# Arithmetic Progression

26) Which term of the AP: 120, 116, 112.... is its first negative term?

2012/2015 [2 marks)

Let nth term of the APbe zero. Then, $a_n = 0 \Rightarrow \mathrm{a} + (\mathrm{n} - 1)d = 0$

So, $120 + (n - 1)(-4) = 0$

Or$4n = 124$

Or$n = 31$

∴ The first negative term of the AP is $(n+1)^{th}$ term =$32nd$ term.

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27) The ratio of the $5^{th}$ and $3^{rd}$ terms of AP $2:5$ Find the ratio of the $15^{th}$ and $7^{th}$ terms.

2014/2015 (4 Marks)

13:1

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28) In an AP, $6^{th}$ term is half the $4^{th}$ term and $3^{rd}$ term is 15. How many terms are needed to give a sum that is equal to 66?

2012/2014/2015 (4 Marks)

$$a_6 = \frac{1}{2}a_4$$

$a + 5d = \frac{1}{2}(a + 3d) \Rightarrow \quad a + 7d = 0$ ..................(1)

Also, $\qquad a_3 = a + 2d = 15$ ...................(2)

From (1) $and$ (2),$5d = -15 \Rightarrow \quad d = -3$

Putting, $d = -3 in$ (2), $we\ get$

$$a = 15 - 2 \quad (\quad) \quad 15 \quad 6 \quad 21$$

Now$66 = \frac{n}{2}[2(21) + (n - 1)(-3)]$

$$\Rightarrow \qquad 132 = n(42 - 3n + 3)$$

$\Rightarrow \quad 132 = 45n - 3n^2$

$$\Rightarrow \quad n2 - 15n + 44 = 0$$

$$\Rightarrow \quad (n - 11)(n - 4) = 0 \Rightarrow \quad n = 11 \quad or \quad n = 4.$$

So, terms needed are 4 $or$ 11.

In this case, the sum of $5^{th}$to $11^{th}$ terms will be zero.

So, only 5 terms are needed for the purpose.

29) In a garden bed, there are 23 roseplants in the first row, 21 are in the $2^{nd}$, 19 in $3^{rd}$ row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also, find the total number of rose plants in the garden.

2012/2014/2015 (4Marks)



The number of rose plants in the $1^{st}$, $2^{nd}$, $3^{rd}$ ......, last rows are 23, 21, 19.....,$a = 23, d = -2$ $and$ $a_n = 5$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 5 = 23 + (n-1)(-2)$$

$$\Rightarrow \quad n = \frac{5-23}{-2} + 1 = 9 + 1 = 10.$$

Total number of rose plants in the flower bed

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

i.e. $$s_{10} = 5(46 - 18) = 140.$$

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30) The sum of first six terms of an AP is 42. The ratio of $10^{th}$ term to its $30^{th}$ term is $1:3$. Calculate the first term and the $13^{th}$ term of AP.

Let the first term be a and common difference be d. Then,

$$s_6 = \frac{6}{2}[2a + (6-1)d] = 42$$

$$\Rightarrow 6a + 15d = 42 \quad \ldots\ldots(1)$$

According to the equation,

$$\frac{a+9d}{a+29d} = \frac{1}{3}$$

$$\Rightarrow 3a + 27d = a + 29d$$

⇒a=d.

Putting $d = a\ in\ (1), we\ get\ 21a = 42 \Rightarrow a = 2 = d.$

Therefore $a13 = a + 12d = 2 + 12$ .

31) The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of houses preceding the house numbered x is equal to the sum of the number of houses following it. Find the value of x.

Here, we are given that

$\{1 + 2 + 3 + \dots + (x - 1)\} = \{(x + 1) + (x + 2) + (x + 3) + \cdots + 49\}$

$\Rightarrow \left(\frac{x-1}{2}\right)\{2 \times 1 + (x - 1 - 1) \times 1\} = \left(\frac{49-x}{2}\right)\{2(x + 1) + (49 - x - 1) \times 1\}$

$\Rightarrow \quad (\quad)(\quad) \quad (\quad)(\quad)$

$\Rightarrow \quad x \quad 1 \quad x \quad 49 \quad x \quad 50 \quad x$

$\Rightarrow \quad {}^2 - x \quad 2450 \quad 49x \quad 50x \quad {}^2$

$\Rightarrow \quad 2 \quad {}^2 = 2450 \Rightarrow x^2 = 1225$

$\Rightarrow \quad x \quad \sqrt{\quad} \quad 35$

Thus, value of is

32) If the sum of the first terms of an AP is and the sum of first n terms is

Let the AP be, $a, a + d, a + 2d, \dots$

So, $\frac{m}{2}[2a + (m - 1)d] = n$

$\Rightarrow \quad 2am + m(m - 1)d] = 2n \dots (1)$

Also, $\frac{n}{2}[2a + (n - 1)d] = m$

$\Rightarrow \quad [2an + n(n - 1)d] = 2m \quad \dots\dots(2)$

Subtracting (2)from (1), We have:

$$2a(m - n) + [(m^2 - n^2) - (m - n)]d = 2(n - m)$$

$\Rightarrow \quad 2a + (m + n - 1)d = -2$

So, $S_{m+n} = \frac{m+n}{2}[2a + (m + n - 1)d]$

$= \frac{m+n}{2}(-2) = -(m + n).$

Thus, it shows that the sum of the first $(m + n)$ terms of the AP is – $(m + n)$

$$\Rightarrow \quad (x-1)(2+x-2) = (49-x)(2x+2+49-x-1)$$

$$\Rightarrow \quad (x-1)(x) = (49-x)(50+x)$$

$$\Rightarrow \quad x^2 - x = 2450 + 49x - 50x - x^2$$

$$\Rightarrow \quad 2x^2 = 2450 \Rightarrow x^2 = 1225$$

$$\Rightarrow \quad x = \sqrt{1225} = 35$$

Thus, value of $x$ is 35.

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32) If the sum of the first $m$ terms of an AP is $n$ and the sum of first n terms is m, then show that the sum of its first$(m+n)$ terms is $-(m+n)$.

Let the AP be,$a, a+d, a+2d, \dots$

So, $\quad \frac{m}{2}[2a+(m-1)d] = n$

$\Rightarrow \quad 2am + m(m-1)d] = 2n \dots.(1)$

Also, $\quad \frac{n}{2}[2a+(n-1)d] = m$

$\Rightarrow \quad [2an + n(n-1)d] = 2m$ .......(2)

Subtracting (2)from (1), We have:

$$2a(m-n) + [(m^2 - n^2) - (m-n)]d = 2(n-m)$$

$\Rightarrow \quad 2a + (m+n-1)d = -2$

So, $\quad S_{m+n} = \frac{m+n}{2}\ [2a + (m+n-1)d]$

$= \frac{m+n}{2}(-2) = -(m+n).$

Thus, it shows that the sum of the first $(m+n)$ terms of the AP is – $(m+n)$