DBC = 120°

To prove: BC + BD = BO, i.e., BO = 2BC

Construction: Join OB, OC and OD.



Proof: In $\triangle$OBC and $\triangle$OBD, we have

OB = OB [Common]

OC = OD [Radii of same circle]

BC = BD [Tangents from an external point are equal in length] ...(i)

$\therefore \triangle OBC \cong \triangle OBD$ [By SSS criterion of congruence]

$\Rightarrow \angle$OBC = $\angle$OBD (CPCT)

$\therefore \angle OBC = \frac{1}{2} \angle DBC = \frac{1}{2} \times 120°$ [$\because \angle$CBD = 120° given]

$\Rightarrow \angle OBC = 60°$

OC and BC are radius and tangent respectively at contact point C.

Hence, $\angle OCB = 90°$

Now, in right angle $\triangle OCB$, $\angle OBC = 60°$

$\therefore \cos 60° = \frac{BC}{BO}$

$\Rightarrow \frac{1}{2} = \frac{BC}{BO}$

$\Rightarrow OB = 2BC$

$\Rightarrow OB = BC + BC$

$\Rightarrow OB = BC + BD$ [$\because$ BC = BD from (i)]

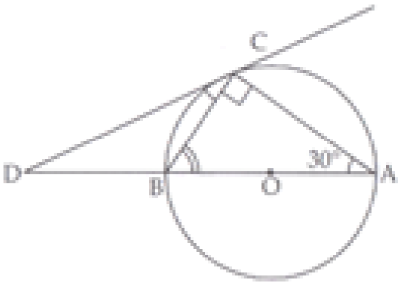
Hence, proved.

OR

Given: A circle with centre O. A tangent CD at C.

Diameter AB is produced to D.

BC and AC chords are joined, ∠BAC = 30°



To prove: BC = BD

Proof: DC is tangent at C and, CB is chord at C.

Therefore, ∠DCB = ∠BAC [∠s in alternate segment of a circle]

⇒ ∠DCB = 30° ...(i) [∵ ∠BAC = 30° (Given)]

AOB is diameter. [Given]

Therefore, ∠BCA = 90° [Angle in s semi circle]

Therefore, ∠ABC = 180° - 90° - 30° = 60°

In ΔBDC,

Exterior ∠B = ∠D + ∠BCD

⇒ 60° = ∠D + 30°

⇒ ∠D = 30° ...(ii)

Therefore, ∠DCB = ∠D = 30° [From (i), (ii)]

⇒ BD = BC [∵ Sides opposite to equal angles are equal in a triangle]

Hence, proved.

13. i. According to the figure,

    $\sin 42^{o} = \frac{x}{8}$

    $0.669 = \frac{x}{8}$

    x = 5.4'

    So, height of the tree is 5.4 foot.

    ii. Given: x = 5.4' and $\theta = 30^{o}$

    Let length of the wire be y foot.

    Now, $\sin 30^{o} = \frac{5.4}{y}$

    $y = \frac{5.4}{0.5} = 10.8$

    So, length of the wire is 10.8 foot.

14. Let the required number of rows be n.

    According to question:-

    20 + 19 + 18 + ... to n terms = 200 ...(1)

    The equation (1) is an arithmetic series in which

    a = 20,d=(19 - 20) = -1 and $S_n = 200$

    We know that sum of n terms of an A.P. is given by :-

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$ ...(A)

$\therefore \frac{n}{2}\{2 \times 20 + (n - 1) \times (-1)\} = 200$

$\Rightarrow n(41 - n) = 400$

$\Rightarrow -n^2 + 41n - 400 = 0$

$\Rightarrow n^2 - 41n + 400 = 0$ (taking minus sign common)

$\Rightarrow n^2 - 25n - 16n + 400 = 0$

$\Rightarrow n(n - 25) - 16(n - 25) = 0$

$\Rightarrow (n - 25)(n - 16) = 0$

$\Rightarrow n - 25 = 0$ or $n - 16 = 0$

$\Rightarrow n = 25$ or $n = 16$

Now, nth term of an A.P. is given by :-

$T_n = a + (n - 1)d$ ...(B)

$\Rightarrow T_{25} = (a + 24d) = 20 + 24 \times (-1) = -4$

This is meaningless as the number of logs cannot be negative. So, we reject the value n = 25

$\therefore$ n = 16.

Thus, there are 16 rows in the whole stack.

Now, again using formula B for 16th term of A.P., we get :-

$T_{16} = (a + 15d) = 20 + 15 \times (-1) = 20 - 15 = 5$

Hence, there are 5 logs in the top row.