CBSE Class 11 Mathematics

Sample Papers 08 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Justify whether the given information is a 'Set' or 'Not'? A collection of most dangerous animals of the world.

OR

Write down the subsets of the set: F = {2, {3}}

2. If a point lies on the z-axis then find its x-coordinate and y-coordinate.
3. Find the degree measure corresponding to the given radian measure: $\left(\frac{2\pi}{15}\right)^c$.

OR

Find the radian measure to the degree measure: -47°30'

4. Show that: $\left\{i^{18} + \left(\frac{1}{i}\right)^{24}\right\}^3 = 0$.
5. How many different words can be formed with the letters of the word EQUATION so that the words begin with E?

OR

Compute: $\frac{20!}{18!}$.

6. The first and last terms of an AP are 1 and 11 respectively. If the sum of its terms is 36, find the number of terms.
7. Find the domain of function given by $f(x) = \frac{3x}{2x-8}$.

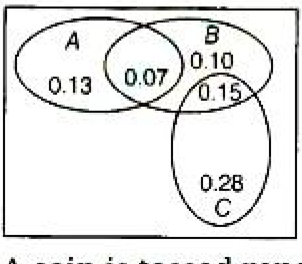
OR

Find the domain and range: $R = \left\{\left(x, \frac{1}{x}\right)\right.$: x is an integer, 0 < x < 5}.

8. Find the centre and radius of the circle: $x^2 + (y - 1)^2 = 2$
9. A card is selected from a pack of 52 cards. How many points are there in the sample space?

OR

The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, P (A ∩ B) = 0.07). Determine P (A)



10. A coin is tossed repeatedly until a tail comes up for the first time. Write the sample space for this experiment.

11. In which octant does the given point (-4, -1, -6) lie.
12. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that repetition of the digits is allowed
13. Express angle in degree: $\left(\frac{5}{6}\right)^\circ$
14. Prove that: $\frac{\tan(x+y)}{\cot(x-y)} = \frac{\sin^2 x-\sin^2 y}{\cos^2 x-\sin^2 y}$.
15. Find the values of: $\cos\frac{\pi}{8}$
16. Let R be a relation from N to N defined by R = {(a, b) : a, b ∈ N and a = b^2}. Check whether $(a,a) \in R$ for all $a \in N$? Justify your answer.

Section - II

17. **Read the Case study given below and attempt any 4 subparts:**

In science practical class, students used to perform experiments, one-day group A of class 11th have the following experiment results: A solution of 10% boric acid is to be diluted by adding a 4% boric acid solution to it. The resulting mixture is to be more than 5% but less than 8% boric acid. If we have 750 litres of the 10% solution, then:

i. The quantity of the 4% solution that has to be added will lie between
   a. 370 liters and 3750 liters
   b. 375 liters and 3750 liters
   c. 320 liters and 1280 liters
   d. 370 liters and 3700 liters

ii. In an experiment, a solution of hydrochloric acid is to be kept between 30 and 35 degrees Celcius. What is the range of temperature in degree Fahrenheit if the conversion formula is given by, C = $\frac{5}{9}$(F-32) where C and F represent the temperature in degree Celsius and degree Fahrenheit, respectively
   a. 30 < F < 35
   b. 86 < F < 95

c. 62 < F < 67

d. 77 < F < 86

iii. Solve for x: −8x+3≥27 And −13x+5≥57

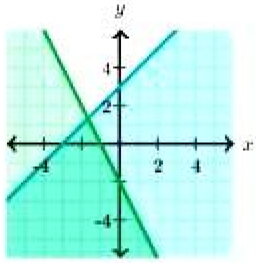
a. x ≤ −4

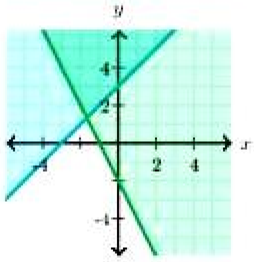
b. x ≤ −3

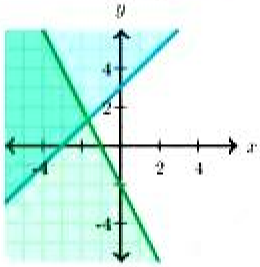
c. -4 ≤ x ≤ −3

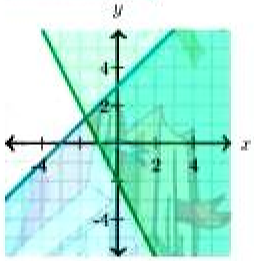
d. There are no solutions

iv. y≤x+3 and y≥−2x−2, Which graph represents the system of inequalities?

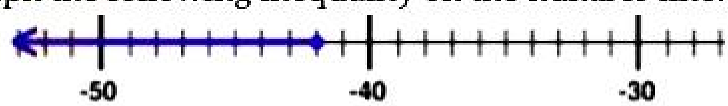
a. 

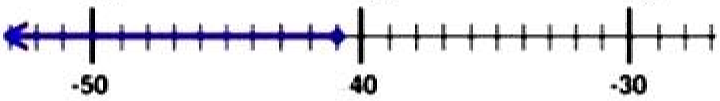
b. 

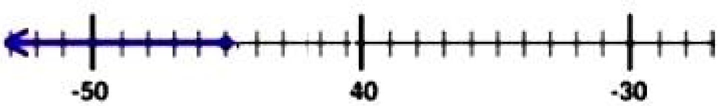
c. 

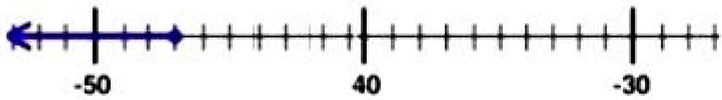
d. 

v. Graph the following inequality on the number line: x ≤ −42

a. 

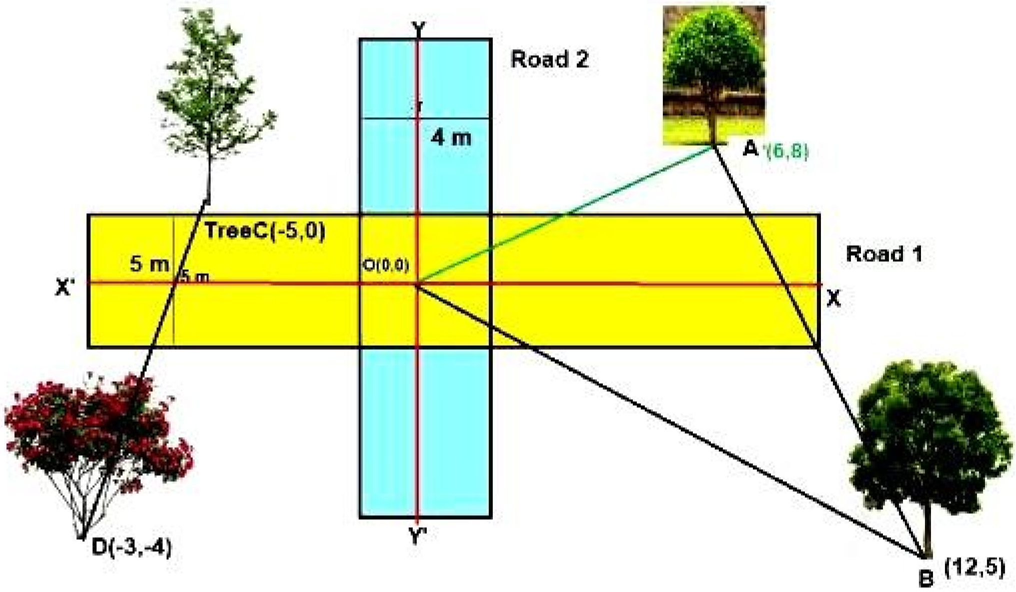
b. 

c. 

d. 

18. **Read the Case study given below and attempt any 4 sub parts:**

In a park Road 1 and road 2 of width 5 m and 4 m are crossing at centre point O(0, 0). As shown in the following figure:



For trees A, B, C and D are situated in four quadrants of the Cartesian system of coordinate. The coordinates of the trees A, B, C and D are (6, 8), (12, 5), (-5, 0) and (-3, 4) respectively.

Now answer the following questions:

i. What is the distance of Tree C from the Origin?
   a. 5 m
   b. 10 m
   c. 15 m
   d. 25 m

ii. What is the equation of line AB?
   a. $2x + y = 22$
   b. $x - 2y = -6$
   c. $x + 2y - 22 = 0$
   d. $x + 2y = 6$

iii. What is the slope of line CD?
   a. $\frac{2}{1}$

b. $\frac{1}{2}$
c. $\frac{-1}{2}$
d. $\frac{3}{2}$

iv. What is the slope of line OA?

a. $\frac{3}{4}$
b. 1
c. $\frac{4}{3}$
d. $\frac{6}{8}$

v. What is the distance of point B from the origin?

a. 13 m
b. 15 m
c. 12 m
d. 5 m

Part - B Section - III

19. Are the C = {x : x ∈ N, x < 3} and D = {x : x ∈ W, x < 3} pairs of equivalent sets?

20. Let $A \times B$ = {(a, b) : b = 3a - 2}. If (x, -5) and (2, y) belong to $A \times B$, find the values of x and y.

OR

Let S be the set of all real numbers and let R be a binary relation on S defined by (a, b) $\in R \Leftrightarrow 1 + ab > 0$ for all a, b ∈ S. Show that R is reflexive as well as symmetric. Give an example to show that R is not transitive.

21. Prove that: $\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A}$ = tan 3A.

22. Find the value of $\left[i^{19}+\left(\frac{1}{i}\right)^{25}\right]^2$

23. If $(x + iy)^{1/3} = (a + ib)$ then prove that: $\frac{x}{a}+\frac{y}{b} = 4(a^2 - b^2)$

OR

Prove that: $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}=0$

24. Let R be a relation on Z, defined by (x, y) $\in R \Leftrightarrow x^2 + y^2 = 9$. Then, write R as set of ordered pairs. What is its domain?

25. Evaluate: $\lim_{x \to 4} \frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$.

26. Differentiate the function $\frac{2x+3}{3x+2}$ with respect to x from first principle.

27. In a group of 950 persons, 750 can speak Hindi and 460 can speak English. Find: how many can speak both Hindi and English.

28. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

OR

If $z = (\sqrt{2} - \sqrt{-3})$, find Re(z), Im(z), $\bar{z}$ and |z|.

Section - IV

29. Differentiate $\frac{x^2+1}{x}$ from first principle.

30. In an entrance test that is graded on the basis of two examination, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

31. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

OR

The side of a given square is 10 cm. The midpoints of its sides are joined to form a new square. Again, the midpoints of the sides of this new square are joined to form another square. This process is continued indefinitely. Find the sum of the areas of the squares.

32. Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola $16x^2 - 9y^2 = -144$.

33. Find r if ${}^5P_r = 2\,{}^6P_{r-1}$

34. If X = {4^n - 3n - 1 : n∈N} and Y = {9(n - 1):n∈N}, prove that X⊂Y.

OR

Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7}. Verify : A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

35. If $f(x) = 3x^4 - 5x^2 + 9$ find f(x-1).

Section - V

36. Find four numbers in GP, whose sum is 85 and product is 4096.

OR

If a, b, c, d are in G.P., prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

37. Calculate the mean and standard deviation for the following table, given the age distribution of a group of people:

Age:	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of persons:	3	51	122	141	130	51	2

OR

The mean and variance of five observations are 6 and 4 respectively. If three of these are 5, 7 and 9, find the other two observations.

38. Solve the following equations graphically:

$x + y \geq 5, 2x + 3 \geq 3y$

$0 \leq x \leq 4, 0 \leq y \leq 2$

OR

Solve the linear inequality $\frac{-3x+10}{x+1} > 0.$

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Solution

Part - A Section - I

1. A collection of most dangerous animals of the world is not well defined because opinions about most dangerous animals' vary from person to person and hence it does not form a set.

OR

Let x = {3}
Then, F = {2, x}
Thus,the subsets of {2, x} are ϕ , {2}, {x}, {2, x}
i.e ϕ , {2}, {{3}}, {2,{3}}

2. x and y coordinates of a point are its distance from the origin along or parallel to the horizontal x-axis and y-axis.
To measure the x and y coordinates, you must move either to the left of the origin or to its right.
In case of a point on the z-axis, you do not move to the right or to the left of the origin.
Hence x and y coordinates are 0 for a point on the z-axis.
If point lie on z-axis its x and y coordinates are zero.

3. $\left(\frac{2\pi}{15}\right)^c = \left(\frac{2\pi}{15} \times \frac{180}{\pi}\right)^\circ$ = 24 .... [$\because 1^c = \left(\frac{180}{\pi}\right)^\circ$]

OR

To convert degree measures into radians, we multiply by $\frac{\pi}{180}$
-47°30'
$= -\left(47\frac{30}{60}\right)^\circ = -\left(\frac{95}{2}\right)^\circ$
$= -\left(\frac{95}{2} \times \frac{\pi}{180}\right)^C = -\left(\frac{19\pi}{72}\right)^C$

4. $\left\{i^{18} + \left(\frac{1}{i}\right)^{24}\right\}^3 = \left\{i^{18} + \frac{1}{i^{24}}\right\}^3 = \left(i^2 + \frac{1}{1}\right)^3$ = $(-1 + 1)^3$ = 0 [$i^2 = -1$]

5. Clearly, the given word contains 8 letters out of which 5 are vowels and 3 consonants.

Since all words must begin with E. So, we fix E at the first place. Now, remaining 7 letters can be arranged in ${}^7P_7$ = 7! ways.

So, total number of words = 7!

OR

We have,

$\frac{20!}{18!} = \frac{20(19!)}{18!} = \frac{20\times19\times18!}{18!}$ [∵ n! = n × (N - 1)!]

= 20 × 19 = 380

6. Given: the sum of terms of AP is 36, the first and last terms of an AP are 1 and 11

To find: number of terms

Sum of AP using first and last terms is given by

$S_n = \frac{n}{2}(a + 1)$

36 × 2 = n(1 + 11)

Therefore, n = 6

7. Here we have, $f(x) = \frac{3x}{28-x}$

Clearly, f(x) is not defined, if 28 - x = 0

$\Rightarrow \quad x \neq 28$

∴ Domain of f = R - {28}

OR

Here we have,

$R = \{(x, \frac{1}{x})$: x is an integer, 0 < x < 5}

$R = \{(1,1), (2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4})\}$

Hence, Domain = {1, 2, 3, 4} and Range $= \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$

8. The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

h = 0 , k = 1, $r^2 = 2$

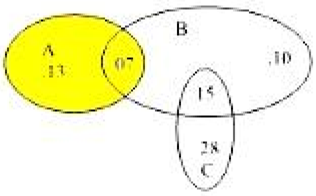
⇒ centre of the circle= (0, 1) and radius of the circle = $\sqrt{2}$ units.

9. We have to draw One card from a pack of 52 cards.

   ⇒ Number of points in the sample space S = n(S) = 52.

OR

To find P(A) we have from diagram



P(A) = 0.13 + 0.7 = 0.20.

10. A coin is tossed and if the outcome is tail the experiment is over, and, if the outcome is Head then the coin is tossed again.

    In the second toss also if the outcome is tail then the experiment is over, otherwise, the coin is tossed again.

    This process continues indefinitely.

    So, The sample space for this experiment is S = {T, HT, HHT, HHHT, HHHHT...}

    Hence, S is the sample space for the given experiment.

11. Point (-4, -1, -6) lies in octant VII

12. The unit place can be filled by any one of the digits 1, 2, 3, 4 and 5. So the unit place can be filled in 5 ways. Similarly, the tens place and hundreds place can be filled in 5 ways each because the repetition of digits is allowed.

    ∴Total number of 3-digits numbers = $5 \times 5 \times 5$= 125

13. We know that, Angle in degrees = Angle in radians $\times \frac{180}{\pi}$

    The angle in minutes = Decimal of angle in radian $\times$ 60'

    The angle in seconds = Decimal of angle in minutes $\times$ 60''

    Therefore, Angle in degrees $= \frac{5}{6} \times \frac{180}{\pi} = \frac{150}{22/7} = 47.7272^\circ$

    Angle in minutes = $0.7272 \times 60' = 43.632'$

    Angle in seconds $= 0.632 \times 60'' = 37.92''$

    Final angle $= 47^\circ 43' 38''$

14. Take LHS

    $= \frac{\tan(x+y)}{\cot(x-y)} = \frac{\sin(x+y)}{\cos(x+y)} \frac{\sin(x-y)}{\cos(x-y)}$

    $= \frac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y} = RHS$

15. Let $y = \cos\frac{\pi}{8}$, then

$y = \sqrt{\frac{1+\cos\pi/4}{2}} = \sqrt{\frac{1+1/\sqrt{2}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$ ... [$\because \cos\frac{\pi}{8}$ is positive]

16. We have, $R = \{(a, b): a, b \in N \text{ and } a = b^2\}$

$(a, a) \in R$, for all $a \in N$

As we can see that $3 \in N$ but $3 \neq 3^2 = 9$

Hence, the statement is not true.

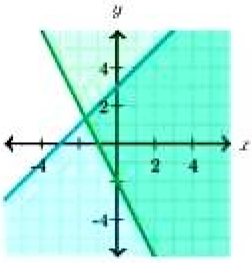
Section - II

17. i. (b) 375 liters and 3750 liters

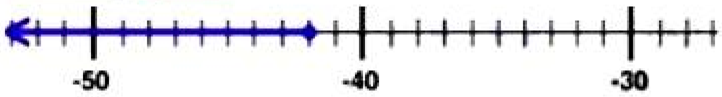
ii. (b) 86 < F < 95

iii. (a) $x \le -4$

iv. (d)



v. (a)



18. i. (a) 5 m

ii. (c) x + 2y - 22 = 0

iii. (b) $\frac{1}{2}$

iv. (c) $\frac{4}{3}$

v. (a) 13 m

Part - B Section - III

19. Equivalent Sets can have different or same elements but have the same number of elements.

We have, $C = \{x : x \in N, x < 3\}$

Natural numbers = 1, 2, 3, 4, ...

Natural numbers less than 3 (x < 3) = 1, 2

Therefore, C = {1, 2}

and $D = \{x : x \in W, x < 3\}$

Whole numbers = 0, 1, 2, 3, 4, ...

Whole numbers less than 3 (x < 3) = 0, 1, 2

Therefore, D = {0, 1, 2}

$\therefore$ C and D are not equivalent sets because their cardinality is not same.

20. Here we have, $A \times B$ ={(a, b): b = 3a - 2} and {(x, 5), (2, y)} $\in A \times B$

For (x, -5) $\in A \times B$, we have

b = 3a - 2 $\Rightarrow$ -5 = 3(x) - 2 $\Rightarrow$ - 3 = 3x $\Rightarrow$ x = -1

For (2, y) $\in A \times B$, we have

b = 3a - 2 $\Rightarrow$ y = 3(2) - 2 $\Rightarrow$y = 4

Hence, the value of x = -1 and y = 4.

OR

i. Let a be an arbitrary real number.

Then, $(1 + a^2) > 0 \Rightarrow$ (a, a) $\in$ R for every a $\in$ S.

$\therefore$ R is reflexive

ii. Let (a, b) $\in$ R. Then,

$(a, b) \in R \Rightarrow$ 1 + ab > 0

$\Rightarrow$ 1 + ba > 0

$\Rightarrow$ (b, a) $\in$ R.

$\therefore (a, b) \in R \Rightarrow (b, a) \in R$

Hence R is symmetric

iii. In order to show that R is not transitive, consider the real number $\left(\frac{-2}{3}\right)$, 1 and 2

Now, $\left(\frac{-2}{3}, 1\right) \in R$, since $\left\{1+\left(\frac{-2}{3}\right) \times 1\right\}=\left(1-\frac{2}{3}\right)=\frac{1}{3}>0$

And, (1, 2) $\in$ R, since $\{1+(1 \times 2)\}=3>0$

But, $\left(\frac{-2}{3}, 2\right) \notin R$, since $\left\{1+\left(\frac{-2}{3}\right) \times 2\right\}=\left(1-\frac{4}{3}\right)=-\frac{1}{3}<0$

Thus, $\left(\frac{-2}{3}, 1\right) \in R$ (1, 2) $\in$ R and $\left(\frac{-2}{3}, 2\right) \notin R$

This shows that R is not transitive.

21. To prove: $\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A}$ = tan 3A

Now, L.H.S = $\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A}$

$= \frac{(\sin 5A+\sin A)+\sin 3A}{(\cos 5A+\cos A)+\cos 3A}$

$= \frac{2 \sin \frac{5A+A}{2} \cos \frac{5A-A}{2}+\sin 3A}{2 \cos \frac{5A+3A}{2} \cos \frac{5A-3A}{2} \sin 3A}$

$= \frac{2 \sin 3A \cos A+\sin 3A}{\cos 3A \cos A+\sin 3A}$

$= \frac{\sin 3A(2 \cos A+1)}{\cos 3A(2 \cos A+1)}$

= tan 3A

= RHS

Hence, RHS = LHS

22. We have, $\left[i^{19}+\left(\frac{1}{i}\right)^{25}\right]^{2}=\left[i^{16}\cdot i^{3}+\frac{1}{(i)^{25}}\right]^{2}$ [∵ $i^{19} = i^{16} . i^3$]

= $\left[\left(i^{4}\right)^{4}\cdot i^{2}\cdot i+\frac{1}{(i)^{24}\cdot i}\right]^{2}=\left[(1)^{4}(-i)+\frac{1}{\left(i^{4}\right)^{6}\cdot i}\right]^{2}$ [∵ $i^4 = 1$ and $i^2 = -1$]

= $\left[-i+\frac{1}{(1)^{6}\cdot i}\right]^{2}=\left[-i+\frac{1}{i}\right]^{2}=\left[\frac{-i^{2}+1}{i}\right]^{2}=\left[\frac{1+1}{i}\right]^{2}=\left(\frac{2}{i}\right)^{2}$

= $\frac{4}{i^{2}}=\frac{4}{-1}$ = - 4 [∵ $i^2 = 1$]

23. We have

$(x + iy)^{1/3} = (a + ib)$

⇒ $(x + iy) = (a + ib)^3$ [taking cube on both sides]

⇒ $(x + iy) = a^3 + i^3b^3 + 3iab (a + ib)$

$a^3 - ib^3 + 3a^2bi - 3ab^2 = (a3 - 3ab2) + i(3a^2b - b^3)$

⇒ $x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

[on equating real and imaginary parts separately]

⇒ $\frac{x}{a}=\left(a^{2}-3b^{2}\right)$ and $\frac{y}{b}=\left(3a^{2}-b^{2}\right)$

⇒ $\left(\frac{x}{a}+\frac{y}{b}\right)=4\left(a^{2}-b^{2}\right)$

Hence, $\frac{x}{a}+\frac{y}{b}=4\left(a^{2}-b^{2}\right)$

OR

Given: $\frac{1}{i}-\frac{1}{i^{2}}+\frac{1}{i^{3}}-\frac{1}{i^{4}}$

To prove: $\frac{1}{i}-\frac{1}{i^{2}}+\frac{1}{i^{3}}-\frac{1}{i^{4}}=0$

⇒ L.H.S.= $i^{-1} - i^{-2} + i^{-3} - i^{-4}$

= $i^{-4\times1+3} - i^{-4\times1+2} + i^{-4\times1+3} - i^{-4\times1}$ [$i^{4n} = 1, i^{4n+1} = i, i^{4n+2} = -1$ and $i^{4n+3} = -i$]

= $i^1 - i^2 + i^3 - 1$

= i + 1 - i - 1

= 0

⇒ L.H.S = R.H.S

Hence Proved.

24. Here it is given that R is a relation on Z, defined by (x, y) $\epsilon R \leftrightarrow x^2 + y^2 = 9$.

Now, $x^2 + y^2 = 9$

Put x = 0, y = 3, $0^2 + 3^2 = 9$

Put x = 3. y = 0, $3^2 + 0^2 = 9$

Therefore, R = {(0, 3), (3, 0), (0, -3), (-3, 0)}

The domain of R is the set of first co-ordinates of R.

Dom (R) = {-3, 0, 3}

The range of R is the set of second co-ordinates of R.

Range (R) = {-3, 0, 3}.

25. We have to find the value of

$\frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$

When x = 4, the expression $\frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$ assumes the form $\frac{0}{0}$. So, (x - 4) is a common factor in numerator and denominator.

Factorising the numerator and denominator, we get

$\lim\limits_{x\to4}\frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$

$= \lim\limits_{x\to4}\frac{[(x-4)(x+3)]^{18}}{[x(x^2-8x+16)]^9} = \lim\limits_{x\to4}\frac{[(x-4)(x+3)]^{18}}{x^9(x-4)^{18}}$

$= \lim\limits_{x\to4}\frac{(x-4)^{18}(x+3)^{18}}{x^9(x-4)^{18}} = \lim\limits_{x\to4}\frac{(x+3)^{18}}{x^9} = \frac{7^{18}}{4^9}$

26. Let f(x) = $\frac{2x+3}{3x+2}$.

Therefore, f (x + h) = $\frac{2(x+h)+3}{3(x+h)+2} = \frac{2x+3+2h}{3x+2+3h}$

$\therefore \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{\frac{2x+3+2h}{3x+2+3h}-\frac{2x+3}{3x+2}}{h}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{(2x+3+2h)(3x+2)-(2x+3)(3x+2+3h)}{h(3x+2)(3x+2+3h)}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{(2x+3)(3x+2)+2h(3x+2)-(2x+3)(3x+2)-3h(2x+3)}{h(3x+2)(3x+2+3h)}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{h(6x+4-6x-9)}{h(3x+2)(3x+2+3h)}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to0}\frac{-5}{(3x+2)(3x+2+3h)} = -\frac{5}{(3x+2)^2}$

27. Let A & B denote the sets of the persons who like Hindi & English, respectively.

Given:

n(A) = 750

n(B) = 460

n(A ∪ B) = 950

We know:

n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

⇒ 950 = 750 + 460 - n(A ∩ B)

⇒ n(A ∩ B) = 260

Thus, 260 persons can speak both Hindi and English.

28. $\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=\left|\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}\right|$

$=\left|\frac{1+i^2+2i-1-i^2+2i}{1-i^2}\right|$

$\left|\frac{4i}{2}\right|=|2i|=\sqrt{4}=2.$

OR

z = $(\sqrt{2}-\sqrt{-3})$

z = $\sqrt{2}-i\sqrt{3}$

$\therefore \quad \mathrm{Re}(z)=\sqrt{2}, \mathrm{Im}(z)=-\sqrt{3}, \bar{z}=\overline{(\sqrt{2}-i\sqrt{3})}=(\sqrt{2}+i\sqrt{3})$

and $|z|^2=\left\{(\sqrt{2})^2+(-\sqrt{3})^2\right\}=$ (2 + 3) = 5

$\Rightarrow |z|=\sqrt{5}$

Section - IV

29. We need to find derivative of f(x) = $\frac{x^2+1}{x}$

Derivative of a function f(x) from first principle is given by

f'(x) = $\lim_{h\to 0}$ = $\frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

∴ derivative of f(x)= $\frac{x^2+1}{x}$ is given as

f'(x) = $\lim_{h\to 0}$ = $\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x)=\lim_{h\to 0}\frac{\frac{(x+h)^2+1}{x+h}-\frac{x^2+1}{x}}{h}$

$\Rightarrow f'(x)=\lim_{h\to 0}\frac{\left\{(x+h)^2+1\right\}x-(x+h)(x^2+1)}{hx(x+h)}$

$\Rightarrow f'(x)=\lim_{h\to 0}\frac{\left\{(x+h)^2+1\right\}x-(x+h)(x^2+1)}{h}\times\lim_{h\to 0}\frac{1}{x(x+h)}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{\left\{(x+h)^2+1\right\}x-(x+h)(x^2+1)}{h}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{\{x^2+h^2+2xh+1\}x-\{x^3+hx^2+x+h\}}{h}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{x^3+h^2x+2x^2h+x-x^3-hx^2-x-h}{h}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{h^2x+x^2h-h}{h}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} \frac{h(hx+x^2-1)}{h}$

$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \to 0} (hx + x^2 - 1)$

$\Rightarrow f'(x) = \frac{1}{x^2} (0 \times x + x^2 - 1) = \frac{x^2-1}{x^2} = 1 - \frac{1}{x^2}$

$\therefore f(x) = 1 - \frac{1}{x^2}$

30. Let A be the event that the student passes the first examination and B be the event that the student passes the second examination.

Given, P(A) = 0.8, P(B) = 0.7, and

Probability of passing at least A or B means $P(A \cup B)$

Given, $P(A \cup B) = 0.95$

Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

So, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

= 0.8+0.7-0.95=1.5 - 0.95 = 0.55.

Hence, the probability of passing both examination is 0.55.

31. Since G.M. between two numbers a and b is $\sqrt{ab}$.

According to question, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = a^{\frac{1}{2}} b^{\frac{1}{2}}$

$\Rightarrow a^{n+1} + b^{n+1} = (a^n + b^n) a^{\frac{1}{2}} b^{\frac{1}{2}}$

$\Rightarrow a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n+\frac{1}{2}}$

$\Rightarrow a^{n+1} - a^{n+\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{n+\frac{1}{2}} - b^{n+1}$

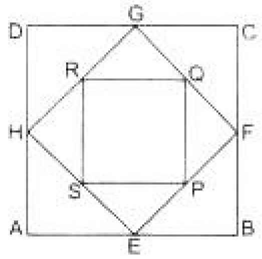
$\Rightarrow a^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) = b^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)$

$\Rightarrow a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}}$

$\Rightarrow \frac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}} = 1$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^{0}$$
$$\Rightarrow n + \frac{1}{2} = 0 \Rightarrow n = -\frac{1}{2}$$

OR



Suppose ABCD be the given square with each side equal to 10 cm. Suppose E, F, G, H be the midpoints of the sides AB, BC, CD and DA respectively. Suppose P, Q, R, S be the midpoints of the sides EF, FG, GH and HE respectively.

$\therefore$ BE = BF = 5 cm

$\Rightarrow$ EF = $\sqrt{BE^2 + BF^2}$ = $\sqrt{25+25}$ cm = $\sqrt{50}$ cm = $5\sqrt{2}$ cm

$\therefore$ FQ = FP = $\frac{1}{2}$ EF = $\frac{5\sqrt{2}}{2}$ cm = $\frac{5}{\sqrt{2}}$ cm

$\Rightarrow$ PQ = $\sqrt{FP^2 + FQ^2}$ = $\sqrt{\frac{25}{2} + \frac{25}{2}}$ cm = $\sqrt{25}$ cm = 5 cm.

Therefore, the sides of the squares are 10 cm, $5\sqrt{2}$ cm, 5 cm, ...

Sum of the areas of the squares formed

= $\{(10)^2 + (5\sqrt{2})^2 + 5^2 + ... + \infty\}$ cm^2

= (100 + 50 + 25 + ... ∞) cm^2

= $\frac{100}{\left(1-\frac{1}{2}\right)}$ cm^2 = 200 cm^2 [taking the sum of infinite GP with a = 100 and r = $\frac{1}{2}$]

32. Equation of the hyperbola: $16x^2 - 9y^2 = -144$

This can be rewritten in the following way:

$$\frac{x^2}{9} - \frac{y^2}{16} = -1$$

Comparing with the standard equation of a hyperbola, where

$a^2 = 9$ and $b^2 = 16 \Rightarrow a = 3$ and $b = 4$

$\Rightarrow a^2 = b^2(e^2 - 1)$

$\Rightarrow 9 = 16(e^2 - 1)$

$\Rightarrow e^2 - 1 = \frac{9}{16}$

$\Rightarrow e^2 = \frac{25}{16}$

$\Rightarrow e = \frac{5}{4}$

now, be = (4) $\frac{5}{4}$ =

Coordinates of foci are given by (0, $\pm$ be) $\Rightarrow$ (0, $\pm$5)

Equation of directrices: $y = \pm\frac{a}{e}$

$\Rightarrow y = \pm\frac{4}{\frac{5}{4}}$

$\Rightarrow$ 5y $\pm$ 16 = 0

Length of the latus rectum of the hyperbola $= \frac{2a^2}{b}$

Length of the latus rectum $= \frac{2\times 9}{4} = \frac{9}{2}$

33. Here ${}^5P_r = 2^6P_{r-1}$

$\frac{5!}{(5-r)!} = 2 \cdot \frac{6!}{(7-r)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{2\times 6\times 5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = \frac{12}{(7-r)(6-r)} \Rightarrow r^2 - 13r + 42 = 12$

$\Rightarrow r^2 - 13r + 30 = 0. \Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r(r-10) - 3(r-10) = 0$

$\Rightarrow (r-10)(r-3) = 0$

$\Rightarrow$ r = 10 or r = 3

Now r = 10 is not possible because r > n

Thus r = 3

34. Let $x_n = 4^n - 3n - 1$, n $\in$ N. Then, $x_1 = 4 - 3 - 1 = 0$

And, for any $n \geq 2$, we have

$x_n = 4^n - 3n - 1 = (1+3)^n - 3n - 1$

$\Rightarrow x_n = ^n C_0 + ^n C_1(3) + ^n C_2(3)^2 + ^n C_3(3)^3 + ^n C_n(3^n) - 3n - 1$

$\Rightarrow x_n = 1 + 3n + ^n C_2(3)^2 + ^n C_3(3)^3 + ^n C_n(3^n) - 3n - 1$

$\Rightarrow x_n = 3^2\{^nC_2 + ^nC_3(3) + ^nC_4(3)^2 + ...+ ^nC_n(3^{n-2})\}$

$\Rightarrow x_n = 9\{^nC_2 + ^nC_3 + ^nC_4(3)^2 + ...+ ^nC_n(3^{n-2})\}$

$\Rightarrow x_n$ is some positive integral multiple of 9 for all n $\geq$ 2.

Thus, X consists of all those positive integral multiples of 9 which are of the form

$9\{^nC_2 + ^nC_3 + ^nC_4\times(3)^2 + ... + ^nC_n\times(3^{n-2})\}$ together with 0.

Clearly, Y = {9(n - 1) : n $\in$ N} consists of all integral multiples of 9 together with 0.

Hence, X $\subset$ Y.

OR

L.H.S.

$(B \cup C) = \{x : x \in B \text{ or } x \in C\}$

$= \{2, 3, 4, 5, 6, 7\}$

$(A \cap (B \cup C)) = \{x : x \in A \text{ and } x \in (B \cup C)\}$

$= \{2, 4, 5\}$

R.H.S:

$(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

$= \{2, 5\}$

$(A \cap C) = \{x : x \in A \text{ and } x \in C\}$

$= \{4, 5\}$

$(A \cap B) \cup (A \cap C) = \{x : x \in (A \cap B) \text{ and } x \in (A \cap C)\}$

$= \{2, 4, 5\}$. Hence proved.

35. We have

$f(x) = 3x^4 - 5x^2 + 9$

$\therefore f(x-1) = 3(x-1)^4 - 5(x-1)^2 + 9$

$= 3x^4 - 12x^3 + 13x^2 - 2x + 7$

Section - V

36. Let the four numbers in GP be

$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3 ...(i)

Product of four numbers = 4096 [given]

$\Rightarrow \left(\frac{a}{r^3}\right)\left(\frac{a}{r}\right)$ (ar) (ar^3) = 4096

$\Rightarrow a^4 = 4096 \Rightarrow a^4 = 8^4$

On comparing the base of the power 4, we get

$\Rightarrow \frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 85$

$\Rightarrow a\left[\frac{1}{r^3} + \frac{1}{r} + r + r^3\right] = 85$

$\Rightarrow 8\,[r^3 + \frac{1}{r^3}] + 8\,[r + \frac{1}{r}] = 85$ [$\because$ a = 8]

$\Rightarrow 8\left[\left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right)\right] + 8\left(r + \frac{1}{r}\right) = 85$ [$\because a^3 + b^3 = (a + b)^3 - 3\,(a + b)$]

$\Rightarrow 8\left(r + \frac{1}{r}\right)^3 - 16\left(r + \frac{1}{r}\right) - 85 = 0$ ...(ii)

On putiing $\left(r + \frac{1}{r}\right)$ = x in Eq. (ii), we get

$8x^3 - 16x - 85 = 0$

$\Rightarrow (2x - 5)(4x^2 + 10x + 17) = 0$

$\Rightarrow 2x - 5 = 0$ [$\because 4x^2 + 10x + 177 = 0$ has imaginary roots]

$\Rightarrow x = \frac{5}{2} \Rightarrow r + \frac{1}{r} = \frac{5}{2}$ [put $x = r + \frac{1}{r}$]

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (r - 2)(2r - 1) = 0$

$\Rightarrow r = 2$ or $r = \frac{1}{2}$

On putting a = 8 and r = 2 or $r = \frac{1}{2}$ in Eq. (i), we obtain four numbers as

$\frac{8}{2^3}, \frac{8}{2}, 8 \times 2, 8 \times 2^3$

or $\frac{8}{(1/2)^3}, \frac{8}{(1/2)}, 8 \times \frac{1}{2}, 8 \times \left(\frac{1}{2}\right)^3$

$\Rightarrow$ 1, 4, 16, 64 or 64, 16, 4, 1.

OR

Given: a, b, c, d are in G.P.

To prove: $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

$\Rightarrow \frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore \frac{b}{a} = k$

$\Rightarrow b = ak$

And $\frac{c}{b} = k$

$\Rightarrow c = bk = (ak)k = ak^2$

Also $\frac{d}{c} = k$

$\Rightarrow d = ck = (ak^2)k = ak^3$

Now, $\frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

$\Rightarrow \frac{(ak)^n+(ak^2)^n}{a^n+(ak)^n} = \frac{(ak^2)^n+(ak^3)^n}{(ak)^n+(ak^2)^n}$

$\Rightarrow \frac{a^nk^n+a^nk^{2n}}{a^n+a^nk^n} = \frac{a^nk^{2n}+a^nk^{3n}}{a^nk^n+a^nk^{2n}}$

$\Rightarrow \frac{a^nk^n(1+k^n)}{a^n(1+k^n)} = \frac{a^nk^{2n}(1+k^n)}{a^nk^n(1+k^n)}$

$\Rightarrow k^n = k^n$

Therefore, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

37. Here A = 55, h = 10

Calculation of Mean and Standard Deviation

Age	mid-values (x_i)	Number of persons (f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
20-30	25	3	-3	-9	9	27
30-40	35	51	-2	-102	4	204
45-50	45	122	-1	-122	1	122
50-60	55	141	0	0	0	0
60-70	65	130	1	130	1	130
70-80	75	51	2	102	4	204
80-90	85	2	3	6	9	18
		$N = \Sigma f_i = 500$		$\Sigma f_i u_i = 5$		$\Sigma f_i u_i^2 = 705$

Here, $N = \Sigma f_i = 500$, $\Sigma f_i u_i = 5$, $\Sigma f_i u_i^2 = 705$

$\therefore \overline{X} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right) = 55 + 10\left(\frac{5}{500}\right) = 55.1$

and, $\sigma^2 = h^2\left\{\left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\}$

$\Rightarrow \sigma^2 = 100\left\{\frac{705}{500} - \left(\frac{5}{500}\right)^2\right\} = 100 \times 1.4099 = 140.99$

$\Rightarrow$ Standard Deviation, $\sigma = \sqrt{140.99} = 11.8739$

OR

Let the other two observations be x and y

Therefore, our observations are 5, 7, 9, x and y

Mean $= \frac{\text{Sum of observations}}{\text{Total number of observations}}$

$6 = \frac{5+7+9+x+y}{5} \Rightarrow 6 \times 5$ = 21 + x + y

$\Rightarrow 30 - 21 = x + y$ or x + y = 9 .... (i)

Now prepare the following table we have,

x_i	$x_1 - \bar{x} = x_1 - 6$	$(x_1 - \bar{x})^2$
5	5 - 6 = -1	$(-1)^2 = 1$
7	7 - 6 = 1	$(1)^2 = 1$
9	9 - 6 = 3	$(3)^2 =$
x	x - 6	$(x - 6)^2$
y	y - 6	$(y - 6)^2$
		$\sum (x_i - \bar{x})^2 = 11 + (x - 6)^2 + (y - 6)^2$

So, Variance, $\sigma^2 = \frac{\sum (x_1 - \bar{x})^2}{n}$

$4 = \frac{11 + (x-6)^2 + (y-6)^2}{5}$

$\Rightarrow 20 = 11 + (x^2 + 36 - 12x) + (y^2 + 36 - 12y)$

$\Rightarrow 20 - 11 = x^2 + 36 - 12x + y^2 + 36 - 12y$

$\Rightarrow x^2 + y^2 + 72 - 12(9) - 9 = 0$ from (i)]

$\Rightarrow x^2 + y^2 + 63 - 108 = 0$

$\Rightarrow x^2 + y^2 = 45$ .... (ii)

From eq.(i)

Now, x + y = 9

$(x + y)^2 = (9)^2$

$\Rightarrow x^2 + y^2 + 2xy = 81$

$\Rightarrow$ 45 + 2xy = 81 [from (ii)]

$\Rightarrow$ 2xy = 81 - 45 $\Rightarrow$ 2xy = 36

$\Rightarrow$ xy =18 $\Rightarrow x = \frac{18}{y}$ .... (iii)

After Putting the value of x in eq. (i) we get

$\Rightarrow \frac{18}{y} + y = 9 \Rightarrow \frac{18+y^2}{y} = 9 \Rightarrow y^2 + 18 = 9y$

$\Rightarrow y^2 - 9y + 18 = 0 \Rightarrow y^2 - 6y - 3y + 18 = 0$

$\Rightarrow$ y(y - 6) - 3(y - 6) = 0 $\Rightarrow$ (y - 3) (y - 6) = 0

$\Rightarrow$ y - 3 = 0 and y - 6 = 0 $\Rightarrow$ y = 3 and y = 6

For y = 3

$x = \frac{18}{y} = \frac{18}{3} = 6$

Hence, x = 6, y = 3 are the remaining two observation

For y = 6

$x = \frac{18}{y} = \frac{18}{6} = 6$

Hence, x = 3, y = 6 are the remaining two observation

Therefore, remaining two observations are 3 and 6

38. First we write the inequalities

$x + y \geq 5$, $2x + 3 \geq 3y$, $0 \leq x \leq 4$, $0 \leq y \leq 2$ in the form of linear equations i.e., x + y = 5, 2x + 3 = 3y, x = 0, x = 4, y = 0 and y = 2, respectively.

Region represented by $x + y \geq 5$

The line x + y = 5 meets the axes at points A (5, 0) and B (0, 5).

x	0	5
y	5	0

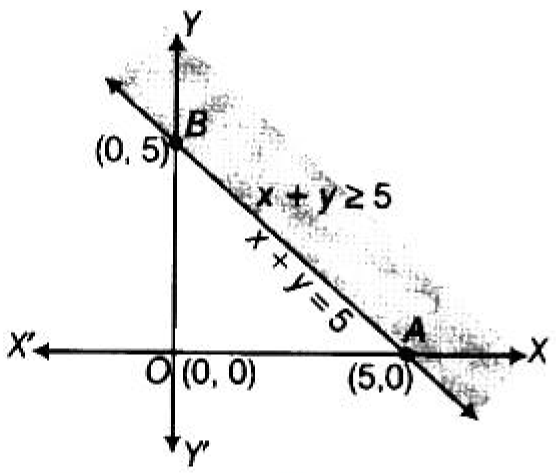
Join these points with a dark line which indicate that the points on this line are included in the solution set.

Now, we check that the line $x + y \geq 5$ contains origin or not.

$0 + 0 \geq 5$

$\Rightarrow 0 \geq 5$, which is false.

So, the region not containing the origin i.e., the region lie above the line x + y = 5 is the solution set of inequality $x + y \geq 5$.



Region represented by $2x + 3 \geq 3y$ or $2x - 3y \geq -3$

Its equation form is $2x + 3 = 3y \Rightarrow 2x - 3y = -3$

The line $2x - 3y = -3$ meets the axes at points P(- 1.5, 0) and Q(0, 1)

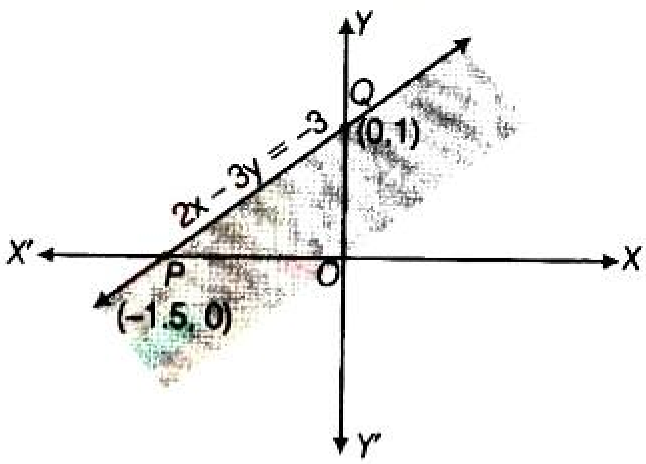
x	0	- 1.5
y	1	0

Join these points with a dark line which indicate that the points on this line are included in the solution set.

Now, we check that the line contains origin or not.

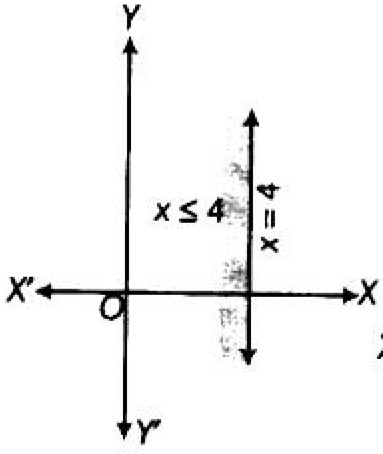
$0 - 0 \geq -3 \Rightarrow 0 \geq -3$, which is true.

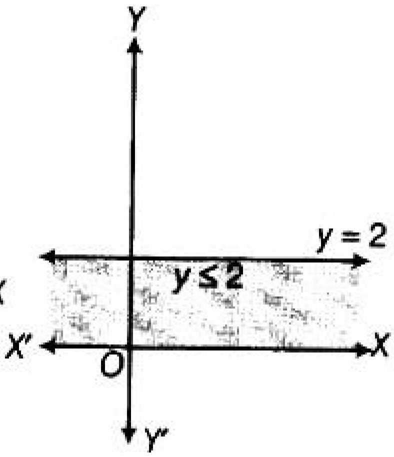
So, the region containing origin i.e., the region below line $2x - 3y = -3$ represents the solution of the inequality $2x - 3y \geq -3$



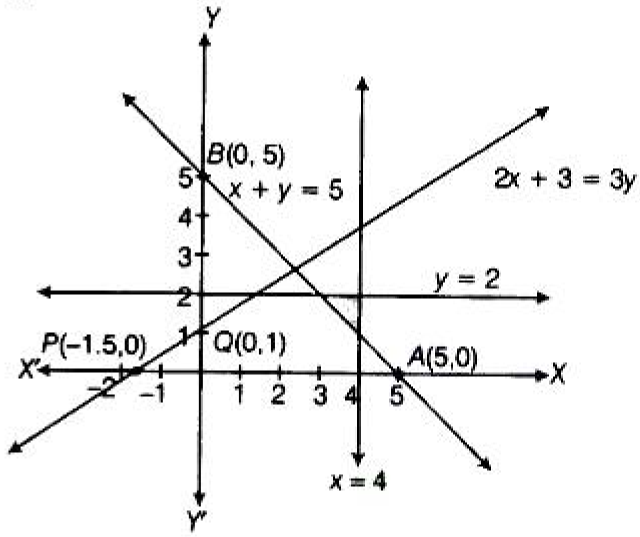
The region represented by $0 \leq x \leq 4$ and $0 \leq y \leq 2$:

$0 \leq x \leq 4$ represents the region on the right side of the Y-axis and left side of the line x = 4 included the points on this line and $0 \leq y \leq 2$ represents the region above the X-axis and below line y = 2 included the points on this line.





Hence, the solution region of given inequalities is obtained by shading the common region.



Any point in this shaded region represents the solution of the given system of inequalities.

OR

We have $\frac{-3x+10}{x+1} > 0$

$\Rightarrow \frac{-3x+10}{x+1} \times (x+1)^2 > 0 \cdot (x+1)^2$ [multiplying both sides by $(x + 1)^2$]

$\Rightarrow$ (-3x + 10)(x + 1)>0

Therefore, Product of (-3x + 10) and (x + 1) will be positive.

Case I: if both are positive.

i.e., (-3x +10) > 0 and (x + 1)>0

$\Rightarrow$ 3x < 10 and x > -1

$\Rightarrow \quad x < \frac{10}{3}$ and $x > -1$

$\Rightarrow \quad -1 < x < \frac{10}{3}$

$\Rightarrow \quad x \in \left(-1, \frac{10}{3}\right)$

Case II: If both are negative.

i.e., (-3x + 10) < 0 and (x + 1) < 0

⇒ -3x < -10 and x < -1

⇒ 3x > 10 and x < -1

$\Rightarrow \quad x > \frac{10}{3}$ and $x < -1$

So, this is impossible. [since, system of inequalities have no common solution]

Thus, the solution is $\left(-1, \frac{10}{3}\right)$.