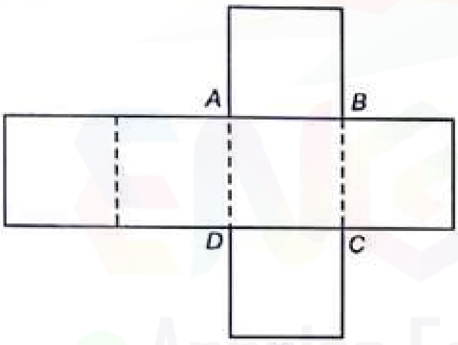
CHAPTER 23

Surface Area and Volume of Solids

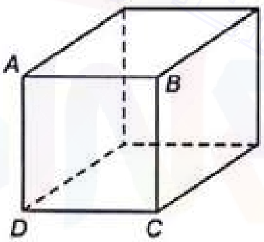
Solid Figures

The objects which occupy space (*i.e.,* they have three dimensions) are called solids. The solid figures can be derived from the plane figures.

e.g., In figure (i), we have a paper cut in the form as shown. It is a plane figure. But when we fold the paper along the dotted lines, a box can be made as shown in figure (ii).



(i)



(ii)

Sol. As the box in Figure (ii) occupies some part of the space, it has more than two dimensions and therefore it fulfils the criteria of being a solid figure.

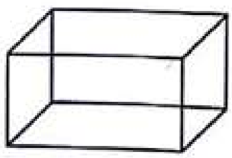
Surface Area

The sum of the areas of the plane figures making up the boundary of a solid figure is called its surface area. *e.g.,* the area of paper in figure (i) is the surface area of box.

Volume

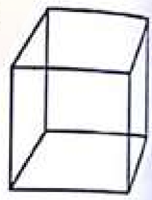
The measure of part of space occupied by a solid is called its volume.

Parallelopiped



A solid bounded by three pairs of parallel plane surfaces (or faces) is called a parallelopiped. A parallelopiped whose faces are rectangles is called a rectangular parallelopiped or a rectangular solid or a cuboid.

Cube



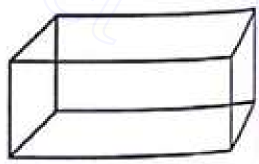
If the faces of a rectangular parallelopiped be squares, then it is called a cube.

- Edge of cube = Length = Breadth = Height
- Volume of a cube $= (\text{Edge})^3$
- Total surface area of a cube $= 6 \times (\text{Edge})^2$
- Diagonal of a cube $= \sqrt{3} \times \text{Edge}$

Cuboid

- A cuboid has six rectangular plane surfaces called faces.
- A cuboid has 8 corners called the vertices.
- Volume of cuboid $= l \times b \times h$

  where l = length, b = breadth
  and h = height



- Whole surface of cuboid $= 2(lb + bh + lh)$
- Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$
- Area of 4 walls $= 2(l + b)h$.

Example 1. If the surface area of a cube is 96 sq cm, then its volume is

(a) 16 cu cm (b) 64 cu cm (c) 12 cu cm (d) 32 cu cm

Sol. (*b*) Surface area of cube $= 6 \times l^2$, where l = edge length

or $6l^2 = 96$

$\therefore$ $l^2 = 16$

$\therefore$ $l = 4$ cm

$\therefore$ Volume of the cube $= l^3 = 4^3 = 64$ cu cm

Example 2. The surface area and the length of the diagonal of the cube. If the volume of a cube is 2197 cu cm, are

(a) 1012 sq cm and 21.516 cm
(b) 1024 sq cm and 24.516 cm
(c) 1014 sq cm and 22.516 cm
(d) None of the above

Sol. (*c*) Volume of cube $=(\text{side})^3 = 2197$ cu cm

$\therefore$ Side of cube = 13 cm

$\therefore$ Surface area of cube $= 6(\text{side})^2$ sq units $= 6(13)^2 = 1014$ sq cm

and length of the diagonal of the cube $= \sqrt{3} \times$ side

$$= \sqrt{3} \times 13 = 22.516 \text{ cm}$$

Example 3. The volume of a cuboid is 880 cm^3, the area of its base is 88 sq cm. Then, its height is

(a) 10 cm (b) 12 cm (c) 14 cm (d) 16 cm

Sol. (a) Height $= \dfrac{\text{Volume of the cuboid}}{\text{Area of the base}} = \dfrac{880}{88} = 10$ cm

Example 4. The length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high is

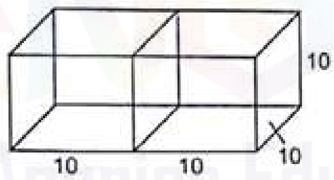
(a) 15 m (b) 16 m (c) 17 m (d) 17.5 m

Sol. (c) Since, the length of longest rod is the diagonal of the room

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(12)^2 + (9)^2 + (8)^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m}$$

Example 5. Two cubes each of 10 cm edge are joined end-to-end. Then, the surface area of the resulting cuboid is



(a) 100 cm^2 (b) 1000 cm^2
(c) 2000 cm^2 (d) None of these

Sol. (*b*) Length of the resulting cuboid = 10 cm + 10 cm = 20 cm

Breadth of resulting cuboid = 10 cm

Height of resulting cuboid = 10 cm

$\therefore$ Surface area of resulting cuboid $= 2(lb + bh + hl)$

$$= 2[20 \times 10 + 10 \times 10 + 20 \times 10]$$
$$= 2[200 + 100 + 200] = 1000 \text{ cm}^2$$

Example 6. The areas of three adjacent faces of a cuboid are x, y and z. If its volume is V, then which is true?

(a) $V = x^3y^2z^2$ (b) $V^2 = xyz$

(c) $V = \sqrt[3]{xyz}$ (d) $V = \dfrac{x^2y}{z}$

Sol. (*b*) Let the dimensions be, l, b, h, then volume of cuboid $V = lbh$

Also, $x = lb, y = bh, z = hl$

$\therefore$ $V = lbh$

$V^2 = l^2b^2h^2 = (lb)(bh)(hl)$ $V^2 = xyz$

Example 7. How many 6 m cubes can be cut from a cuboid measuring 36 m $\times$ 15 m $\times$ 8 m?

(a) 10 (b) 15 (c) 19 (d) 20

Sol. (*d*) Volume of given cuboid $= 36 \times 15 \times 8$ m^3

Volume of the cube to be cut $= 6 \times 6 \times 6$ m^3

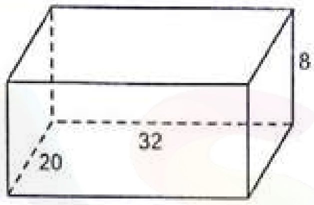
$\therefore$ Number of cubes that can be cut from the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of the cube}} = \frac{36 \times 15 \times 8}{6 \times 6 \times 6} = 20$$

Example 8. A metallic sheet is of rectangular shape with dimensions 48 cm $\times$ 36 cm from each of its corners a square of 8 cm is cut off. An open box is made of the remaining sheet, what is the volume of the box?

(a) 13824 cm^3 (b) 1728 cm^3
(c) 5120 cm^3 (d) None of these

Sol. (c) Length of metallic sheet = 48 cm

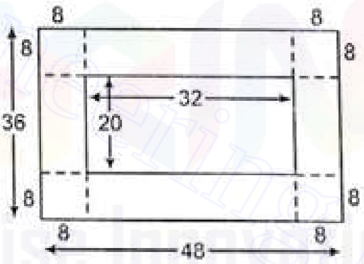


Breadth of metallic sheet = 36 cm

When square of side 8 cm is cut off from each corner and the flaps turned up, we get an open box whose

Length $= 48 - (8 + 8) = 32$ cm

Breadth $= 36 - (8 + 8) = 20$ cm

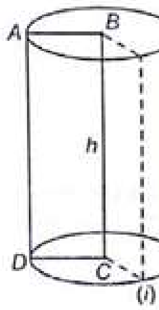


and height = 8 cm

$\therefore$ Volume of the box $= l \times b \times h = 32 \times 20 \times 8 = 5120$ cm^3

Right Circular Cylinder

A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. A cylinder shown in figure (i) is described by revolving the rectangle *ABCD* about the side *BC*. The examples of right circular cylinder are many such as water pipes, powder box, laboratory beakers etc.



(*i*)

- Volume of the cylinder

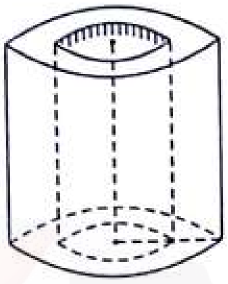
$$= (\text{Area of base}) \times \text{Height}$$
$$= \pi r^2 h \text{ cu units}$$

where, r is the radius of the base and h is the height of the cylinder.

- Curved surface area = Circumference of the base × Height
$= 2\pi rh$ sq units
- Total surface area = Curved surface + Area of two ends
$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Also, Note $h = \dfrac{V}{\pi r^2} \Rightarrow r = \sqrt{\dfrac{V}{\pi h}}$

Hollow Cylinder



The volume of material in a hollow cylinder is the difference between the volume of a cylinder having the external dimensions and the volume of a cylinder having the internal dimensions.

Let R and r be the external and internal radii of the hollow cylinder and h be its height.

- Volume of material $= \pi(R^2 - r^2)h$
- Total surface area $= 2\pi(R + r)(h + R - r)$
- Curved surface area $= 2\pi Rh + 2\pi rh = 2\pi(R + r)h$
- Total outer surface area $= 2\pi rh + \pi R^2 + \pi(R^2 - r^2)$

Example 9. The volume of a cylinder is $448\pi\ \text{cm}^3$ and height 7 cm. Then, its lateral surface area and total surface area are

(a) 349 cm^2 and 753.286 cm^2
(b) 352 cm^2 and 754.286 cm^2
(c) 353 cm^2 and 755.286 cm^2
(d) None of the above

Sol. (b) Volume of the cylinder $= 448\pi\ \text{cm}^3$

Height of the cylinder = 7 cm

Let radius be r, then, $\pi r^2 h = 448\pi$

$\therefore \quad r^2 = \dfrac{448\pi}{h\pi} = \dfrac{448}{7} = 64 \Rightarrow r = 8$ cm

$\therefore$ Lateral surface area of the cylinder $= 2\pi rh$

$= 2 \times \dfrac{22}{7} \times 8 \times 7 = 352\ \text{cm}^2$

Total surface area of the cylinder $= 2\pi r(h + r) = 2 \times \dfrac{22}{7} \times 8(7 + 8)$

$= 2 \times \dfrac{22}{7} \times 8 \times 15 = \dfrac{5280}{7} = 754.286\ \text{cm}^2$

Example 10. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then, the ratio of their volumes is

(a) 4 : 9 (b) 16 : 25
(c) 20 : 27 (d) None of these

Sol. (c) For first cylinder

Let radius $= r_1$, height $= h_1$, volume $= V_1$

For second cylinder

Let radius $= r_2$, height $= h_2$ and volume $= V_2$

Then, $\quad \dfrac{r_1}{r_2} = \dfrac{2}{3}$ and $\dfrac{h_1}{h_2} = \dfrac{5}{3} \Rightarrow r_1 = \dfrac{2r_2}{3}$ and $h_1 = \dfrac{5h_2}{3}$

Required ratio of their volumes be $V_1 : V_2$.

$\Rightarrow \quad \pi r_1^2 h_1 : \pi r_2^2 h_2 \Rightarrow r_1^2 h_1 : r_2^2 h_2$

$\Rightarrow \quad \dfrac{4}{9} r_2^2 \dfrac{5h_2}{3} : r_2^2 h_2 \Rightarrow \dfrac{20}{27} : 1 \Rightarrow 20 : 27$

Example 11. A cylindrical bucket of diameter 28 cm and height 12 cm is full of water. The water is emptied into a rectangular tub of length 66 cm and breadth 28 cm. The height to which water rises in the tub is

(a) 2 cm (b) 4 cm (c) 6 cm (d) 8 cm

Sol. (b) Volume of water in the bucket

$= \pi r^2 h = \dfrac{22}{7} \times 14 \times 14 \times 12 = 7392$ cu cm

Let h be the height to which water rises in the tub.

$\therefore$ Volume of water in the tub $= 66 \times 28 \times h$ cu cm

According to the question, $66 \times 28 \times h = 7392$

or $\quad h = \dfrac{7392}{66 \times 28} = 4$ cm

$\therefore$ Water rises to a height of 4 cm in the tub.

Example 12. A hollow cylindrical tube, open at both ends is made of iron 1 cm thick. The volume of iron used in making the tube, if the external diameter is 12 cm and the length of tube is 70 cm is

(a) 2420 cm^2 (b) 2520 cm^2
(c) 2720 cm^2 (d) 2900 cm^2

Sol. (a) Here, external radius $(R_1) = 6$ cm

and internal radius $(R_2) = 6 - 1 = 5$ cm

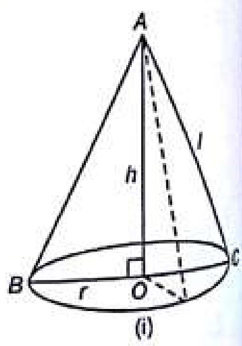
height $(h) = 70$ cm

$\therefore$ Volume of iron used = External volume − Internal volume

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$

$= \dfrac{22}{7} \times 70 \times (36 - 25)$ cu cm $= 220 \times 11$ cu cm $= 2420$ cu cm

Right Circular Cone



(i)

A right circular cone is a solid generated by the revolution of a right angled triangle about one of its sides containing the right angle as axis. In the adjoining figure (i) a cone of height 'h' and radius 'r' is generated by revolving the right ΔAOB along AO.

The slant height of the cone is

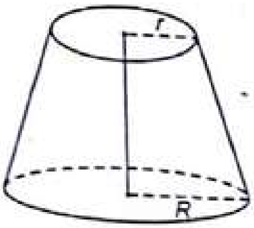
$$l = AC = \sqrt{r^2 + h^2}$$

- Volume of cone $= \dfrac{1}{3}\pi r^2 h$ cu units
- Curved surface area of cone $= \pi rl$ sq units
- Total surface area of a cone $= \pi r(l + r)$ sq units

Frustum of a Cone

If a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the frustum of the cone.

Let R, r be the radii of base and top of the frustum of a cone. Let h be the height, then



- Volume of frustum of right circular cone
$$=\frac{\pi h}{3}[R^2+r^2+Rr]\text{ cm}^3$$
- Lateral surface area of frustum of right circular cone $=\pi l(R+r)$ sq units where slant height $l^2=h^2+(R-r)^2$
- Total surface area of frustum of right circular cone
$=$ Area of base + Area of top + Lateral surface area
$=\pi[R^2+r^2+l(R+r)]$
- Total surface Area of bucket $=\pi[(R+r)l+r^2]$
[∵ it is open at the bigger end]

Example 13. The radius and vertical height of a cone are 5 cm and 12 cm, respectively. Then its lateral surface area is

(a) 202 cm^2 (b) 203.1 cm^2
(c) 204.2 cm^2 (d) 204.3 cm^2

Sol. (d) Radius of cone $(r)=5$ cm

Height of cone $(h)=12$ cm

Slant height $(l)=\sqrt{r^2+h^2}=\sqrt{5^2+12^2}=\sqrt{169}=13$ cm

∴ Lateral/curved surface area $=\pi rl$
$$=\frac{22}{7}\times5\times13=\frac{1430}{7}=204.3\text{ cm}^2$$

Example 14. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m?

(a) 100 m (b) 105 m (c) 109 m (d) 110 m

Sol. (d) Radius of base $(r)=7$ m

Vertical height of tent $(h)=24$ m

∴ Slant height $(l)=\sqrt{r^2+h^2}=\sqrt{24^2+7^2}=\sqrt{625}=25$ m

∴ Curved surface area $=\pi rl=\frac{22}{7}\times7\times25=550\text{ m}^2$

Width of cloth $=5$ m

Length required to make conical tent $=\frac{550}{5}=110$ m

Example 15. The radius and the height of a right circular cone are in the ratio 5 : 12. If its volume is 314 m^3, the slant height and the radius are

(a) 12, 5 m (b) 13, 4 m (c) 1, 4 m (d) 13, 5 m

Sol. (d) Let radius of cone $=5x$ and the height of the cone $=12x$

Volume of cone $=314=\frac{1}{3}\pi r^2h$

$\Rightarrow\quad \frac{1}{3}\times3.14\times(5x)^2\times(12x)=314x^3$

$314=314x^3\Rightarrow x^3=1\Rightarrow x=1$

∴ Radius $=5$ m and height $=12$ m

∴ Slant height $=\sqrt{r^2+h^2}=\sqrt{25+144}=\sqrt{169}=13$ m

∴ Radius $=5$ m and slant height $=13$ m

Example 16. The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Then, its capacity is

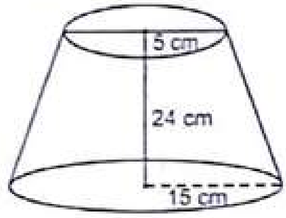
(a) 8000 cm^3 (b) 8100 cm^3
(c) 8171.43 cm^3 (d) 8200.43 cm^3

Sol. (c) Capacity of bucket

$=$ Volume of frustum of a cone

$=\frac{\pi h}{3}[R^2+r^2+Rr]$

$=\frac{22}{7}\times\frac{24}{3}[(15)^2+5^2+15\times5]\text{ cm}^3$



$$=\frac{22}{7}\times8[225+25+75]\text{ cm}^3=\frac{176}{7}(325)\text{ cm}^3=8171.43\text{ cm}^3$$

Example 17. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4. The ratio of their curved surface areas is

(a) 4 : 5 (b) 3 : 5 (c) 5 : 3 (d) 5 : 4

Sol. (d) As diameters are equal ⇒ Radii are also equal.

Let r be radius of each cone and let slant height be $5x$ and $4x$.

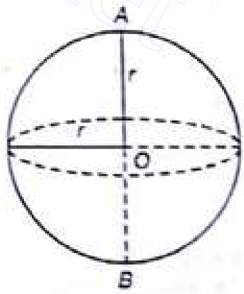
∴ Curved surface area of first cone $=\pi r\times5x$

Curved surface area of second cone $=\pi r\times4x$

∴ The required ratio $=\frac{\pi r\times5x}{\pi r\times4x}=5:4$

Sphere

A sphere is a solid generated by the revolution of a semi-circle about its diameter.



- A sphere is the locus of a point which moves in space such that its distance from a fixed point in space remains constant. The fixed point is called the centre of sphere and the constant distance is called the radius of sphere.
- Here, 'O' is centre and r is radius.
- All radii are equal.
- The section of a sphere cut by any plane is a circle.
- If the cutting plane passes through the centre of the sphere, the section is called a great circle.
- Volume of sphere $=\frac{4}{3}\pi r^3$ cu units
- Surface area of sphere $=4\pi r^2$ sq units
- Volume of a hollow sphere $=\frac{4}{3}\pi(R^3-r^3)$ cu units
where $r=$ inner radius and $R=$ outer radius.

Hemisphere

A plane passing through the centre cuts the sphere in two equal parts each called a hemisphere.

- Volume of hemisphere $=\frac{2}{3}\pi r^3$ cu units
- Curved surface area of hemisphere $=2\pi r^2$ sq units
- Total surface area $=2\pi r^2+\pi r^2=3\pi r^2$ sq units

Example 18. Given that the volume of a metal sphere is 38808 cm^3. Then, its radius and its surface area are

(a) 7 cm and 616 cm^2 (b) 21 cm and 5544 cm^2
(c) 14 cm and 2464 cm^2 (d) None of these

Sol. (*b*) Volume of the metal sphere $=38808\ cm^3$

But volume of sphere $=\frac{4}{3}\pi r^3$

$$\therefore \quad \frac{4}{3}\pi r^3=38808$$

$$r^3=38808\times\frac{3}{4}\times\frac{7}{22}=9261\Rightarrow r=(9261)^{1/3}=21\text{ cm}$$

$\therefore$ Surface area of the sphere $=4\pi r^2$

$$=4\times\frac{22}{7}\times21\times21=5544\text{ cm}^2$$

Example 19. If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetre in its volume. What is the diameter of the sphere?

(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Sol. (*d*) Volume of a sphere $=\frac{4}{3}\pi r^3$, $r=$ radius

Surface area $=4\pi r^2\Rightarrow4\pi r^2=\frac{4}{3}\pi r^3$ (by condition)

$$\Rightarrow \quad r=3$$

$\therefore$ Diameter of the sphere $=3\times2=6$ cm

Example 20. The volume of two hemisphere are in the ratio 8:27. What is the ratio of their radii?

(a) 2 : 3 (b) 3 : 2
(c) 1 : 2 (d) 2 : 1

Sol. (*a*) Let volumes be V_1 and V_2.

$$\therefore \quad V_1:V_2=8:27\Rightarrow\frac{2}{3}\pi r_1^3:\frac{2}{3}\pi r_2^3=8:27$$

$$\Rightarrow \quad r_1^3:r_2^3=8:27\Rightarrow r_1:r_2=2:3$$

Example 21. How many balls each of radius 2 cm can be made by melting a big ball whose radius is 8 cm.

(a) 4 balls (b) 16 balls
(c) 64 balls (d) 128 balls

Sol. (*c*) Radius of big ball $(R)=8$ cm

$\therefore$ Volume of big ball $=\frac{4}{3}\pi R^3=\frac{4}{3}\pi(8)^3$

Radius of small ball $(r)=2$ cm

$\therefore$ Volume of small balls $=\frac{4}{3}\pi(2)^3$

$\therefore$ Required number of balls $=\frac{\text{Volume of big ball}}{\text{Volume of small ball}}$

$$=\frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(2)^3}=\frac{8\times8\times8}{2\times2\times2}=64\text{ balls}$$

Example 22. The number of lead balls of diameter 1 cm each, that can be made from a sphere of diameter 16 cm is

(a) 2024 (b) 4096 (c) 5012 (d) 6000

Sol. (*d*) Radius of the sphere = 8 cm

Volume of the sphere $=\left(\frac{4}{3}\pi\times8\times8\times8\right)\text{cm}^3$

Radius of each lead ball $=\frac{1}{2}$ cm

Volume of each lead ball $=\left(\frac{4}{3}\pi\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)=\frac{\pi}{6}\text{ cm}^3$

$\therefore$ Number of lead balls $=\left(\frac{4}{3}\pi\times8\times8\times8\times\frac{6}{\pi}\right)=4096$

Example 23. A copper sphere of diameter 18 cm is drawn into a wire of diameter 44 mm. Then, the length of the wire is

(a) 243 m (b) 343 m (c) 443 m (d) 972 m

Sol. (*a*) Let the length of wire be *h* cm.

Volume of sphere = Volume of wire (by condition)

$$\Rightarrow \quad \left(\frac{4}{3}\pi\times9\times9\times9\right)=\left(\pi\times\frac{2}{10}\times\frac{2}{10}\times h\right)$$

$$\Rightarrow \quad \frac{h}{25}=972\Rightarrow h=(972\times25)\text{ cm}=\frac{972\times25}{100}=243\text{ m}$$

Hence, the length of the wire is 243 m.

Miscellaneous

- If the side of a cube is increased by $x\%$, then its volume increased by $\left[\left(1+\frac{x}{100}\right)^3-1\right]\times100\%$.
- If the length, breadth and height of cuboid are made x, y and z times respectively, its volume is increased by $(xyz-1)\times100\%$.
- If the length, breadth and height of a cuboid are increased by $x\%$, $y\%$ and $z\%$ respectively, then its volume is increased by $\left[x+y+z+\frac{xy+yz+zx}{100}+\frac{xyz}{(100)^2}\right]\%$.
- If the sides and diagonal of a cuboid are given, then the total surface area in terms of diagonal and sides is given by
- Total surface area $=(\text{Sum of the sides})^2-(\text{Diagonal})^2$.
- If the side of a cube is increased by $x\%$, the surface area is increased by $\left(2x+\frac{x^2}{100}\right)\%$.
- If each side of a cube is doubled, its volume becomes 8 times *i.e.*, volume is increased by 700%.

Exercise

Level I

1. Three equal cubes are placed adjacently in a row. The ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes is
(a) 3 : 1 (b) 6 : 5 (c) 7 : 9 (d) 6 : 7

2. 100 persons can sleep in a room 25 m by 9.8 m. If each person requires 12.25 cm^3 of air. The height of the room is
(a) 5 m (b) 6 m (c) 7 m (d) 8 m

3. A solid cube is cut into two cuboids of equal volumes. The ratio of the total surface area of the given cube and that of one of the cuboids
(a) 4 : 3 (b) 3 : 2 (c) 2 : 3 (d) 3 : 4

4. The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each person requires 150 m^3 of air?
(a) 550 (b) 500 (c) 400 (d) 600

5. A 4 cm cube is cut into 1 cm cubes. What is the ratio of surface area of small cubes to that of the large cube?
(a) 4 : 1 (b) 4 : 3 (c) 1 : 4 (d) 2 : 3

6. A class room is 7 m long, 6.5 m wide and 4 m high. It has one door $3\,m \times 1.4\,m$ and three windows each measuring $2\,m \times 1\,m$. The interior walls are to be coloured washed. The contractor charges ₹ 5.25 per sq m. The cost of colour washing is
(a) ₹ 519.45 (b) ₹ 159.45 (c) ₹ 513.45 (d) ₹ 419.45

7. The dimensions of a field are 12 m × 10 m. A pit 5 m long, 4 m wide and 2 m deep is dug in one corner of the field and the Earth removed has been evenly spread over the remaining area of the field. The level of the field is raised by
(a) 30 cm (b) 35 cm (c) 38 cm (d) 40 cm

8. Each edge of a cube is increased by 50%. Then, the percentage increase in its surface area is
(a) 125% (b) 150% (c) 175% (d) 180%

9. Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted to form a cube whose diagonal is $12\sqrt{3}$ cm. The edges of the three cubes are
(a) 15 cm, 20 cm, 25 cm (b) 9 cm, 12 cm, 15 cm
(c) 12 cm, 16 cm, 20 cm (d) 6 cm, 8 cm, 10 cm

10. A granary is in the shape of a cuboid of size $8\,m \times 6\,m \times 3\,m$. If a bag of grain occupies a space of 0.65 m^3, how many bags can be stored in the granary?
(a) 219 (b) 220 (c) 221 (d) 222

11. How many bricks each measuring 25 cm × 15 cm × 8 cm will be required to build a wall 10 m × 4 dm × 5 m when $\frac{1}{10}$ of its volume is occupied by mortar?
(a) 5000 b) 5500 (c) 6000 (d) 6500

12. A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of base are 15 cm and 12 cm. Then, the rise in water level in the vessel is
(a) 4.05 cm (b) 4 cm (c) 3.5 cm (d) 3 cm

13. A rectangular block measuring 18 cm × 15 cm × 12 cm is cut into exact number of cubes, the least possible number of cubes will be
(a) 150 (b) 120 (c) 60 (d) 90

14. One cubic metre piece of copper is melted and recast into a square cross-section bar, 36 m long. An exact cube is cut off from this bar. If 1 cu m of copper cost is ₹ 108, the cost of this cube is
(a) 30 paise (b) 40 paise (c) 50 paise (d) ₹ 1

15. If the diameter of the base of a closed right circular cylinder be equal to its height 'h', then its total surface area is
(a) $\frac{2}{3}\pi h^2$ (b) $\frac{1}{3}\pi h^2$ (c) $\frac{3}{2}\pi h^2$ (d) $\frac{5}{2}\pi h^2$

16. The curved surface area of cylinder whose height is 21 cm and the base radius is 5 cm is
(a) 540 cm^2 (b) 560 cm^2 (c) 640 cm^2 (d) 660 cm^2

17. The curved surface area of a cylinder is 1320 cm^2 and its base has diameter 21 cm, then the height of the cylinder is
(a) 10 cm (b) 20 cm (c) 22 cm (d) 25 cm

18. Two cylindrical cans have bases of the same size. The diameter of each is 14 cm. One of the cans is 10 cm high and the other is 20 cm high. The ratio of their volume is
(a) 1 : 2 (b) 2 : 3 (c) 1 : 3 (d) 3 : 2

19. A right circular cylinder tunnel of diameter 2 m and length 40 m is to be instructed from a sheet of iron. The area of the iron sheet required in m^2 is
(a) 40 π (b) 60 π (c) 80 π (d) 100 π

20. The radius and the height of a cylinder are in the ratio 5 : 7 and its volume is 550 cm^3. Then, its radius is
(a) 3 cm (b) 4 cm (c) $\frac{4}{3}$ cm (d) 5 cm

21. The ratio of total surface area to the lateral surface area of a cylinder with base radius 70 cm and height 30 cm is
(a) 3 : 7 (b) 7 : 3 (c) 10 : 3 (d) 3 : 10

22. A solid cylinder has a total surface area of 231 m^2. Its curved surface area is $\frac{2}{3}$ of the total surface area. The volume of the cylinder is
(a) $269\frac{1}{2}$ m^3 (b) $259\frac{1}{2}$ m^3 (c) $249\frac{1}{2}$ m^3 (d) $239\frac{1}{2}$ m^3

23. The radii of two right circular cylinder are in the ratio 2 : 3 and their height are in the ratio 5 : 4. The ratio of their curved surface area is
(a) 5 : 9 (b) 4 : 3 (c) 5 : 6 (d) 4 : 9

24. The radii of two right circular cylinder in the ratio 2 : 3 and the ratio of their volumes is 5 : 9, then the ratio of their heights is
(a) 5 : 4 (b) 9 : 5 (c) 3 : 4 (d) 4 : 3

25. A cylindrical vessel can hold 154 gm of water. If the radius of its base is 3.5 cm and 1 cm^3 of water weights 1 g. The depth of the water
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

26. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions?
(a) 11 m^2 (b) 22 m^2 (c) 44 m^2 (d) 66 m^2

27. The diameter of the roller 120 cm long is 84 cm. If it takes 500 complete revolutions to level a playground the cost of levelling at the rate of 30 paise per m^2 is
(a) ₹ 575.20 (b) ₹ 475.20 (c) ₹ 375.20 (d) ₹ 485.20

28. Two circular cylinders of equal volume have their heights in the ratio 1 : 2. Ratio of their radii is
(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$ (c) 1 : 2 (d) 2 : 1

29. The radius of a wire is decreased to one-third. If volume remains the same, the length will become
(a) 1 time (b) 2 times (c) 3 times (d) 9 times

30. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The diameter of the pillar is
(a) 3 m (b) 6 m (c) 7 m (d) 14 m

31. The cost of sinking a tube well 280 m deep having diameter 3 m at the rate of ₹ 3.60 per m^2 is
(a) ₹ 7128 (b) ₹ 7218 (c) ₹ 2718 (d) ₹ 8172

32. If the radius of the base of a right circular cylinder is halved, keeping the height same. The ratio of the volume of the reduced cylinder to that of the original is
(a) 2 : 3 (b) 3 : 5 (c) 1 : 4 (d) 4 : 3

33. A conical tent of a diameter 24 m at the base and its height 16 m. The canvas required to make it is
(a) $\frac{5280}{7}$ m^2 (b) $\frac{5180}{7}$ m^2 (c) $\frac{4180}{7}$ m^2 (d) $\frac{3480}{7}$ m^2

34. The circumference of the base of a 9 m high wooden solid cone is 44 m. The slant height of the cone is
(a) $\sqrt{120}$ m (b) $\sqrt{130}$ m (c) $\sqrt{150}$ m (d) $7\sqrt{5}$ m

35. The diameter of a right circular cone is 12 m and the slant height is 10 m. The total surface area of cone is
(a) $\frac{2412}{7}$ m^2 (b) $\frac{2312}{7}$ m^2 (c) $\frac{2112}{7}$ m^2 (d) $\frac{2012}{7}$ m^2

36. The radius and the slant height of a cone are in the ratio of 4 : 7. If its curved surface area is 792 cm^2. Then, the radius is
(a) 10 cm (b) 11 cm (c) 12 cm (d) 13 cm

37. The height of a right circular cone is 21 cm and the diameter of the base is 2r cm, then its volume (in cm^3) is
(a) $21\pi r^2$ (b) $14\pi r^2$ (c) $7\pi r^2$ (d) $9\pi r^2$

38. How many metres of cloth 50 m wide will be req... to make a conical tent, the radius of whose base is ... and whose height is 24 m?
(a) 9 m (b) 11 m (c) 12 m (d) 13 m

39. If the radius of the base of a right circular cone ... and its height is equal to the radius of the base ... its volume is
(a) $\frac{1}{3}\pi r^3$ (b) $\frac{2}{3}\pi r^3$ (c) $\frac{4}{3}\pi r^3$ (d) $3\pi r^3$

40. The ratio of volume of two cones is 4 : 5 and the ... of the radii of their bases is 2 : 3. The ratio of ... vertical heights is
(a) 9 : 5 (b) 4 : 9 (c) 16 : 25 (d) 8 : ...

41. The radius and height of a right circular cone are in the ratio of 5 : 12 and its volume is 2512 cm^3 ... slant height of the cone is
(a) 24 cm (b) 25 cm (c) 26 cm (d) 27 cm

42. If the slant height and the radius of the base of a right circular cone are H and r respectively, then the ratio of the areas of the lateral surface and the base is
(a) $2H:r$ (b) $H:r$ (c) $H:2r$ (d) H^2 : ...

43. If the height of a cone is doubled, then its volume is increased by
(a) 100% (b) 200% (c) 300% (d) 400%

44. If the height and the radius of a cone are doubled, the volume of the cone becomes
(a) 2 times (b) 4 times (c) 6 times (d) 8 times

45. A sector containing an angle of 90° is cut from a circle of radius 42 cm and folded into a cone. The curved surface area of cone is
(a) 1396 cm^2 (b) 1386 cm^2 (c) 1376 cm^2 (d) 1366 cm^2

46. Two cones have their heights in the ratio 1 : 3 and the radii of their bases in the ratio 3 : 1. The ratio of their volumes is
(a) 2 : 3 (b) 1 : 3 (c) 3 : 1 (d) 3 : [illegible]

47. The diameter of two cones are equal. If their slant heights are in the ratio 5 : 4. The ratio of their curved surface areas is
(a) 5 : 4 (b) 9 : 5 (c) 3 : 2 (d) 2 : 3

48. Two right circular cones X and Y are made, X having three times the radius of Y and Y having half the volume of X. The ratio of heights of X and Y
(a) 2 : 3 (b) 2 : 9
(c) 4 : 9 (d) 4 : 5

49. It is required to make a hollow cone 24 cm high whose base radius is 7 cm. The area of sheet required including the base is
(a) 700 cm^2 (b) 704 cm^2
(c) 708 cm^2 (d) 710 cm^2

50. The area of the base of cone is 770 cm^2 and the area of the curved surface is 814 cm^2. Then, the volume of cone is
(a) $616\sqrt{5}$ cm^3 (b) $516\sqrt{3}$ cm^3
(c) $316\sqrt{7}$ cm^3 (d) None of these

51. If the base radius and the height of a right circular cone are increased by 20%, then the percentage increase in volume is approximately
(a) 60 (b) 68 (c) 73 (d) 78

52. A conical tent has 60° angle at the vertex. The ratio of its radius and slant height is
(a) 1 : 2 (b) 1 : 3 (c) $1 : \sqrt{2}$ (d) $1 : \sqrt{3}$

53. If h, s, V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 - s^2h^2 + 9V^2)$ is equal to
(a) 2 (b) 0 (c) 2π (d) $3\pi^2$

54. The volume of two hemisphere are in the ratio 8 : 27. The ratio of their radii is
(a) 4 : 9 (b) 8 : 27 (c) 3 : 4 (d) 2 : 3

55. A hollow sphere of internal and external diameters 4 cm and 8 cm, respectively is melted into a cone of base diameter 8 cm. The height of the cone is
(a) 11 cm (b) 12 cm (c) 14 cm (d) 16 cm

56. If the ratio of volumes of two spheres is 1 : 8, then the ratio of their surface areas is
(a) 1 : 2 (b) 1 : 4 (c) 1 : 6 (d) 1 : 8

57. If the surface areas of two spheres are in the ratio of 4 : 25, then the ratio of their volumes is
(a) 2 : 25 (b) 4 : 75 (c) 8 : 125 (d) 16 : 125

58. If the radius of a sphere is doubled, then the percentage increase in its volume is
(a) 500 (b) 400 (c) 700 (d) 800

59. Three spherical metal balls of radii 6 cm, 8 cm, and r cm are melted to form a solid sphere of radius 12 cm. Then, the value of r is
(a) 10 cm (b) 9 cm (c) 8 cm (d) 6 cm

60. If the volume and the surface area of a sphere are numerically equal, then its radius is
(a) 1 unit (b) 2 units (c) 3 units (d) 4 units

61. If the volume of a sphere is double that of the other, then the ratio of their radii is
(a) $2\sqrt{2} : 1$ (b) $\sqrt[3]{2} : 1$ (c) $1 : \sqrt[3]{2}$ (d) 2 : 1

62. 64 balls are melted to cast a big ball of radius 8 cm, then the radius of small balls be
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

63. The volumes of two sphere are in the ratio 64 : 27. If the sum of their radii is 21 cm, so their radii are (in cm)
(a) 12, 9 (b) 11, 10 (c) 18, 3 (d) 17, 4

64. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm, respectively. The total area to be painted is
(a) $\frac{13211}{7}$ cm^2 (b) $\frac{26961}{14}$ cm^2
(c) $\frac{6961}{14}$ cm^2 (d) $\frac{16951}{14}$ cm^2

65. A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is
(a) 1 : 3 (b) 3 : 4 (c) 2 : 1 (d) 3 : 2

66. A sphere and a cube have the same surface area. Then, the ratio of the volume of sphere to that of the cube is
(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{6} : \sqrt{\pi}$
(c) $4 : \pi$ (d) None of these

67. A cone and a cylinder having the same area of the base have also the same area of curved surfaces. If the height of sthe cylinder be 2 m, the slant height of the cone is
(a) 2 m (b) 4 m (c) 6 m (d) 8 m

68. A cone and a cylinder are of the same height. Their radii of the bases are in ratio of 2 : 1. The ratio of their volumes is
(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 1 : 3

69. The volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm is
(a) $\frac{2673}{14}$ cm^3 (b) $\frac{2683}{14}$ cm^3
(c) $\frac{2693}{14}$ cm^3 (d) None of these

70. A solid metallic cylinder of base 3 cm and height 5 cm is melted to make n solid cones of height 1 cm and base radius 1 mm. The value of n is
(a) 450 (b) 1350 (c) 4500 (d) 13500

71. If the height and diameter of a right circular cylinder are 32 cm and 6 cm respectively, then the radius of the sphere whose volume is equal to the volume of the cylinder is
(a) 3 cm (b) 4 cm (c) 6 cm (d) 8 cm

72. A right circular cylinder and a sphere have the same volume and same radius. The ratio of the areas of their curved surfaces is
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 2 : 3

73. If a solid sphere of radius r is melted and cast into the shape of a solid cone of height r, then the radius of the base of the cone is
(a) r (b) $2r$ (c) $3r$ (d) $4r$

74. The ratio of the volume of a right cylinder and a right circular cone of the same base and height will be
(a) 1 : 3 (b) 3 : 1 (c) 4 : 3 (d) 3 : 4

75. If the height of two cones are in the ratio of 1 : 4 and their radii of their bases are in the ratio 4 : 1, then the ratio of their volumes is
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 1

76. The ratio between the volume of a sphere and the volume of the circumscribing right cylinder is
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

77. If a sphere is inscribed in a cube, then the ratio of the volume of the sphere to the volume of the cube is
(a) $\pi : 6$ (b) $\pi : 4$
(c) $\pi : 3$ (d) $\pi : 2$

78. If the diameter of a sphere is doubled, how does its surface area change? **(CDS 2011 II)**
(a) It increases two times (b) It increases three times
(c) It increases four times (d) It increases eight times

79. A sphere is inscribed in a cubical box such that the sphere is tangent to all six faces of the box. What is the ratio of the volume of the cubical box to the volume of sphere? (CDS 2011 II)
(a) 6π (b) 36π (c) $\frac{4\pi}{3}$ (d) $\frac{6}{\pi}$

80. The curved surface of a cylinder is 1000 sq cm. A wire of diameter 5 mm is wound around it, so as to cover it completely. What is the length of the wire used?
(a) 22 m (b) 20 m (CDS 2011 II)
(c) 18 m (d) None of these

81. From a solid cylinder of height 4 cm and radius 3 cm, a conical cavity of height 4 cm and of base radius 3 cm is hollowed out. What is the total surface area of the remaining solid? (CDS 2008 I)
(a) 15π sq cm (b) 22π sq cm
(c) 33π sq cm (d) 48π sq cm

82. From a solid cube of edge 3 m, a solid of largest sphere is curved out. What is the volume of solid left? (CDS 2008 II)
(a) $(27 - 2.25\pi)$ cu m (b) $(27 - 4.5\pi)$ cu m
(c) 2.25π cu m (d) 4.5π cu m

83. A lead pencil is in the shape of a cylinder. The pencil is 21 cm long with radius 0.4 cm and its lead is of radius 0.1 cm. What is the volume of wood in the pencil? (CDS 2009 I)
(a) 9 cu cm (b) 9.4 cu cm
(c) 9.9 cu cm (d) 10.1 cu cm

84. The paint in a certain container is sufficient to point an area equal to 5.875 m^2. How many bricks of dimensions 12.5 cm × 10 cm × 7.5 cm can be painted out of this container? (CDS 2009 II)
(a) 225 (b) 180 (c) 150 (d) 100

85. Two solid spheres of gold having diameters 3 cm and 4 cm are molten and then cast into one big sphere of gold. If the radius of this sphere is x, then what is the value of x^3? (CDS 2007 I)
(a) 125 cu cm (b) 15.625 cu cm
(c) 11.375 cu cm (d) 9.875 cu cm

86. **Assertion (A)** When eight drops of water combine to form a single drop, the surface area of all the eight drops is greater than the surface area of big drop.
Reason (R) Square of volume of a spherical body is directly proportional to cube of its surface area. (CDS 2007 I)
(a) A and R are true and R is the correct explanation of A.
(b) A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

87. Consider the following
The length of a side of a cube is 1 cm. Which of the following can be the distance between any two vertices?
I. 1 cm II. $\sqrt{2}$ cm III. $\sqrt{3}$ cm
Select the correct answer using the code given below
(a) Only I (b) Only II
(c) Only III (d) I, II and III

88. Smaller lead shots are to be prepared by using material of a spherical lead shot of radius 1 cm. Some possibilities are listed in the statements given below. (CDS 2011)
I. The material is just sufficient to prepare 8 shots each of radius 0.5 cm.
II. A shot of radius 0.75 cm and a second shot of radius 0.8 cm can be prepared from the available material.
Which of the above statement is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Directions (Q. Nos. 89-90) *The following two questions consists of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below*
(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

89. **Assertion (A)** The volume of a cuboid is the product of the lengths of its coterminous edges.
Reason (R) The surface area of a cuboid is twice the sum of the products of lengths of its coterminous edges taken two at a time.

90. **Assertion (A)** The curved surface area of a right circular cone of base radius r and height h is given by $\pi r(\sqrt{h^2 + r^2})$.
Reason (R) The right circular cone of base radius r and height h when cut opened along the slant height forms a rectangle of length πr and breadth $\sqrt{h^2 + r^2}$. (CDS 2008 II)

91. The total surface area of a cone, whose generator is equal to the radius R of its base, is S. If A is the area of a circle of radius $2R$, then which one of the following is correct? (CDS 2007 II)
(a) $A = S$ (b) $A = 2S$ (c) $A = S/2$ (d) $A = 4S$

92. In order to fix an electric pole along a roadside, a pit with dimensions 50 cm × 50 cm is dug with the help of a spade. The pit is prepared by removing Earth by 250 strokes of spade. If one stroke of spade removes 500 cm^3 of Earth, then what is the depth of the pit? (CDS 2010 II)
(a) 2 m (b) 1 m (c) 0.75 m (d) 0.5 m

93. A figure is formed by revolving a rectangular sheet of dimensions 7 cm × 4 cm about its length. What is the volume of the figure thus formed? (CDS 2010 II)
(a) 352 cu cm (b) 296 cu cm
(c) 176 cu cm (d) 616 cu cm

94. A roller of diameter 70 cm and length 2 m is rolling on the ground. What is the area covered by the roller in 50 revolutions? **(CDS 2009 I)**
(a) 180 sq m (b) 200 sq m (c) 220 sq m (d) 240 sq m

95. A cylinder having base of circumference 60 cm is rolling without sliding at a rate of 5 rounds per s. How much distance will the cylinder roll in 5 s?
(a) 15 m (b) 1.5 m (c) 30 m (d) 3 m

96. A cistern 6 m long and 4 m wide contains water to a depth of 1.25 m. What is the area of wetted surface? **(CDS 2011 I)**
(a) 40 sq m (b) 45 sq m (c) 49 sq m (d) 73 sq m

97. The outer and inner diameters of a circular pipe are 6 cm and 4 cm, respectively. If its length is 10 cm, then what is the total surface area in sq cm?
(a) 35π (b) 110π **(CDS 2011 I)**
(c) 150π (d) None of these

98. The volume of a cone is equal to that of a sphere. If the diameter of base of cone is equal to the diameter of the sphere, what is the ratio of height of cone to the diameter of the sphere? **(CDS 2010 I)**
(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 4 : 1

99. The diameter of the Moon is approximately one-fourth of that of the Earth. What is the (approximate) ratio of the volume of the Moon to that of Earth? **(CDS 2009 I)**
(a) 1/16 (b) 1/32 (c) 1/48 (d) 1/64

100. 27 drops of water form a big drop of water. If the radius of each smaller drop is 0.2 cm, then what is the raidus of the bigger drop? **(CDS 2007 II)**
(a) 0.4 cm (b) 0.6 cm
(c) 0.8 cm (d) 1.0 cm

Level II

1. The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rate of ₹ 4 and ₹ 4.50 per m^2 is ₹ 416. Then, length of box is
(a) 16 m (b) 8 m (c) 26 m (d) 12 m

2. The outer dimensions of a closed wooden box are 10 cm by 8 cm by 7 cm. Thickness of the wood is 1 cm. The total cost of wood required to make box, if 1 cm^3 of wood cost ₹ 2 is
(a) ₹ 540 (b) ₹ 640 (c) ₹ 740 (d) ₹ 780

3. A rectangular tank is $80 \times 40\ cm^3$. Water flows into it through a pipe $40\ cm^2$ are the opening at the speed of 10 km/h. The rise in the level of water in the tank in $\frac{1}{2}$ h is
(a) $\frac{3}{2}$ cm (b) $\frac{4}{3}$ cm
(c) $\frac{5}{8}$ cm (d) 6 cm

4. 50 students set in a classroom, each student requiring 9 m^2 on floor and 108 m^2 in sapce. If the breadth of the room is 18 m, then its length and height are respectively
(a) 25 m, 12 m (b) 30 m, 10 m
(c) 12 m, 10 m (d) 25 m, 15 m

5. A circus tent is made of canvas and is in the form of a right circular cylinder and a right circular cone above it the diameter and height of the cylindrical part of the tent are 126 m and 5 m, respectively. The total height of the tent is 21 m. Then, the cost of the canvas used for tent at the rate of ₹ 12 per m^2
(a) ₹ 14850 (b) ₹ 168200
(c) ₹ 178200 (d) ₹ 112000

6. A solid sphere of radius 5 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm. The uniform thickness of the cylinder is
(a) 1.5 cm (b) 3 cm (c) 1.2 cm (d) 1 cm

7. Water is flowing at the rate of 3 km/h through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how many time will the cistern be filled?
(a) 1 h 40 min (b) 1 h 30 min
(c) 1 h 10 min (d) 2 h

8. The volume of a sphere and a right cylinder are equal and the diameter of the sphere equals the diameter of the base of the cylinder. Then, the height of the cylinder in terms of diameter of the sphere is
(a) 3 of diameter of the sphere
(b) $\frac{2}{3}$ of diameter of sphere
(c) $\frac{1}{3}$ of diameter of sphere
(d) $\frac{3}{2}$ of diameter of sphere

9. A tank is 225 m long, 162 m broad with what velocity per sec must water flow into it through an aperture 60 cm by 45 cm that the level may be raised by 20 cm in 5 h?
(a) 2.5 m/s (b) 0.25 m/s (c) 1.5 m/s (d) 3.5 m/s

10. A cylindrical water tank of diameter 1.4 m and height 2.1 m is being fed by a pipe of diameter 3.5 cm through which water flows at the rate of 2 m/s. The time it take to fill the tank (in min) is
(a) 20 (b) 24 (c) 26 (d) 28

11. The difference between the inner and outer surfaces of a cylinder 14 cm long is 88 cm^2. If the volume of the tube be 176 cm^2, then its inner and outer radii are, respectively
(a) 1, 3 (b) 1.5, 2.5
(c) 3, 4 (d) 1.5, 3.5

12. A solid is hemi spherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of its radius and the height of its [illegible] part is
(a) 1 : 3 (b) [illegible]
(c) 1 : 1 (d) [illegible]

13. The lateral surface of a cylinder is developed into a square whose diagonal is $\sqrt{5}$ cm. The area of the base of the cylinder (in m^2) is
(a) $\frac{5}{8\pi}$ (b) $\frac{3}{8\pi}$ (c) $\frac{8}{5\pi}$ (d) $\frac{8}{3\pi}$

14. A rod of copper 400 cm in length and 0.6 cm in radius is drawn into a root of length 720 cm. The new radius is
(a) $\sqrt{0.35}$ cm (b) $\sqrt{0.40}$ cm (c) $\sqrt{0.2}$ cm (d) 0.5 cm

15. 2.2 dm^3 of brass is to be drawn into a wire $\frac{1}{4}$ cm in diameter. The length of the wire is
(a) 106 m (b) 112 m (c) 212 m (d) 448 m

16. Given a solid cylinder of radius 10 cm and length 1000 cm a cylindrical hole is made into it to obtain a cylindrical shell of uniform thickness and having volume equal to one-fourth of the original volume of the original cylinder. The thickness of the cylindrical shell is
(a) $5(\sqrt{5}-2)$ cm (b) $7(2-\sqrt{3})$ cm
(c) 10 cm (d) $5\sqrt{2}$ cm

17. The difference between outside and inside surface of a cylindrical metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cm^3 of metal, the outer and inter radii are, respectively
(a) 3.5 cm, 3 cm (b) 2.5 cm, 2 cm
(c) 4.5 cm, 4 cm (d) 4.75 cm, 4.25 cm

18. A vessel in the form of a hemisphere surrounded by a cylinder (open at the other end) of same radius is full of liquid of whose volume is 432 π cm^3. If water is filled in it to a level which is 1 cm below the top of vessel the volume of the water is 396 π cm^3. The radius of the circular end is
(a) 8 cm (b) 7 cm (c) 6 cm (d) 5 cm

19. A tent is of the shape of right circular cylinder upto a height of 3 m and then becomes a right circular cone with a maximum height of 13.5 m above the ground. The cost of painting the inner side of the tent at the rate of ₹ 2 per m^2, if the radius of the base is 14 m
(a) ₹ 2048 (b) ₹ 2068 (c) ₹ 2008 (d) ₹ 2088

20. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm, respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm the diameter of the cylinder is
(a) 7 cm (b) 9.5 cm (c) 14 cm (d) 15.5 cm

21. A conical tent has an angle of 30° at the vertex. If the curved surface is 100 m^2, then the volume of the tent is
(a) $\frac{500}{\sqrt{6}\pi}$ m^3 (b) $\frac{400}{\sqrt{6}\pi}$ m^3 (c) $\frac{500}{\sqrt{6}}$ m^3 (d) $\frac{500}{\sqrt{\pi}}$ m^3

22. If from a circular sheet of paper of radius 10 cm, a sector of area 40% is removed and the remaining is used to make a conical surface, then the angle at the vertex will be
(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$ (c) $2\sin^{-1}\left(\frac{4}{5}\right)$ (d) $2\sin^{-1}\left(\frac{3}{5}\right)$

23. A measuring jar of internal diameter 10 cm is partly filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in the water completely. The change in the level of water in the jar is
(a) $\frac{16}{65}$ cm (b) $\frac{15}{16}$ cm (c) $\frac{16}{75}$ cm (d) None of these

24. A cylindrical vessel open at the top contains water upto $\frac{1}{3}$ to its height. A heavy sphere whose diameter is equal to the height of the cylinder is palced into the vessel touching its curved surface from all sides. Then the correct statement is
(a) water stands at the top of the vessel
(b) water stands at $\frac{3}{4}$ the height of the vessel
(c) the water overflows out of the vessel
(d) water stands at half the height of the vessel

25. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is
(a) 1 : 2 (b) 1 : 4 (c) 1 : 6 (d) 1 : 8

26. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, then the height above the base where the section is made is
(a) 7 cm (b) 6 cm (c) 10 cm (d) 12 cm

27. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. If it is 6 m deep its capacity is
(a) 166 m^3 (b) 176 m^3 (c) 186 m^3 (d) 196 m^3

28. A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top which is open is 5 cm. It is filled with water upto the rim. When lead shots each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of water flows out. The number of lead shots dropped into the vessel.
(a) 50 (b) 75 (c) 85 (d) 100

29. A toy is in the form of a cone mounted on a hemisphere such that the diameter of the base of the cone is equal to that of the hemisphere. If the diameter of the base of the cone is 6 cm and its height is 4 cm, what is the surface area of the toy in sq cm? (Take $\pi = 3.14$) [CDS 2011 I]
(a) 93.62 (b) 103.62 (c) 113.62 (d) 115.50

30. The diagonals of the three faces of a cuboid are x, y, z respectively. What is the volume of the cuboid? [CDS 2010 II]
(a) $\frac{xyz}{2\sqrt{2}}$
(b) $\frac{\sqrt{(y^2+z^2)(z^2+x^2)(x^2+y^2)}}{2\sqrt{2}}$
(c) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$
(d) None of the above

31. Half of a large cylindrical tank open at the top is filled with water and identical heavy spherical balls are to be dropped into the tank without spilling water out. If the radius and the height of the tank are equal and each is four times the radius of a ball, what is the maximum number of balls that can be dropped? **(CDS 2010 II)**
(a) 12 (b) 24 (c) 36 (d) 48

32. The length, breadth and height of a rectangular parallelopiped are in ratio 6 : 3 : 1. If the surface area of a cube is equal to the surface area of this parallelopiped, then what is the ratio of the volume of the cube to the volume of the parallelopiped? **(CDS 2010 I)**
(a) 1 : 1 (b) 5 : 4 (c) 7 : 5 (d) 3 : 2

33. A semi-circular thin sheet of a metal of diameter 28 cm is bent and an open conical cup is made. What is the capacity of the cup? **(CDS 2010 I)**
(a) $\frac{1000}{3}\sqrt{3}$ cu cm (b) $300\sqrt{3}$ cu cm
(c) $\frac{700}{3}\sqrt{3}$ cu cm (d) $\frac{1078}{3}\sqrt{3}$ cu cm

34. From a cylindrical log whose height is equal to its diameter, the greatest possible sphere has been taken out. What is the fraction of the original log which is cut away? **(CDS 2009 II)**
(a) 1/2 (b) 1/3 (c) 1/4 (d) 2/3

35. A container is in the form of a right circular cylinder surmounted by a hemisphere of the same radius 15 cm as the cylinder. If the volume of the container is 32400π cm^3, then the height h of the container satisfies which one of the following? **(CDS 2008 II)**
(a) 135 cm < h < 150 cm (b) 140 cm < h < 147 cm
(c) 145 cm < h < 148 cm (d) 139 cm < h < 145 cm

36. A conical flask of base radius r and height h is full of milk. The milk is now poured into a cylinderical flask of radius $2r$. What is the height to which the milk will rise in the flask? **(CDS 2008 I)**
(a) $\frac{h}{3}$ (b) $\frac{h}{6}$ (c) $\frac{h}{9}$ (d) $\frac{h}{12}$

37. From a wooden cylindrical block, whose diameter is equal to its height, a sphere of maximum possible volume is curved out. What is the ratio of the volume of the utilised wood to that of the wasted wood? **(CDS 2008 I)**
(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 3 : 2

38. An iron block is in the form of a cylinder of 1.5 m diameter and 3.5 m length. The block is to be rolled into the form of a bar, having a square section of side 5 cm. What will be the length of the bar?
(a) 2375 m (b) 2475 m (c) 2575 m (d) 2600 m

39. The base diameter of a right circular cylinder is 3 cm. There is a section making an angle of 30° with the cross section. what is its area? **(CDS 2007 II)**
(a) $\frac{9\pi}{4}$ sq cm (b) $\frac{3\sqrt{3}\pi}{2}$ sq cm
(c) $\frac{9\pi}{8}$ sq cm (d) $\frac{9\sqrt{3}\pi}{8}$ sq cm

40. A cone is inscribed in a hemisphere such that their bases are common. If C is the volume of the cone and H that of the hemisphere, then what is the value of $C : H$? **(CDS 2007 II)**
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5

41. If the diameter of a wire is decreased by 10%, by how much per cent (approximately) will the length be increased to keep the volume constant? **(CDS 2007 I)**
(a) 5% (b) 17% (c) 20% (d) 23%

42. The diameter of a solid metallic right circular cylinder is equal to its height. After cutting out the largest possible solid sphere S from this cylinder, the remaining material is recast to form a solid sphere S_1. What is the ratio of the radius of sphere S to that of sphere S_1? **(CDS 2007 I)**
(a) $1 : 2^{\frac{1}{3}}$ (b) $2^{\frac{1}{3}} : 1$ (c) $2^{\frac{1}{3}} : 3^{\frac{1}{3}}$ (d) $3^{\frac{1}{2}} : 2^{\frac{1}{2}}$

43. A square has its side equal to the radius of a sphere. The square revolves round a side to generate a surface of total area S. If A be the surface area of the sphere, which one of the following is correct? **(CDS 2007 I)**
(a) $A = 3S$ (b) $A = 2S$ (c) $A = S$ (d) $A < S$

44. A swimming pool is 24 m long and 15 m broad. When x number of men dive into the pool, the height of the water rises by 1 cm. If the average amount of water displaced by one man is 0.1 m^3, then what is the value of x? **(CDS 2007 I)**
(a) 36 (b) 72 (c) 108 (d) 360

45. Water is distributed to a town of 50000 inhabitants from a rectangular reservoir consisting of 3 equal compartments. Each compartment has length and breadth 200 m, 100 m, respectively and 12 m depth of water in the beginning. The allowance is 20 L per head per day. For how many days will the supply of water hold out? **(CDS 2007 I)**
(a) 240 days (b) 720 days (c) 800 days (d) 900 days

46. A right circular cylinder and a right circular cone have equal bases and equal volumes. But the lateral surface area of the right circular cone is 15/8 times the lateral surface area of the right circular cylinder. What is the ratio of radius to height of the cylinder? **(CDS 2007 I)**
(a) 3 : 4 (b) 9 : 4 (c) 15 : 8 (d) 8 : 15

47. The volume of a cuboid whose sides are in the ratio of 1 : 2 : 4 is same as that of a cube. What is the ratio of length of diagonal of cuboid to that of cube? **(CDS 2007 I)**
(a) $\sqrt{1.25}$ (b) $\sqrt{1.75}$ (c) $\sqrt{2}$ (d) $\sqrt{3.5}$

48. A right ΔABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. What is the volume of the solid so obtained? **(CDS 2009 II)**
(a) 50π cu cm (b) 100π cu cm
(c) 125π cu cm (d) 150π cu cm

49. A field is 125 m long and 15 m wide. A tank 10 m × 7.5 m × 6 m was dug in it and the Earth, thus dug out was spread equally on the remaining field. The level of the field thus raised is equal to which one of the following? **(CDS 2008 II)**
(a) 15 cm (b) 20 cm (c) 25 cm (d) 30 cm

50. If C_1 is a right circular cone with base radius r_1 cm and height h_1 cm and C_2 is a right circular cylinder with base radius r_2 cm and height h_2 cm and if $r_1 : r_2 = 1 : n$ (where n is a positive integer) and their volumes are equal, then which one of the following is correct? **(CDS 2008 II)**

(a) $h_1 = 3nh_2$ (b) $h_1 = 3n^2h_2$ (c) $h_1 = 3h_2$ (d) $h_1 = n^2h_2$

51. A right circular cone is cut by a plane parallel to its base in such a way that the slant heights of the original and the smaller cone thus obtained are in the ratio 2 : 1. If V_1 and V_2 are respectively the volumes of the original cone and of the new cone, then what is $V_1 : V_2$? **(CDS 2008 II)**

(a) 2 : 1 (b) 3 : 1 (c) 4 : 1 (d) 8 : 1

52. A rectangular tank whose length and breadth are 2.5 m and 1.5 m, respectively is half full of water. If 750 L more water is poured into the tank, what is the height through which water level further goes up? **(CDS 2010 I)**

(a) 20 cm (b) 18 cm (c) 15 cm (d) 200 cm

53. How many litres of water flow out of a pipe having an area of cross-section of 5 cm^2 in one minute, if the speed of water in the pipe is 30 cm/s? **(CDS 2009 II)**

(a) 90 L (b) 15 L (c) 9 L (d) 15...

54. If a sphere of radius 10 cm is intersected by a plane a distance 8 cm from its centre, what is the radius of the curve of intersection of the plane and the sphere? **(CDS 2009...)**

(a) 8 cm (b) 6 cm (c) 5 cm (d) 4 cm

55. A cylindrical vessel of base radius 14 cm is filled with water to same height. If a rectangular solid of dimensions 22 cm × 7 cm × 5 cm is immersed in it, what is the rise in water level? **(CDS 2009...)**

(a) 0.5 cm (b) 1.0 cm
(c) 1.25 cm (d) 1.5 cm

56. A hemisphere is made of a sheet of a metal 1 cm thick. If the outer radius is 5 cm. What is the weight of the hemisphere (1 cm^3 of the metal weighs 9 g)? **(CDS 2010...)**

(a) 54 π g (b) 366 π g
(c) 122 π g (d) 108 π g

57. The radius and height of a right circular cone are in the ratio 3 : 4 and its volume is 96 π cm^3. What is the lateral surface area? **(CDS 2008...)**

(a) 24 π cm^2 (b) 36 π cm^2 (c) 48 π cm^2 (d) 60 π cm...

Answers

Level I

1. (c)	2. (a)	3. (b)	4. (d)	5. (a)	6. (c)	7. (d)	8. (a)	9. (d)	10. (c
11. (c)	12. (a)	13. (b)	14. (c)	15. (c)	16. (d)	17. (b)	18. (a)	19. (c)	20. (c
21. (c)	22. (a)	23. (c)	24. (a)	25. (c)	26. (c)	27. (b)	28. (a)	29. (d)	30. (c
31. (a)	32. (c)	33. (a)	34. (b)	35. (c)	36. (c)	37. (c)	38. (b)	39. (d)	40. (a
41. (c)	42. (b)	43. (a)	44. (d)	45. (b)	46. (c)	47. (a)	48. (b)	49. (b)	50. (a
51. (c)	52. (a)	53. (b)	54. (d)	55. (c)	56. (b)	57. (c)	58. (c)	59. (a)	60. (c
61. (c)	62. (b)	63. (a)	64. (b)	65. (c)	66. (b)	67. (b)	68. (c)	69. (a)	70. (c
71. (c)	72. (d)	73. (b)	74. (b)	75. (d)	76. (d)	77. (a)	78. (c)	79. (d)	80. (
81. (d)	82. (b)	83. (c)	84. (d)	85. (c)	86. (b)	87. (c)	88. (a)	89. (b)	90. (c
91. (b)	92. (d)	93. (a)	94. (c)	95. (a)	96. (c)	97. (b)	98. (a)	99. (d)	100. (b

Level II

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)	6. (d)	7. (a)	8. (b)	9. (c)	10. (c
11. (b)	12. (b)	13. (a)	14. (c)	15. (d)	16. (a)	17. (b)	18. (c)	19. (b)	20. (c
21. (a)	22. (d)	23. (c)	24. (a)	25. (d)	26. (c)	27. (b)	28. (d)	29. (b)	30. (c
31. (b)	32. (d)	33. (d)	34. (d)	35. (a)	36. (d)	37. (a)	38. (b)	39. (b)	40. (a
41. (d)	42. (b)	43. (c)	44. (a)	45. (b)	46. (b)	47. (b)	48. (b)	49. (c)	50. (b
51. (d)	52. (a)	53. (c)	54. (b)	55. (c)	56. (b)	57. (d)			

Hints and Solutions

Level I

1. Let 'a' cm be the edge of three equal cubes.

Surface area of a single cube = $6(\text{Edge})^2 = 6a^2$ cm^2

∴ Sum of the surface areas of the three cube is
$$= 3(6a^2) = 18a^2 \text{ cm}^2$$

Length of resulting cuboid = $3a$
Breadth of resulting cuboid = a cm
Height of resulting cuboid = a cm

∴ Total surface area of cuboid $= 2(lb + bh + hl)$
$$= 2(3a^2 + a^2 + 3a^2) = 14a^2 \text{ cm}^2$$

Required ratio $= \dfrac{14a^2}{18a^2} = \dfrac{7}{9}$, i.e., 7 : 9

2. Let height of the room be h m.

So, volume of the room $= 25 \times 9.8 \times h$ m^3 $= 245 \times h$ m^3

$$245 \times h = 12.25 \times 100 \quad \text{(by condition)}$$

But

$$\Rightarrow \quad h = \frac{100 \times 12.25}{245} = 5\text{ m}$$

3. Let edge of the solid cube be 'a' cm.
The dimensions of each of the cuboids will be a cm, a cm, $\frac{a}{2}$ cm.
Total surface area of one cuboid

$$= 2\left(a \times a + a \times \frac{a}{2} + \frac{a}{2} \times a\right) = 2\left(a^2 + \frac{a^2}{2} + \frac{a^2}{2}\right) = 4a^2$$

Total surface area of the cube $= 6(\text{Edge})^2 = 6a^2\text{ cm}^2$
So, ratio of the total surface area of the given cube and that of one of the cuboid $= \frac{6a^2}{4a^2} = \frac{3}{2} = 3:2$

4. Volume of air in cinema hall $= 100 \times 50 \times 18 = 90000\text{ m}^3$
Number of persons which can sit in the hall $= \frac{90000}{150} = 600$

5. Number of small cubes $= \frac{\text{Volume of large cube}}{\text{Volume of a small cube}} = \frac{(4)^3}{1^3} = 64$
Total surface area of the large cube $= 6(4^2) = 96\text{ cm}^2$
Total surface area of 1 small cube $= 6(1)^2 = 6\text{ cm}^2$
Total surface area of 64 small cube $= 64 \times 6 = 384\text{ cm}^2$
Required ratio $= 384 : 96$ or $4:1$

6. Area of the walls to be coloured washed
$=$ Area of four wall $-$ (Area of door $+$ Area of three windows)
$= 2 \times 4(7 + 6.5) - (3 \times 1.4 + 3 \times 2 \times 1) = 108 - 10.2 = 97.8\text{ m}^2$
Total cost of colour washing $=$ ₹ $(97.8 \times 5.25) =$ ₹ 513.45

7. Area of the field $=$ length $\times$ breadth $= 12 \times 10 = 120\text{ m}^2$
Area of the pit's surface $= 5 \times 4 = 20\text{ m}^2$
Area on which the Earth is to be spread $= 120 - 20 = 100\text{ cm}^2$
Volume of Earth dug out $= 5 \times 4 \times 2 = 40\text{ cm}^3$
$\therefore$ Level of field raised $= \frac{40}{100} = \frac{2}{5}\text{ m} = \frac{2}{5} \times 100 = 40\text{ cm}$

8. Let the edge of the cube be x cm
$\therefore$ Increased edge $= x + \frac{x}{2} = \frac{3x}{2}$
Initial surface area $= 6x^2$

$$\text{Increased surface area} = 6\left(\frac{3x}{2}\right)^2$$

$$\text{Increased in surface area} = 6\left[\frac{9x^2}{4} - x^2\right] = \frac{6 \times 5x^2}{4}$$

$$\text{Percentage increased} = \frac{30x^2 \times 100}{4 \times 6x^2} = 125\%$$

9. Let the edges be $3x$, $4x$ and $5x$, respectively.
$\therefore$ The total volume of the three cubes
$= (3x)^3 + (4x)^3 + (5x)^3 = 27x^3 + 64x^3 + 125x^3 = 216x^3$

Let the edge of the newly formed cube be 'a' cm.
$\therefore \quad$ Diagonal $= \sqrt{3}a = 12\sqrt{3} \Rightarrow a = 12$
$\therefore$ Volume of new cube $= 12^3 = 1728\text{ cm}^3$
But $\quad 216x^3 = 1728$
$\therefore \quad x^3 = 8 \Rightarrow x = 2$
$\therefore$ Edges are 6 cm, 8 cm and 10 cm.

10. Volume of granary $= 8 \times 6 \times 3 = 144\text{ m}^3$ [as it is cuboid]
Volume of each bag of grain $= 0.65\text{ m}^3$
$\therefore$ Required number of bags which can be stored in the granary

$$= \frac{144}{0.65} = \frac{144 \times 100}{65} = 221.53$$

$\therefore$ 221 bags can be stored in the granary.

11. Volume of the wall $= 10\text{ m} \times \frac{4}{10}\text{ m} \times 5\text{ m} = 20\text{ m}^3$
Volume occupied by mortar $= \frac{1}{10} \times 20\text{ m}^3 = 2\text{ m}^3$
$\therefore$ Volume occupied by bricks $= 20\text{ m}^3 - 2\text{ m}^3 = 18\text{ m}^3$
Volume of each brick $= \frac{25}{100}\text{ m} \times \frac{15}{100}\text{ m} \times \frac{8}{100}\text{ m} = \frac{3}{1000}\text{ m}^3$
$\therefore$ Number of bricks required to build the wall

$$= \frac{18}{\frac{3}{1000}} = \frac{18 \times 1000}{3} = 6000$$

12. Edge of cube $= 9$ cm
For rectangular vessel
Length $= 15$ cm, Breadth $= 12$ cm
Let the rise in the water level is x cm

$$\text{Volume of cube} = 9^3 = 9 \times 9 \times 9$$

$$\text{Volume of vessel} = 15 \times 12 \times x$$

$$\Rightarrow \quad 15 \times 12 \times x = 9 \times 9 \times 9$$

$$x = \frac{9 \times 9 \times 9}{15 \times 12} = \frac{81}{20} = 4.05\text{ cm}$$

13. Volume of the block $= (18 \times 15 \times 12)\text{ cm}^3 = 3240\text{ cm}^3$
The side of the largest cube that can be accommodated in the block
$=$ HCF of 18 cm, 15 cm, 12 cm $= 3$ cm
$\therefore \quad$ Volume of cube $= 27\text{ cm}^3$
Number of cube $= \frac{\text{Volume of block}}{\text{Volume of cube}} = \frac{3240}{27} = 120$

14. Volume of bar $=$ Volume of piece of copper $= 1\text{ m}^3$
Length of bar $= 36$ m
$\therefore$ Area of square cross-section of bar $= \frac{\text{Volume}}{\text{Length}} = \frac{1}{36}\text{ m}^2$
$\therefore$ Side of square cross-section $= \sqrt{\frac{1}{36}} = \frac{1}{6}$ m
Volume of exact cube which is cut off from bar

$$= \left(\frac{1}{6}\right)^3 = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}\text{ m}^3$$

Cost of cube $=$ ₹ $\frac{1}{216} \times 108 =$ ₹ $\frac{1}{2} = 50$ paise

15. Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$ $\left(\because r = \frac{h}{2}\right)$

$$= 2\pi \frac{h}{2}\left(h + \frac{h}{2}\right) = \frac{3}{2}\pi h^2$$

16. Curved surface area of cylinder

$$= 2\pi rh = 2 \times \frac{22}{7} \times 5 \times 21 = 660 \text{ cm}^2$$

17. Diameter of the base of the cylinder = 21 cm

$\therefore$ Radius $= \frac{21}{2}$ cm

As curved surface area $= 2\pi rh = 1320$

$\Rightarrow$ $2 \times \frac{22}{7} \times \frac{21}{2} \times h = 1320$

$$h = \frac{1320}{22 \times 3} = 20 \text{ cm}$$

18. Volume of first cylinder

i.e. $V_1 = \pi r_1^2 h_1 = \pi \times 7 \times 7 \times 10 = 490\pi \text{ cm}^3$

Volume of second cylinder

i.e. $V_2 = \pi r_2^2 h_2 = \pi \times 7 \times 7 \times 20 = 980\pi \text{ cm}^3$

$\therefore$ $\frac{V_1}{V_2} = \frac{490\pi}{980\pi} = \frac{1}{2}$

Here, $V_1 : V_2 = 1 : 2$

> If the radius of base is same, then ratio of volume = ratio of lateral surface area = ratio of heights

19. Area of iron sheet required = Curved surface area of tunnel

$$= 2\pi rh$$
$$= (2 \times \pi \times 1 \times 40) \text{ m}^2 = 80\pi \text{ m}^2$$

20. Let radius of cylinder be $5x$ and height be $7x$.

$\therefore$ Volume $= \pi r^2 h = 550$

$\frac{22}{7} \times (5x)^2 \times 7x = 550$ (given)

$22 \times 25x^3 = 550$

$$x^3 = \frac{550}{22 \times 25} = 1 \Rightarrow x = 1$$

Here, radius $= 5x = 5 \times 1 = 5$ cm

21. $\frac{\text{Total surface area}}{\text{Curved surface area}} = \frac{2\pi r(h + r)}{2\pi rh} = \frac{h + r}{h}$

$\therefore$ $\frac{\text{Total surface area}}{\text{Curved surface area}} = \left(\frac{h + r}{h}\right) = \left(\frac{30 + 70}{30}\right) = \frac{10}{3}$

22. Given, total surface area $= 231 \text{m}^2$

and curved surface area $= \frac{2}{3} \times$ total surface area $= \frac{2}{3} \times 231$

$$= 154 \text{m}^2$$

Now, $\frac{\text{Total surface area}}{\text{Curved surface area}} = \frac{231}{154}$

$\Rightarrow$ $\frac{2\pi rh + 2\pi r^2}{2\pi rh} = \frac{3}{2} \Rightarrow \frac{h + r}{h} = \frac{3}{2}$

$\Rightarrow$ $2h + 2r = 3h \Rightarrow h = 2r$

$\because$ Curved surface area $= 2\pi rh = 154$

$\Rightarrow$ $2\pi r(2r) = 154 \Rightarrow r = \frac{7}{2}$ m

$\therefore$ $h = 7$ m

$\therefore$ Required volume $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 = \frac{539}{2} = 269\frac{1}{2} \text{ m}^3$$

23. $\frac{\text{Curved surface area of Ist cylinder}}{\text{Curved surface area of 2nd cylinder}} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2}$

$$\frac{s_1}{s_2} = \frac{r_1 h_1}{r_2 h_2} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$$

$\therefore$ $s_1 : s_2 = 5 : 6$

24. $\frac{\text{Volume of Ist cylinder}}{\text{Volume of 2nd cylinder}} = \frac{\pi(2r)^2 \times h_1}{\pi(3r)^2 \times h_2}$

$\because$ $\frac{V_1}{V_2} = \frac{5}{9}$

$\therefore$ $\frac{5}{9} = \frac{4h_1}{9h_2} \Rightarrow \frac{h_1}{h_2} = \frac{5}{4}$

$\therefore$ $h_1 : h_2 = 5 : 4$

25. $\because$ 1 cm^3 of water weights 1 g

$\therefore$ Volume of 154 g of water $= 154 \text{ cm}^3$

$\therefore$ Volume of cylinder $= \pi r^2 h = 154 \text{ cm}^3$

$\therefore$ $\frac{22}{7}(3.5)^2 \times h = 154$

$\Rightarrow$ $\frac{77h}{2} = 154 \Rightarrow h = \frac{154 \times 2}{77} = 4$ cm

26. Radius of roller $= \frac{1.4}{2} = 0.7$ m

Height = 2 m

Area covered in 1 revolution $= 2\pi rh$

Area covered in 5 revolutions $= 5 \times 2 \times \frac{22}{7} \times 0.7 \times 2 = 44 \text{m}^2$

27. Radius of roller = 42 cm = 0.42 m

Height of roller = 120 cm = 1.2 m

$\therefore$ Lateral surface area of roller $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.42 \times 1.2 = 3.168 \text{ m}^2$$

$\therefore$ Area covered in 500 revolutions $= 500 \times 3.168 = 1584 \text{m}^2$

$\therefore$ Cost of levelling $= 0.30 \times 1584 =$ ₹ 475.20

28. Let the radii be r_1 and r_2 and heights be h and $2h$

$\therefore$ $V_1 = V_2 \Rightarrow \pi r_1^2 \times h = \pi r_2^2 \times 2h$ (by condition)

$$\frac{r_1}{r_2} = \frac{\sqrt{2}}{1}$$

29. Let the radius be R and $\frac{R}{3}$ and heights be h_1 and h_2.

Here, $V_1 = V_2$

$$\pi R^2 h_1 = \pi \times \left(\frac{R}{3}\right)^2 h_2$$

$\therefore$ $$h_1 = \frac{h_2}{9} \Rightarrow h_2 = 9h_1$$

$\therefore$ So, length will becomes 9 times.

30. Here, $2\pi rh = 264\ m^2$ and $\pi r^2 h = 924\ m^3$ (given)

$\therefore$ $$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{7}{2} \Rightarrow r = 7\ m$$

$\therefore$ Diameter of pillar $= r \times 2 = 7 \times 2 = 14$ m

31. Volume $= \pi r^2 h = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 280\ m^3 = 1980\ m^3$

$\therefore$ Total cost of sinking tube well at the rate of ₹ 3.60 per m^3
$= 1980 \times 3.60 =$ ₹ 7128

32. Let r be the radius of base and h be the height.

$\therefore$ Volume of given cylinder $= \pi r^2 h$

Radius of reduced cylinder $= \frac{r}{2}$

$\therefore$ Volume of reduced cylinder $= \pi\left(\frac{r}{2}\right)^2 h = \pi \frac{r^2}{4} h$

$\therefore$ Required ratio $= \dfrac{\frac{1}{4}\pi r^2 h}{\pi r^2 h} = \frac{1}{4} = 1 : 4$

33. Slant height $= \sqrt{12^2 + 16^2} = \sqrt{400} = 20$ m

Canvas required = Curved surface area of cone

$= \pi rl = \frac{22}{7} \times 12 \times 20 = \frac{5280}{7}\ m^2$

34. Here, $2\pi r = 44$ (given)

$\Rightarrow$ $$r = \frac{44}{2\pi} = 7\ m$$

Slant height $= \sqrt{r^2 + h^2} = \sqrt{49 + 81} = \sqrt{130}$ m

35. Total surface area $= \pi r(l + r) = \frac{22}{7} \times 6 \times (10 + 6) = \frac{2112}{7}\ m^2$

36. Let radius $= 4x$, slant height $= 7x$

Curved surface area $= 792$ (given)

$\therefore$ $$\frac{22}{7} \times 4x \times 7x = 792$$

$\Rightarrow$ $$\frac{88}{7}x^2 = 792 \Rightarrow x^2 = \frac{792}{88} = 9$$

$\therefore$ $$x = 3$$

$\Rightarrow$ Radius of cone $= 4x = 4 \times 3 = 12$ cm

37. Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times r^2 \times 21 = 7\pi r^2$

38. Slant height $= \sqrt{r^2 + h^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ m

Curved surface area $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550\ m^2$

Width of cloth $= 50$ m

Length required $= \frac{550}{50} = 11$ m

39. Volume of the cone $= \frac{1}{3}\pi R^2 h$, $R = 3r = \frac{1}{3}\pi(3r)^2 \times r = 3\pi r^3$

40. Let radii of two cones be $2x$ and $3x$ and their vertical height be h_1 and h_2

$\therefore$ $$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{\pi(2x)^2 h_1}{\pi(3x)^2 h_2} = \frac{h_1}{h_2}$$

$$\frac{4}{5} = \frac{4h_1}{9h_2} \Rightarrow \frac{h_1}{h_2} = \frac{9}{5}$$ (given)

41. Here, $\frac{1}{3}\pi r^2 h = 2512$ (given)

and $$r = 5x \text{ and } h = 12x$$

$$\frac{1}{3} \times 3.14 \times (5x)^2 \times 12x = 2512$$

$$x^3 = \frac{2512}{314} = 8 \Rightarrow x = 2$$

$\therefore$ Radius = 10 cm, height = 24 cm

$\therefore$ Slant height, $h = \sqrt{r^2 + h^2} = \sqrt{10^2 + 24^2} = \sqrt{676} = 26$ cm

42. $\dfrac{\text{Lateral surface area of cone}}{\text{Area of base}} = \dfrac{\pi r H}{\pi r^2} = \dfrac{H}{r}$

43. Let r and H be the radius and height of the cone.

$\therefore$ Original volume $= \frac{1}{3}\pi r^2 h = V$

New radius $= r$ and new height $= 2h$

New volume $= \frac{1}{3}\pi r^2 \times 2h = 2V$

$\therefore$ Increase in volume $= 2V - V = V$

Increase percentage of volume $= \left(\frac{V}{V} \times 100\right)\% = 100\%$

44. Let original radius $= r$ and original height $= h$

New radius $= 2r$ and new height $= 2h$

$$\frac{\text{New volume}}{\text{Original volume}} = \frac{\frac{1}{3}\pi(2r)^2 \times 2h}{\frac{1}{3}\pi r^2 \times h} = \frac{8}{1}$$

45. Curved surface area of cone = Area of sector

$= \frac{\theta}{360}\pi r^2 = \frac{90}{360} \times \frac{22}{7} \times 42 \times 42 = 1386\ cm^2$

46. Let the radii of their bases be $3r$ and r respectively and their heights be x and $3x$, respectively.

Now, $$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(3r)^2 \times x}{\frac{1}{3}\pi(r^2) \times 3x} = \frac{9r^2 \times x}{r^2 \times 3x} = \frac{3}{1}$$

$\therefore$ Ratio of their volumes $= 3 : 1$

47. As diameters are equal so radii are equal.
Let the slant heights be $5l$ and $4l$, respectively.
$\therefore$ Curved surface area of first cone $= \pi r(5l) = 5\pi rl$
And curved surface area of second cone $= \pi r(4l) = 4\pi rl$

$\therefore$ Required ratio $= \dfrac{5\pi rl}{4\pi rl} = 5 : 4$

48. For cone X : radius $= 3r$, height $= h_1$, volume $= V_1$

For cone Y : radius $= r$, height $= h_2$, volume $= V_2$

Also, $V_2 = \frac{1}{2}V_1$ (by condition)

$$\therefore \quad \frac{V_1}{V_2} = \frac{V_1}{\frac{1}{2}V_1} = \frac{\frac{1}{3}\pi(3r)^2h_1}{\frac{1}{3}\pi r^2h_2}$$

$$\Rightarrow \quad \frac{2}{1} = \frac{9h_1}{h_2} \Rightarrow \frac{h_1}{h_2} = \frac{2}{9} \Rightarrow h_1 : h_2 = 2:9$$

49. Height of cone, $h = 24$ cm

Radius of base $= 7$ cm

$\therefore$ Slant height, $l = \sqrt{r^2+h^2} = \sqrt{24^2+7^2} = \sqrt{576+49}$

$= \sqrt{625} = 25$ cm

Area of metal sheet required = Total surface area of cone

$= \pi r(l+r) = \frac{22}{7} \times 7(7+25) = 22 \times 32 = 704 \text{ cm}^2$

50. Let the radius be r, height be h and slant height be l, then

$\pi r^2 = 770 \text{ cm}^2$

$\frac{22}{7} \times r^2 = 770 \Rightarrow r^2 = \frac{770 \times 7}{22} = 245$

$\Rightarrow \quad r = 7\sqrt{5}$ cm

Area of the curved surface $= \pi rl = \frac{22}{7} \times 7\sqrt{5} \times l = 814 \text{ cm}^2$

$l = \frac{814 \times 7}{22 \times 7\sqrt{5}} = \frac{37}{\sqrt{5}}$

$\therefore \quad h^2 = l^2 - r^2 = \left(\frac{37}{\sqrt{5}}\right)^2 - (7\sqrt{5})^2 = \frac{1369 - 1225}{5} = \frac{144}{5}$

$\therefore \quad h = \frac{12}{\sqrt{5}}$ cm

$\therefore$ Volume of cone $= \frac{1}{3} \times \pi r^2 \times h$

$= \frac{1}{3} \times 770 \times \frac{12}{\sqrt{5}} = \frac{1}{3} \times \frac{770 \times 12\sqrt{5}}{5}$

$= 616\sqrt{5} \text{ cm}^3$

51. Let original radius of base $= R$ and

Original height $= H$

New radius $= \left(\frac{120}{100}R\right) = \frac{6}{5}R$

and New height $= \frac{6}{5}H$

Original volume, $V_1 = \frac{1}{3}\pi R^2 H$

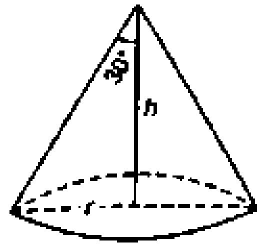
New volume $V_2 = \frac{1}{3}\pi \times \left(\frac{6}{5}R\right)^2 \times \frac{6}{5}h = \frac{216}{125}V_1$

$\therefore$ Increase in volume $= \frac{216}{125}V_1 - V_1 = \frac{91}{125}V_1$

Percentage increase in volume $= \frac{91}{125}V_1 \times \frac{1}{V_1} \times 100\%$

$= 72.8\% \approx 73\%$ (approximately)

52. Here, $\frac{\text{Radius of base}}{\text{Slant height}} = \sin 30° = \frac{1}{2}$



53. $\because V = \frac{1}{3}\pi r^2 h$, $s = \pi r\sqrt{r^2+h^2}$

$\Rightarrow \quad s^2 = \pi^2 r^2(r^2+h^2)$

$\therefore 3\pi Vh^3 - s^2h^2 + 9V^2$

$= 3\pi h^3 \frac{1}{3}\pi r^2 h - \pi^2 r^2 h^2(r^2+h^2) + 9\frac{1}{9}\pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0$

54. Let their volume be V_1 and V_2.

$V_1 : V_2 = 8:27$

$\frac{\frac{2}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3} = \frac{8}{27} = \frac{r_1^3}{r_2^3} = \frac{8}{27}$

$\frac{r_1}{r_2} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3} \quad \therefore r_1 : r_2 = 2:3$

55. Inner radius $(r) = \frac{4}{2} = 2$ cm

Outer radius $(R) = \frac{8}{2} = 4$ cm

Volume of metal of the sphere $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(4^3 - 2^3) = \frac{4}{3}\pi \times 56 \text{ cm}^3$

Radius of base of cone $(x) = \frac{8}{2} = 4$ cm

$\therefore \quad \frac{1}{3}\pi 4^2 h = \frac{4}{3}\pi 56$ (by condition)

$\Rightarrow \quad h = \frac{\frac{4}{3} \times 56 \times 3}{16} = 14$ cm

56. Let the radii of the two spheres be r and R, respectively.

So, $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{1}{8} \Rightarrow \left(\frac{r}{R}\right)^3 = \frac{1}{8}$

$\therefore \quad \frac{r}{R} = \frac{1}{2}$

Now, ratio of surface areas $= \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

57. Ratio of surface areas $= \dfrac{4\pi r^2}{4\pi R^2} = \left(\dfrac{4}{25}\right) = \left(\dfrac{2}{5}\right)^2$

$$\Rightarrow \quad \frac{r}{R} = \frac{2}{5}$$

Now, ratio of volumes $= \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \left(\dfrac{r}{R}\right)^3 = \left(\dfrac{2}{5}\right)^3 = \dfrac{8}{125}$

58. Let original radius be R.

$\therefore$ Original volume, $V = \dfrac{4}{3}\pi R$

New radius $= 2R$

$\therefore$ New volume $= \dfrac{4}{3}\pi (2R)^3 = \dfrac{32}{3}\pi R^3$

$\therefore$ Increase in volume $= \dfrac{32}{3}\pi R^3 - \dfrac{4}{3}\pi R^3 = \dfrac{28}{3}\pi R^3$

Percentage increase in volume $= \left(\dfrac{\frac{28}{3}\pi R^3}{\frac{4}{3}\pi R^3}\right) 100\%$

$$= \frac{28}{4} \times 100 = 700\%$$

59. Here, $\dfrac{4}{3}\pi(6)^3 + \dfrac{4}{3}\pi(8)^3 + \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(12)^3$ (by condition)

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3 - \frac{4}{3}\pi(6)^3 - \frac{4}{3}\pi(8)^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi[12^3 - 6^3 - 8^3]$$

$$r^3 = 1000 \Rightarrow r = 10 \text{ cm}$$

60. Let r be the radius.

But volume of sphere = Surface area (by condition)

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\frac{r}{3} = 1 \Rightarrow r = 3 \text{ units}$$

61. Let r_1 and r_2 be the radius and V_1, V_2 be its volume, respectively.

$$V_1 = \frac{4}{3}\pi r_1^3, V_2 = \frac{4}{3}\pi r_2^3$$

$$V_1 = 2V_2$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{2}{1} \quad \text{(by condition)}$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{2}{1} \therefore \frac{r_1}{r_2} = \frac{\sqrt[3]{2}}{1}$$

62. Let r be the radius of small ball.

$$\therefore \quad 64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3$$

$$r^3 = \frac{8^3}{64} = 8$$

$$r^3 = 2^3 \Rightarrow r = 2 \text{ cm}$$

63. $\dfrac{V_1}{V_2} = \dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{64}{27}$ (given)

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \quad \text{...(i)}$$

Let $r_1 = 4x$ and $r_2 = 3x$

$\therefore$ $4x + 3x = 21$ (given)

$\Rightarrow$ $7x = 21 \Rightarrow x = 3$

$\therefore$ Radius of first sphere $= 4 \times 3 = 12$ cm

Radius of second sphere $= 3 \times 3 = 9$ cm

64. Internal radius $(r) = 12$ cm, External radius $(R) = \dfrac{25}{2}$ cm

$\therefore$ Area to be painted = Internal area + External area + Area of edge

$$= 2\pi r^2 + 2\pi R^2 + \pi(R^2 - r^2)$$

$$= 2 \times \frac{22}{7} \times 12 \times 12 + 2 \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} + \frac{22}{7}\left(\frac{25}{2} \times \frac{25}{2} - 12 \times 12\right)$$

$$= \frac{6336}{7} + \frac{6875}{7} + \frac{539}{14} = \frac{26422}{14} + \frac{539}{14} = \frac{26961}{14} \text{ cm}^2$$

65. Let radius of cone be r and height be h.

$\therefore$ Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Volume of the hemisphere $= \dfrac{2}{3}\pi r^3$

But $\dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi r^3$ (by condition)

$$h = 2r \Rightarrow \frac{h}{r} = \frac{2}{1}$$

66. Let the radius of sphere be r and the edge of the cube $= x$

$\therefore$ By given case surface area are equal.

So, $4\pi r^2 = 6x^2$

$$\therefore \quad \frac{r}{x} = \sqrt{\frac{3}{2\pi}}$$

$$\therefore \quad \frac{\text{Volume of sphere}}{\text{Volume of cone}} = \frac{\frac{4}{3}\pi r^3}{x^3} = \frac{4}{3}\pi\left(\frac{r}{x}\right)^3$$

$$= \frac{4}{3}\pi \frac{3}{2\pi} \cdot \frac{\sqrt{3}}{\sqrt{2\pi}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

67. As base area of the cylinder and cone are equal, so radius is equal let it be 'r'.

and $h = 2$ m but $2\pi rh = \pi rl$ (by condition)

$\Rightarrow$ $2h = l \Rightarrow l = 2 \times 2 = 4$ m

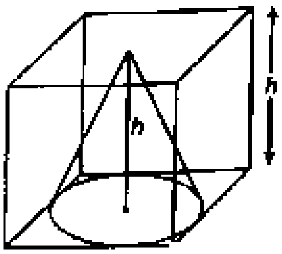
68. Let radius of cylinder $= x$ and radius of cone $= 2x$

Height of each $= h$

$\therefore$ Required ratio $= \dfrac{\text{Volume of cone}}{\text{Volume of cylinder}} = \dfrac{\frac{1}{3}\pi 4x^2 h}{\pi x^2 h} = \dfrac{4}{3}$

69. Radius of base $(r) = \frac{9}{2}$ cm

Height of cone $(h) = 9$ cm



$\therefore$ Volume of cone

$$= \frac{1}{3} \times \pi \times \frac{9}{2} \times \frac{9}{2} \times 9$$

$$= \frac{2673}{14}\ \text{cm}^3$$

70. Number of cones $= \frac{\text{Volume of solid cylinder}}{\text{Volume of 1 cone}}$

$$= \frac{\pi \times 3 \times 3 \times 5}{\frac{1}{3}\pi \times \frac{1}{10} \times \frac{1}{10} \times 1} = 13500$$

71. Volume of sphere = Volume of cylinder

$\therefore$ $\frac{4}{3}\pi R^3 = \pi \times 3 \times 3 \times 32$

$\Rightarrow$ $R^3 = 3 \times 3 \times 3 \times 8 \Rightarrow R = 6$ cm

72. As volumes are equal

$\Rightarrow$ $\pi R^2 H = \frac{4}{3}\pi R^3 \Rightarrow \frac{H}{R} = \frac{4}{3}$

Ratio of the curved surface areas $= \frac{2\pi RH}{4\pi R^2} = \frac{1}{2} \cdot \frac{H}{R}$

$$= \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}, \text{ i.e., } 2:3$$

73. Let the radius of the cone be R, then

$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$ $(\because h = r)$

$\Rightarrow$ $R^2 = 4r^2 \Rightarrow R = 2r$

74. Let radius of cone and cylinder be r and their height be 'h'

Then, $\frac{\text{volume of cylinder}}{\text{volume of cone}} = \frac{\pi r^2 \cdot h}{\frac{1}{3}\pi r^2 h} = \frac{3}{1}$

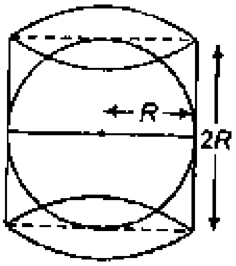
75. Let the height of cones be x and $4x$ and their radii be $4y$ and y.

$\therefore$ The required ratio of volume $= \frac{\frac{1}{3}\pi (4y)^2 \times x}{\frac{1}{3}\pi y^2 \times 4x} = \frac{4}{1}$

76. Let radius of cylinder $= R$

$\therefore$ Radius of sphere $= R$

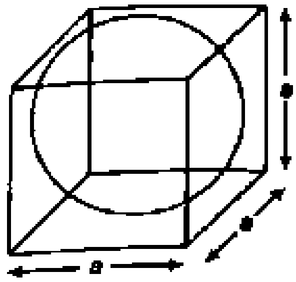
Height of cylinder $= 2R$



Required ratio $= \frac{\frac{4}{3}\pi R^3}{\pi R^2 \times 2R} = \frac{2}{3}$

77. Let edge of the cube be a.

$\Rightarrow$ Diameter of sphere = Edge of cube $= a$



$\therefore$ $\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi \times \left(\frac{a}{2}\right)^3}{a^3} = \frac{\pi}{6}$

78. Surface area of sphere, $S_1 = 4\pi r^2$

If radius is $2r$, then surface area of sphere,

$$S_2 = 4\pi (2r)^2 = 16\pi r^2$$

$\therefore$ $S_2 = 4S_1$

Hence, it increases four times.

79. Let side of a cube be 'a' unit.

Then, radius of sphere is $\frac{a}{2}$ unit.

$\therefore$ $\frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{(a)^3}{\frac{4\pi}{3}\left(\frac{a}{2}\right)^3} = \frac{6}{\pi}$

80. Curved surface of a cylinder $= 1000\ \text{cm}^2$

$$2\pi rh = 1000$$

Length of wire used in around = Perimeter of cylinder's base $= 2\pi r$

and number of rounds $= \frac{\text{Height of cylinder}}{\text{Diameter of wire}} = \frac{h}{0.5}$

$\therefore$ Required length of wire $= 2\pi r \times \frac{h}{0.5} = \frac{2\pi rh}{0.5}$

$$= \frac{1000}{0.5} = 2000 \text{ cm or } 20 \text{ m}$$

81. Total surface area = Curved surface area of cylinder + Curved surface area of cone + Top surface area of cylinder

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi\left[2 \times 3 \times 4 + 3\sqrt{3^2 + 4^2} + 3^2\right] = 48\pi \text{ sq cm}$$

82. The maximum diameter of a sphere in a cube is of 3 m.

$\therefore$ Volume of sphere, $V_1 = \frac{4}{3}\pi (1.5)^3 = 4.5\pi$ cu m

$\therefore$ Volume of cube $V_2 = (3)^3 = 27$ cu m

$\therefore$ Volume of solid left $= V_2 - V_1 = (27 - 4.5\pi)$ cu m

83. Volume of wood = Volume of lead pencil − Volume of lead

$$= \pi (0.4)^2\, 21 - \pi (0.1)^2 \times 21$$

$$= 21 \times \frac{22}{7}(0.16 - 0.01) = 66\,(0.15) = 9.9 \text{ cu cm}$$

84. Let $l = 12.5$ cm

$b = 10$ cm, $h = 7.5$ cm

$\therefore$ Area of a brick $= 2(lb + bh + hl)$

$$= 2(12.5 \times 10 + 10 \times 7.5 + 12.5 \times 7.5) = 587.50 \text{ sq cm}$$

Area to be painted $= 5.875 = 5.875 \times 10^4 = 58750$ sq cm

$\therefore$ Number of bricks $= \dfrac{58750}{587.50} = 100$

85. Volume of first sphere $= \dfrac{4}{3}\pi\left(\dfrac{3}{2}\right)^3$

and volume of second sphere $= \dfrac{4}{3}\pi(2)^3$

$\therefore$ Volume of big sphere $= \dfrac{4}{3}\pi\left\{\left(\dfrac{3}{2}\right)^3 + (2)^3\right\}$ (by condition)

$$\Rightarrow \quad \frac{4}{3}\pi x^3 = \frac{4}{3}\pi\left\{\frac{27}{8} + 8\right\}$$

$$x^3 = 91/8 = 11.375 \text{ cu cm}$$

86. 'A' is true that the surface area of all the eight drops is greater than the surface area of big drop.

'R' Let radius of spherical body be 'a'.

$\therefore$ Volume of sphere, $V = \dfrac{4}{3}\pi a^3$

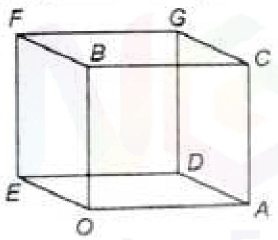
$\therefore$ Surface area of sphere, $S = 4\pi a^2$

$$\therefore \quad V^2 = \frac{16}{9}\pi^2 a^6 \propto S^3$$

Hence, both A and R are true but R is not a correct explanation of A.

87. The distance between vertices B and C is 1 cm.

The distance between A and B is $\sqrt{1^2 + 1^2} = \sqrt{2}$ cm



The distanc between diagonal B and D is

$$\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \text{ cm}$$

88. Volume of spherical lead shot $= \dfrac{4}{3}\pi(1)^3 = \dfrac{4}{3}\pi$ cu cm

I. Volume of 8 shots $= \dfrac{4}{3}\pi(0.5)^3 \times 8 = \dfrac{4}{3}\pi$ cu cm

II. Volume of both shots $= \dfrac{4}{3}\pi(0.75)^3 + \dfrac{4}{3}\pi(0.8)^3$

$$= \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + \left(\frac{4}{5}\right)^3\right] = \frac{4}{3}\pi\left[\frac{27}{64} + \frac{64}{125}\right]$$

$$= \frac{4}{3}\pi\left[\frac{3375 + 4096}{8000}\right] = \frac{4}{3}\pi\left(\frac{7471}{8000}\right) = \frac{4}{3}\pi(0.93) \text{ cu cm}$$

Hence, only statement I is true.

89. Both statements are true but (R) is not a correct explanation of (A).

90. 'A' is a true statement.

'R' is a false statement.

Because the length may be πr but the breadth is always less than $\sqrt{h^2 + r^2}$ of formed rectangle.

91. Given, $l = R$

$\therefore$ Total surface area of cone, $S = \pi R(R + l) = \pi R(R + R) = 2\pi R^2$

$\therefore$ Area of circle, $A = \pi(2R)^2 = 4\pi R^2$

$\therefore \quad A = 2S$

92. Let h be the depth of the pit.

$\therefore$ Volume of Earth dug $= 500 \times 250$ cm^3 $= 125000$ cu cm

But volume of pit $= 50 \times 50 \times h$

$$\therefore \quad h = \frac{125000}{50 \times 50} = 50 \text{ cm} = 0.5 \text{ m}$$

93. $\because$ Sheet is revolved about its length

$h = 7$ cm and $r = 4$ cm

$\therefore$ Volume of the figure, thus formed

$$= \pi r^2 h = \frac{22}{7} \times 4 \times 4 \times 7 = 352 \text{ cu cm}$$

94. The radius of roller is 0.35 m.

The area covered in one revolution = Curved surface area of roller

$$= 2 \times \frac{22}{7} \times 0.35 \times 2 = 4.4 \text{ sq m}$$

$\therefore$ Total area covered in 50 revolutions $= 4.4 \times 50 = 220$ sq m

95. In one round, distance covered by cylinder = 60 cm

In one second, distance covered by cylinder $= 60 \times 5 = 300$ cm

In five seconds, distance covered by cylinder $= 300 \times 5$

$= 1500$ cm $= 15$ m

96. Given, $l = 6$ m, $b = 4$ m and $h = 1.25$ m

Area of wetted surface $= 2(l \times h + b \times h) + 6 \times 4$

$= 2(7.5 + 5) + 24 = 25 + 24 = 49$ sq m

97. Given, $R = 3$ cm, $r = 2$ cm, $h = 10$ cm

Total surface area $= 2\pi(R + r)(R + h - r)$

$= 2\pi(3 + 2)(3 + 10 - 2) = 110\pi$ sq cm

98. Let the radius of cone and sphere be r.

By given conditions,

Volume of cone = Volume of sphere

$$\therefore \quad \frac{1}{3}\pi r^2 h_1 = \frac{4}{3}\pi(r)^3 \Rightarrow \frac{h_1}{2r} = \frac{2}{1}$$

99. Let x be the diameter of a moon.

Then, $\dfrac{\text{Volume of Moon}}{\text{Volume of Earth}} = \dfrac{\frac{4}{3}\pi\left(\frac{x}{8}\right)^3}{\frac{4}{3}\pi\left(\frac{x}{2}\right)^3} = \dfrac{1}{64}$

100. By given condition,

$27 \times$ Volume of smaller drops = Volume of bigger drop

$$\therefore \quad 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\therefore \quad 27 \times (0.2)^3 = R^3$$

$$\Rightarrow \quad (3 \times 0.2)^3 = R^3 \Rightarrow R = 0.6 \text{ cm}$$

Level II

1. Let the dimensions be $2x$ m, $3x$ m and $4x$ m.
   Total surface area of the box
   $= 2(2x \times 3x + 3x \times 4x + 4x \times 2x) = 52x^2 \text{ m}^2$
   Cost of covering with sheet of paper at the rate of ₹ 4 per $m^2 = 52x^2 \times 4 =$ ₹ $208x^2$
   cost of covering with sheet of paper at the rate of ₹ 4.50 per $m^2 = 52x^2 \times 4.50 =$ ₹ $234x^2$
   Difference between rates $= 234x^2 - 208x^2 =$ ₹ $26x^2$
   $\Rightarrow \quad 26x^2 = 416$ (given)
   $\Rightarrow \quad x^2 = 16 \Rightarrow x = 4$
   Hence, length $= 2 \times x = 2 \times 4 = 8$ m

2. External dimensions of box are, length = 10 cm, breadth = 8 cm and height = 7 cm.
   $\therefore$ External volume of the box $= 10 \times 8 \times 7 = 560 \text{ cm}^3$
   Internal dimensions as thickness of wood = 1 cm
   Internal length = 10 − 2 = 8 cm
   Internal breadth = 8 − 2 = 6 cm
   Internal height = 7 − 2 = 5 cm
   $\therefore$ Internal volume $= 8 \times 6 \times 5 = 240 \text{ cm}^3$
   Volume of wood = External volume − Internal volume
   $= 560 - 240 = 320 \text{ cm}^3$
   $\therefore$ Total cost of wood required to make the box
   $= 320 \times 2 =$ ₹ 640

3. Length of water flowing in 1 h = 10 km
   $\therefore$ Length of the water flowing in $\frac{1}{2}$ h = 5 km = 5000 m
   Area of pipe of opening $= 40 \text{ cm}^2 = \frac{40}{10000} \text{ m}^2 = \frac{1}{250} \text{ m}^2$
   $\therefore$ Volume of water flowing in $\frac{1}{2}$ h $= 5000 \times \frac{1}{250} = 20 \text{ m}^3$
   Area of bottom of tank $= 80 \times 40 = 3200 \text{ m}^2$
   $\therefore$ Depth of water $= \frac{\text{Volume}}{\text{Area}} = \frac{20}{3200} = \frac{1}{160}$ m
   $= \frac{1}{160} \times 100 \text{ cm} = \frac{5}{8}$ cm

4. Floor area for each student $= 9 \text{ m}^2$
   $\therefore$ Floor area for 50 students $= 9 \times 50 = 450 \text{ m}^2$
   Let length of room be l m, then
   $l \times 18 = 450 \text{ m}^2 \Rightarrow l = \frac{450}{18} = 25$ m
   Space required for each student $= 108 \text{ m}^3$
   Space required for 50 students $= 50 \times 108 = 5400 \text{ m}^3$
   $\therefore$ Volume of classroom $= 5400 \text{ m}^3$
   $\Rightarrow \quad l \times b \times h = 5400$
   $\Rightarrow \quad 25 \times 18 \times h = 5400$
   $h = \frac{5400}{25 \times 18} = 12$ m
   $\therefore$ Length = 25 m and height = 12 m

5. Common radius $= \frac{126}{2} = 63$ m
   Curved surface area of cylindrical part $= 2\pi rh$
   $= 2 \times \frac{22}{7} \times 63 \times 5 = 1980 \text{ m}^2$
   Curved surface area of conical part $= \pi rl$
   $= \frac{22}{7} \times 63 \times \sqrt{(63)^2 + (16)^2} = \frac{22}{7} \times 63 \times \sqrt{3969 + 256}$
   $= \frac{22}{7} \times 63 \times 65 = 12870 \text{ m}^2$
   $\therefore$ Total surface area $= 1980 + 12870 = 14850 \text{ m}^2$
   $\therefore$ Total cost of canvas used $= 14850 \times 12 =$ ₹ 178200

6. Radius of cylinder = 6 cm
   Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 6 \times 6 \times 6$
   $= \frac{88 \times 2 \times 36}{7} \text{ cm}^3$
   Let r be its internal radius, R be its external radius, then material used to cast the cylinder $= \pi h(R^2 - r^2) = \frac{22}{7} \times 32(25 - r^2)$
   Hence, $\frac{22}{7} \times 32 \times (25 - r^2) = \frac{88 \times 2 \times 36}{7}$
   $\Rightarrow \quad (25 - r^2) = \frac{88 \times 2 \times 36 \times 7}{22 \times 32 \times 7} = 9$
   $r^2 = 25 - 9 = 16 \Rightarrow r = 4$ cm
   Thickness of cylinder $= R - r = 5 - 4 = 1$ cm

7. Radius of cistern $= \frac{10}{2}$ m = 5 m
   Depth of cistern = 2 m
   Volume of the cistern $= \frac{22}{7} \times 5^2 \times 2 = \frac{1100}{7} \text{ m}^3$
   Internal radius of pipe $= \frac{20}{2}$ cm = 10 cm $= \frac{1}{10}$ m
   Water is flowing through a circular pipe in the form of a right circular cylinder, length of this cylinder (in 1 h)
   = 3 km = 3000 m
   Volume of water poured in 1 h
   $= \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times 3000 = \frac{660}{7} \text{ m}^2$
   Time taken to fill the cistern $= \frac{1100}{7} \div \frac{660}{7} = \frac{1100}{660} = \frac{5}{3}$ h
   = 1 h and 40 min

8. Let the radius of cylinder be r, the radius of sphere $= r$
   $\therefore \quad$ Volume of cylinder $= \pi r^2 h$
   and $\quad$ Volume of sphere $= \frac{4}{3}\pi r^3$
   But $\quad \pi r^2 h = \frac{4}{3}\pi r^3 \Rightarrow h = \frac{4\pi r^3}{3\pi r^2} = \frac{4}{3} r$
   $\therefore \quad h = \frac{2}{3}(2r) = \frac{2}{3} \times$ diameter of sphere
   $\therefore$ Height of cylinder $= \frac{2}{3} \times$ diameter of sphere

9. Length of tank = 225 m, Breadth = 162 m

The volume of water accumulated in tank in 5 h

$$= 225 \times 162 \times \frac{20}{100} \text{ m}^3$$

Volume of water which flows in 1 h through the aperture

$$= \frac{1}{5} \times 225 \times 162 \times \frac{20}{100} = 1458 \text{ m}^3$$

The area of the cross-section of aperture $= \frac{60}{100} \times \frac{45}{100} = \frac{27}{100} \text{ m}^3$

∴ Velocity of flow of water per hour

$$= \frac{\text{The volume of water which flows in 1 h}}{\text{The area of the cross-section}}$$

$$= \frac{1458}{27/100} \text{ m} = \frac{1458 \times 100}{27} \text{ m} = 5400 \text{ m/h}$$

∴ The velocity per second $= \frac{5400}{60 \times 60} = 1.5 \text{ m/s}$

10. Radius of water tank $= \frac{1.4 \times 100}{2} \text{ cm} = 70 \text{ cm}$

Height of tank = 2.1 m = 210 cm

∴ Volume of water tank $= \pi r^2 h$

$$= \frac{22}{7} \times (70)^2 \times 210 \text{ cm}^3 = 22 \times 70 \times 210 \text{ cm}^3$$

Radius of pipe $= \frac{3.5}{2} \text{ cm} = \frac{7}{4} \text{ cm}$

Length of water which flows from the pipe in 1 s = 2 m = 200 cm

∴ Volume of water delivered in 1 s

$$= \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 200 = 77 \times 25 \text{ cm}^3$$

∴ Time to fill the tank by the pipe

$$= \frac{22 \times 700 \times 210}{77 \times 25} = 1680 = \frac{1680}{60} = 28 \text{ min}$$

11. Let inner and outer radii be r_1 and r_2 cm, respectively.

Length of tube = 14 cm

Inner lateral surface area $= 2\pi r_1 \times 14 \text{ cm}^2$

Outer lateral surface area $= 2\pi r_2 \times 14 \text{ cm}^2$

Difference of surfaces $= 2\pi (r_2 - r_1) \times 14$ sq cm

But difference of surface areas is 88 cm².

⇒ $88 = 2 \times \frac{22}{7} \times 14 (r_2 - r)$

⇒ $(r_2 - r_1) = 1$ ...(i)

Volume of tube = Outer volume − Inner volume

$$= \pi r_2^2 \times 14 - \pi r_1^2 \times 14 = \frac{22}{7} \times 14 (r_2^2 - r_1^2)$$

But volume of tube = 176 cm³

⇒ $176 = \frac{22 \times 14}{7} (r_2^2 - r_1^2)$

$r_2^2 - r_1^2 = 4 \Rightarrow (r_2 - r_1)(r_2 + r_1) = 4$

$r_2 + r_1 = 4$ [from Eq. (i)]...(ii)

Adding Eqs. (i) and (ii), we get

⇒ $2r_2 = 5 \Rightarrow r_2 = 2.5$ cm

$r_1 = 1.5$ cm

∴ Inner radius = 1.5 cm and outer radius = 2.5 cm

12. As, here radius of cone = Radius of hemisphere = R

Then, $2\pi R^2 = \pi R \sqrt{R^2 + H^2}$. (by condition)

Here, H = height of cone

∴ $2R = \sqrt{R^2 + H^2}$

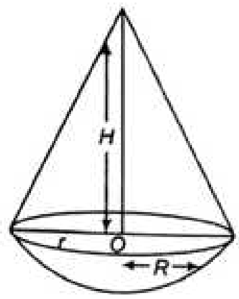
$4R^2 = R^2 + H^2$

$3R^2 = H^2$

$\frac{R^2}{H^2} = \frac{1}{3}$

∴ $\frac{R}{H} = \frac{1}{\sqrt{3}}$

$R : H = 1 : \sqrt{3}$



13. ∴ Lateral surface area of cylinder = Area of the square

∴ $2\pi rh = \frac{1}{2} \times (\sqrt{5})^2$ (by condition)

$2\pi rh = \frac{5}{2}$ (but $2\pi r = h$)

∴ $2\pi r \times 2\pi r = \frac{5}{2} \Rightarrow \pi r^2 = \frac{5}{8\pi}$

> Cylinderical surface is conustible into a square when $2\pi r = h$ or Circumference of base = height

14. Let r_1 and h_1 and r_2 and h_2 be the old and new radii and heights.

So, $\pi r_1^2 h_1 = \pi r_2^2 h_2$ [∵ volume does not changed]

∴ $(0.6)^2 \times 400 = r_2^2 \times 720$

$r_2^2 = \frac{0.36 \times 400}{720} = 0.2 \Rightarrow r_2 = \sqrt{0.2}$ cm

15. Volume of wire = 2.2 dm³ = 2.2 × 10 × 10 × 10 cm³ = 2200 cm³

Let length of wire be h

then $\frac{22}{7} \times \frac{1}{8} \times \frac{1}{8} \times h = 2200$; $\frac{h \times 22}{7 \times 64} = 2200$

$h = \frac{2200 \times 7 \times 64}{22} = 44800$ cm

Length of wire = 448 m

16. Volume of original cylinder $= [\pi \times (10)^2 \times 1000] \text{ cm}^3$

Volume of shell $= \frac{1}{4} \times \pi \times 100000 = 25000\pi \text{ cm}^3$

$\pi (R^2 - 10^2) \times 1000 = 25000\pi$ (by condition)

⇒ $(R^2 - 10^2) = 25$

⇒ $R^2 = 100 + 25 = 125 \Rightarrow R = \sqrt{125}$

∴ Thickness $= R - 10 = \sqrt{125} - 10 = 5\sqrt{5} - 5 \times 2$

$= 5(\sqrt{5} - 2)$ cm

17. Let R_1 is inner radius and R_2 is outer radius of cylinder, then

$2\pi R_2 h - 2\pi R_1 h = 44$ (given)

$2 \times \frac{22}{7} \times R_2 \times 14 - 2 \times \frac{22}{7} \times R_1 \times 14 = 44$

⇒ $2 \times \frac{22}{7} \times 14 (R_2 - R_1) = 44$

$$\Rightarrow \quad R_2 - R_1 = \frac{1}{2} \quad \text{...(i)}$$

Volume of pipe $= 99\,cm^3$ (given)

$$\pi R_2^2 h - \pi R_1^2 h = 99$$

$$\Rightarrow \quad R_2^2 - R_1^2 = 99 \times \frac{7}{22} \times \frac{1}{14} \Rightarrow (R_2 + R_1)(R_2 - R_1) = \frac{9}{4}$$

$$(R_2 + R_1)\frac{1}{2} = \frac{9}{4} \quad \text{[from Eq. (i)]}$$

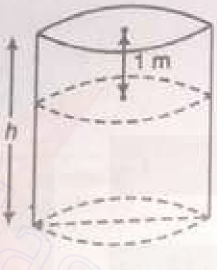
$$R_2 + R_1 = \frac{9}{2} \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get $R_2 = \frac{5}{2} = 2.5$ cm

$$\therefore \quad \frac{5}{2} + R_1 = \frac{9}{2} \Rightarrow R_1 = 2$$

$$\therefore \quad R_2 = 2.5 \text{ cm}, R_1 = 2 \text{ cm}$$

18. Volume of liquid in it $= \frac{2}{3}\pi r^3 + \pi r^2 h$



$$\frac{2}{3}\pi r^3 + \pi r^2 h = 432\pi \quad \text{(given)}$$

$$\frac{2r^3}{3} + r^2 h = 432 \quad \text{...(i)}$$

When liquid is filled into a level which is 1 cm bellow the top of vessel.

So, $$\frac{2}{3}\pi r^3 + \pi r^2 (h-1) = 396\pi \quad \text{(given)}$$

$$\frac{2}{3}r^3 + r^2(h-1) = 396 \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$r^2 = 36 \Rightarrow r = 6 \text{ cm}$$

19. Curved surface area of cylinder $= 2 \times \frac{22}{7} \times 14 \times 3 = 264\,m^2$

Height of cone $= 13.5 - 3 = 10.5$ m

Slant height of cone $= \sqrt{(10.5)^2 + (14)^2}$

$$= \sqrt{110.25 + 196} = \sqrt{306.65} = 17.5 \text{ m}$$

Curved surface area of cone $= \pi r l$

$$= \frac{22}{7} \times 14 \times 17.5 = 770\,m^2$$

Total area to be painted $= 264 + 770 = 1034\,m^2$

$\therefore$ Cost of painting $= 1034 \times 2 =$ ₹ 2068

20. Internal volume of shell $= \frac{4}{3}\pi(3)^3\,cm^3$

External volume of shell $= \frac{4}{3}\pi(5)^3\,cm^3$

$\therefore$ Volume of metal $= \frac{4}{3}\pi 5^3 - \frac{4}{3}\pi 3^3 = \frac{4}{3}\pi(125 - 27) = \frac{4}{3}\pi \times 98$

Length of cylinder $= \frac{8}{3}$ cm

$\therefore$ Volume of cylinder formed $= \frac{4}{3}\pi \times 98$

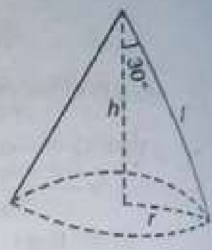
$$\pi r^2 h = \frac{4}{3}\pi \times 98 \quad \text{(by condition)}$$

$$r^2 = \frac{4}{3} \times 98 \times \frac{3}{8}$$

$$r^2 = 49 \Rightarrow r = 7 \text{ cm}$$

Here, diameter of cylinder $= 2 \times 7 = 14$ cm

21. Here, $\frac{r}{l} = \sin 30° = \frac{1}{2}$



$$\Rightarrow \quad l = 2r$$

Curved surface area $= \pi r l$

$$= \pi r \times 2r = 100$$

$$\Rightarrow \quad \pi r^2 = 50 \text{ or } r = \frac{5\sqrt{2}}{\sqrt{\pi}}$$

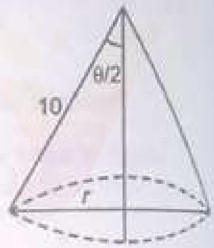
Also, $$\frac{h}{r} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}.r = \frac{\sqrt{3} \times 5\sqrt{2}}{\sqrt{\pi}} = \frac{5\sqrt{6}}{\sqrt{\pi}}$$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\frac{50 \times 5\sqrt{6}}{\sqrt{\pi}}$

$$= \frac{500}{\sqrt{6\pi}}\,m^3$$

22. Area of sheet $= \pi \times 10^2 = 100\pi\,cm^2$



Remaining area = 60% of 100π

$$= \frac{60}{100} \times 100\pi = 60\pi$$

Here, $l = 10$ cm

Lateral surface area $= 60\pi$

$$\therefore \quad \pi r \times 10 = 60\pi \Rightarrow r = 6$$

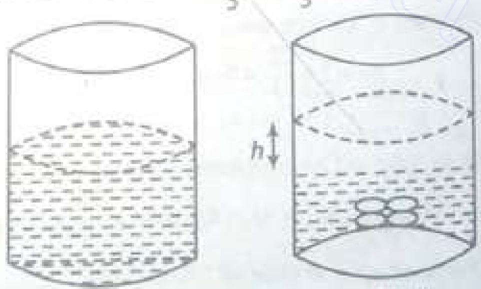
$$\frac{r}{l} = \sin\frac{\theta}{2} \Rightarrow \sin\frac{\theta}{2} = \frac{6}{10} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{3}{5}\right) \Rightarrow \theta = 2\sin^{-1}\left(\frac{3}{5}\right)$$

23. Radius of balls = 1 cm

Volume of each ball $= \frac{4}{3} \times \pi \times 1 \times 1 = \frac{4}{3}\pi\,cm^3$

$\therefore$ Volume of 4 balls $= 4 \times \frac{4}{3}\pi = \frac{16}{3}\pi\,cm^3$



Volume of water increased = Volume of balls

Area of base $\times$ height $= \frac{16}{3}\pi$

$$\pi \times 5 \times 5 \times h = \frac{16}{3}\pi$$

$$h = \frac{16}{3 \times 25} = \frac{16}{75} \text{ cm}$$

24. Radius of the sphere = Radius of cylinder $= R$

Height of the cylinder $= 2R$

Volume of water + Volume of sphere

$$= \left[\pi R^2 \times \left(\frac{1}{3} \text{ of } 2R\right) + \frac{4}{3}\pi R^3\right] = 2\pi R^3$$

$\therefore$ Volume of cylinder $= \pi R^2 \times 2R = 2\pi R^3$

$\therefore$ Volume of cylinder = Volume of water + Volume of sphere

So, water stands at the top of the vessel.

25. Let R be the radius and H be the height of the cone.

Its volume $= \frac{1}{3}\pi R^2 H = V$ (say)

Radius of upper part $= \frac{R}{2}$ and height $= \frac{H}{2}$

Volume of upper part $= \frac{1}{3}\pi \times \left(\frac{R}{2}\right)^2 \times \frac{H}{2} = \frac{1}{24}\pi R^2 H = \frac{V}{8}$

Required ratio $= \dfrac{\frac{1}{8}V}{V} = \frac{1}{8}$

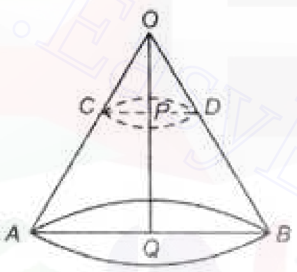
26. Volume of original cone $= \frac{1}{3}\pi R^2 H$

$= \frac{1}{3} \times \pi R^2 \times 30 = 10\pi R^2$ cm^3

Volume of small cone OCD

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{27}$ of volume cone AOB



$\frac{1}{3}\pi r^2 h = \frac{1}{27} \times 10\pi R^2$

$\Rightarrow \quad h = \frac{10\pi R^2}{27} \times \frac{3}{\pi r^2} \Rightarrow h = \frac{10}{9}\left(\frac{R}{r}\right)^2$

$\because \quad \frac{R}{r} = \frac{30}{h}$ $\quad$ $[\because \Delta OQB \sim \Delta OPD]$

$\therefore \quad h = \frac{10}{9}\left(\frac{30}{h}\right)^2 = \frac{10}{9} \times \frac{900}{h^2}$

$\therefore \quad h^3 = 10 \times 100 \Rightarrow h = \sqrt[3]{1000} = 10$ cm

27. Capacity of a frustum $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$$= \frac{22}{7} \times \frac{1}{3} \times 6(16 + 4 + 8) = 176 \text{ m}^3$$

28. Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5^2 \times 8 = \frac{200}{3}\pi$ cm^3

$\frac{1}{4}$th volume of the cone $= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi$ cm^3

Volume of one lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 = \frac{\pi}{6}$ cm^3

Let n leads be dropped.

$\therefore$ Volume of n leads shots $= \frac{1}{4}$th volume of the cone

$\therefore \quad n \times \frac{\pi}{6} = \frac{50}{3}\pi \Rightarrow n = \frac{50 \times 6}{3} = 100$

29. For conical part, $r = \frac{6}{2} = 3$ cm, $h = 4$ cm, $l = \sqrt{h^2 + r^2} = 5$ cm

Surface area of conical part $= \pi r l = 3.14 \times 3 \times 5 = 47.1$ cm^2

For hemispherical part, $r = \frac{6}{2} = 3$ cm

Surface area of hemispherical part $= 2\pi r^2$

$= 2 \times 3.14 \times 3 \times 3 = 56.52$ sq cm

$\therefore$ Surface area of toy $= 47.1 + 56.52 = 103.62$ sq cm

30. Let l, b and h be the sides of cuboid,

$l^2 + b^2 = x^2$ ...(i)

$b^2 + h^2 = y^2$ ...(ii)

and $h^2 + l^2 = z^2$ ...(iii)

$2(l^2 + b^2 + h^2) = x^2 + y^2 + z^2$

[from Eqs. (i) (ii) and (iii)]

$\Rightarrow \quad l^2 + b^2 + h^2 = \frac{1}{2}(x^2 + y^2 + z^2)$ ...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}$, $l = \sqrt{\frac{z^2 + x^2 - y^2}{2}}$ and $b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$

Hence, volume of cuboid $= lbh$

$= \sqrt{\frac{(y^2 + z^2 - x^2)(z^2 + x^2 - y^2)(x^2 + y^2 - z^2)}{2 \times 2 \times 2}}$

$= \frac{1}{2\sqrt{2}}\sqrt{(y^2 + z^2 - x^2)(z^2 + x^2 - y^2)(x^2 + y^2 - z^2)}$

31. Let the radius of ball $= r$

$\therefore$ Radius of base of cylinder $= 4r$

and height of cylinder $= 4r$

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi r^3$

and volume of water $= \pi(4r)^2(2r) = 32\pi r^3$

Also, volume of remaining portion of cylinder $= 32\pi r^3$

Let number of spherical balls $= n$

$\therefore \quad 32\pi r^3 = n \times \frac{4}{3}\pi r^3 \Rightarrow n = 8 \times 3 = 24$

32. Let the length, breadth and height of a rectangular parallelopiped be $6x, 3x$ and x.

Also, let the side of a cube be a.

By given condition,

Surface area of a cube = Surface area of rectangular parallelopiped

$\Rightarrow \quad 6(a)^2 = 2(6x \times 3x + 3x \times x + x \times 6x)$

$\Rightarrow \quad 6a^2 = 2(18x^2 + 3x^2 + 6x^2)$

$\Rightarrow \quad 6a^2 = 54x^2 \quad \therefore \quad a = 3x$

Now, $\dfrac{\text{Volume of cube}}{\text{Volume of rectangular parallelopiped}}$

$$= \frac{a^3}{6x \times 3x \times x} = \frac{(3x)^3}{18x^3} = \frac{3}{2}$$

33. Let $r = 14$ cm

For conical cup, $l = 14$ cm

By given condition,

Circumference of base cone = Circumference of semi-circle

$\Rightarrow \quad 2\pi R = \pi r \Rightarrow 2R = r \Rightarrow 2R = 14 \Rightarrow R = 7$ cm

$\because \quad l^2 = R^2 + h^2 \Rightarrow (14)^2 = (7)^2 + h^2$

$\Rightarrow \quad h^2 = 196 - 49 = 147 \Rightarrow h = 7\sqrt{3}$

$\therefore$ Capacity of cup $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\sqrt{3} = \frac{1078}{3}\sqrt{3} \text{ cu cm}$$

34. Let diameter of cylindrical log $= d$

Then, height of cylindrical log $= d$

Diameter of greatest possible sphere $= d$

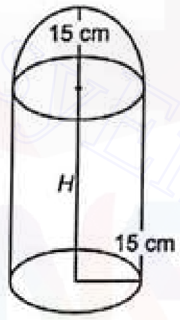
$$\text{Radius of sphere } = \frac{d}{2}$$

$$\text{Volume of cylindrical log} = \pi r^2 h = \pi\left(\frac{d}{2}\right)^2 d = \frac{\pi d^3}{4}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6} = \frac{\pi d^3}{4} \times \frac{4}{6}$$

$$= \frac{2}{3} \text{ volume of cylinder}$$

35. Let height of the cylinder be H.



By given condition,

Volume of hemisphere + Volume of cylinder = Volume of container

$\therefore \quad \frac{2}{3}\pi r^3 + \pi r^2 H = 32400\pi$

$\Rightarrow \quad \frac{2}{3}\pi \times 3375 + \pi \times 225H = 32400\pi$

$\Rightarrow \quad 2\pi \times 1125 + \pi \times 225H = 32400\pi$

$\Rightarrow \quad 10 + H = 144 \Rightarrow H = 134$

$\therefore \quad$ Height of container $= 15 + 134 = 149$

Hence, option (a) is correct.

36. Since, milk is in a conical flask whose base radius and height are r and h, respectively.

$\therefore \quad$ Volume of milk $= \frac{1}{3}\pi r^2 h$

Now, this milk is poured into a cylindrical flask whose base radius is $2r$. Let H be the height of cylindrical flask.

$\therefore$ Volume of milk = Volume of cylindrical flask

$$= \pi(2r)^2 H = 4\pi r^2 H \Rightarrow \frac{1}{3}\pi r^2 h = 4\pi r^2 H \Rightarrow H = \frac{h}{12}$$

37. Let r be the radius of cylindrical block, then height will be $2r$.

Volume of block $= \pi(r^2)(2r) = 2\pi r^3$

A sphere of maximum possible volume is curved out whose radius will be r.

$\therefore \quad$ Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore \quad$ Volume of utilised wood $= \frac{4}{3}\pi r^3$

and Volume of wasted wood

$$= 2\pi r^3 - \frac{4}{3}\pi r^3 = \frac{6\pi r^3 - 4\pi r^3}{3} = \frac{2\pi r^3}{3}$$

$\therefore$ Required ratio $= \frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 2:1$

38. By given condition,

Volume of cylinder = Volume of bar

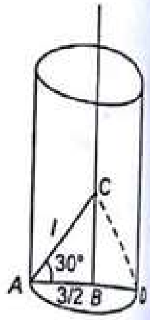
$\pi r^2 h$ = Base area of block × length

$$\therefore \quad \frac{22}{7} \times \left(\frac{1.5}{2}\right)^2 \times 3.5 = \frac{5}{100} \times \frac{5}{100} \times L$$

$$\frac{22}{7} \times \frac{2.25}{4} \times 3.5 \times \frac{100}{5} \times \frac{100}{5} = L$$

$\therefore \quad L = 2475$ m

39. In ΔABC,



$$\cos 30° = \frac{3/2}{l}$$

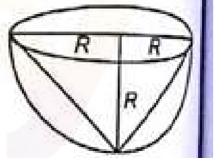
$$\Rightarrow \quad l = \frac{3/2}{\sqrt{3}/2} = \sqrt{3} \text{ cm}$$

$\therefore$ Area of cone $ACD = \pi r l$

$$= \pi \times \frac{3}{2} \times \sqrt{3}$$

$$= \frac{3\sqrt{3}\,\pi}{2} \text{ sq cm}$$

40. Volume of cone, $C = \frac{1}{3}\pi R^2 H$



$$= \frac{1}{3}\pi R^3 \quad (\because H = R)$$

Volume of hemisphere, $H = \frac{2}{3}\pi R^3$

$$\therefore \quad C : H = \frac{1}{3}\pi R^3 : \frac{2}{3}\pi R^3 = 1:2$$

41. $\therefore$ Volume of wire $= \pi r^2 h$

Now, new radius of the wire $= \frac{r \times 90}{100} = \frac{9r}{10}$

Let new length of the wire be L.

$\therefore$ Volume of new wire $= \pi\left(\frac{9r}{10}\right)^2 \times L = \frac{81}{100}\pi r^2 L$

By given condition,

$$\pi r^2 h = \frac{81}{100}\pi r^2 L$$

$$\Rightarrow \quad L = \frac{100}{81}h$$

Increases in length $= \frac{100}{81}h - h = \frac{19}{81}h$

Percentage increase $= \frac{19/81h}{h} \times 100\% = 23.45\%$

$= 23\%$ (approx)

42. Let the height of cylinder $= h$

Then, radius of cylinder $= \frac{h}{2}$

Also, radius of the sphere, $r = \frac{h}{2}$

$\therefore$ Volume of cylinder $= \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{4}$

and volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 = \frac{\pi h^3}{6}$

$\therefore$ Volume of remaining material $= \frac{\pi h^3}{4} - \frac{\pi h^3}{6} = \frac{\pi h^3}{12}$

$\therefore$ Volume of sphere in remaining material, $S_1 = \frac{4}{3}\pi R^3$

By given condition, $\frac{4}{3}\pi R^3 = \frac{\pi h^3}{12} \Rightarrow R^3 = \frac{h^3}{16} \Rightarrow R = \frac{h}{2\cdot 2^{1/3}}$

$\therefore$ Required ratio $= \frac{r}{R} = \frac{h/2}{h/2\cdot 2^{1/3}} = 2^{1/3}$

$\Rightarrow$ $r : R = 2^{1/3} : 1$

43. Let side of square $= x$

$\therefore$ Radius of sphere $= x$

Surface area of sphere, $A = 4\pi x^2$

Since, square revolves round a side to generate a cylinder whose height and radius are x and x, respectively.

$\therefore$ $S = 2\pi x(x + x) = 4\pi x^2$

$\therefore$ It is clear that $A = S$

44. Length of pool $= 24$ m

Breadth of pool $= 15$m

Rise in height of water $= 1$ m $= 0.01$m

Volume of water displaced by x men $= 24 \times 15 \times 0.01$
$= 3.6$ cu m

But volume of water displaced by x men $= 0.1 \times m^3$

$\therefore$ $0.1x = 3.6 \Rightarrow x = \frac{3.6}{0.1} = 36$

45. Length of one compartment $= 200$ m

Breadth of one compartment $= 100$ m

and height of water in one compartment $= 12$ m

Volume of one compartment $= 200 \times 100 \times 12 = 240000$ cu m

$\therefore$ Volume of 3 compartment $= 3 \times 240000$
$= 720000$ cu m $= 720000000$ L

Total requirement of water in 50000 inhabitants
$= 50000 \times 20 = 10000000$ L

$\therefore$ Required number of days $= \frac{720000000}{1000000} = 720$ days

46. Let r and h be the radius and height of the cone and r and H be the radius and height of the cylinder,

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

and volume of cylinder $= \pi r^2 h$

By given condition, $\frac{1}{3}\pi r^2 h = \pi r^2 H \Rightarrow h = 3H$ ...(i)

Lateral surface area of cone $= \pi r l$

and lateral surface area of cylinder $= 2\pi r H$

By given condition $\pi r l = \frac{15}{8} \times 2\pi r H$

$\Rightarrow$ $l = \frac{15}{4}H \Rightarrow l^2 = \frac{225}{16}H^2 \Rightarrow r^2 + h^2 = \frac{225}{16}H^2$

$\Rightarrow$ $r^2 + 9H^2 = \frac{225}{16}H^2$ [from Eq. (i)]

$\Rightarrow$ $r^2 = \frac{81}{16}H^2 \Rightarrow \frac{r^2}{H^2} = \frac{81}{16} \Rightarrow \frac{r}{H} = \frac{9}{4} \Rightarrow r : H = 9 : 4$

47. Let the sides of the cuboid are x, $2x$ and $4x$ and the side of th cube is y.

$\therefore$ Volume of cuboid $= x \times 2x \times 4x = 8x^3$

and volume of cube $= y^3$

By given condition,

Volume of cuboid = Volume of cube

$\therefore$ $8x^3 = y^3 \Rightarrow \left(\frac{x}{y}\right)^3 = \left(\frac{1}{2}\right)^3 \Rightarrow \frac{x}{y} = \frac{1}{2}$

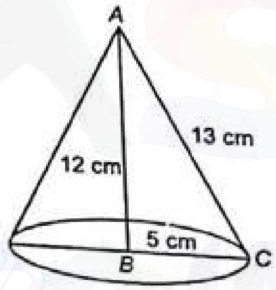
$\Rightarrow$ $y = 2x$ ...(

$\therefore$ Diagonal of cuboid $= \sqrt{x^2 + 4x^2 + 16x^2} = \sqrt{21}x$

and diagonal of cube $= y\sqrt{3} = 2x\sqrt{3}$ [from Eq. (i

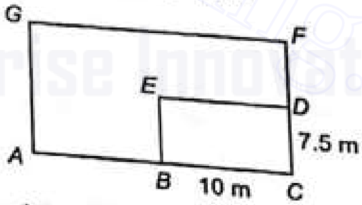
Hence, required ratio $= \frac{\sqrt{21}x}{2x\sqrt{3}} = \sqrt{\frac{21}{4\times 3}} = \sqrt{1.75}$

48. $\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$



$= \frac{1}{3}\pi \times 5^2 \times 12 = 100\pi$ cu cm

49. Area of tank, $BCDE = 10 \times 7.5 = 75\,m^2$



Area of remaining field $ABEDFGA = 125 \times 15 - 75$
$= 1800$ sq m

Volume of Earth dug $= 10 \times 7.5 \times 6 = 450$ cu m

By given condition, $1800 \times h = 450$

$\Rightarrow$ $h = \frac{1}{4}$m $= \frac{1}{4} \times 100$ cm $= 25$ cm

50. Let $r_1 = 1k$ and $r_2 = nk$

Since,

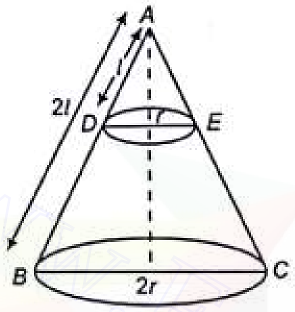
$V_1 = V_2$

$\therefore$ $\frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow$ $\frac{1}{3}\pi k^2 \times h_1 = \pi n^2 k^2 h_2$

$\Rightarrow$ $h_1 = 3n^2 h_2$

51. Since, ΔABC and ΔADE are similar.



$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \times 4r^2 \times \sqrt{(2l)^2 - (2r)^2}}{\frac{1}{3}\pi \times r^2 \times \sqrt{l^2 - r^2}} = \frac{8}{1}$$

52. Increase in the height of water level

$$= \frac{0.75}{2.5 \times 15}\text{ m} = 0.2\text{ m} = 20\text{ cm}$$

53. Height of water in a second = 30 cm

Height of water in 60s = 30 × 60

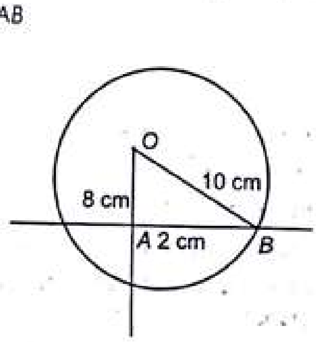
i.e., $h = 1800$ cm

Area of cross-section, $\pi r^2 = 5$ sq cm

Volume of water flow in one minute $= \pi r^2 h$

$$= 5 \times 1800 = 9000\text{ cu cm} = \frac{9000}{1000} = 9\text{L}$$

54. In right ΔOAB



$$AB^2 = OB^2 - OA^2 = 10^2 - 8^2 = 100 - 64 = 36$$

$$\therefore \quad AB = 6\text{ m}$$

55. Volume of solid $= l \times b \times h = 22 \times 7 \times 5 = 770$ cu cm

Let the water rise in height $= h$

$\because$ Volume of water rise in vessel = Volume of solid

$$\pi r^2 h = 770 \Rightarrow \frac{22}{7} \times 14 \times 14 \times h = 770$$

$$\therefore \quad h = \frac{770 \times 7}{22 \times 14 \times 14} = \frac{5}{4} = 1.25\text{ cm}$$

56. Volume of hemisphere $= \frac{2}{3}\pi(5^3 - 4^3) = \frac{2}{3}\pi(125 - 64)$

$$= \frac{2}{3}\pi \times 61\text{ cu cm}$$

$$= \frac{2}{3}\pi \times 61 \times 9\text{ g} = 366\pi\text{ g} \quad (\because 1\text{ cu cm} = 1\text{ g given})$$

57. Let radius and height of a cone be r and h.

$$\because \quad \frac{r}{h} = \frac{3}{4}$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \quad 96\pi = \frac{1}{3}\pi \times r^2 \times \frac{4r}{3} \Rightarrow r^3 = \frac{96 \times 3 \times 3}{4} = 216$$

$$\Rightarrow \quad r = 6\text{ cm and } h = 8\text{ cm}$$

$\therefore$ Lateral surface area $= \pi r\sqrt{r^2 + h^2}$

$$= \pi \times 6\sqrt{36 + 64} = 60\pi\text{ sq cm}$$