Chapter – 11

Waves

Multiple Choice Questions

Question 1.
A student tunes his guitar by striking a 120 Hertz with a tuning fork, and simultaneously plays the 4^{th} string on his guitar. By keen observation, he hears the amplitude of the combined sound oscillating thrice per second. Which of the following frequencies is the most likely the frequency of the 4^{th} string on his guitar?
(a) 130
(b) 117
(c) 110
(d) 120

Answer:
(b) 117

Question 2.
A transverse wave moves from a medium A to a medium B. In medium A, the velocity of the transverse wave is 500 ms^{-1} and the wavelength is 5 m. The frequency and the wavelength of the wave in medium B when its velocity is 600 ms^{-1}, respectively are
(a) 120 Hz and 5 m
(b) 100 Hz and 5 m
(c) 120 Hz and 6 m
(d) 100 Hz and 6 m

Answer:
(d) 100 Hz and 6 m

Question 3.
For a particular tube, among six harmonic frequencies below 1000 Hz, only four harmonic frequencies are given : 300 Hz, 600 Hz, 750 Hz and 900 Hz. What are the two other frequencies missing from this list?
(a) 100 Hz, 150 Hz
(b) 150 Hz, 450 Hz

(c) 450 Hz, 700 Hz
(d) 700 Hz, 800 Hz

Answer:
(b) 150 Hz, 450 Hz

Hint:
If the tube is open at both ends so the harmonic frequencies are based on 150 Hz.
1^{st} = 150 Hz ; 2^{nd} = 300 Hz ; 3^{rd} = 450 Hz ; 4^{th} = 600 Hz ; 5^{th} = 750 Hz ; 6^{th} = 900 Hz
The above frequencies the missing frequency in the list 150 Hz, 450 Hz

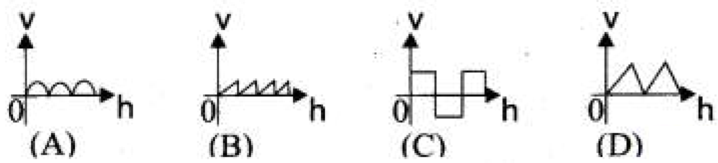
Question 4.
Which of the following options is correct?

A		B	
(1)	Quality	(A)	Intensity
(2)	Pitch	(B)	Waveform
(3)	Loudness	(C)	Frequency

Options for (1), (2) and (3), respectively are
(a) (B), (C) and (A)
(b) (C), (A) and (B)
(c) (A), (B) and (C)
(d) (B), (A) and (C)

Answer:
(a) (B), (C) and (A)

Question 5.
Compare the velocities of the wave forms given below, and choose the correct option.



where, v_A, v_B, v_C and v_D are velocities given in (A), (B), (C) and (D), respectively.

(a) $V_A > V_B > V_D > V_C$
(b) $V_A < V_B < V_D < V_C$
(c) $V_A = V_B = V_D = V_C$
(d) $V_A > V_B = V_D > V_C$

Answer:
(c) $V_A = V_B = V_D = V_C$

Question 6.
A sound wave whose frequency is 5000 Hz travels in air and then hits the water surface. The ratio of its wavelengths in water and air is ....
(a) 4.30
(b) 0.23
(c) 5.30
(d) 1.23

Answer:
(a) 4.30
Hint.
Frequency of sound, f = 5000 Hz

Velocity of sound in air, $v_a = 343\ ms^{-1}$
Velocity of sound in water, $v_w = 1480\ ms^{-1}$

The ratio of wavelength $\frac{\lambda_a}{\lambda_w} = \frac{v_w}{f} \times \frac{f}{v_a} = \frac{1480}{5000} \times \frac{5000}{343} = \frac{1480}{343}$

$= 4.31$

Question 7.
A person standing between two parallel hills fires a gun and hears the first echo after t_1 sec and the second echo after t_2 sec. The distance between the two hills is .....

(a) $\frac{v(t_1 - t_2)}{2}$ (b) $\frac{v(t_1 t_2)}{2(t_1 + t_2)}$ (c) $v(t_1 + t_2)$ (d) $\frac{v(t_1 + t_2)}{2}$

Answer:

(d) $\frac{v(t_1 + t_2)}{2}$

Hint:

Distance between man and hill 1, $d_1 = v \times \frac{t_1}{2}$

Distance between man and hill 2, $d_2 = v \times \frac{t_2}{2}$

Distance between the two hills, $d = d_1 + d_2$

$$d = \frac{v(t_1 + t_2)}{2}$$

Question 8.

An air column in a pipe which is closed at one end, will be in resonance with the vibrating body of frequency 83 Hz. Then the length of the air column is ………

(a) 1.5 m
(b) 0.5 m
(c) 1.0 m
(d) 2.0 m

Answer:

(c) 1.0 m

Hint:

Frequency, $f = \frac{v}{\lambda}$

$$83 = \frac{v}{\lambda}$$

For air column in a pipe closed at one end,

$$l_1 = \frac{\lambda}{4} \Rightarrow \lambda = 4l_1$$

$$\therefore \quad 83 = \frac{v}{4l_1}$$

Velocity of sound in air y = 343 ms^{-1}

$$83 = \frac{343}{4l_1}$$

$$\therefore \quad l_1 = \frac{343}{332} = 1.033\text{m}$$

Question 9.
The displacement y of a wave travelling in the x direction is given by $y = (2 \times 10^{-3}) \sin\left(300t - 2x + \frac{\pi}{4}\right)$, where x and y are measured in metres and t in second. The speed of the wave is ………
(a) 150 ms^{-1}
(b) 300 ms^{-1}
(c) 450 ms^{-1}
(d) 600 ms^{-1}

Answer:
(a) 150 ms^{-1}

Hint:
From the standard equation of wave,

$$y = a \sin(\omega t - kx + \phi)$$

Here, $\omega = 600$ and $k = 2$

So, speed of wave is, $v = \frac{\omega}{k} = \frac{600}{2}$; $v = 150$ m/s

Question 10.
Consider two uniform wires vibrating simultaneously in their fundamental notes. The tensions, densities, lengths and diameter of the two wires are in the ratio 8 : 1, 1 : 2, x : y and 4 : 1 respectively. If the note of the higher pitch has a frequency of 360 Hz and the number of beats produced per second is 10, then the value of x : y is ……….
(a) 36 : 35
(b) 35 : 36
(c) 1 : 1
(d) 1 : 2

Answer:
(a) 36 : 35

Question 11.
Which of the following represents a wave?
(a) $(x - vt)^3$
(b) $x(x + vt)$
(c) $\frac{1}{(x+vt)}$
(d) $\sin (x + vt)$

Answer:
(d) $\sin (x + vt)$

Question 12.
A man sitting on a swing which is moving to an angle of 60° from the vertical is blowing a whistle which has a frequency of 2.0 k Hz. The whistle is 2.0 m from the fixed support point of the swing. A sound detector which detects the whistle sound is kept in front of the swing. The maximum frequency the sound detector detected is ....
(a) 2.027 kHz
(b) 1.947 kHz
(c) 9.74 kHz
(d) 1.011 kHz

Answer:
(a) 2.027 kHz

Question 13.
Let. $y = \frac{1}{1+x^2}$ at t = 0s be the amplitude of the wave propagating in the positive x-direction. At t = 2s, the amplitude of the wave propagating becomes $y = \frac{1}{1+(x-2)^2}$ Assume that the shape of the wave does not change during propagation. The velocity of the wave is .....
(a) 0.5 ms^{-1}
(b) 1.0 ms^{-1}
(c) 1.5 ms^{-1}
(d) 2.0 ms^{-1}

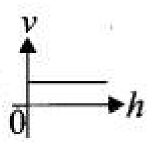
Answer:
(b) 1.0 ms^{-1}
Hint.
The general expression y in terms of x

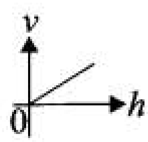
$$y = \frac{1}{1+(x-vt)^2}$$

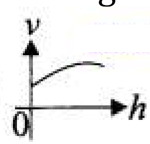
The shape of wave does not change, also wave move in 2 sec, 2m in positive ‘x’ direction. So, wave moves 2m in 2 sec.

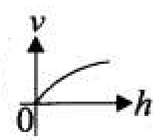
$$\therefore \text{ The velocity of the wave } = \frac{\text{displacement}}{\text{time}} = \frac{2}{2}\ ;\ v = 1\ ms^{-1}$$

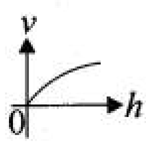
Question 14.
A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. Which of the following plots shows the correct variation of speed v with height h from the lower end?

(a) 

(b) 

(c) 

(d) 

Answer:
(d) 

Question 15.
An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is ....

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

Answer:
(c) 1/6

Hint:
(c) 1/6

Short Answer Questions

Question 1.
What is meant by waves?

Answer:
The disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a wave.

Question 2.
Write down the types of waves.

Answer:
Waves can be classified into two types:
(a) Transverse waves
(b) Longitudinal waves

Question 3.
What are transverse waves? Give one example.

Answer:
In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves.
Example: light (electromagnetic waves)

Question 4.
What are longitudinal waves? Give one example.

Answer:
In longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves.
Example: Sound waves travelling in air.

Question 5.
Define wavelength.

Answer:
For transverse waves, the distance between two neighbouring crests or troughs is known as the wavelength. For longitudinal waves, the distance between two neighbouring compressions or rarefactions is known as the wavelength. The SI unit of wavelength is meter.

Question 6.
Write down the relation between frequency, wavelength and velocity of a wave.

Answer:
Frequency, $f = \frac{1}{\text{Time period}}$, which implies that the dimension of frequency is,

$$[f] = \frac{1}{[T]} = T^{-1} \Rightarrow [\lambda f] = [\lambda][f] = LT^{-1} = [\text{Velocity}]$$

Therefore, Velocity, $\lambda f = v$

Question 7.
What is meant by interference of waves?

Answer:
Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

Question 8.
Explain the beat phenomenon.

Answer:
When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two. sources, then their difference in frequency gives the beat frequency. Number of beats per second $n = | f_1 - f_2|$ per second

Question 9.
Define intensity of sound and loudness of sound.

Answer:

1. The loudness of sound is defined as "the degree of sensation of sound produced in the ear or the perception of sound by the listener".

2. The intensity of sound is defined as "the sound power transmitted per unit area taken normal to the propagation of the sound wave".

Question 10.
Explain Doppler Effect.

Answer:
When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.

Question 11.
Explain red shift and blue shift in Doppler Effect.

Answer:
If the spectral lines of the star are found to shift towards red end of the spectrum (called as red shift) then the star is receding away from the Earth. Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum (called as blue shift) then the star is approaching Earth.

Question 12.
What is meant by end correction in resonance air column apparatus?

Answer:
The antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is

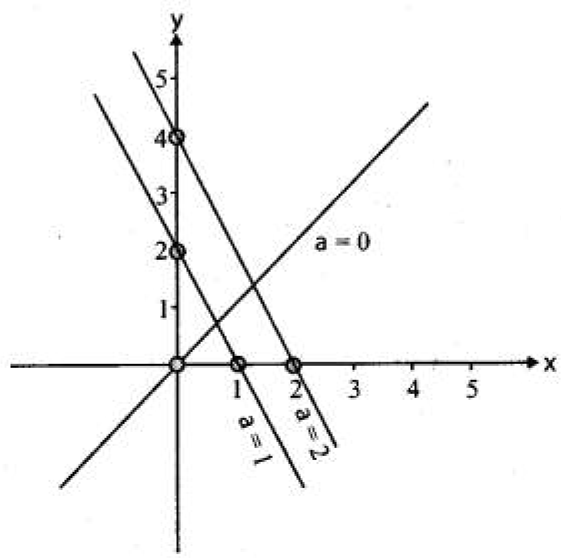
$\frac{1}{4}\lambda = L_1 + e$

Again taking end correction into account, we have

$\frac{3}{4}\lambda = L_2 + e$

Question 13.
Sketch the function Y = x + a. Explain your sketch

Answer:
When a = 0, y = x
when a = 1; x = 1: y = 1 + 1 = 2
when a = 2; x = 2; y = 2 + 2 = 4



Explanation: This implies, when increasing the value of a, the line shifts towards right side at a = 0, and line shifts towards left side at a = 1, 2, ….For a = vt, y = x – vt satisfies the differential equation. Though this function satisfies the differential equation, it is not finite for all values of x and t. Hence it does not represent a waves.

Question 14.
Write down the factors affecting velocity of sound in gases.

Answer:
(a) Effect of pressure
(b) Effect of temperature

(c) Effect of density
(e) Effect of wind

Question 15.
What is meant by an echo? Explain.

Answer:
Echo: An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces.

Explanation: The speed of sound in air at 20°C is 344 m s^{-1}. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{\text{th}}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{Velocity} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2d}{t}$$

$2d = 344 \times 0.1 = 34.4\text{m}$; $d = 17.2\text{m}$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

Long Answer Questions

Question 1.
Discuss how ripples are formed in still water.

Answer:
Suppose we drop a stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface. We find that this disturbance spreads out (diverges out) in the form of concentric circles of ever-increasing radii (ripples) and strike the boundary of the trough. This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually, the particles of the water (medium) themselves do not move outward with the disturbance. This can be observed by keeping a paper strip on the water surface. The strip moves up and down

when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their – mean positions.

Question 2.
Briefly explain the difference between travelling waves and standing waves.

Answer:

S.No.	Progressive waves	Stationary waves
1.	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2.	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same. particles. The amplitude is minimum or zero at nodes and maximum at antinodes.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different
3.	These wave carry energy while propagating.	These waves do not transport energy.

Question 3.
Show that the velocity of a travelling wave produced in a string is $v = \sqrt{\frac{T}{\mu}}$

Answer:
Velocity of transverse waves in a stretched string: Let us compute the velocity of transverse travelling waves on a string. When a jerk is given at one end (left end) of the rope, the wave pulses move towards right end with a velocity v. This means that the pulses move with a velocity v with respect to an observer who is at rest frame.

Suppose an observer also moves with same velocity v in the direction of motion of the wave pulse, then that observer will notice that the wave pulse is stationary and the rope is moving with pulse with the same velocity v. Consider an elemental segment in the string. Let A and B be two points on the string at an instant of time. Let dl and dm be the length and mass of the elemental string, respectively. By definition, linear mass density, μ is

$$\mu = \frac{dm}{dl} \qquad ...(1)$$

$$dm = \mu dl \qquad ...(2)$$

The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ at the origin O. The angle θ can be written in terms of arc length and radius as $\theta = \frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is

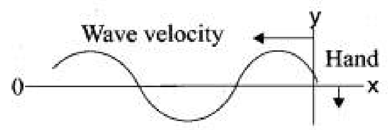
$$a_{cp} = \frac{v^2}{R} \qquad ...(3)$$

Then, centripetal force can be obtained when mass of the string (dm) is included in equation (3)

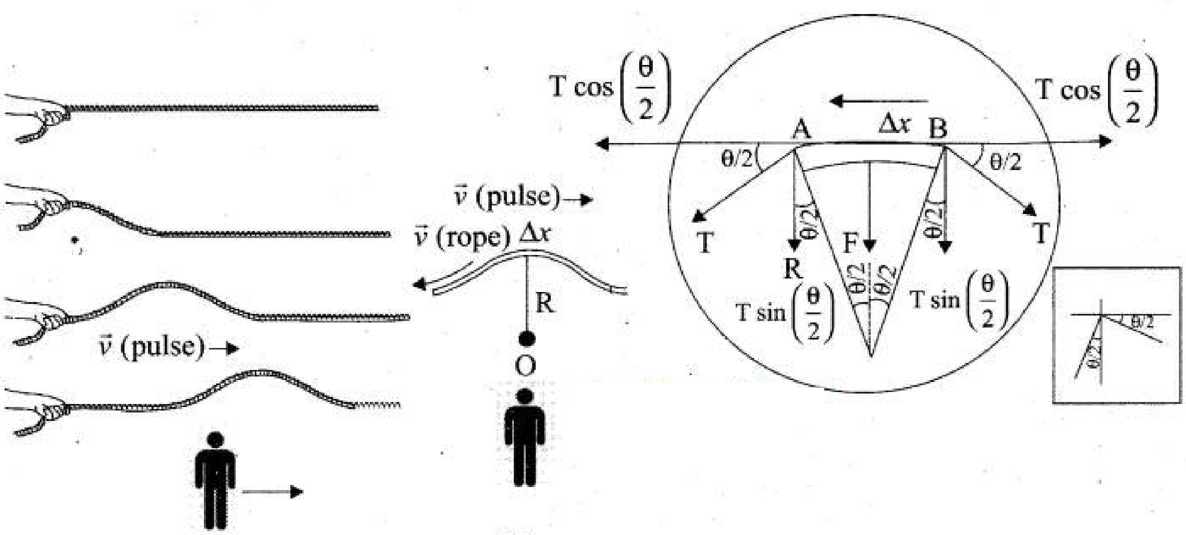
$$F_{cp} = \frac{(dm)v^2}{R} \qquad ...(4)$$

The centripetal force experienced by elemental string can be calculated by substituting equation (2) in equation (4) we get

$$\frac{(dm)v^2}{R} = \frac{\mu v^2 dl}{R} \qquad ...(5)$$



(a)
Transverse waves in a stretched string



(b)
Elemental segment in a stretched string is zoomed and the pulse seen from an observer frame who moves with velocity v

The tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored. We can resolve T into horizontal component T cos $\left(\frac{\theta}{2}\right)$ and vertical component T sin $\left(\frac{\theta}{2}\right)$ The horizontal component at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to acts Vertical towards the centre of the arc and hence, they add up. The net radial force F_r is

$$F_r = 2T\sin\left(\frac{\theta}{2}\right) \quad ...(6)$$

Since the amplitude of the wave is very small when it is compared with the length of the spring, the sine of small angle is approximated as sin $\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$.

Hence equation (6) can be written as

$$F_r = 2T\times\frac{\theta}{2} = T\theta \quad ...(7)$$

But $\theta = \frac{dl}{R}$, , therefore substituting in equation (7), we get

$$F_r = T\frac{dl}{R} \quad ...(8)$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (5)

and equation (8), we have $T\frac{dl}{R} = \mu v^2\frac{dl}{R}$

$$v = \sqrt{\frac{T}{\mu}} \text{ measured in } ms^{-1} \quad ...(9)$$

Question 4.

Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.

Answer:

Newton's formula for speed of sound waves in air: Sir Isaac Newton assumed that when sound propagates in air, the formation of compression and

rarefaction takes place in a very slow manner so that the process is isothermal in nature. That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law, Mathematically
PV = constant ...(1)

Differentiating equation (1), we get

$$P\,dV + V\,dP = 0 \text{ (or) } P = -V\frac{dP}{dV} = B_T \qquad ...(2)$$

where, B_T is an isothermal bulk modulus of air. Substituting equation (2) in equation the speed of sound in air is

$$v_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad ...(3)$$

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8)\ \mathrm{N\,m^{-2}}$$
$$\rho = 1.293\ \mathrm{kg\,m^{-3}}$$

Here ρ is density of air, then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$v_T = \sqrt{\frac{(0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}}$$
$$= 279.80\ \mathrm{ms^{-1}} \approx 280\ \mathrm{ms^{-1}} \text{ (theoretical value)}$$

But the speed of sound in air at 0°C is experimentally observed as 332 m s^{-1} which is close upto 16% more than theoretical value (Percentage error is $\frac{(332-280)}{332} \times 100\% = 15.6\%$). This error is not small.

Laplace's correction: In 1816, Laplace satisfactorily corrected this discrepancy by assuming that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat. Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$PV^{\gamma} = \text{constant} \qquad ...(4)$$

where, $\gamma = \frac{C_P}{C_v}$ which is the ratio between specific heat at constant pressure and specific heat at constant volume. Differentiating equation (4) on both the sides, we get

$$V^{\gamma}\, dP + P(\gamma V^{\gamma-1}\, dV) = 0 \text{ (or) } \gamma P = -V\frac{dP}{dV} = B_A \qquad ...(5)$$

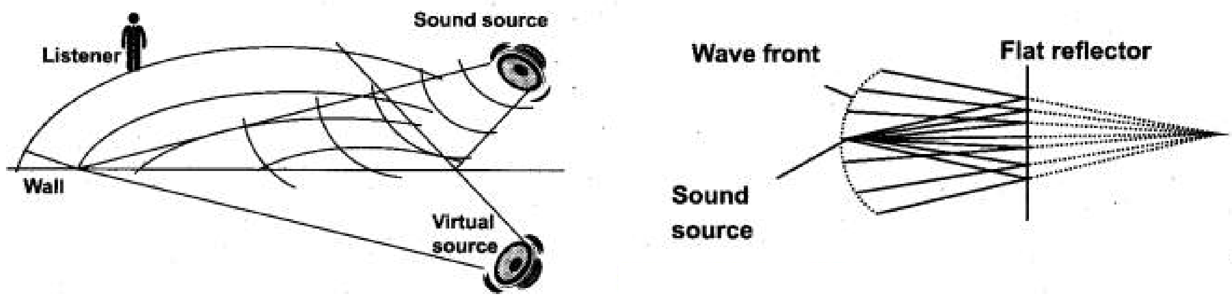
where, B_A is the adiabatic bulk modulus of air. Now, substituting equation (5) in equation $v = \sqrt{\frac{B}{\rho}}$, the speed of sound in air is $v_A = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma}\, v_T$

Since air contains mainly, nitrogen, oxygen, hydrogen etc, (diatomic gas), we take $\gamma = 1.47$.

Hence, speed of sound in air is $v_A = \left(\sqrt{1.4}\right)$ (280 m s^{-1}) = 331.30 m s^{-1}, which is very much closer to experimental data.

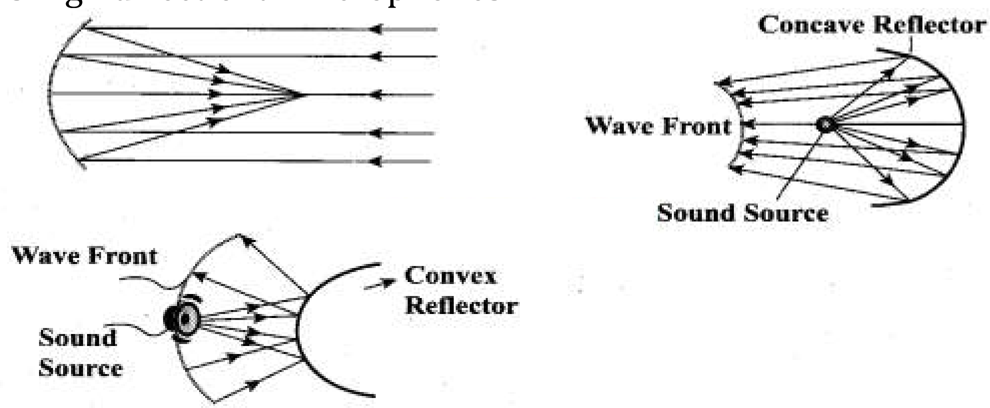
Question 5.
Write short notes on reflection of sound waves from plane and curved surfaces. Reflection of sound through the plane surface

Answer:
When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front). Therefore, the reflected wave front on the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker) which can be assumed to be at a position behind the plane surface.



Reflection of sound through the plane surface

Reflection of sound through the curved surface: The behaviour of sound is different when – it is reflected from different surfaces-convex or concave or plane. The sound reflected from a convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easily amplified. The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones.



Reflection of sound through the curved surface

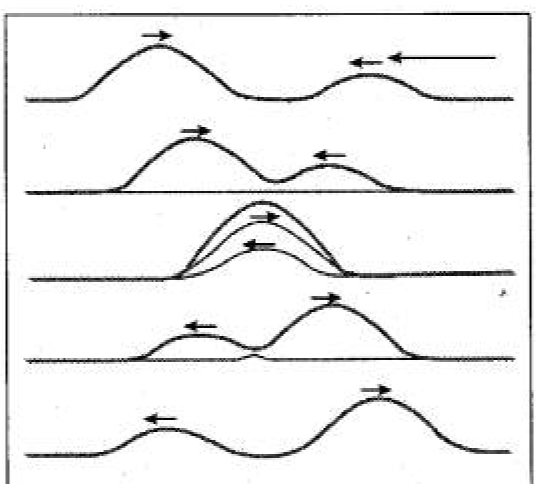
We know that any surface (smooth or rough) can absorb sound. For example, the sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats etc. To avoid such losses, a curved sound board (concave board) is kept in front of the speaker, so that the board reflects the sound waves of the speaker towards the audience.

This method will minimize the spreading of sound waves in all possible direction in that hall and also enhances the uniform distribution of sound throughout the hall. That is why a person sitting at any position in that hall can hear the sound without any disturbance.

Question 6.
Briefly explain the concept of superposition principle.

Answer:
Superposition Principle: When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity. Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape.

When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses. Whereas, if the two pulses have same amplitude but shapes are 180° out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing.


Superposition of two waves

Only waves can possess such a peculiar property and it is called superposition of waves. This means that the principle of superposition explains the net behaviour of the waves when they overlap. Generalizing to any number of waves i.e., if two are more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements. We know that the waves satisfy the wave equation which is a linear second order homogeneous partial differential equation in both space

coordinates and time. Hence, their linear combination (often called as linear superposition of waves) will also satisfy the same differential equation. To understand mathematically, let us consider two functions which characterize the displacement of the waves, for example,
$y_1 = A_1 \sin (kx - \omega t)$ and $y_2 = A_2 \cos (kx - \omega t)$

Since, both y_1 and y_2 satisfy the wave equation (solutions of wave equation) then their algebraic sum
$y = y_1 + y_2$
also satisfies the wave equation. This means, the displacements are additive. Suppose we multiply y_1 and y_2 with some constant then their amplitude is scaled by that constant Further, if C_1 and C_2 are used to multiply the displacements y_1 and y_2, respectively, then, their net displacement y is
$C = C_1y_1 + C_2y_2$

This can be generalized to any number of waves. In the case of n such waves in more than one dimension the displacements are written using vector notation. Here, the net displacement $\vec{y}$ is $\vec{y} = \sum_{i=1}^{n} C_i \vec{y}_i$

The principle of superposition can explain the following :
(a) Space (or spatial) Interference (also known as Interference)
(b) Time (or Temporal) Interference (also known as Beats)
(c) Concept of stationary waves

Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves. These violate the linear superposition principle, e.g., laser. In this chapter, we will focus our attention only on linear waves.

Question 7.
Explain how the interference of waves is formed.

Answer:
Consider two harmonic waves having identical frequencies, constant phase difference cp and same wave form (can be treated as coherent source), hut having amplitudes A_1 and A_2, then

$$y_1 = A_1 \sin(kx - \omega t) \quad ...(1)$$
$$y_2 = A_2 \sin(kx - \omega t + \phi) \quad ...(2)$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves). Mathematically
$y = y_1 + y_2$ .... (3)

Therefore, substituting equation (1) and equation (3) in equation (3), we get

$$y = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \text{Times New Roma})$$

Using trigonometric identity $\sin(\alpha + \beta) = (\sin\alpha \cos\beta + \cos\alpha \sin\beta)$, we get

$$y = A_1 \sin(kx - \omega t) + A_2 [\sin(kx - \omega t)\cos\phi + \cos(kx - \omega t)\sin\phi]$$
$$y = \sin(kx - \omega t)(A_1 + A_2\cos\phi) + A_2 \sin\phi \cos(kx - \omega t) \quad ...(4)$$

Let us re-define $A\cos\theta = (A_1 + A_2\cos\phi)$ ...(5)

and $A\sin\theta = A_2\sin\phi$ ...(6)

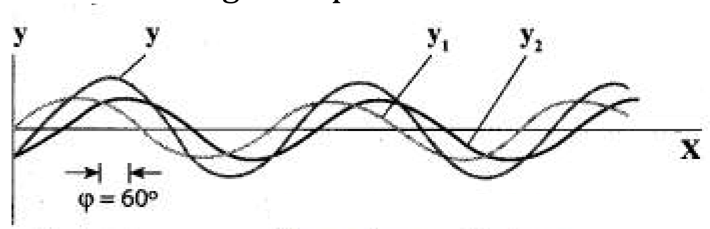
then equation (4) can be rewritten as $y = A\sin(kx - \omega t)\cos\theta + A\cos(kx - \omega t)\sin\theta$

$$y = A(\sin(kx - \omega t)\cos\theta + \sin\theta\cos(kx - \omega t))$$
$$y = A\sin(kx - \omega t + \theta) \quad (7)$$

By squaring and adding equation (5) and equation (6), we get
$$A_2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \quad (8)$$
Since, intensity is square of the amplitude ($I = A_2$), we have
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad ...(9)$$
This means the resultant intensity at any point depends on the phase difference at that point.

(a) For constructive interference:

When crests of one wave overlap with crests of another wave, their amplitudes will add up and we get constructive interference. The resultant wave has a larger amplitude than the individual waves as shown in figure (a).



Interference of two sinusoidal waves

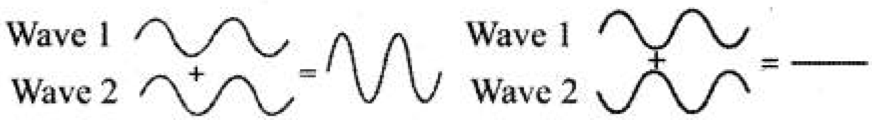
The constructive interference at a point occurs if there is maximum intensity at that point, which means that
$\cos\varphi = +1 \Rightarrow \varphi = 0, 2\pi, 4\pi, \ldots = 2n\pi$, where $n = 0,1,2,\ldots$

This is the phase difference in which two waves overlap to give constructive interference. Therefore, for this resultant wave,

$$I_{maximum} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$$

Hence, the resultant amplitude

$$A = A_1 + A_2$$



Constructive interference (*a*) Destructive interference (*b*)

(b) For destructive interference: When the trough of one wave overlaps with the crest of another wave, their amplitudes "cancel" each other and we get destructive interference as shown in figure (b). The resultant amplitude is nearly zero. The destructive interference occurs if there is minimum intensity at that point, which means $\cos \varphi = -1 \Rightarrow \varphi = \pi, 3\pi, 5\pi, \ldots = (2n - 1)$ K, where $n = 0, 1, 2, \ldots$. i.e. This is the phase difference in which two waves overlap to give destructive interference. Therefore,

$$I_{minimum} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = (A_1 - A_2)^2$$

Hence, the resultant amplitude $A = |A_1 - A_2|$

Question 8.
Describe the formation of beats.

Answer:
Formation of beats: When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency. Number of beats per second
$n = |f_1 - f_2|$

Question 9.
What are stationary waves?. Explain the formation of stationary waves and also write down the characteristics of stationary waves.

Answer:
Explanation of stationary waves: When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves.

Explanation: Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is
$y_1 = A \sin (kx - \omega t)$ (waves move toward right) ...(1)
and the displacement of the second wave (reflected wave) is

$y_2 = A \sin (kx + \omega t)$ (waves move toward left) ...(2)
both will interfere with each other by the principle of superposition, the net displacement is

$y = y_1 + y_2$ ...... (3)
Substituting equation (1) and equation (2) in equation (3), we get
$y = A \sin (kx - \omega t) + A \sin (kx + \omega t)$ ...(4)

Using trigonometric identity, we rewrite equation (4) as
$y(x, t) = 2A \cos (\omega t) \sin (kx)$ ...(5)

This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (5) can be written in more compact form,
$y(x, t) = A' \cos (\omega t)$

where, A' = 2A sin (foe), implying that the particular element of the string executes simple harmonic motion with amplitude equals to A'. The maximum of this amplitude occurs at positions for which

$$\sin (kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots.. = m\pi$$

where m takes half integer or half-integral values. The position of maximum amplitude is known as antinode. Expressing wave number in terms of wavelength, we can represent the anti-nodal positions as

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, } m = 0,1,2... \qquad ...(6)$$

For $m = 0$ we have maximum at $x_0 = \frac{\lambda}{2}$

For $m = 1$ we have maximum at $x_1 = \frac{3\lambda}{4}$

For $m = 2$ we have maximum at $x_2 = \frac{5\lambda}{4}$ and so on.

The distance between two successive antinodes can be computed by

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Similarly, the minimum of the amplitude A' also occurs at some points in the space, and these points can be determined by setting

$\sin(kx) = 0 \Rightarrow kx = 0, \pi, 2\pi, 3\pi, \ldots = n\pi$

where n takes integer or integral values. Note that the elements at these points do not vibrate (not move), and the points are called nodes. The n^{th} nodal positions is given by,

$$x_n = n\frac{\lambda}{2} \text{ where, } n = 0,1,2,... \qquad ...(7)$$

For $n = 0$ we have minimum at $x_0 = 0$

For $n = 1$ we have minimum at $x_1 = \frac{\lambda}{2}$

For n = 2 we have maximum at $x_2 = \lambda$ and so on.

The distance between any two successive nodes can be calculated as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Characteristics of stationary waves:

1. Stationary waves are characterised by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called "stationary waves or standing waves".

2. Certain points in the region in which the wave exists have maximum amplitude, called as anti-nodes and at certain points the amplitude is minimum or zero, called as nodes.

3. The distance between two consecutive nodes (or) anti-nodes is $\lambda/2$
4. The distance between a node and its neighbouring anti-node is $\lambda/4$
5. The transfer of energy along the standing wave is zero.

Question 10.
Discuss the law of transverse vibrations in stretched strings.

Answer:
Laws of transverse vibrations in stretched strings: There are three laws of transverse vibrations of stretched strings which are given as follows:

(i) The law of length: For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

$\Rightarrow l \times f = C$, where C is a constant

(ii) The law of tension: For a given vibrating length l (fixed) and mass per unit length p , (fixed) the frequency varies directly with the square root of the tension T,

$$f \propto \sqrt{T}$$

$$\Rightarrow \quad f = A\sqrt{T}, \text{ where A is a constant}$$

(iii) The law of mass: For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ,

$$f \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow \quad f = \frac{B}{\sqrt{\mu}}, \text{ where B is a constant}$$

Question 11.
Explain the concepts of fundamental frequency, harmonics and overtones in detail. Fundamental frequency and overtones in detail.

Answer:

Fundamental frequency and overtones: Let us now keep the rigid boundaries at x = 0 and x = L and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary must satisfy the following conditions

$x(x = 0, t) = 0$ and $y(x = L, t) = 0$

Since, the nodes formed at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left(\frac{\lambda_n}{2}\right) = \mathrm{L}$, where n is an integer, L is the length between the two boundaries and λ_n is the specific wavelength that satisfy the specified boundary conditions. Hence,

$$\lambda_n = \left(\frac{2L}{n}\right) \qquad \ldots(2)$$

Therefore, not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions, i.e., for n = 1, the first mode of vibration has specific wavelength $\lambda_1 = 2L$. Similarly for n = 2, the second mode of vibration has specific wavelength

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

For n = 3, the third mode of vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right) \text{ and so on.}$$

The frequency of each mode of vibration (called natural frequency) can be calculated.

We have,
$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) \qquad \ldots(3)$$

The lowest natural frequency is called the fundamental frequency.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right) \qquad \ldots(4)$$

The second natural frequency is called the first over tone.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

The third natural frequency is called the second over tone.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right) \text{ and so on.}$$

Therefore, the n^{th} natural frequency can be computed as integral (or integer)

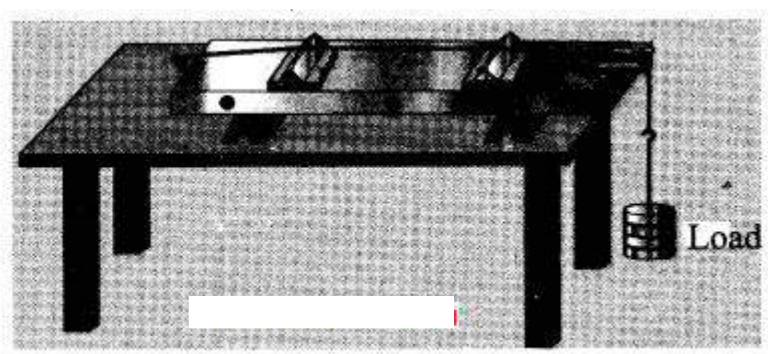
multiple of fundamental frequency, i.e.,
$f_n = nf_1$ where n is an integer ...(5)

If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is $f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$ etc.

Question 12.
What is a sonometer? Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

Answer:
Stationary waves in sonometer: Sono means sound related, and sonometer implies sound-related measurements.

It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string. Therefore, using this device, we can determine the following quantities:



Sonometer

(a) the frequency of the tuning fork or frequency of alternating current
(b) the tension in the string
(c) the unknown hanging mass

Construction: The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in figure. Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire. Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.

Working: A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed. If the length of the vibrating element is then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let f be the frequency of the vibrating element, T the tension of in the string and p the mass per unit length of the string. Then using equation, we get

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \text{ in Hertz} \quad ....(1)$$

Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ

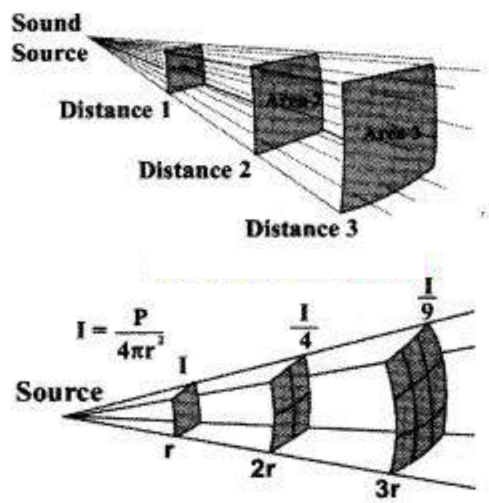
$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}$$

$$\text{Frequency } f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}}$$

$$\therefore \quad f = \frac{1}{ld}\sqrt{\frac{T}{\pi \rho}} \quad ...(2)$$

Question 13.
Write short notes on intensity and loudness.

Answer:
Intensity and loudness: Consider a source and two observers (listeners). The source emits sound waves which carry energy. The sound energy emitted by the source is same regardless of whoever measures it, i.e., it is independent of any observers standing in that region. But the sound received by the two observers may be different; this is due to some factors like sensitivity of ears, etc. To quantify such thing, we define two different quantities known as intensity and loudness of sound.



Intensity of sound waves

Intensity of sound: When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or transmitted per unit time or per second is called sound power.

Therefore, the intensity of sound is defined as “the sound power transmitted per unit area taken normal to the propagation of the sound wave ”.
For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{\text{Power of the source}}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

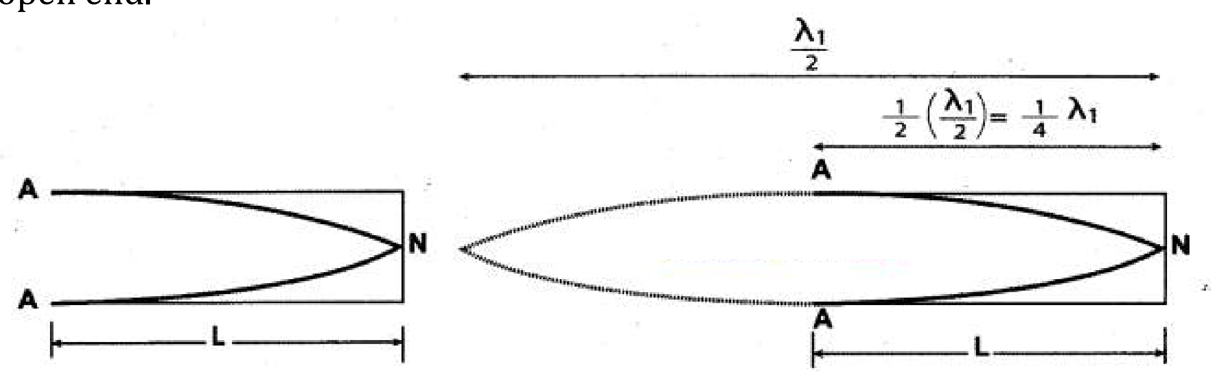
This is known as inverse square law of sound intensity.
the power output does not depend on the observer and depends on the baby.

Therefore, Loudness of sound: Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market. Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases.

But additionally, not only does intensity matter, the internal and subjective experience of “how loud a sound is” i.e., the sensitivity of the listener also matters here. This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound does not depend on the observer. The loudness of sound is defined as "the degree of sensation of sound produced in the ear or the perception of sound by the listener".

Question 14.
Explain how overtones are produced in a:
(a) Closed organ pipe
(b) Open organ pipe

Answer:
(a) Closed organ pipes: Clarinet is an example of a closed organ pipe. It is a pipe with one end closed and the other end open. If one end of a pipe is closed, the wave reflected at this closed end is 180° out of phase with the incoming wave. Thus there is no displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.



(*a*) No motion of particles which leads to nodes at closed end and antinodes at open end (fundamental mode) (N-node, A-antinode)

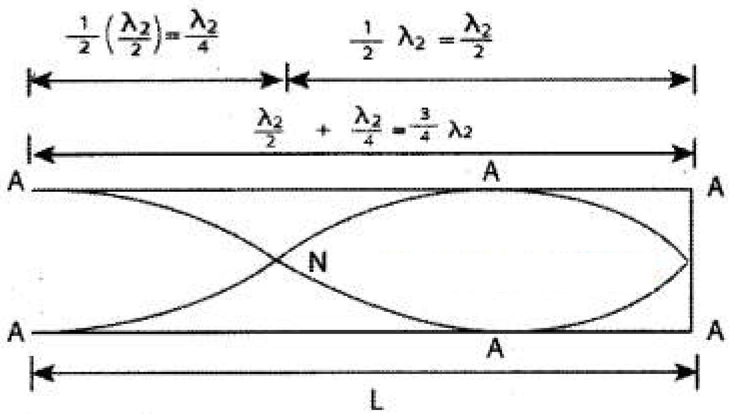
Let us consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the figure, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L \qquad ...(1)$$

The frequency of the note emitted is

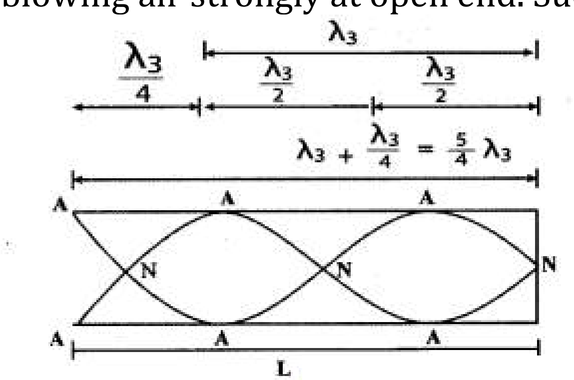
$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \qquad ...(2)$$

which is called the fundamental note



(b) Second mode of vibration having two nodes and two anti-nodes

The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.



(c) Third mode of vibration having three nodes and three anti-nodes

The figure (b) shows the second mode of vibration having two nodes and two antinodes,

$$4L = 3\lambda_2$$

$$L = \frac{3\lambda_2}{4} \text{ or } \lambda_2 = \frac{4L}{3}$$

The frequency for this, $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$

is called first over tone, since here, the frequency is three times the fundamental frequency it is called third harmonic.

The figure (c) shows third mode of vibration having three nodes and three anti-nodes.

we have, $4L = 5\lambda_3$

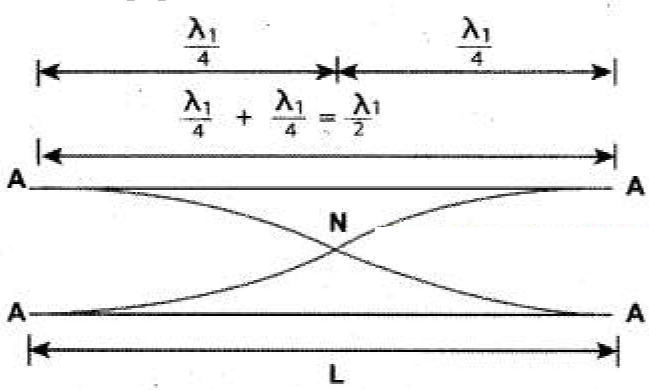
$$L = \frac{5\lambda_3}{4} \text{ or } \lambda_3 = \frac{4L}{5}$$

The frequency, $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$

is called second over tone, and since n = 5 here, this is called fifth harmonic. Hence, the closed organ pipe has only odd harmonics and frequency of the n^{th} harmonic is $f_n = (2n + 1)f_1$. Therefore, the frequencies of harmonics are in the ratio

$f_1 : f_2 : f_3 : f_4 \ldots\ldots = 1 : 3 : 5 : 7 : \ldots\ldots$ .... (3)

(b) Open organ pipes: Flute is an example of open organ pipe. It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.



(*d*) Antinodes are formed at the open end and a node is formed of the middle of the pipe

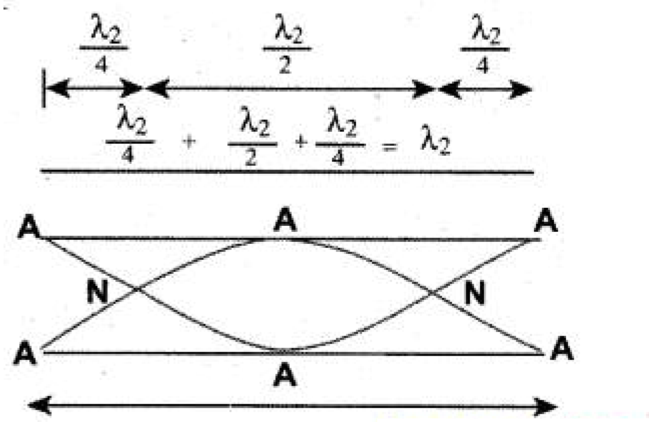
From figure (d), if L be the length of the tube, the wavelength of the wave produced is given by

$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L \qquad \ldots(4)$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_2} = \frac{v}{2L} \qquad \ldots(5)$$

which is called the fundamental note. The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.



(*e*) Second mode of vibration in open pipes having two nodes and three anti-nodes

The Figure (e) shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore,

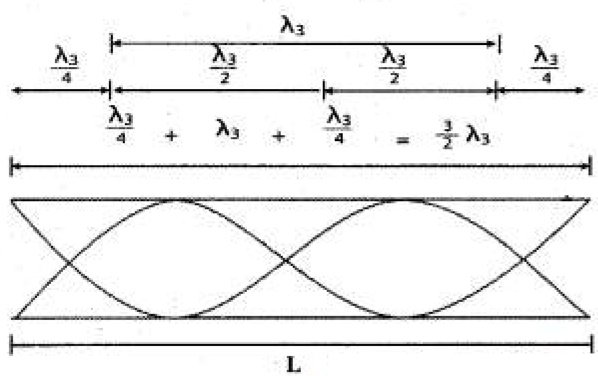
$$L = \lambda_2 \text{ or } \lambda_2 = L$$

$$\text{The frequency } = \frac{v}{\lambda_1} = \frac{v}{L} = 2 \times \frac{v}{2L} = 2f_1$$

is called first over tone. Since n = 2 here, it is called second harmonic.
The Figure (f) above shows the third mode of vibration having three nodes and four anti-nodes.

$$L = \frac{3}{2}\lambda_3 \text{ or } \lambda_3 = \frac{2L}{3}$$

$$\text{The frequency, } \quad f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$$



(*f*) Third mode of vibration having three nodes and four antinodes

is called second over tone. Since n = 3 here, it is called the third harmonic.
Hence, the open organ pipe has all the – harmonics and frequency of nth

harmonic is $f_n = nf_1$. Therefore, the frequencies of harmonics are in the ratio
$f_1 : f_2 : f_3 : f_4 \ldots = 1 : 2 : 3 : 4 : \ldots$ ...(6)

Question 15.
How will you determine the velocity of sound using resonance air column apparatus?

Answer:
Resonance air column apparatus:
The resonance air column apparatus and first, second and third resonance
The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it.

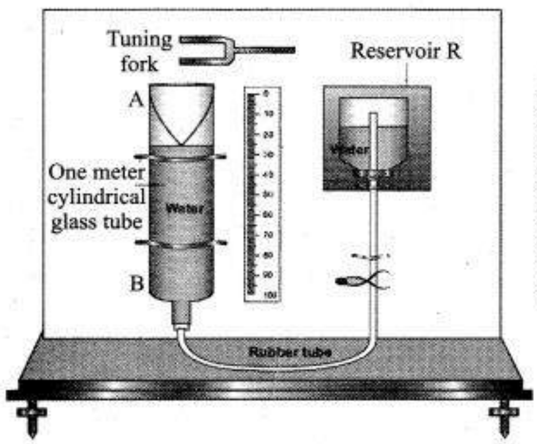
The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed erid and other as the open end. Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end. When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column.

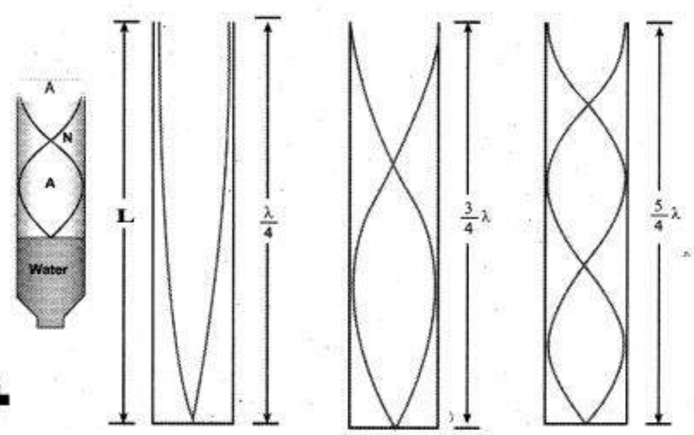
These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of Waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork).

At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced. Let the first resonance occur at length L_1, then

$$\frac{1}{4}\lambda = L_1 \qquad \ldots(1)$$

But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is





$$\frac{1}{4}\lambda = L_1 + e \qquad \text{...(2)}$$

Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which the second resonance occurs. Again taking end correction into account, we have

$$\frac{3}{4}\lambda = L_2 + e \qquad \text{...(3)}$$

In order to avoid end correction, let us take the difference of equation (3) and equation (2)

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e) \Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L \Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\Delta L$$

Further, to compute the end correction, we use equation (2) and equation (3), we get

$$e = \frac{L_2 - 3L_1}{2}$$

Question 16.
What is meant by Doppler effect? Discuss the following cases
(1) Source in motion and Observer at rest
(a) Source moves towards observer

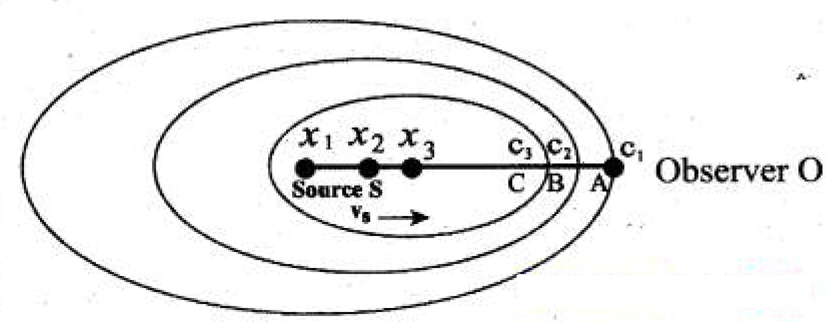
(b) Source moves away from the observer

(2) Observer in motion and Source at rest.
(a) Observer moves towards Source
(b) Observer resides away from the Source

(3) Both are in motion
(a) Source and Observer approach each other
(b) Source and Observer resides from each other
(c) Source chases Observer
(d) Observer chases Source

Answer:
Doppler Effect: When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.

1. Source in motion and the observer at rest
(a) Source moves towards the observer: Suppose a source S moves to the right (as shown in figure) with a velocity v_s and let the frequency of the sound waves produced by the source be fs. We assume the velocity of sound in a medium is v.



Source S moves towards an observer O (right) with velocity

The compression (sound wave front) produced by the source S at three successive instants of time are shown in the figure. When S is at position x_1 the compression is at C_1. When S is at position x_2, the compression is at C_2 and similarly for x_3 and C_3. Assume that if reaches the observer's position A then at that instant C_2 reaches the point B and C_3 reaches the point C as shown in the figure. It is obvious to see that the distance between compressions C_2 and C_3 is shorter than distance between C_1 and C_2. This means the wavelength decreases when the source S moves towards the observer O (since sound travels longitudinally and wavelength is the distance between two consecutive compressions). But frequency is inversely related to wavelength and therefore, frequency increases.

Let λ be the wavelength of the source S as measured by the observer when S is at position x_1 and λ' be wavelength of the source observed by the observer when S moves to position x_2. Then the change in wavelength is $\Delta\lambda = \lambda - \lambda' = v_s t$, where t is the time taken by the source to travel between x_1 and x_2. Therefore,

$$\lambda' = \lambda - v_s t \qquad \text{...(1)}$$

But

$$t = \frac{\lambda}{v} \qquad \text{...(2)}$$

On substituting equation (2) in equation (1), we get

$$\lambda' = \lambda\left(1 - \frac{v_s}{v}\right)$$

Since frequency is inversely proportional to wavelength, we have $f' = \frac{v_s}{\lambda'}$ and $f = \frac{v_s}{\lambda}$

Hence,

$$f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)} \qquad \text{...(3)}$$

Since, $\frac{v_s}{v} << 1$, we use the binomial expansion and retaining only first order in $\frac{v_s}{v}$, we get

$$f' = f\left(1 + \frac{v_s}{v}\right)v \qquad \text{...(4)}$$

(b) Source moves away from the observer: Since the velocity here of the source is opposite in direction when compared to case (a), therefore, changing the sign of the velocity of the source in the above case i.e, by substituting ($v_s \rightarrow -v_s$) in equation (1), we get

$$f' = \frac{f}{\left(1 + \frac{v_s}{v}\right)} \qquad \text{...(5)}$$

Using binomial expansion again, we get,

$$f' = f\left(1 - \frac{v_s}{v}\right) \qquad \text{...(6)}$$

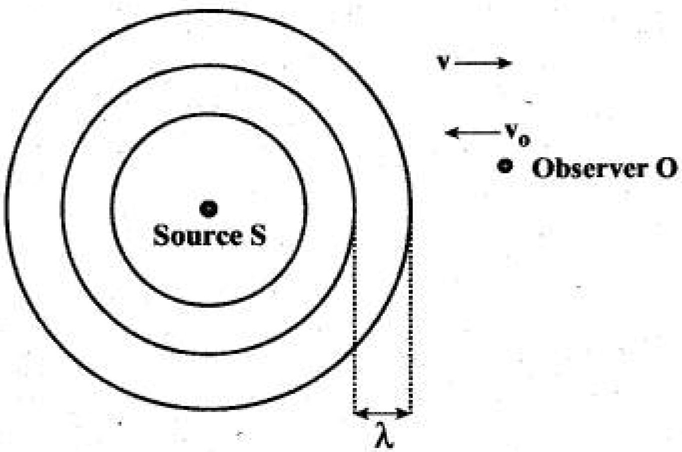
2. Observer in motion and source at rest:

(a) Observer moves towards Source:

Let us assume that the observer O moves towards the source S with velocity v_o. The source S is at rest and the velocity of sound waves (with respect to the medium) produced by the source is v. From the figure, we observe that both vo and v are in opposite direction. Then, their relative velocity is $v_r = v + v_o$.

The wavelength of the sound wave is $\lambda = \frac{v}{f}$, which means the frequency observed by the observer O is f' = . Then

$$f' = \frac{v_r}{\lambda} = \left(\frac{v + v_o}{v}\right) f = f\left(1 + \frac{v_o}{v}\right) \quad ...(7)$$



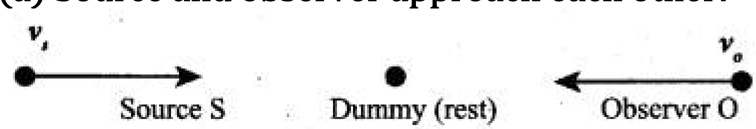
Observer moves towards source

(b) Observer recedes away from the Source: If the observer O is moving away (receding away) from the source S, then velocity v_0 and v moves in the same direction. Therefore, their relative velocity is $v_r = v - v_r$. Hence, the frequency observed by the observer O is

$$f' = \frac{v_r}{\lambda} = \left(\frac{v - v_o}{v}\right) f = f\left(1 - \frac{v_o}{v}\right) \quad ...(8)$$

3. Both are in motion:

(a) Source and observer approach each other:



Source and observer approach towards each other

Let v_s and v_o be the respective velocities of source and observer approaching each other as shown in figure. In order to calculate the apparent frequency observed by the observer, as a simple calculation, let us have a dummy (behaving as observer or source) in between the source and observer. Since the dummy is at rest, the dummy (observer) observes the apparent frequency due to approaching source as given in equation (3) as

$$f_d = \frac{f}{\left(1 - \frac{v_s}{v}\right)} \qquad \text{...(9)}$$

At that instant of time, the true observer approaches the dummy from the other side. Since the source (true source) comes in a direction opposite to true observer, the dummy (source) is treated as stationary source for the true observer at that instant. Hence, apparent frequency when the true observer approaches the stationary source (dummy source), from equation (7) is

$$f' = f_d\left(1 + \frac{v_0}{v}\right) \Rightarrow f_d = \frac{f'}{\left(1 + \frac{v_0}{v}\right)} \qquad \text{...(10)}$$

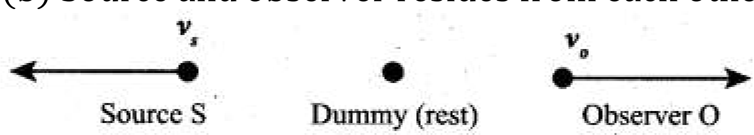
Since this is true for any arbitrary time, therefore, comparing equation (9) and equation (10), we get

$$\frac{f}{\left(1 - \frac{v_s}{v}\right)} = \frac{f'}{\left(1 + \frac{v_0}{v}\right)} \Rightarrow \frac{vf'}{(v + v_0)} = \frac{vf}{(v - v_s)}$$

Hence, the apparent frequency as seen by the observer is

$$f' = \left(\frac{v \quad v}{v \quad v}\right) f \qquad \text{...(11)}$$

(b) Source and observer resides from each other



Source and observer resides from each other

Here, we can derive the result as in the previous case. Instead of a detailed calculation, by inspection from figure, we notice that the velocity of the source and the observer each point in opposite directions with respect to the case in (a) and hence, we substitute ($v_s \rightarrow -v_s$) and ($v_0 \rightarrow -v_o$) in equation (11), and therefore, the apparent frequency observed by the observer when the source and observer recede from each other is

$$f' = \left(\frac{v - v_0}{v + v_s}\right) f \qquad \text{...(12)}$$

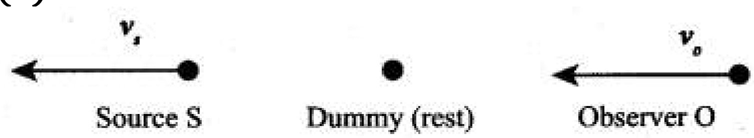
(c) Source chases the observer

Source chases observer

Only the observer's velocity is oppositely directed when compared to case (a).
Therefore, substituting ($v_0 \rightarrow -v_0$) in equation (11), we get

$$f' = \left(\frac{v - v_0}{v - v_s}\right) f \qquad \text{...(13)}$$

(d) Observer chases the source



Observer chases the source

Only the source velocity is oppositely directed when compared to case (a).
Therefore, substituting $v_s \rightarrow -v_s$ in equation (12), we get

$$f' = \left(\frac{v + v_0}{v + v_s}\right) f \qquad \text{...(14)}$$

Numerical Problems

Question 1.
The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?

Answer:

Speed of the wave in medium v = 900 ms^{-1}

$$\text{Frequency}(n) = \frac{\text{Number of waves}}{\text{Time}} = \frac{3000}{2 \times 60} = 25 s^{-1}$$

$$\text{Wavelength } \lambda = \frac{v}{n} = \frac{900}{25} \ ; \lambda = 36 \text{ m}$$

Question 2.
Consider a mixture of 2 mol of helium and 4 mol of oxygen. Compute the speed of sound in this gas mixture at 300 K.

Answer:
The mixture of helium and oxygen.

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{(2 \times 4) + (4 \times 32)}{2 + 4} = \frac{136}{6}$$

$$M_{mix} = 22.6 \times 10^{-3} \text{ kg/mol}$$

Helium is an mono atomic, $C_{V_1}, = \frac{3R}{2}$

Oxygen is an diatomic, $C_{V_2} = \frac{5R}{2}$

$$(C_V)_{mix} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \frac{\left(2 \times \frac{3R}{2}\right) + \left(4 \times \frac{5R}{2}\right)}{2 + 4}$$

$$(C_V)_{mix} = \frac{13R}{6}$$

$$(C_p)_{mix} = (C_V)_{mix} + R \qquad \gamma_{mix} = \frac{(C_P)_{mix}}{(C_V)_{mix}} = \frac{19R}{6} \times \frac{6}{13R}$$

$$= \frac{13R}{6} + R = \frac{19R}{6} \qquad \gamma_{mix} = \frac{19}{13}$$

According to Laplace correction, Speed of sound,

Extend root length litre this $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{19}{13} \times \frac{8.31 \times 300}{22.6 \times 10^{-3}}} \ ; v = 400.9 \text{ ms}^{-1}$

Question 3.
A ship in a sea sends SONAR waves straight down into the seawater from the

bottom of the ship. The signal reflects from the deep bottom bed rock and returns to the ship after 3.5 s. After the ship moves to 100 km it sends another signal which returns back after 2s. Calculate the depth of the sea in each case and also compute the difference in height between two cases.

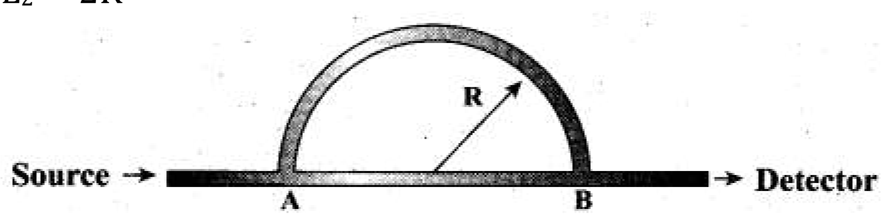
Answer:
Speed of SONAR waves in water $c = 1500 \text{ ms}^{-1}$
Time taken to reflect from the bottom of the sea, $2t = 3.5$ sec
$\therefore t = 1.75$ sec

Distance covered in forward and reflected backward $(d_1) = c \times t$
$d_2 = 1500 \times 1.75 = 2625$ m
After ship moves in a distance = 150 km
Time taken to reflect by the waves $2t = 2s$
$t = 1s$

Distance covered by the waves $(d_2) = c \times t = 1500 \times 1 = 1500$ m
The different between the height of two cases = 2625 – 1500
$h_{difference} = 1124$ m

Question 4.
A sound wave is transmitted into a tube as shown in figure. The sound wave splits into two waves at the point A which recombine at point B. Let R be the radius of the semicircle which is varied until the first minimum. Calculate the radius of the semi-circle if the wavelength of the sound is 50.0 m

Answer:
The sound travelling in the curved path distance = πR
$L_1 = \pi R$
The sound travelling in the straight path distance = $2R$
$L_2 = 2R$



The path distance of straight and curved path AP = $L_1 - L_2$

$$\Delta P = \pi R - 2R = R(\pi - 2) \qquad ...(1)$$

The different in the path length of the sound waves,

$$\Delta P = \frac{\lambda}{2} \qquad ...(2)$$

Equating (1) and (2), $\frac{\lambda}{2} = R(\pi - 2)$; $\lambda = 50$ m

$$R = \frac{\lambda}{2(\pi - 2)} = \frac{50}{2(3.14 - 2)} = \frac{50}{2.28}$$

$$R = 21.9 \text{ m}$$

Question 5.

N tuning forks are arranged in order of increasing frequency and any two successive tuning forks give n beats per second when sounded together. If the last fork gives double the frequency of the first (called as octave), Show that the frequency of the first tuning fork is f = (N – 1)n.

Answer:

Total number of fork = N

The frequency of the 1st fork = f

The frequency of the last fork = 2f

$\therefore a_n = a + (n - 1)d$

2f = f + (N – 1)n

2f – f = (N – 1)n

$\therefore$ f = (N – 1)n

Question 6.

Let the source propagate a sound wave whose intensity at a point (initially) be I. Suppose we consider a case when the amplitude of the sound wave is doubled and the frequency is reduced to one-fourth. Calculate now the new intensity of sound at the same point?

Answer:

Intensity of sound wave (old) = I_1

Amplitude of sound wave (A_2) = $2A_1$

Frequency of the sound wave I_2 = ?

$I_1 \propto f_1^2 A_1^2$; $I_2 \propto f_2^2 A_2^2$

$$\frac{I_1}{I_2} = \frac{f_1^2 \cdot A_1^2}{f_2^2 . A_2^2} = \frac{f_1^2\ A_1^2}{\frac{1}{16} f_1^2 \cdot 4A_1^2} = \frac{16}{4} = 4$$

$$\boxed{I_2 = \frac{1}{4} I_1}$$

Question 7.
Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

Answer:
Fundamental frequency of closed organ pipe

$$f_c = \frac{v}{4l} = 250 \text{ Hz}$$

Fundamental frequency:f open organ pipe $f_o = \frac{v}{2l} = ?$

$$\frac{f_c}{f_o} = \frac{v}{4l} \times \frac{2l}{v} = \frac{1}{2}$$

$$f_o = 2f_c = 2 \times 250$$

$$\boxed{f_o = 500 \text{ Hz}}$$

Question 8.
A police in a siren car moving with a velocity 20 ms⁻ chases a thief who is moving in a car with a velocity v_0 ms^{-1}. The police car sounds at frequency 300 Hz, and both of them move towards a stationary siren of frequency 400 Hz. Calculate the speed in which thief is moving.
(Assume the thief does not observe any beat)

Answer:
Velocity of sound v = 330 ms^{-1}
Velocity of car (v_s) = 20 ms^{-1}
Frequency of car (f_1) = 300 Hz
Frequency of stationary siren (f_2) = 400 Hz

The speed of the thief (v_o) = ?

$$v_1 = f_1\left[\frac{v - v_o}{v - v_s}\right] \text{ and } v_2 = f_2\left[\frac{v + v_o}{v}\right]$$

Both are moving towards stationary siren $v_1 = v_2$

$$300\left[\frac{300 - v_o}{310}\right] = 400\left[\frac{330 + v_o}{330}\right]$$

$$330 - v_o = \frac{4 \times 310}{3}\left[\frac{330 + v_o}{330}\right]$$

$$330 - v_o = \frac{1240}{990}[330 + v_0] = 1.2525(330 + v_0)$$

$330 – v_o = 413.325 + 1.2525v_o$
$2.2525v_o = – 83. 325$
$v_o = – 36.99$
∴ speed of the thief in moving = 36.99 ms^{-1}

Question 9.
Consider the following function:
(a) $y = x^2 + 2 \alpha tx$
(b) $y = (x + vt)^2$ which among the above function can be characterized as a wave?

Answer:
(a) $y = x^2 + 2 \alpha tx$. This expression is not a wave equation.
(b) $y = (x + vt)^2$. This expression is satisfies the wave equation.

Conceptual Questions

Question 1.
Why is it that transverse waves cannot be produced in a gas? Can the transverse waves can be produced in solids and liquids?

Answer:
Transverse waves travel in the form of crests and through. They involve changes in the shape of the medium. As gas has no elasticity of shape, hence

transverse waves cannot be produced in it. So, they can be transmitted through media which sustain shearing stress such as solids, strings and liquid surface.

Question 2.
Why is the roar of our national animal different from the sound of a mosquito?

Answer:
Both sounds travel at the speed of sound. The speed of sound varies according to the density and temperature of the air, but not according to the loudness of the sound, at least not for the levels of loudness we talking about here.
The roaring of the lion will be audible a lot further away, but that's simply because it's louder.

Question 3.
A sound source and listener are both stationary and a strong wind is blowing. Is there a Doppler effect?

Answer:
Yes, It does not matter whether the sound source or the transmission media are in motion, vibrations will be compressed in the direction of convergence and dilated in the direction of divergence.

Question 4.
In an empty room why is it that a tone sounds louder than in the room having things like furniture etc.

Answer:
Because in a furniture room will absorb the sound waves, hence there went be any echo. But in an empty room reflect the sound. Therefore there will be echo hence we hear sound louder.

Question 5.
How do animals sense impending danger of hurricane?

Answer:
Some animals are believed to be sensitive to be low frequency sound waves

emitted by hurricanes, they can also detect the slight drops in air and water pressure that signal a storm's approach.

Question 6.
Is it possible to realize whether a vessel kept under the tap is about to fill with water?

Answer:
The frequency of the note produced by an air column is inversely proportional to its length. As the level of water is the vessel rises, the length of the air column above it decreases. It produces sound of decreasing frequency, i.e., the sound becames shorter. From the shrillness of sound, it is possible to realize whether the vessel is filled which water.
$v_{min} = 11.71\ ms^{-1}$