CBSE Board
Class IX Mathematics
Sample Paper 5

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

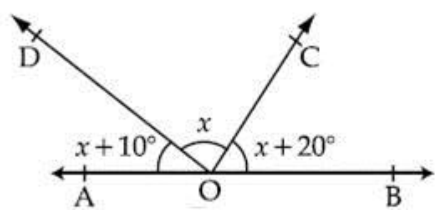
1. What is the decimal form of $\frac{11}{1000}$?

OR

Is zero a rational number? Justify.

2. x + y = 2 and x – y = 4. Find the values of x and y.

3. In the given figure, find the value of x?



4. If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line will lie in which Quadrant?

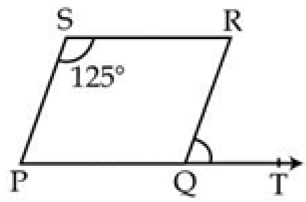
OR

The cost of 5 pencils is equal to the cost of 2 ballpoints. Write a linear equation

d 102.

Determine the Class size?

6. PQRS is a parallelogram in which $\angle PSR = 125°$. Find $\angle RQT$.



Section B
(Questions 7 to 12 carry 2 marks each)

7. If $a = 2 + \sqrt{3}$, find the value of $a + \frac{1}{a}$.

8. The perpendicular distance of a point from the x-axis is 2 units and the perpendicular distance from the y-axis is 5 units. Write the coordinates of such a point if it lies in one of the following quadrants:
(i) I Quadrant (ii) II Quadrant (iii) III Quadrant (iv) IV Quadrant

9. The total surface area of a cube is 294 cm^2. Find its volume.

OR

A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What is the volume of a packet containing 12 such matchboxes?

10. Check which of the following are solutions of the equation $7x - 5y = -3$.
i. $(-1, -2)$
ii. $(-4, -5)$

OR

Explain linear equations in two variables.

11. Find the area of an isosceles triangle with base 10 cm and perimeter 36 cm.

12. Simplify: $(-2x + 5y - 3z)^2$.

Section C
(Questions 13 to 22 carry 3 marks each)

13. Express $0.\overline{001}$ as a fraction in the simplest form.

OR

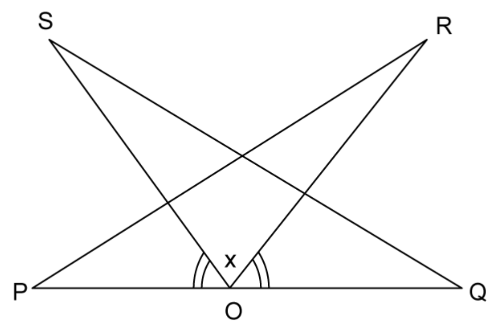
Simplify $\frac{\sqrt{a^2 - b^2} + a}{\sqrt{a^2 + b^2} + b} \div \frac{\sqrt{a^2 + b^2} - b}{a - \sqrt{a^2 - b^2}}$

14. Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

OR

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$.

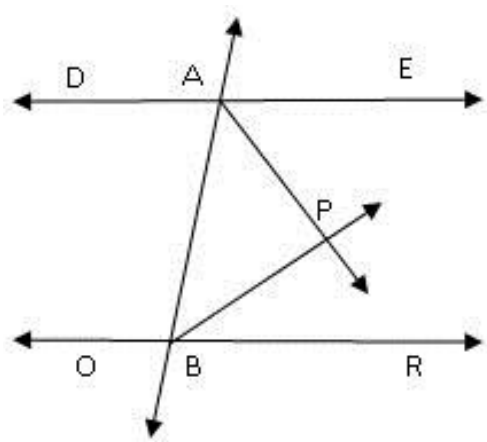
15. In the figure, PQ is a line segment and O is the mid-point of PQ. R and S are on the same side of PQ such that ∠PQS = ∠QPR and ∠POS = ∠QOR. Prove that



(i) ΔPQR ≅ ΔQOS
(ii) PR = QS

16. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, then find the value of a.

17. In the given figure, DE ∥ OR and AP and BP are bisectors of ∠EAB and ∠RBA, respectively. Find ∠APB.



18. The following frequency distribution table gives the weights of 38 students of a class.

Weight in kg	Number of students
30 – 35	10
35 – 40	5
40 – 45	15
45 – 50	5
50 – 55	1
55 – 60	2
Total	38

Find the probability that the weight of students is

i. more than or equal to 45 Kg

ii. less than 30 kg

iii. more than or equal to 30 Kg but less than 60 Kg

19. Find the area of the triangle formed by A (0, 4), O(0, 0) and B(3, 0).

20. If $x = 1+\sqrt{2}$, find the value of $\left(x - \frac{1}{x}\right)^3$.

OR

Find the value of $x^3 + y^3 - 12xy + 64$ when $x + y = -4$.

21. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm and its base is 7 cm, find the total surface area of the article.

OR

How much cardboard is required to make 35 penholders in the shape of cylinders, each of radius 3 cm and height 10.5 cm?

22. The length of 40 leaves of a plant are measured correct to one millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

i. Draw a histogram to represent the given data.
ii. Is there any other suitable graphical representation for the same data?
iii. Is it correct to conclude that maximum leaves are 153 mm long? Why?

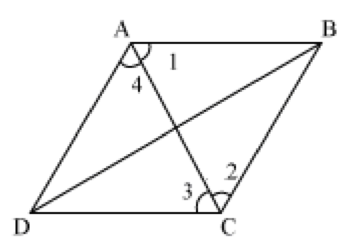
Section D
(Questions 23 to 30 carry 4 marks each)

23. Simplify: $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

OR

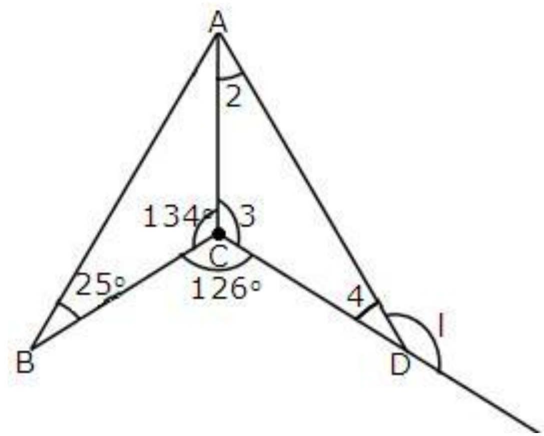
Express $\frac{3}{\sqrt{3}-\sqrt{2}+\sqrt{5}}$ with rational denominator.

24. ABCD is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.



25. If the polynomial $x^4 + mx^3 - 25x^2 - 16x + n$ is exactly divisible by $x^2 - 4$, then what are the values of m and n?

26. In the given figure, AC is the bisector of ΔBAD. Find the measures of $\angle 1, \angle 2, \angle 3$ and $\angle 4$.



27. A cube and cuboid have the same volume. The dimensions of the cuboid are in the ratio 1 : 2 : 4. If the difference between the cost of polishing the cube and cuboid at the rate of Rs. 5 per m^2 is Rs. 80, find their volumes.

OR

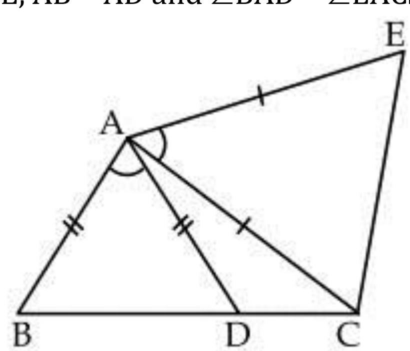
30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the other to form a cylindrical solid. Find the total surface area and volume of the cylinder formed.

28. If O is point lying inside ΔXYZ, then show that (OX + OY + OZ) cannot be less than the semi-perimeter of ΔXYZ.

OR

Show that the bisectors of the base angles of a triangle can never enclose a right angle.

29. In the given figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Prove that BC = DE.



30. If $m = \frac{1}{2-\sqrt{3}}$ and $n = \frac{1}{2+\sqrt{3}}$, then what is the value of $7m^2 + 11mn - 7n^2$?

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Sample Paper 5 – Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. $\frac{11}{1000} = 0.011$

OR

Yes, because 0 can be written as $\frac{0}{1}$ which is of the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

2. $x + y = 2$ ……(i)
and $x - y = 4$ ……(ii)
Adding (i) and (ii), we get
$x = 3$
Putting $x = 3$ in the equation (i), we get
$y = -1$
$\therefore$ $x = 3$ and $y = -1$

3. Since AOB is a Straight Line,
$\therefore \angle AOB = 180°$
$\therefore \angle AOD + \angle COD + \angle COB = 180°$
$\therefore x + 10° + x + x + 20° = 180°$
$\therefore 3x = 150°$
$\therefore x = 50°$

4. If for one of the solutions of the equation $ax + by + c = 0$, x is negative and y is positive, then a portion of the above line definitely lies in the II Quadrant.
As in the II Quadrant the x-axis contains only negative numbers and y-axis contains only positive numbers.

OR

Let the cost of a pencil to be Rs. x and that of a ballpoint to be Rs. y.
Hence, the linear equation is $5x = 2y$
$5x - 2y = 0$.

5. Here the class marks are uniformly spaced,
Therefore,
Class size is the difference between any two consecutive class marks.
$\therefore$ Class Size = 52 – 47 = 5

6. $\angle PSR = \angle RQP = 125°$ (since PQRS is a parallelogram, then the opposite angles will be equal)
$\therefore \angle PQT = 180°$…………….. (PQT is a straight line)
$\therefore \angle PQR + \angle RQT = 180°$
$\therefore 125° + \angle RQT = 180°$
$\therefore \angle RQT = 55°$

Section B

7. $a = 2 + \sqrt{3}$

$$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$= 2 - \sqrt{3}$$

So, $a + \frac{1}{a} = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

8. (i) I quadrant: (5, 2)
(ii) II quadrant: (–5, 2)
(iii) III quadrant: (–5, –2)
(iv) IV quadrant: (5, –2)

9. Let ‘l’ be the length of the cube.
Now, T.S.A. of the cube = 294 cm^2 …(given)
$\therefore 6l^2 = 294$
$\therefore$ Side, l = 7 cm
Volume of cube = l × l × l = 7 × 7 × 7 = 343 cm^3

OR

According to the question,
Volume of one matchbox = 4 cm × 2.5 cm × 1.5 cm = 15 cm^3
Volume of 12 such matchboxes = 15 × 12 = 180 cm^3

10. Given equation is $7x - 5y = -3$

i. $(-1, -2)$

Putting $x = -1$ and $y = -2$ in the L.H.S. of the given equation, we get

$7x - 5y = 7(-1) - 5(-2) = -7 + 10 = 3 \neq$ R.H.S.

$\therefore$ L.H.S. $\neq$ R.H.S.

Hence, $(-1, -2)$ is not a solution of this equation.

ii. $(-4, -5)$

Putting $x = -4$ and $y = -5$ in the L.H.S. of the given equation, we get

$7x - 5y = 7(-4) - 5(-5) = -28 + 25 = -3 =$ R.H.S.

$\therefore$ L.H.S. = R.H.S.

Hence, $(-4, -5)$ is a solution of this equation.

OR

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers such that $a \neq 0$ and $b \neq 0$, is called a linear equation in two variables x and y.

11. Perimeter = 36 cm = 10 cm + 2(Length of an equal side)

$\Rightarrow$ Length of an equal side = 13 cm

Here, $s = \frac{36}{2} = 18$, and the sides are 10, 13 and 13.

By Heron's formula,

Area = $\sqrt{18 \times 8 \times 5 \times 5} = 60$ sq. cm

12. $(-2x + 5y - 3z)^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-2x)(-3z)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

Section C

13. Let $x = 0.\overline{001}$

Then, x= 0.001001001……….. (i)

Therefore, 1000x = 1.001001001…………… (ii)

Subtracting (i) from (ii), we get,

$999x = 1 \Rightarrow x = \frac{1}{999}$

Hence, $0.\overline{001} = \frac{1}{999}$

OR

$$\frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \div \frac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}} = \frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \times \frac{a-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-b}$$

$$= \frac{a^2-\left(\sqrt{a^2-b^2}\right)^2}{\left(\sqrt{a^2+b^2}\right)^2-b^2}$$

$$= \frac{a^2-\left(a^2-b^2\right)}{\left(a^2+b^2\right)-b^2}$$

$$= \frac{a^2-a^2+b^2}{a^2+b^2-b^2}$$

$$= \frac{b^2}{a^2}$$

14. Given: $x = 2y + 6$ or $x - 2y - 6 = 0$

We know that if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3xyz$

Therefore, we have:

$(x)^3 + (-2y)^3 + (-6)^3 = 3x(-2y)(-6)$

Or, $x^3 - 8y^3 - 36xy - 216 = 0$

OR

Let $p(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x + \pi$

Now, $g(x) = 0$

$x + \pi = 0$

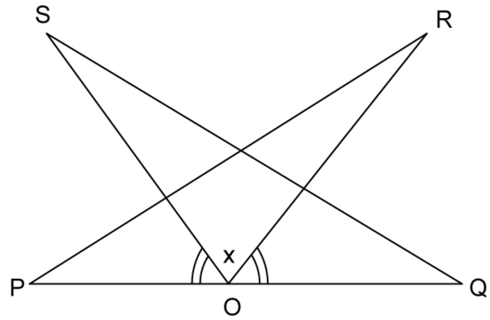
$x = -\pi$

By the remainder theorem, when $p(x)$ is divided by $x + \pi$ then the remainder is $p(-\pi)$.

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 = -\pi^3 + 3\pi^2 - 3\pi + 1$

Hence, remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.

15.



In ΔPOR and ΔQOS

$\angle QPR = \angle PQS$ (given)

OP = OQ (O is the mid-point of PQ)

$\angle POS = \angle QOR$ (given)

$\angle POS + x^o = \angle QOR + x^o$

$\angle POR = \angle QOS$

By ASA congruence rule,

$\Delta PQR \cong \Delta QOS$

$\Rightarrow$ PR = QS (By CPCT)

16. Let $p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$

When p(z) is divided by z − 3, the remainder is given by

$p(3) = a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4$

$= 27a + 36 + 9 - 4$

$= 27a + 41$ ....(i)

When q(z) is divided by z − 3, the remainder is given by

$q(3) = 3^3 - 4 \times 3 + a$

$= 27 - 12 + a$

$= 15 + a$ ....(ii)

Given that p(3) = q(3).

So, from (i) and (ii), we have

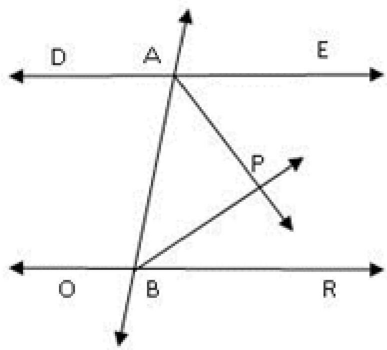
$27a + 41 = 15 + a$

$27a - a = -41 + 15$

$26a = -26$

$a = -1$

17. Since interior angles on the same side of transversal are supplementary.



Therefore, $\angle EAB + \angle RBA = 180°$

$\frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = \frac{1}{2} \times 180°$ ....(i)

As AP and BP are bisectors of $\angle EAB$ and $\angle RBA$ respectively,

$\angle PAB = \frac{1}{2}\angle EAB$ and $\angle PBA = \frac{1}{2}\angle RBA$ ....(ii)

From (i) and (ii), we get

$\angle PAB + \angle PBA = 90°$

In $\triangle APB$, we have

$\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow 90° + \angle APB = 180°$

$\Rightarrow \angle APB = 180° - 90° = 90°$

18. Total number of students = 38

i. Number of students weighing more than or equal to 45 kg $= 5 + 1 + 2 = 8$

$\therefore$ P(a student weighing more than or equal to 45 kg) $= \frac{8}{38} = \frac{4}{19}$

ii. Number of students weighing less than 30 kg $= 0$

$\therefore$ P(a student weighing less than 30 kg) $= \frac{0}{38} = 0$

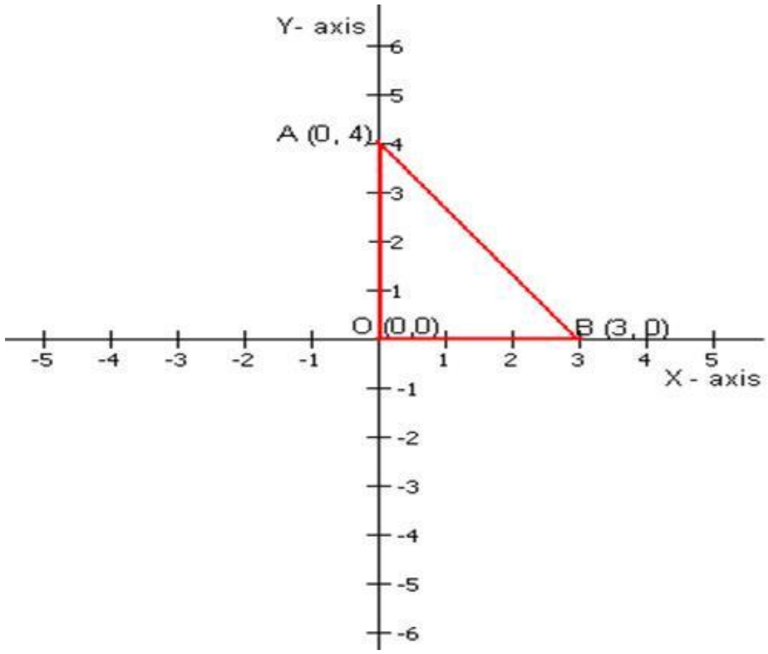
iii. Number of students weighing more than or equal to 30 kg but less than 60 kg

$= 10 + 5 + 15 + 5 + 1 + 2$

$= 38$

$\therefore$ Required probability $= \frac{38}{38} = 1$

19. The given points A(0, 4), O(0, 0), B(3, 0) can be plotted as follows:



Clearly, AOB is a right-angled triangle.

OA = 4 units, OB = 3 units.

Area of $\Delta AOB = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 3 \times 4 = 6$ square units

20. Given, $x = 1 + \sqrt{2}$

And, $\frac{1}{x} = \frac{1}{1+\sqrt{2}}$

$= \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \frac{1-\sqrt{2}}{1^2 - \left(\sqrt{2}\right)^2}$

$= -1 + \sqrt{2}$

Now, $\left(x - \frac{1}{x}\right) = \sqrt{2} + 1 - (\sqrt{2} - 1) = 2$

$\therefore \left(x - \frac{1}{x}\right)^3 = 2^3 = 8$

OR

$x^3 + y^3 - 12xy + 64$

$= x^3 + y^3 + 4^3 - 3xy \times 4$

$= x^3 + y^3 + z^3 - 3xyz$ where $z = 4$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$ $\because z = 4$

$= (-4 + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$ $\because x + y = 4$

$= 0$

Hence, $x^3 + y^3 - 12xy + 64 = 0$.

21. Base radius, $r = \frac{7}{2} = 3.5$ cm

Height of the cylinder, h = 10 cm

Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \pi \times 3.5 \times 10$

$= 70\pi \text{ cm}^2$

Curved surface area of two hemisphere $= 2 \times 2\pi r^2$

$= 4 \times \pi \times 3.5^2$

$= 49\pi \text{ cm}^2$

$\therefore$ Total surface area = $70\pi + 49\pi = 374 \text{ cm}^2$

OR

Cardboard required to make 1 penholder = Curved surface area + area of the base

$= 2\pi rh + \pi r^2$

$= \pi r (2h + r)$

$= \frac{22}{7} \times 3 \times \left(2 \times \frac{21}{2} + 3\right)$

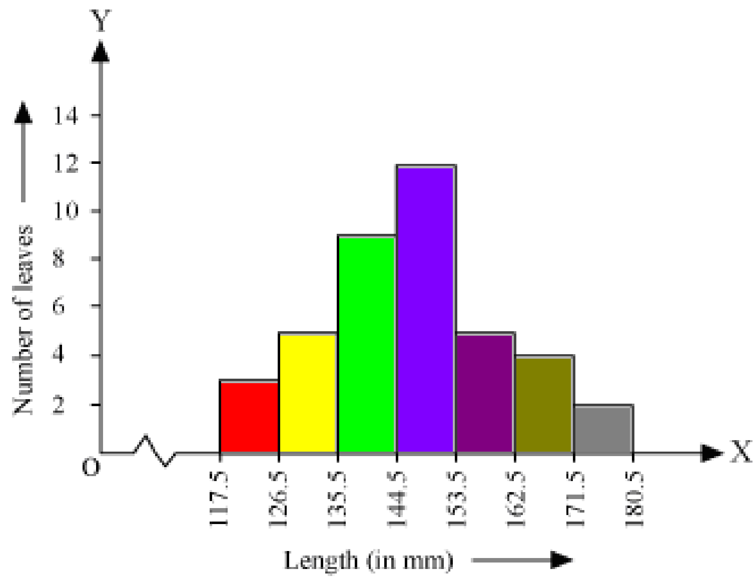
$= \frac{22}{7} \times 3 \times 24$

Cardboard required to make 35 penholders. = $\frac{22}{7} \times 3 \times 24 \times 35$ = 7920 cm^2

22. Lengths of the leaves are represented in discontinuous class intervals. Hence we have to add 0.5 mm to each upper class limit and also have to subtract 0.5 mm from the lower class limits so as to make our class intervals continuous.

Length (in mm)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

i. Now taking the length of leaves on the x-axis and number of leaves on the y-axis we can draw the histogram of this information as below:



Here 1 unit on the y-axis represents 2 leaves.

ii. Other suitable graphical representation of this data could be a frequency polygon.

iii. No, as maximum numbers of leaves (i.e. 12) have their length in between of 144.5 mm and 153.5 mm. It is not necessary that all have a length of 153 mm.

Section D

23.

We have,

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)}+\frac{\sqrt{12}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$

$$=\frac{\left(3\sqrt{2}\right)^2+\left(2\sqrt{3}\right)^2-2\times3\sqrt{2}\times2\sqrt{3}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{2\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{18+12-12\sqrt{6}}{18-12}+\frac{6+2\sqrt{6}}{3-2}$$

$$=\frac{30-12\sqrt{6}}{6}+6+2\sqrt{6}$$

$$=\frac{6\left(5-2\sqrt{6}\right)}{6}+6+2\sqrt{6}$$

$$=5-2\sqrt{6}+6+2\sqrt{6}$$

$$=11$$

$$\therefore \quad \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}=11$$

OR

$$\frac{3}{\sqrt{3}-\sqrt{2}+\sqrt{5}} = \frac{3}{\sqrt{3}-\sqrt{2}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{2}-\sqrt{5}}{\sqrt{3}-\sqrt{2}-\sqrt{5}}$$

$$= \frac{3\left(\sqrt{3}-\sqrt{2}-\sqrt{5}\right)}{\left(\sqrt{3}-\sqrt{2}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{3\left(\sqrt{3}-\sqrt{2}-\sqrt{5}\right)}{\left(3+2-2\sqrt{6}\right)-5}$$

$$= \frac{3\left(\sqrt{3}-\sqrt{2}-\sqrt{5}\right)}{-2\sqrt{6}}$$

$$= \frac{3\left(\sqrt{3}-\sqrt{2}-\sqrt{5}\right)}{-2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

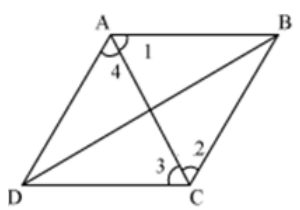
$$= \frac{3\left(\sqrt{18}-\sqrt{12}-\sqrt{30}\right)}{-12}$$

$$= \frac{3\left(3\sqrt{2}-2\sqrt{3}-\sqrt{30}\right)}{-12}$$

$$= \frac{3\sqrt{2}-2\sqrt{3}-\sqrt{30}}{-4}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{4}$$

24. Let us join AC.



In ΔABC

BC = AB (sides of a rhombus)

$\therefore \angle 1 = \angle 2$ (angles opposite to equal sides of a triangle are equal)

But $\angle 1 = \angle 3$ (alternate interior angles)

$\Rightarrow \angle 2 = \angle 3$

So, AC bisects $\angle C$.

Also, $\angle 2 = \angle 4$ (alternate interior angles)

$\Rightarrow \angle 1 = \angle 4$

So, AC bisects $\angle A$

Similarly, we can prove that BD bisects $\angle B$ and $\angle D$ as well.

25. Let $p(x) = x^4 + mx^3 - 25x^2 - 16x + n$ and $g(x) = x^2 - 4 = (x + 2)(x - 2)$

Since p(x) is exactly divisible by g(x), p(x) is divisible by both (x + 2) and (x – 2).

Using factor therom, we have

$p(-2) = 0$

$\Rightarrow (-2)^4 + m(-2)^3 - 25(-2)^2 - 16(-2) + n = 0$

$\Rightarrow 16 - 8m - 100 + 32 + n = 0$

$\Rightarrow n - 8m - 52 = 0$ ....(1)

Also, $p(2) = 0$

$\Rightarrow (2)^4 + m(2)^3 - 25(2)^2 - 16(2) + n = 0$

$\Rightarrow 16 + 8m - 100 - 32 + n = 0$

$\Rightarrow n + 8m - 116 = 0$ ...(2)

From equation (1) and (2), we get

$n - 8m - 52 = n + 8m - 116$ (Each is 0)

$\Rightarrow -8m - 52 = 8m - 116$

$\Rightarrow 116 - 52 = 16m$

$\Rightarrow m = \frac{64}{16} = 4$

Substituting the value of m in equation (1),

$n - 8(4) - 52 = 0$

$\Rightarrow n - 32 - 52 = 0$

$\Rightarrow n - 84 = 0$

$\Rightarrow n = 84$

Thus, the values of m and n are 4 and 84 respectively.

26. Applying angle sum property in ΔABC, we have

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\Rightarrow 25° + 134° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 1590 = 210$

Now, AC is the bisector of $\angle BAD$.

$\therefore \angle BAC = \angle CAD$

$\Rightarrow \angle 2 = 21°$ $(\because \angle BAC = 210)$

We know that the measure of one complete angle is 3600.

$\therefore \angle BCD + \angle BCA + \angle ACD = 360°$

$\Rightarrow 126° + 134° + \angle 3 = 360°$

$\Rightarrow \angle 3 = 360° - 260° = 100°$

By exterior angle property of triangles, we have

$\angle 1 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = 21° + 100° = 121°$

By applying angle sum property in ΔACD, we have:

$\angle ACD + \angle CDA + \angle CAD = 180°$

$\Rightarrow 100° + \angle 4 + 21° = 180°$

$\Rightarrow \angle 4 = 180° - 121° = 59°$

27. Let the dimensions of the cuboid are x, 2x and 4x.

Hence, volume of cuboid = $x \times 2x \times 4x = 8x^3$

Volume of cube = Volume of cuboid

∴ Volume of cube = $8x^3 = \text{side}^3$

∴ Side of cube = 2x

Total Surface Area of cuboid = $2(lb + bh + hl) = 2(2x^2 + 8x^2 + 4x^2) = 28x^2$

Cost of polishing the cuboid = Rs. $5 \times 28x^2 = 140x^2$

Total Surface Area of cube = $6(\text{side})^2 = 6(2x)^2 = 24x^2$.

Cost of polishing the cube = Rs. $5 \times 24x^2 = 120x^2$

The difference between cost of polishing the cube and the cuboid is Rs. 80.

Hence, $140x^2 - 120x^2 = 80$

$\Rightarrow 20x^2 = 80$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2$

Volume of cuboid = Volume of cube = $8 \times (2)^3\ m^3 = 64\ m^3$.

OR

The cylinder formed as base radius, r = 14 cm and height = h = 3 × 30 = 90 cm

The total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 14 + (90 + 14)$$

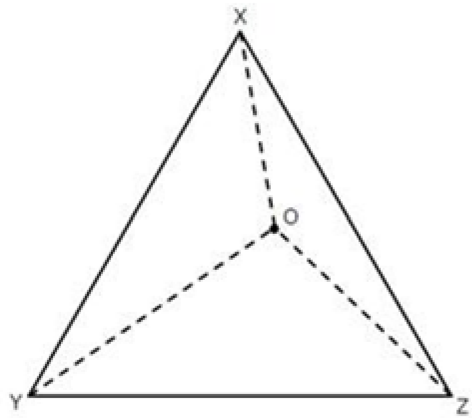
$$= 9152\ cm^2$$

Volume of the cylinder formed = $\pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 90$$

$$= 55440\ cm^3$$

28. The sum of any two sides of a triangle is always greater than the third side.



In ΔOXY, OX + OY > XY ....(1)

In ΔOYZ, OY + OZ > YZ ....(2)

In ΔOXZ, OX + OZ > XZ ....(3)

Adding equations (1), (2) and (3),

(OX + OY) + (OY + OZ) + (OX + OZ) > XY + YZ + ZX

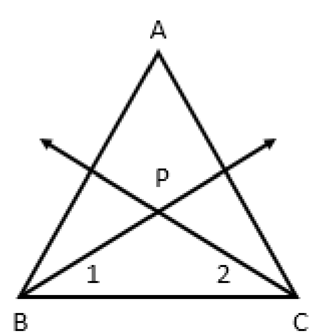
2(OX + OY + OZ) > XY + YZ + ZX

$OX + OY + OZ > \frac{1}{2}(XY + YZ + ZX)$

OX + OY + OZ > Semi-perimeter of ΔXYZ.

Hence, (OX + OY + OZ) cannot be less than the semi-perimeter of ΔXYZ.

OR



In ΔABC, BP and CP are bisectors of angles B and C respectively.

Hence, $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 2\angle 1 + 2\angle 2 = 180^\circ$

$2\angle 1 + 2\angle 2 = 180^\circ - \angle A$

$\angle 1 + \angle 2 = 90^\circ - \angle A/2$........ (1)

In ΔPBC,

∠P + ∠1 + ∠2 = 180˚

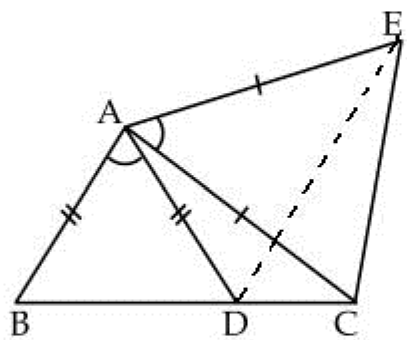
∠P + [90˚ – ∠A/2] = 180˚ from (i)

∠P = 180˚ – [90˚ – ∠A/2] = 90˚ + ∠A/2

Hence, angle P is always greater than 90˚.

Thus, PBC can never be a right angled triangle.

29. Join DE.



∠ BAD = ∠ EAC

∠ BAD + ∠ DAC = ∠ EAC + ∠ DAC

∠ BAC = ∠ DAE --------- (i)

In ΔABC & Δ ADE

AB = AD (Given)

∠ BAC = ∠ DAE (from (i))

AC =AE

So, By SAS congrience criteria

ΔABC ≅ ΔADE

BC = DE (CPCT)

30.

$$m=\frac{1}{2-\sqrt{3}} \text{ and } n=\frac{1}{2+\sqrt{3}}$$

$$\therefore\ 7m^2+11mn-7n^2$$

$$=7\times\left(\frac{1}{2-\sqrt{3}}\right)^2+11\left(\frac{1}{2-\sqrt{3}}\right)\left(\frac{1}{2+\sqrt{3}}\right)-7\left(\frac{1}{2+\sqrt{3}}\right)^2$$

$$=\frac{7}{4+3-4\sqrt{3}}+\frac{11}{4-3}-\frac{7}{4+3+4\sqrt{3}}$$

$$=\frac{7}{7-4\sqrt{3}}+11-\frac{7}{7+4\sqrt{3}}$$

$$=\frac{7}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}+11-\frac{7}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$=\frac{7\left(7+4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}+11-\frac{7\left(7-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}$$

$$=\frac{7\left(7+4\sqrt{3}\right)}{49-48}+11-\frac{7\left(7-4\sqrt{3}\right)}{49-48}$$

$$=49+28\sqrt{3}+11-49+28\sqrt{3}$$

$$=11+56\sqrt{3}$$