Number of Questions: 25 **Section Marks: 30**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Which of the following is TRUE for Turing Machines?
(i) Turing machines accepts Type-0 languages.
(ii) Turing machines are used for computing functions.
(iii) Turing machines are used for determining the undecidability of certain languages.
(A) (i), (ii) only (B) (i), (iii) only
(C) (ii), (iii) only (D) (i), (ii), (iii)

2. Which of the following operations are not performed on Turing machines?
(A) Writing a new symbol in the cell being currently scanned.
(B) Moving to the cell left of the present cell.
(C) Moving to the cell right of the present cell.
(D) None of the above

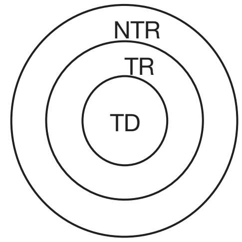
3. If the languages L_1 and L_2 are decidable then which of the following are also decidable?
(i) $\overline{L_1} \cup \overline{L_2}$ (ii) $\overline{L_1} \cap \overline{L_2}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

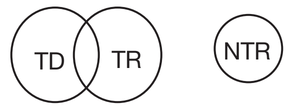
4. If L_1 and L_2 are Turing-recognizable languages then which of the following are not Turing-recognizable?
(i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$
(iii) $L_1 - L_2$ (iv) $\overline{L_1}$
(v) L_1^* (vi) $L_1 \text{ o } L_2$
(A) (iv), (v) (B) (iii), (iv)
(C) (i), (ii), (vi) (D) (ii), (iv), (v)

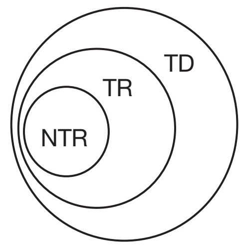
5. Which of the following statement is FALSE?
(A) The set of decidable languages is closed under symmetric difference.
(B) If L_1 is undecidable and L_2 is decidable then the symmetric difference of L_1 and L_2 is undecidable.
(C) The intersection of a recognizable language and an unrecognizable language is always unrecognizable.
(D) All of the above

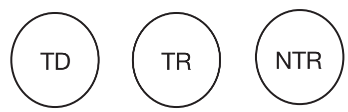
6. If L_1 and L_2 are recursive languages, then which of the following also belongs to recursive languages?
(i) $L_1 \cup L_2$
(ii) $L_1 L_2$
(iii) $\overline{L_1}$
(A) (i) only (B) (i), (ii)
(C) (ii), (iii) (D) (i), (ii), (iii)

7. Which of the following correctly specifies the relation between Turing decidable (TD), Turing recognizable (TR) and 'not Turing recognizable' (NTR) languages?

(A)


(B)


(C)


(D)


8. Which of the following DFA problems are Decidable?
(A) The language of all DFA's with an empty language.
(B) The language of all pairs of DFA's that have the same language.
(C) $A_{DFA} = \{<D, \omega>|D$ is a DFA, ω is a word and D accepts $\omega\}$
(D) All of the above

9. Which of the following language is accepted by a Turing machine?
(i) $\{a^n \# a^n \# a^n : n \geq 0\}$ and $\Sigma = \{a, \#\}$
(ii) $\{a^n \# b^{2^n} : n \geq 0\}, \Sigma = \{a, b\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

10. For which of the following languages, no Turing-machines will exist?
(A) Recursive languages
(B) Recursively-enumerable languages
(C) Non-Recursively enumerable.
(D) None of the above

11. If L_1 is a recursive language and L_2 is a recursively enumerable language then which of the following is also recursively enumerable?
(i) $L_1 \cup L_2$ (ii) $L_1 \cap L_2$
(iii) $L_1 L_2$ (iv) $L_1 - L_2$
(v) $L_2 - L_1$
(A) (i), (ii), (iii) (B) (i), (iii), (iv)
(C) (iii), (iv), (v) (D) (i), (ii), (iii), (v)

12. Which of the following is False, if A is mapping reducible to B?
(A) If B is decidable then A is decidable.
(B) If B is recursively enumerable then A is recursively enumerable.
(C) If A is undecidable then B is undecidable.
(D) None of the above

13. Which of the following language is Recursive?
(A) $L_1 = \{<M> | M$ is a TM and there exists an input on which M halts in less than $|<M>|$ steps.$\}$
(B) $L_2 = \{<M> \mid M$ is a TM and $|L(M)| \leq 3\}$
(C) $L_3 = \{<M> \mid M$ is a TM and $|L(M)| \geq 3\}$
(D) All of the above

14. The language $\{<M> \mid M$ is a Turing machine and $L(M)$ is finite$\}$ is
(A) recursive
(B) recursively enumerable
(C) not recursively-enumerable
(D) decidable

15. Which of the following problem is Recursive?
(i) $\{<M> \mid M$ is a DFA and $L(M)$ is finite$\}$
(ii) $\{<M> \mid M$ is a DFA and $L(M) = \Sigma^*\}$
(iii) $\{<M, x> \mid M$ is a DFA and M accepts $x\}$
(A) (i), (ii) (B) (iii) only
(C) (ii), (iii) (D) (i), (ii), (iii)

16. Which of the following is not accepted by a PDA but accepted by a TM?
(A) $\{a^n b^n c a^n b^n \mid n > 0\}$
(B) $\{a^n b^{2n} c^{3n} \mid n \geq 0\}$
(C) $\{a^n b^{n+m} c^m d^m \mid m, n \geq 0\}$
(D) All of the above

17. Which of the following has same power as Turing machine?
(A) NDFA (B) 2-PDA
(C) k-tape TM (D) Both (B) and (C)

18. The language $L = \{a^n b^n c^n \mid n \geq 0\}$ can be accepted by a:
(i) 2-PDA (ii) TM
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

19. If $A \leq_m B$ (i.e., A is reducible to B) and if B is regular language then does it implies that A is a regular language?
(A) Yes
(B) No
(C) Data insufficient
(D) Reduction is not possible with regular languages.

20. If a language L is recursively enumerable and $L \leq_m \bar{L}$ then, which of the following is TRUE?
(A) L is recursive
(B) L may or may not be recursive
(C) L is undecidable
(D) None of these

21. Which of the following language(s) is/are undecidable?
$L_1 = \{<M> \mid M$ is a TM and $L(M)$ is a CFL$\}$
$L_2 = \{<M> \mid M$ is a TM and $L(M)$ is finite$\}$
(A) L_1 only (B) L_2 only
(C) Both L_1 and L_2 (D) Neither L_1 nor L_2

22. For which of the following language(s) a Turing machine exists?
(i) $\varnothing$ (ii) $\{\varepsilon\}$
(A) (i) only (B) (ii) only
(C) Both (i) and (ii) (D) Neither (i) nor (ii)

23. A language L is recursive if
(A) L is recursively enumerable.
(B) $\bar{L}$ is recursively enumerable.
(C) Both L and $\bar{L}$ are recursively enumerable.
(D) L is not recursively enumerable.

24. If a Language L is recursively enumerable but not recursive then $\bar{L}$
(A) is recursive
(B) is recursively enumerable
(C) is not recursively enumerable
(D) is decidable

25. A decision problem is undecidable if
(i) it has no algorithm to solve
(ii) it has no TM that halts on every input
(iii) it specifies a language which is not recursive.
(A) (i), (ii) (B) (ii) only
(C) (i), (iii) (D) (i), (ii), (iii)

Answer Keys

1. D	**2.** D	**3.** C	**4.** B	**5.** C	**6.** D	**7.** A	**8.** D	**9.** C	**10.** C
11. D	**12.** D	**13.** A	**14.** C	**15.** D	**16.** D	**17.** D	**18.** C	**19.** B	**20.** A
21. C	**22.** C	**23.** C	**24.** C	**25.** D					

Hints and Explanations

1. Turing machines accepts type-0 languages. They are used for computing functions and for determining the undecidability of languages. Choice (D)
2. A Turing machine can write into some cell, move left or right. Choice (D)
3. If a language is decidable then its complement is also decidable.
The union and intersection of two decidable languages is also decidable. Choice (C)
4. Turing-recognizable languages (i.e. Recursively-enumerable) are not closed under subtraction and complement operations. Choice (B)
5. Decidable languages are closed under symmetric difference.
If L_1 is undecidable and L_2 is decidable then the difference of L_1 and L_2 is undecidable.
Let L_1 is a recognizable language and $L_1 = \varnothing$, L_2 is unrecognizable. Then $L_1 \cap L_2 = \varnothing$ which is recognizable. Choice (C)
6. Recursive languages are closed under union, intersection, complementation, concatenation and Kleene closure. Choice (D)
7. Regular $\subset$ Context-free $\subset$ TD $\subset$ TR $\subset$ NTR. Choice (A)
8. Some of the decidable properties of DFA are
   - Universality
   - Kleene-closedness
   - Inclusion
   - Disjointness
   - Membership
   - Emptyness
   - Equivalance

   $\therefore$ All the problems in given choices are Decidable. Choice (D)
9. A TM can be designed to accept, $\{a^n \# a^n \# a^n\}$. In the input write x for an 'a'. Traverse till # and again write x for an 'a' and traverse till next # and write x for an 'a'. Traverse left till left most x and repeat this for next 'a'. If no more a's left in any part, accept the language.
Similarly, a TM can be constructed for $\{a^n \# b^{2^n} : n \geq 0\}$. Replace a with x and traverse till # and replace n b's with x and traverse left till x and repeat this. If no more symbols left then accept the language. Choice (C)
10. Recursive: TM always halts.
Recursively enumerable: TM either halts or halts in non-final state or loops.
Non-Recursively-enumerable: No TM exists. Choice (C)
11. $L_1 - L_2$ is not recursively enumerable. Choice (D)
12. If A is a mapping reducible to B, then based on some basic Theorems,
    - B is decidable then A is decidable
    - A is undecidable then B is undecidable
    - B is Turing-recognizable then A is Turing recognizable.

    $\therefore$ All are TRUE. Choice (D)
13. L_1 is a recursive language. First find the length of M and store it. Then it runs M on all inputs of length atmost $|<M>|$ steps and accepts if M accepts atleast one of the strings within the specified number of steps. Choice (A)
14. Given language is not recursively enumerable and undecidable. Choice (C)
15. All the given problems are decidable and hence recursive. Choice (D)
16. $\{a^n\, b^n\, ca^n\, b^n.|n > 0\}$. PDA can check equality of number of a's and b's.
But it can't able to compare this number of a's and b's with next a's and b's.
But this can be accepted by a TM. Similarly for $\{a^n\, b^{2n}\, c^{3n}|n \geq 0\}$ and $\{a^n\, b^{n+m}\, c^m\, d^m|\, m, n \geq 0\}$. Choice (D)
17. A TM has same power as k-tape TM ($k \geq 1$) and 2-PDA has same power as TM. Choice (D)
18. A 2-PDA (i.e., 2-stack PDA) has same power as TM. A TM can accept L. Choice (C)
19. Given $A \leq_m B$.
B is regular then it does not imply that A is regular.
Ex: $\{a^n\, b^n \mid n \geq 0\} \leq_m \{a^n \mid n \geq 0\}$. Choice (B)
20. Given L is recursively enumerable and $L \leq_m \bar{L}$.
As $L \leq_m \bar{L}$ then $\bar{L} \leq_m L$.
($\because A \leq_m B$ then $\bar{A} \leq_m \bar{B}$).
L is RE, so $\bar{L}$ is also RE. As both L and $\bar{L}$ are RE then L is recursive. Choice (A)
21. $\{<M, w>$: M is a TM and M accepts input string $w\}$ is a known undecidable problem. We can reduce this to L_1 and L_2 also. So both L_1 and L_2 are undecidable. Choice (C)
22. A TM can exist for both languages. (i) rejects everything. (ii) accepts '$\in$' only. Choice (C)
23. If a language L and its complement $\bar{L}$ are recursively enumerable, then L is recursive. Choice (C)
24. Both L and $\bar{L}$ are recursively enumerable if L is recursive. Choice (C)
25. An undecidable problem has no algorithm, no TM and is also not recursive. Choice (D)