CHAPTER - 3

# Playing with Numbers

## 3.1 INTRODUCTION

Let us observe the situation.

Hasini wants to distribute chocolates to her classmates on her birthday. Her father brought a box of 125 chocolates. There are 25 students in her class.

She decided to distribute all the chocolates such that each one would get equal number of chocolates. First, she thought of giving 2 chocolates each but found that some chocolates were remaining. Then again she tried of giving 3 each, but again some chocolates were remaining. Finally, she thought of giving 5 chocolates each. Now, she found that no chocolates were remaining.

Is there any easy way to find the no.of chocolates equally distributed among her classmates ? Think. Of course she can divide 125 by 25. In the previous classes you have become familiar with rules which tell us whether a given number is divisible by 2, 3, 5, 6, 9 and 10. In this chapter we will recollect these tests. Further, we will also discover the rules of divisibility for 4, 8 and 11.

## 3.2 DIVISIBILITY RULE

Let us consider 29. When you divide 29 by 4, it leaves remainder 1 and gives quotient 7. Can you say that 29 is completely divisible by 4? Why?

Find the quotient and remainder when 24 is divided by 4?

Is 24 completely divisible by 4? Why?

So, we see that a number is completely divisible by another number, when it leaves zero as remainder.

The process of checking whether a number is divisible by a given number or not without actual division is called divisibility rule for that number.

Let us review the tests of divisibility studied in the previous classes.

### 3.2.1 Divisibility by 2

Let us look at the number chart given below.

**Number Chart**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Now cross all the multiples of 2. Do you see any pattern in the ones place of these numbers?

These numbers have only the digits 0, 2, 4, 6, 8 in the ones place. Looking at there observations we can say that **a number is divisible by 2 if it has any of the digits 0,2,4,6 or 8 in its ones place.**

## Do This

Are 953, 9534, 900, 452 divisibile by 2? Also check by actual division.

### 3.2.2 Divisibility by 3

Now encircle all the multiples of 3 in the above chart. You must have encircled numbers like 21, 27, 36, 54 etc. Do you see any pattern in the ones place of these numbers. No! Because numbers with the same digit in ones place may or may not be divisible by 3. For example, both 27 and 37 have 7 in ones place. Are they both divisible by 3?

Let us now add the digits of 21, 36, 54, 63, 72, 117

$2 + 1 = 3$ $\quad$ $5 + 4 =$ ______ $\quad$ $7 + 2 =$ ______

$3 + 6 =$ _____ $\quad$ $6 + 3 =$ ______ $\quad$ $1 + 1 + 7 =$ ______

All these sums are divisible by 3.

Thus we can say that **if the sum of the digits is divisible by of 3, then the number is divisible by 3.** Check this rule for other circled numbers.

## DO THIS

Check whether the following numbers are divisible by 3?

i. 45986 ii. 36129 iii. 7874

### 3.2.3 Divisibility by 6

Put a cross on the numbers which are multiples of 6 in the number chart.

Do you notice anything special about them.

Yes, they are divisible by both 2 and 3.

**If a number is divisible by both 2 and 3 then it is also divisible by 6 .**

## TRY THESE

1. Is 7224 divisible by 6? Why?
2. Give two examples of 4 digit numbers which are divisible by 6.
3. Can you give an example of a number which is divisible by 6 but not by 2 and 3. Why?

### 3.2.4 Divisibility by 9

Put a ☐ (box) on the numbers which are multiples of 9 in the number chart.

Now try to find a pattern or rule for checking the divisibility of 9. (Hint : Sum of digits)

Sum of digits in these numbers are also divisible by 9.

For example If we take 81, 8 + 1 = 9 similarly 99, 9 + 9 = 18 divisible by 9.

**A number is divisible by 9, if the sum of the digits of the number is divisible by 9.**

## DO THIS

1. Test whether 9846 is divisible by 9?
2. Without actual division, find whether 8998794 is divisible by 9?
3. Check whether 786 is divisible by both 3 and 9?

### 3.2.5 Divisibility by 5

Are all the numbers 20, 25, 30, 35, 40, 45, 50 divisible by 5?

Is 53 divisible by 5? Why?

Can you say that all the numbers with zero and five at ones place is divisible by 5?

Consider the numbers 5785, 6021, 1000, 101010, 9005. Guess which are divisible by 5 and verify by actual division.

### 3.2.6 Divisibility by 10

Mark all the numbers divisible by 10 in the number chart.

What do you notice?

1. All of them have 0 at their ones place.
2. All of them are divisible by both 5 and 2.

## Exercise - 3.1

1. Which of the following numbers are divisible by 2, by 3 and by 6?

   (i) 321729 (ii) 197232 (iii) 972132 (iv) 1790184

   (v) 312792 (vi) 800552 (vii) 4335 (viii) 726352

2. Determine which of the following numbers are divisible by 5 and by 10.

   25, 125, 250, 1250, 10205, 70985, 45880

   Check whether the numbers that are divisible by 10 are also divisible by2 and 5.

3. Fill the table using divisibility test for 3 and 9.

| Number | Sum of the digits in the number | Divisible by | |
|---|---|---|---|
| | | 3 | 9 |
| 72 | ................................................................ | | |
| 197 | ................................................................ | | |
| 4689 | ................................................................ | | |
| 79875 | ................................................................ | | |
| 988974 | 9 + 8 + 8 + 9 + 7 + 4 = 45 | Yes | Yes |

4. Make 3 different 3 digit numbers using 1, 9 and 8, where each digit can be used only once. Check which of these numbers are divisible by 9.

5. Which numbers among 2, 3, 5, 6, 9 divides 12345 exactly?

   Write 12345 in reverse order and test now which numbers divide it exactly?

6. Write different 2 digit numbers using digits 3, 4 and 5. Check whether these numbers are divisible by 2, 3, 5, 6 and 9?

7. Write the smallest digit and the greatest possible digit in the blank space of each of the following numbers so that the numbers formed are divisible by 3.

   i. __ 6724 ii. 4765__ 2 iii. 7221__ 5

8. Find the smallest number that must be added to 123, so that it becomes exactly divisible by 5?

9. Find the smallest number that has to be subtracted from 256, so that it becomes exactly divisible by 10?

## 3.3 Factors

We have studied the divisibility and discovered tests of divisibility for 2, 3, 5, 6, 9 and 10. Now we will learn the concepts of factors.

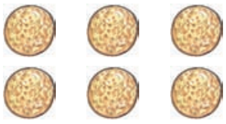
Let us observe a situation:

Devi has 6 coins with her. She wants to arrange them in columns in such a way that each column has the same number of coins. She arranges them in many ways using all the 6 coins.

**Case (i)** 1 coin in each column

number of columns = 6

Total number of coins = 1 × 6 = 6



**Case (ii)** 2 coins in each column

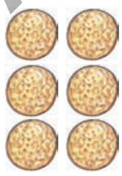
Number of columns = 3

Total number of coins = 2 × 3 = 6

**Case (iii)** 3 coins in each column

Number of columns = 2

Total number of coins = 3 × 2 = 6



**Case (iv)** 6 coins in each column

Number of column = 1

Total number of coins = 6 × 1 = 6

These are the only possible arrangements using all the 6 coins.

From these arrangements, Devi observes that 6 can be written as a product of two numbers in different ways as

6 = 1 × 6 6 = 2 × 3 6 = 3 × 2 6 = 6 × 1

From 6 = 2 × 3 it can be said that 2 and 3 exactly divide 6. So, 2 and 3 are factors of 6. From the other product 6 = 1 × 6, thus 6 and 1 are also factors of 6.

1, 2, 3 and 6 are the only factors of 6.

A number which divides the other number exactly is called a **factor** of that number. In other words, every number is completely divisible by its factors. Here 1, 2, 3 and 6 are all factors of 6. Similarly 1 and 19 are factors of 19. Number 5 is not a factor of 16. Why?

Observe the following table:

| Number | Factors |
|---|---|
| 12 | 1, 2, 3, 4, 6, 12 |
| 18 | 1, 2, 3, 6, 9, 18 |
| 20 | 1, 2, 4, 5, 10, 20 |
| 24 | 1, 2, 3, 4, 6, 8, 12, 24 |

From the above table we can notice that;

1. 1 is a factor of every number and is the smallest of all factors.

2. Every number is a factor of itself and is the greatest of its factors.
3. Every factor of a number is less than or equal to the given number.
4. Number of factors of a given number are countable.

## Do This

1. Find the factors of 80.
2. Do all the factors of a given number divide the number exactly? Find the factors of 28 and verify by division.
3. 3 is a factor of 15 and 24. Is 3 a factor of their difference also?

## 3.4 Prime and Composite Numbers

Let us observe the number of factors of a few numbers as shown below:

| Number | Factors | Number of Factors |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1, 2 | 2* |
| 3 | 1, 3 | 2* |
| 4 | 1, 2, 4 | 3 |
| 5 | 1, 5 | 2* |
| 6 | 1, 2, 3, 6 | 4 |
| 7 | 1, 7 | 2* |

From the table say which numbers have only two factors?

There are four numbers 2, 3, 5 and 7, having exactly two factors (shown with*)

i.e. 1 and the number itself. These numbers whose only factors are 1 and the number itself are called **prime numbers**.

Which numbers have more than two factors?

Numbers having more than two factors like 4, 6 and so on are called **composite numbers**. Give 5 examples of composite numbers greater than 10.

Which number has only one factor?

The number 1 has only one factor (i.e. itself) so, **1 is neither prime nor composite.**

## Try These

1. What is the smallest Prime number?
2. What is the smallest composite number?
3. What is the smallest odd composite number?
4. Give 5 odd and 5 even composite numbers?
5. Is 1 prime or composite and why?

Without actually checking the factors of a number, we can find prime numbers from 1 to 100 with an easy method. This method was given by the Greek Mathematician **Eratosthenes**, in the third century BC. Let us see the method. List all the numbers from 1 to 100, as shown below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

***Step-1*:** Cross out 1 because it is neither prime nor composite.

***Step-2*:** Encircle 2, cross out all the other multiples of 2, i.e. 4, 6, 8 and so on.

***Step-3*:** You will find that the next uncrossed number is 3. Encircle 3 and cross out all the other multiples of 3.

***Step-4*:** The next uncrossed number is 5. Encircle 5 and cross out all the other multiples of 5.

***Step-5*:** Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

### Try These

1. Can you guess a prime number which when on reversing its digits, gives another prime number? (***Hint*** **:** Take a 2 digit prime number)
2. You know 311 is a prime number. Can you find the other two prime numbers just by rearranging the digits?

### 3.4.1 Co-prime or relative prime

Observe the numbers 3 and 8.

The factors of 3 are 1 and 3

The factors of 8 are 1, 2, 4, 8

The common factor for both 3 and 8 is 1 only.

Thus, the numbers which have only 1 as the common factor are called **co-primes** or **relatively prime.** Write two pairs of co-primes, by finding the common factor.

***Example-1.*** Consider two co-prime numbers 4 and 5. Are both of them prime numbers?

***Solution*:** No, 4 is not a prime. Only 5 is a prime.

We can say that **"Only two primes are co-primes but all the co-primes need not be primes."**

### 3.4.2 Twin primes

Twin primes are prime numbers that differ from each other by two e.g. (3, 5), (5, 7), (11, 13), (41, 43) etc.

Are all twin primes relatively prime? Discuss

## Do This

From the following numbers identify different pairs of co-primes
2, 3, 4, 5, 6, 7, 8, 9 and 10

## Exercise - 3.2

1. Write all the factors of the following numbers.
   i. 36 ii. 23 iii. 96 iv. 115
2. Which of the following pairs are co-prime?
   i. 18 and 35 ii. 216 and 215
   iii. 30 and 415 iv. 17 and 68
3. What is the greatest prime number between 1 and 20?
4. Find the prime and composite numbers between 10 and 30?
5. The numbers 17 and 71 are prime numbers. Both these numbers have same digits 1 and 7. Find 2 more such pairs of prime numbers below 100?
6. Write three pairs of twin primes below 20?
7. Write two prime numbers whose product is 35?
8. Express 36 as the sum of two odd primes?
9. Write seven consecutive composite numbers less than 100.
10. Express 53 as the sum of three primes?
11. Write two prime numbers whose difference is 10?
12. Write three pairs of prime numbers less than 20 whose sum is divisible by 5?

## 3.5 Prime Factorization

When a number is expressed as a product of its factors, we say that the number has been factorized. The process of finding the factors is called **factorization**.

There may be several ways in which a number can be factorized. For example, the number 24 can be factorized as:

i) $24 = 1 \times 24$ ii) $24 = 2 \times 12$ iii) $24 = 3 \times 8$

iv) $24 = 4 \times 6$ v) $24 = 2 \times 2 \times 2 \times 3$

In (ii) and (iii) one factor is prime, and the other factor is a composite number. In (iv) both the factors are composite numbers. However in (v) all the factors are **prime numbers**. In (i) one factor is composite.

Factorization of the type (v), where all the factors are prime numbers, is known as **prime factorization**.

Thus, in prime factorization, the factors obtained can not be further factorized.

### 3.5.1 Methods of Prime Factorization

1. ***Division Method*** **:** Prime factorisation of 42 using division method we proceed as follow:

| | |
|---|---|
| 2 | 42 |
| 3 | 21 |
| 7 | 7 |
| | 1 |

Start dividing by the least prime factor. Continue division till the resulting number to be divided is 1.

$\therefore$ Prime factorisation of 42 is $2 \times 3 \times 7$

2. ***Factor Tree Method*** **:** We can find the prime factorization of 60 by drawing a factor tree.

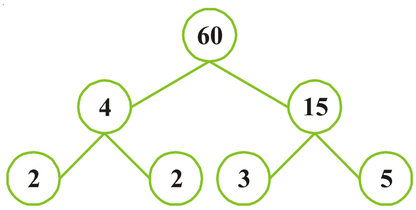
To find the prime factorization of 60 using factor tree method, we proceed as follow:

***Step-1*** **:** Express 60 as a product of two numbers.

***Step-2*** **:** Factorise 4 and 15 further, since they are composite numbers.

***Step-3*** **:** Continue till all the factors are prime numbers.

Prime factorisation of 60 = 2 x 2 x 3 x 5



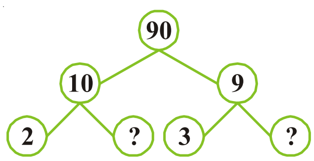

## Do This

1. Write the prime factors of 28 and 36 through division method.
2. Write the prime factors of 42 by factor tree method.

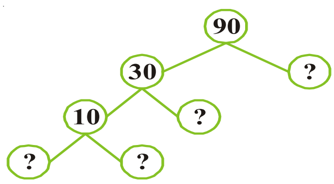
## Exercise - 3.3

1. Write the missing numbers in the factor tree for 90?

i.




ii.



2. Factorise 84 by division method?

3. Write the greatest 4 digit number and express it in the form of its prime factors?

4. I am the smallest number, having four different prime factors. Can you find me?

## 3.6 Common Factors

Observe the following table:

| Number | 12 | 18 |
|---|---|---|
| **Factors of the number** | 1, 2, 3, 4, 6, 12 | 1, 2, 3, 6, 9, 18 |

Common factors of 12 and 18 are 1, 2, 3 and 6

**Common factors are those numbers which are factors of all the given numbers**.

Now find common factor of 20 and 24.

### 3.6.1 Highest Common Factor (HCF)

From the above table we found that common factors of 12 and 18 are 1, 2, 3 and 6.

What is the highest of these common factors? It is 6. So we can say that the Highest Common Factor (HCF) of 12 and 18 is 6.

**The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also called as Greatest Common Divisor (GCD)**

### 3.6.2 Method of finding HCF

**1. Prime Factorisation Method**

The HCF of 12, 30 and 36 can also be found by prime factorisation as follows:

| | |
|---|---|
| 2 | 12 |
| 2 | 6 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 2 | 30 |
| 3 | 15 |
| 5 | 5 |
| | 1 |

| | |
|---|---|
| 3 | 36 |
| 3 | 12 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Thus $12 = \boxed{2} \times \boxed{3} \times 2$

$30 = \boxed{2} \times \boxed{3} \times 5$

$36 = \boxed{2} \times \boxed{3} \times 2 \times 3$

The common factor of 12, 30 and 36 is 2 x 3 = 6.

Hence, HCF of 12, 30 and 36 is 6.

## Do This

Find the HCF of 12, 16 and 28

## 2. HCF by Continued Division Method

This method of division was invented by the famous Greek mathematician **Euclid**. Divide the larger number by the smaller and then divide the previous divisor by the remainder until the remainder is 0. The **last divisor is the HCF** of the numbers.

***Example-2.*** Find the HCF of 56 and 64

***Solution*:**

```
               56)  64 (1
                   -56
Last divisor       8) 56 ( 7
                     -56
    Remainder          0
```

Last divisor is 8 when remainder becomes 0. Thus, HCF of 56 and 64 is 8.

This method is useful to find the HCF of larger numbers.

***Example-3.*** Find the HCF of 40, 56 and 60.

***Solution*:**

***Step-1*:** First find the HCF of any two numbers. Let us find the HCF of 40 and 56.

```
               40)  56 (1
                   -40
Remainder     16)  40  (2
                  -32
Last Divisor       8)  16 (2
                      -16
    Remainder           0
```

HCF of 40 and 56 is 8.

***Step-2*:** Then, find the HCF of the third number and the HCF of first two numbers.

Let us find the HCF of 60 and 8.

```
               8)  60  (7
                  -56
Last Divisor      4)  8  (2
                     -8
Remainder             0
```

HCF of 8 and 60 is 4.

***Step-3*:** This number is the HCF of the given three numbers.

Thus HCF of 40, 56 and 60 is 4.

## Do This

Find the HCF of 28, 35 and 49.

## Think Discuss and Write

What is the HCF of any two;

(i) Consecutive numbers? (ii) Consecutive even numbers?

(iii) Consecutive odd numbers?

What do you observe? Discuss with your friends.

***Example-4.*** Two tankers contain 850 litres and 680 litres of kerosene oil, respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

***Solution*:** The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover this capacity should be **maximum**. Thus the maximum capacity of such a container will be the HCF of 850 and 680. The **HCF** of 850 and 680 is 170.

Therefore, maximum capacity of the required container is 170 litres. It will fill the first container in 5 and the second in 4 refills.

## Exercise - 3.4

1. Find the HCF of the following numbers by prime factorisation and continued division method?
   - i. 18, 27, 36
   - ii. 106, 159, 265
   - iii. 10, 35, 40
   - iv. 32, 64, 96, 128
2. Find the largest number which is a factor of each of the numbers 504, 792 and 1080?
3. The length, breadth and height of a room are 12m, 15m and 18m respectively. Determine the length of longest stick that can measure all the dimensions of the room in exact number of times ?
4. HCF of co-prime numbers 4 and 15 was found as follows by factorisation:
   4 = 2 x 2 and 15 = 3 x 5 Since there is no common prime factor, HCF of 4 and 15 is 0. Is the answer correct? If not, what is the correct HCF?
5. What is the capacity of the largest vessel which can empty the oil from three vessels containing 32 litres, 24 litres and 48 litres an exact number of times?

## 3.7 Common Multiples

The multiples of 4 and 6 are

Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, ....., ....., .....

Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, ....., ....., .....

Common multiples of both 4 and 6 = 12, 24, 36, ....., ....., .....

### 3.7.1 Least common Multiple (LCM)

Common multiples of both 4 and 6 are 12, 24, 36, ....., ....., .....

Least of them is 12.

That means 12 is the lowest among the common multiples of both 4 and 6.

$\therefore$ Lowest Common Multiple (LCM) of 4 and 6 is 12

***Example-5.*** Two bells ring together. If the bells ring at every 3 minutes and 4 minutes respectively. After what interval of time will they ring together again?

***Solution*:** First bell rings after every 3 minutes.

i.e. First bell rings at 3 min, 6, 9, 12, 15, 18, 21, 24, ....., ..... (multiples of 3)

Second bell rings after every 4 minutes.

i.e., Second bell rings at 4 min, 8, 12, 16, 20, 24, ....., ....., (multiples of 4)

both bells ring together after 12 min., 24 min, ....., ...., (common multiples of both 3 and 4)

Least of them (LCM) is 12 min. That means after 12 minutes they ring together again.

Thus, we can say that

**The least common multiple of two or more given numbers is the lowest (or smallest or least) of their common multiples.**

Instead of writing all the common multiples of the given numbers every time to identify the least one of them, we can just find the LCM of those numbers directly.

### 3.7.2 Methods of Finding LCM

**1. Prime Factorization Method**

The LCM of 36 and 60 can be found by prime factorization method as follows:-

***Step-1*:** Express each number as a product of prime factors.

Factors of 36 $= \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$

Factors of 60 $= \boxed{2} \times \boxed{2} \times \boxed{3} \times 5$

***Step-2*:** Take the common factors of both: $2 \times 2 \times 3$

***Step-3*:** Take the extra factors of both 36 and 60 i.e. 3 and 5.

***Step-4*:** LCM is found by the product of all common prime factors of two numbers and extra prime factors of both.

Hence, the LCM of 36 and 60 = $(2 \times 2 \times 3) \times 3 \times 5 = 180$

## Try This

1. Find LCM of

| | | | | | |
|---|---|---|---|---|---|
| i. | 3, 4 | ii. | 10, 11 | iii. | 5, 6, 7 |
| iv. | 10, 30 | v. | 4, 12, 24 | vi. | 3, 12 |

What do you observe?

**If one of the two given numbers is a multiple of the other, then the greater number is the LCM of the given numbers.**

**2. Division Method**

To find the LCM of 24 and 90:

***Step-1*:** Arrange the given numbers in a row.

***Step-2*:** Then divide by a least prime number which divides at least two of the given numbers and carry forward the numbers which are not divisible by that number if any.

***Step-3*:** Repeat the process till numbers have no common factor other than 1.

***Step-4*:** LCM is the product of the divisors and the remaining numbers.

| | |
|---|---|
| 2 | 24, 90 |
| 3 | 12, 45 |
| | 4, 15 |

Thus, the LCM of 24 and 90 is $2 \times 3 \times 4 \times 15 = 360$

***Example-6.*** Find the LCM of 21, 35 and 42.

***Solution*:**

| | |
|---|---|
| 7 | 21, 35, 42 |
| 3 | 3, 5, 6 |
| | 1, 5, 2 |

Thus, the LCM of 21, 35 and 42 is $7 \times 3 \times 5 \times 2 = 210$

## Think, Discuss and Write

When will the LCM of two or more numbers be their own product?

## Exercise - 3.5

1. Find the LCM of the following numbers by prime factorisation method.

   i. 12 and 15 ii. 15 and 25 iii. 14 and 21

   iv. 18 and 27 v. 48, 56 and 72 vi. 26, 14 and 91.

2. Find the LCM of the following numbers by division method.

   i. 84, 112, 196 ii. 102, 119, 153 iii. 45, 99, 132, 165

3. Find the smallest number which when added to 5 is exactly divisible by 12, 14 and 18.

4. Find the greatest 3 digit number which when divided by 75, 45 and 60 leaves:

   i. no remainder ii. the remainder 4 in each case.

5. Prasad and Raju met in the market on $1^{st}$ of this month. Prasad goes to the market every $3^{rd}$ day and Raju goes every $4^{th}$ day. On what day of the month will they meet again?

## 3.8 Relationship between LCM and HCF

Consider the numbers 18 and 27.

Factors of 18 = 2 × 3 × 3;  Factors of 27 = 3 × 3 × 3

LCM of 18 and 27  is 3 × 3 × 3 × 2 = 54

HCF of 18 and 27  is 3 × 3 = 9

LCM × HCF = 54 × 9 = 486

Product of 18 and 27 = 18 × 27 = 486

What do you notice?

We see that

**Product of LCM and HCF of the two numbers = Product of the two numbers.**

***Example 7.*** Find the LCM of 8 and 12 and then find their HCF using the above relation

***Solution*:** LCM of 8 and 12 = 2 × 3 × 4 = 24

| | |
|---|---|
| 4 | 8, 12 |
| | 2, 3 |

We know, LCM × HCF = product of the two numbers

$$\text{HCF} = \frac{\text{Product of the two numbers}}{\text{LCM}}$$

$$= \frac{8 \times 12}{24} = 4$$

Hence, HCF of 8 and 12 = 4

### Think, Discuss and Write

1. What is the LCM and HCF of twin-prime numbers?
2. Interpet relationship between LCM and HCF of any two numbers?

### Exercise - 3.6

1. Find the LCM and HCF of the following numbers?
   i. 15, 24  ii. 8, 25  iii. 12, 48
   Check their relationship.
2. If the LCM of two numbers is 216 and their product is 7776, what will be the HCF?
3. The product of two numbers is 3276. If their HCF is 6, find their LCM?
4. The HCF of two numbers is 6 and their LCM is 36. If one of the numbers is 12, find the other?

## 3.9 Divisibility rules for 4, 8 and 11

We have learnt the divisibility rules for 2, 3, 5, 6, 9 and 10. Now, we derive the divisibility rule for 4, 8 and 11.

### 3.9.1 Divisibility Rule for 4

Observe the pattern

| Number | Can be written as | Whether divisible by 4? |
|---|---|---|
| 100 | 100 | Yes |
| 600 | 6 × 100 | Yes |
| 1000 | 10 × 100 | Yes |
| 10000 | 100 × 100 | Yes |
| 100000 | 1000 × 100 | Yes |

From the above table, we can observe that 100 is divisible by four. Here 600, 1000, 10000, 100000 can be expressed as a multiple of 100. So, these numbers are also divisible by 4.

You know that all even numbers are divisible by 2.

Are all even numbers also divisible by 4?

Let us verify.

126 is an even number divisible by 2. Is 126 divisible by 4?

126 can be written as 126 = 100 + 26

you know that 100 is divisible by 4. But 26 is not divisible by 4.

Hence, we can say that all even numbers are not necessarily divisible by 4.

For example consider 76532

76532 can be written as 70000 + 6000 + 500 + 30 + 2. You know that 100, 1000, 10000 are multiples of 100, and 100 is divisible by 4. So we need not test them every time. So, it is enough to test the last two digits of the given number i.e. 32. Is 32 divisible by 4? Yes. It is divisible by 4. Hence, we can say that 76532 is also divisible by 4.

You already know that odd numbers are not divisible by 4.

**A number is divisible by 4, if the number formed by its last two digits (i.e. tens and ones) is divisible by 4.**

*Note:* This rule works for number greater than hundred. For smaller numbers (1 or 2 digit numbers) we have to do actual division.

***Example-8.*** Verify whether 56496 is divisible by 4?

***Solution*:** 56496 = 50000 + 6000 + 400 + 96

We already know that 50000, 6000, 400 are all multiples of 100, they are completely divisible by 4.

We need to test whether 96 (the last two digits) is divisible by 4 or not.

96 is divisible by 4.

So, the given number 56496 is also divisible by 4

## Do This

1. Is 100000 divisible by 4? Why?
2. Give an example of a 2 digit number that is divisible by 2 but not divisible by 4?

### 3.9.2 Divisibility Rule for 8

We have learnt the divisiblity rule for 4. It is based on expanding the number. Since 10 is not divisible by 4 so we consider 100 and any number greater than 100 can be written as multiple of 100, so if the last two digits are divisible by four it will be divisible by 4. Similarly since 10 is not divisible by 8, we think of 100.

Is 100 divisible by 8? No

Is 1000 divisible by 8? Yes

We know that any number greater than 1000 can be written as something added to multiple of 1000. For example 4825 = 4 x 1000 + 825.

Thus we can say that if last three digits of a number is divisible by 8 then the number will be divisible by 8. Let us see an example-

***Example-9.*** Verify whether 93624 is divisible by 8?

***Solution*:** 93624 = 90000 + 3000 + 600 + 20 + 4

We know that 1000 is divisible by 8.

Here, 90000 and 3000 are multiples of 1000, they are certainly divisible by 8.

So, it is enough to test the divisibility of the last three digits of the number.

Is 624 divisible by 8? Yes.

Hence, the given number 93624 is also divisible by 8.

**A number with 4 or more digits is divisible by 8, if the number formed by its last three digits is divisible by 8. The divisibility for numbers with 1, 2 or 3 digits by 8 has to be checked by actual division.**

## Do This

1. Is 76104 divisible by 8?
2. Write the numbers that are divisible by 8 & lie between 100 and 200?

### 3.9.3 Divisibility Rule for 11

Fill the blanks and complete the table

| Number | Sum of the digits at odd places (from the right) | Sum of the digits at even places (from the right) | Difference | Is the difference divisible by 11 |
|---|---|---|---|---|
| 29843 | | | | |
| 90002 | | | | |
| 80927 | | | | |
| 19091908 | 8+9+9+9=35 | 0+1+0+1=2 | 35-2=33 | Yes |
| 83568 | | | | |

What do you observe from the table?

We observe that in each case the difference is either 0 or divisible by 11. All these numbers are also divisible by 11.

For the number 83568, the difference is 12 which is not divisible by 11. The number 83568 is also not divisible by 11.

**A given number is divisible by 11, if the difference between the sum of the digits at odd places and the sum of the digits at even places (from the right) is either 0 or divisible by 11.**

***Example-10.*** Is 6535 divisible by 11?

***Solutioin*:** Sum of the digits at odd places = 5 + 5 = 10

Sum of the digits at even places = 3 + 6 = 9

Their difference = 10 - 9 = 1

Is 1 divisible by 11? No

So, 6535 is not divisible by 11.

***Example-11.*** Is 1221 divisible by 11?

***Solution*:** Sum of the digits at odd places = 1 + 2 = 3

Sum of the digits at even places = 2 + 1 = 3

There difference = 3 - 3 = 0

So, 1221 is divisible by 11.

## Try this

1221 is a **polyndrome number**, which on reversing their digits gives the same number. Thus, every polyndrome number with even number of digits, is always divisible by 11.
Write a polyndrome number of 6 digits and verify whether it is divisible by 11 or not?

## Exercise - 3.7

1. Which of the following numbers are divisible by 4?
   i. 572 ii. 21,084 iii. 14,560
   iv. 1,700 v. 2150
2. Test whether the following numbers are divisible by 8?
   i. 9774 ii. 5,31,048 iii. 5500
   iv. 6136 v. 4152
3. Check whether the following numbers are divisible by 11?
   i. 859484 ii. 10824 iii. 20801
4. Verify whether the following numbers are divisible by 4 and by 8?
   i. 2104 ii. 726352 iii. 1800
5. Find the smallest number that must be added to 289279, so that it is divisible by 8?
6. Find the smallest number that can be subtracted from 1965, so that it becomes divisible by 4?
7. Write all the possible numbers between 1000 and 1100, that are divisible by 11?
8. Write the nearest number to 1240 which is divisible by 11?
9. Write the nearest number to 105 which is divisible by 4?

## What have we discussed?

1. We have discussed multiples, divisors, factors and have seen how to identify factors and multiples.
2. We have discussed the following:
   i. A factor of a number is an exact divisor of that number.
   ii. Every number is a factor of itself. 1 is a factor of every number.
   iii. Every factor of a number is less than or equal to the given number.
   iv. Every number is a multiple of each of its factors.
   v. Every multiple of a given number is greater than or equal to that number.
   vi. Every number is a multiple of itself.

3. We have learnt that:

   i. The number other than 1, with only factors namely 1 and the number itself, is a prime number. Numbers that have more than two factors are called composite numbers. Number 1 is neither prime nor composite.

   ii. The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.

   iii. Two numbers with only 1 as a common factor are called co-prime numbers.

   iv. If a number is divisible by another number then it is divisible by each of the factors of that number.

   v. A number divisible by two co-prime numbers is divisible by their product also.

4. We have discussed how we can find just by looking at a number, whether it is divisible by small numbers 2, 3, 4, 5, 8, 9 and 11. We have explored the relationship between digits of the numbers and their divisibility by different numbers.

   i. Divisibility by 2, 5 and 10 can be seen by just the last digit.

   ii. Divisibility by 3 and 9 is checked by finding the sum of all digits.

   iii. Divisibility by 4 and 8 is checked by the last 2 and 3 digits respectively.

   iv. Divisibility of 11 is checked by comparing the sum of digits at odd and even places.

5. We have discovered that if two numbers are divisible by a number then their sum and difference are also divisible by that number.

6. We have learnt that:

   i. The Highest Common Factor (HCF) of two or more given numbers is the highest of their common factors.

   ii. The Lowest Common Multiple (LCM) of two or more given numbers is the lowest of their common multiples.

7. If one of the two given numbers is a multiple of the other, then the greater number will be their LCM.

8. Relationship between LCM and HCF

   LCM × HCF = Product of the two numbers.



**Dattathreya Ramachandra Kaprekar (India)**

1905 - 1986 AD

He is a teacher, who played with numbers.

6174 is known as Kaprekar's constant.

He generated demlo numbers and self numbers.