Chapter 4

Pass-Band Modulation

CHAPTER HIGHLIGHTS

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Introduction

In the digital pass-band communication, the bit stream is converted into high frequency symbols by switching the amplitude, frequency, or phase of carrier wave.

In pass-band communication, every high frequency symbol is represented as a point in the signal space, whose axes are orthonormal basis function.

If $\phi_1(t)$, $\phi_2(t)$............$\phi_N(t)$ are orthonormal, then

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = 1 \qquad \text{if } i = j$$
$$= 0 \qquad \text{if } i \neq j$$

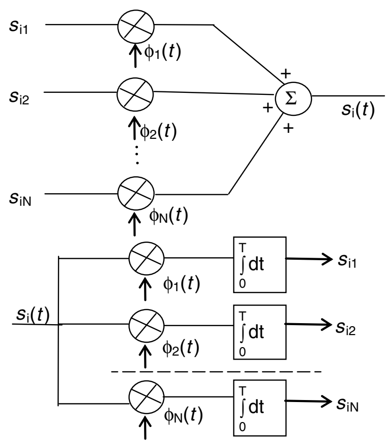
Gram–Schmidt orthogonalization procedure can be used to design the orthonormal basis functions from the high frequency symbols $s_i(t)$.

By using Gram–Schmidt orthogonalization, the symbol $s_i(t)$ can be expressed as

$$s_i(t) = \left[s_{i1}\, s_{i2} \ldots\ldots s_{iN}\right] \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \\ . \\ . \\ \phi_N(t) \end{bmatrix}$$

where s_{ik} is the component of $\phi_K(t)$ in $s_i(t)$.

$$s_{iK} = \int_0^T s_i(t)\,\phi_K(t)\,dt$$

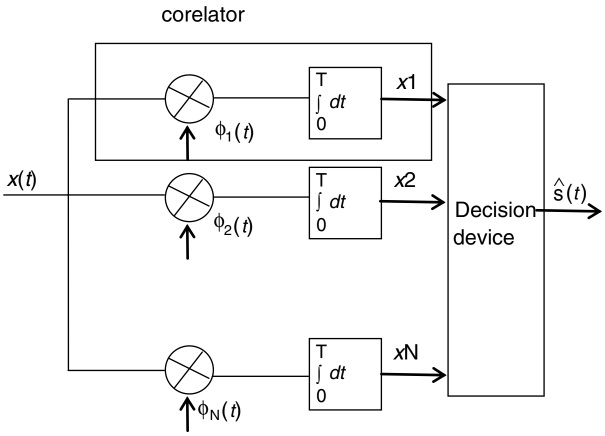


The energy of the symbol $s_i(t)$ is given by

$$E_i = \int_0^T s_i^2(t)\,dt = \sum_{j=1}^{N} s_{ij}^2$$

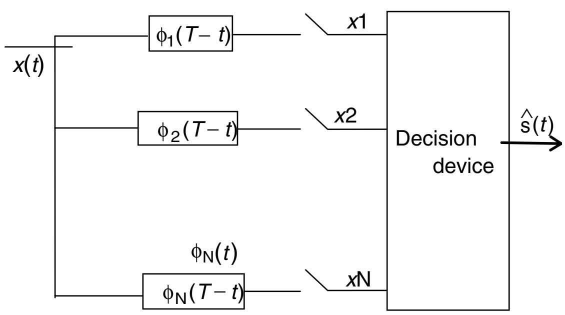
Coherent Detection

If the frequency and phase information of the carriers are available at receiver, this is called coherent detection. In coherent detection, we use correlators at the receivers to detect the signal.



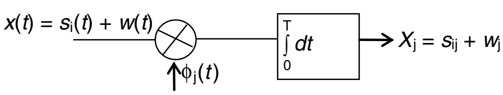
Non-coherent Detection

In the non-coherent detections, carrier frequency and phase information are not available at the receiver. Matched filters are used for non-coherent detection.



$\phi_1(T-t)$, $\phi_2(T-t)$.......are the matched filters matched to specific orthonormal basis function.

Statistics of Correlator Outputs



$$E(x_j) = s_{ij}$$

$$\sigma^2_{xj} = \frac{N_0}{2} \text{ and } \operatorname{cov}[x_j, Y_k] = 0$$

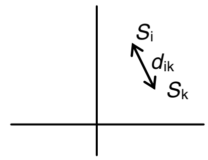
The correlator outputs are Gaussian distributed with mean s_{ij} and variance $\frac{N_0}{2}$.

$$f_{Xj}(x_j) = \frac{1}{\sqrt{\pi N_0}} \exp\left(\frac{-(x_j - s_{ij})^2}{N_0}\right)$$

The covariance between any two correlator outputs is zero, that is, the correlator outputs are statistically independent.

Probability of Symbol Error

If d_{iK} is the distance two symbols s_i and s_k in the signal space diagram.



The probability of symbol error between s_i and s_k by using coherent detection is given by

$$P(S_i, S_k) = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right)$$

Probability of symbol error between s_i and s_k by using non-coherent detection is given by

$$P(S_i, S_k) = \frac{1}{2}\exp\left(\frac{-d_{iK}^{\;2}}{2N_0}\right)$$

Coherent Binary Phase Shift Keying

In BPSK, we use two phases 0 and π to represent binary symbols 1 and 0, respectively.

$$1: \quad s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

$$0: \quad s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$$

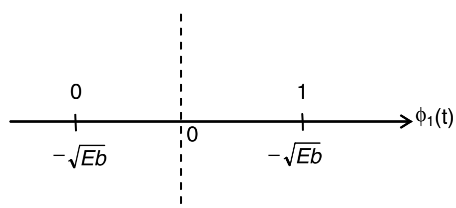
$$= -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

where E_b is the energy per bit and T_b is the bit duration.

If we take $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \quad 0 \le t \le T_b$ as basis function

$s_1(t) = \sqrt{Eb}\ \ \phi_1(t)$ and $s_2(t) = -\sqrt{E_b}\ \ \phi_1(t)$

Signal space diagram for BPSK is given by



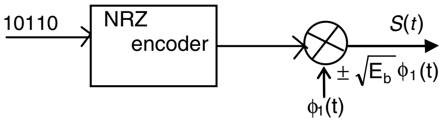
Distance between two symbols $= 2\sqrt{E_b}$

In BPSK, the probability of symbol error = probability of bit error

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{2\sqrt{E_b}}{2\sqrt{N_0}}\right)$$

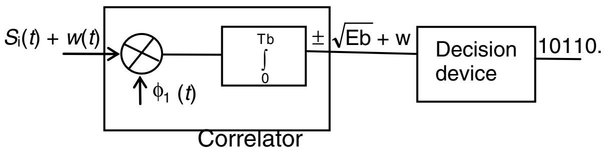
$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BPSK Modulator



The output of NRZ encoder is represented by $+\sqrt{E_b}$ or $-\sqrt{E_b}$ for 1 and '0', respectively.

BPSK Demodulator



Coherent Quadrature Phase Shift Keying

In QPSK, we convert the set of two bits into a high frequency symbol. There exist four combinations of set of two bits and each symbol is represented by a different phase. The symbols are defined as follows:

$$10 \quad : \quad \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{\pi}{4}\right)$$

$$00 \quad : \quad \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{3\pi}{4}\right)$$

$$01 \quad : \quad \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{5\pi}{4}\right)$$

$$11 \quad : \quad \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{7\pi}{4}\right)$$

For $0 \le t \le T$

where E is the energy per symbol and T is the symbol duration. In QPSK, $E = 2E_b$ and $T = 2T_b$

$$\sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{\pi}{4}\right) = \sqrt{\frac{2E}{T}}\left[\cos(2\pi f_c t).\frac{1}{\sqrt{2}} - \sin(2 f_c t).\frac{1}{\sqrt{2}}\right]$$

we can define $\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ as $\phi_1(t)$ and $\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$ as $\phi_2(t)$.

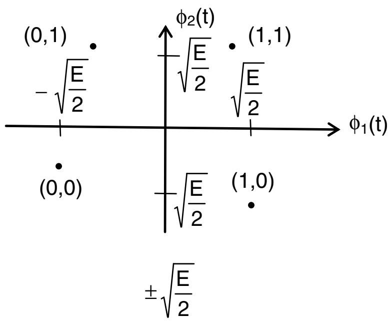
$$10: \quad \left[\sqrt{\frac{E}{2}} \quad -\sqrt{\frac{E}{2}}\right]\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix},$$

$$00: \quad \left[-\sqrt{\frac{E}{2}} \quad -\sqrt{\frac{E}{2}}\right]\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}$$

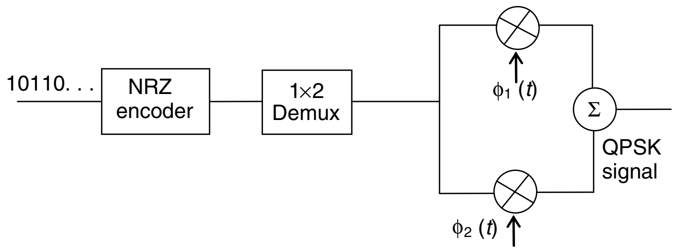
$$11: \quad \left[\sqrt{\frac{E}{2}} \quad \sqrt{\frac{E}{2}}\right]\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}$$

$$01: \quad \left[-\sqrt{\frac{E}{2}} \quad \sqrt{\frac{E}{2}}\right]\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}$$

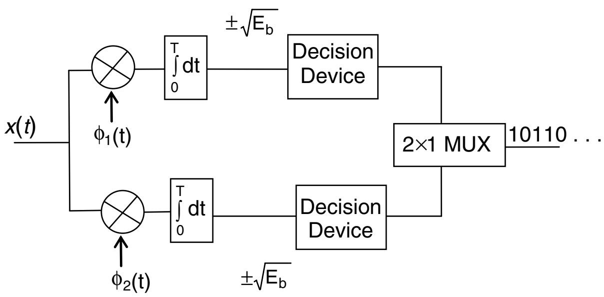
Signal space diagram for QPSK



QPSK modulation



QPSK demodulator



Probability of symbol error in QPSK

$$P_S \approx erfc\left(\sqrt{\frac{E_b}{N_0}}\right)]]$$

Probability of bit error in QPSK

$$P_e \approx \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

P_e in BPSK and P_e in QPSK are equal.

M-Ary Psk Modulation

$M = 2^L$ for some integer L.

In M-ary modulation, a set of L bits are converted into a high frequency symbol.

With L bits, there exists $2^L = M$ combinations; in M-ary PSK modulation, M different phases are allotted to M different symbols. The phases are separated by $\frac{2\pi}{M}$ radians

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1) \cdot \frac{\pi}{M}\right)$$

where $I = 1, 2, 3, \ldots\ldots\ldots M$

To plot signal space diagram, $s_i(t)$ can be written as

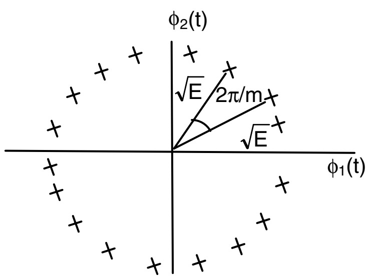
$$\sqrt{\frac{2E}{T}}\left[\cos\left(2\pi f_c t \cos(2i-1)\frac{\pi}{m} - \sin(2\pi f_c t)\sin(2i-1)\frac{\pi}{m}\right)\right]$$

All symbols in M-ary PSK have equal energy E.

If we consider $\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$

$$S_i(t) = \left[\sqrt{E}\cos\left((2i-1)\frac{\pi}{m}\ \ \sqrt{E}\sin\left((2i-1)\frac{\pi}{m}\right)\right)\right]\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}$$

Signal space diagram can be plotted as



The distance between any two adjacent symbols is given by $2\sqrt{E}\sin\left(\frac{\pi}{M}\right)$.

Probability of Symbol Error in M-ary PSK

$$P_S \cong erfc\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

where E is the energy per symbol $= E_b L = E_b \log^{M}{}_2$

Bandwidth Efficiency of PSK Modulations

BW efficiency is the number of bits transmitted per Hz $= \frac{R}{B}$

In the PSK modulations, bandwidth required $= 2 \times$ symbol rate

$$B = 2.\frac{1}{T}$$

where T is the symbol duration $= \log^{M}{}_2 .T_b$

$$B = \frac{2}{\log^{M}{}_2 .T_b} = \frac{2R}{\log^{M}{}_2}$$

Bandwidth efficiency $= \rho = \frac{R}{B} = \frac{\log^{M}{}_2}{2}$

M	ρ
2	0.5
4	1.0
8	1.5
16	2.0
32	2.5
64	3.0

For 64-ary modulation, $\rho = 3$ means we can transmit 3 bits/Hz by using 64-ary modulation, that is, if we have 10 kHz bandwidth by using 64-ary modulations, we can transmit 30 kbps data.

Binary Fsk Modulation

In the binary frequency shift keying, symbol 1 and symbol '0' are represented by two different frequencies

$$1: \quad \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) \qquad 0 \le t \le T_b$$

$$0: \quad \sqrt{\frac{2E_b}{T}}\cos(2\pi f_2 t) \qquad 0 \le t \le T_b$$

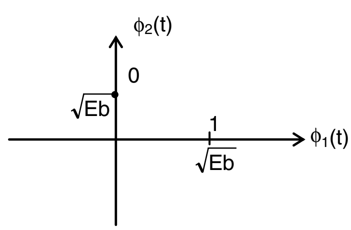
To maintain orthonormality f_1 and f_2 are chosen, such that

$$f_1 = \frac{K}{T_b} \text{ and } f_2 = \frac{K+1}{T_b}, \text{ where } K \text{ is an integer.}$$

If f_1 and f_2 are chosen as abovementioned, the FSK is also called continuous phase FSK or Sunde's FSK.

If we choose $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_1 t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_2 t)$

(both of them are orthonormal in the duration T_b). The signal space diagram is given by



Probability of Bit Error in FSK

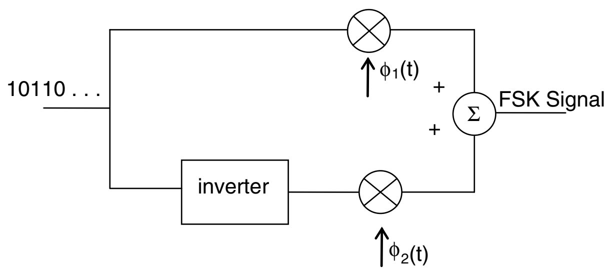
d_{iK} = distance between both symbols in FSK = $\sqrt{2E_b}$

probability of symbol error = probability of bit error = $\frac{1}{2} erfc\left(\frac{d_i k}{2\sqrt{N_0}}\right)$

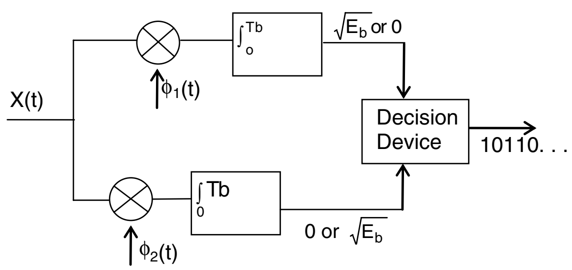
$$p_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

To maintain same P_e in the FSK, we require double the power when compared with PSK. Therefore, PSK is superior to FSK by 3 dB.

FSK Modulator



FSK Demodulator



M-ARY FSK MODULATION

In this modulation, the transmitted signals are defined by

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\frac{2\pi}{T}(n_c + i)t\right) \quad 0 \le t \le T$$

where $i = 1, 2, 3, \ldots\ldots\ldots M$ and n_c is an integer.

In the M-ary FSK, any two successive frequencies differ by $\frac{1}{2T} HZ$. $\frac{1}{2T}$ is the minimum amount of difference required between frequencies to maintain orthogonality.

Probability of symbol error in FSK

$$\approx \frac{(M-1)}{2} erfc\left(\sqrt{\frac{E}{2N_0}}\right)$$

Bandwidth Efficiency in M-ary FSK Modulation

To transmit M-ary FSK signal, bandwidth required = $M . \frac{1}{2T}$

$$B = M . \frac{1}{2\log_2 M . T_b} = \frac{M . R}{2\log_2 M}$$

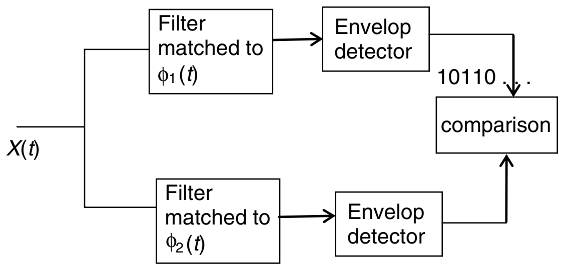
$$\rho = \frac{R}{B} = \frac{2\log_2 M}{M}$$

In the M-ary FSK, as M increases the BW efficiency decreases.

On the other hand in M-ary PSK, as M increases the BW efficiency increases.

Non-coherent Binary FSK

This modulation is similar to coherent FSK except at receivers, matched filters are used to demodulate instead of correlators. Non-coherent demodulator is given by



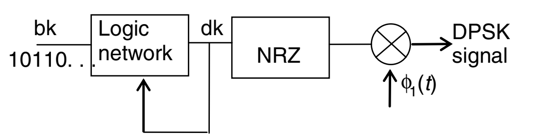
Probability of bit error in non-coherent FSK = $\frac{1}{2}\exp\left(\frac{-E_b}{2N_0}\right)$

DIFFERENTIAL PHASE SHIFT KEYING

FSK modulations can be detected coherently or non-coherently. On the other hand, non-coherent detection of PSK modulation is not possible. DPSK is the modification of BPSK, which can be detected non-coherently.

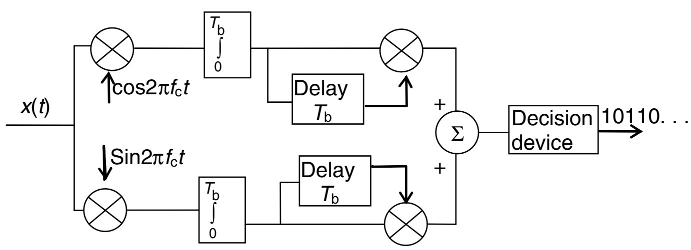
In DPSK, to send symbol '0' we phase advance the current signal waveform by 180°. To send symbol '1', we leave the phase of the current signal wave unchanged.

DPSK Modulator



bk	1	0	1	1	0	1	0	0
dk 1	1	0	0	0	1	1	0	1

DPSK Demodulator



Probability of Bit Error in DPSK

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right)$$

For the same bit error probability, we have to spend double the signal power in non-coherent FSK when compared with DPSK. Therefore, DPSK is superior to non-coherent FSK by 3 dB.

Power Spectrum Of Pass-Band Modulation

If the power spectral density of complex envelope of pass-band signal is $S_B(f)$, the power spectral density of pass-band signal is given by

$$S(f) = \frac{1}{4}\left[S_B(f - fc) + S_B(f + fc)\right]$$

BPSK

For BPSK,

$$S_B(f) = 2E_b \operatorname{sinc}^2(fT_b)$$

$$\therefore \quad S(f) = \frac{E_b}{2}\left[\operatorname{sinc}^2((f - f_c)Tb) + \operatorname{sinc}^2((f + f_c)T_b)\right]$$

QPSK

For QPSK,

$$S_B(f) = 4E_b \operatorname{sinc}^2(2T_b f)$$

$$S(f) = E_b\left[\operatorname{sinc}^2((f - f_c)2T_b) + \sin c^2((f + f_c)2T_b)\right]$$

M-ary PSK

$$S_B(f) = 2E_b \log_2 M \operatorname{sinc}^2(T_b f \log_2 M)$$

$$S(f) = \frac{E_b \log_2 M}{2}\left[\operatorname{sinc}^2((f - f_c)\log_2 M.T_b + \operatorname{sinc}^2((f + f_c)\log_2 M.T_b\right]$$

BFSK

$$S(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b\cos^2(\Pi T_b f)}{\Pi^2(4T_b^2 f^2 - 1)^2}$$

Solved Examples

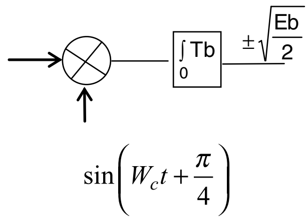
Example 1

A BPSK modulation used over an AWGN channel with noise power spectral density $\frac{N_0}{2}$ uses equiprobable signals $s_1(t) = \sqrt{\frac{2E}{T}}\sin(w_c t)$ and $s_2(t) = -\sqrt{\frac{2E}{T}}\sin(w_c t)$. If the local oscillator in a coherent receiver is ahead in phase by 45° with respect to the received signal, the probability of error in this system is

(A) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$ (B) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$

(C) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{4N_0}}\right)$ (D) $\frac{1}{2}erfc\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Solution



The distance between the two points is $\sqrt{\frac{Eb}{2}} + \sqrt{\frac{Eb}{2}} = \sqrt{2E_b}$

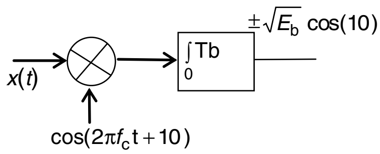
$$P_e = \frac{1}{2}erfc\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Example 2

In a BPSK signal detector, the local oscillator has a fixed phase error of 10°. This phase error deteriorates the SNR at the output by a factor of

(A) cos(10) (B) $\cos^2(10)$ (C) $\cos^2(80)$ (D) cos(80)

Solution



Instead of receiving $\pm\sqrt{E_b}$, an output of $\pm\sqrt{E_b}\cos(10)$ is received.

The SNR at the output deteriorates by a factor of $\cos^2(10)$.

Example 3

For a bit rate of 10 kbps, the best possible values of the transmitted frequencies in a coherent FSK system are

(a) 40 kHz, 60 kHz (B) 40 kHz, 50 kHz

(C) 40 kHz, 45 kHz (D) 40 kHz, 41 kHz

Solution

The frequencies in coherent BFSK should differ by $\frac{1}{T_b}$ = 10 kHz.

The best option available is 40 kHz and 50 kHz.

Example 4

A binary source output of duration 0.1 ms is transmitted using two modulation schemes BPSK and QPSK. The bandwidth required to transmit the data by using BPSK and QPSK is

(A) 20 kHz, 10 kHz (B) 20 kHz, 20 kHz
(C) 40 kHz, 20 kHz (D) 10 kHz, 5 kHz

Solution

By using BPSK,

number of symbols = number of bits

BW = 2 × number of symbols = 2 × 10 K = 20 kHz

By using QPSK,

$$\text{Number of symbols} = \frac{\text{Number of bits}}{2} = 5{,}000 \text{ symbols/s}$$

BW = 2 × 5,000 = 10 kHz

Example 5

A digital communication system employing FSK and 0 and 1 bits are represented by sine waves of 40 kHz and 55 kHz. The waveform will be orthogonal for a bit duration of

(A) 55 µ (B) 40 µs (C) 200 µs (D) 50 µ

Solution

For two frequencies to be orthogonal, the two frequencies should differ by $\frac{n}{T_b}$

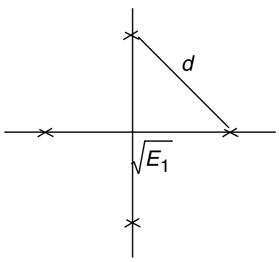
$$f_2 - f_1 = 15 \text{ kHz.}$$

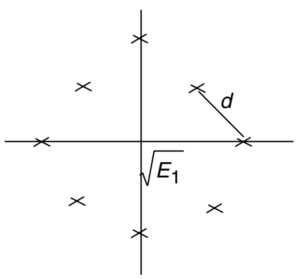
The only option satisfying the condition

$$15 \text{ kHz} = \frac{n}{T_b} \text{ is } T_b = 200\ \mu\text{s.}$$

Example 6

If the probability of symbol error is same in the undermentioned two constellations, the ratio of energy per symbol in 8 PSK to that of 4 PSK is





(A) 2.4 (B) 3.3 (C) 1.8 (D) 4.0

Solution

The probability of symbol error is same in both constellations means distance between two adjacent points is same in both constellations.

$$\sqrt{E_1} = \frac{d}{\sqrt{2}}$$

$$\sqrt{E_2}\sin\left(\frac{\pi}{8}\right) = \frac{d}{2}$$

Equating 'd' in both the equations

$$2E_1 = E_2 .\ 4.\ \sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{E_2}{E_1} = \frac{1}{2\sin^2\left(\frac{\pi}{8}\right)}$$

Example 7

A message bit sequence input to DPSK modulator is 1, 0, 1, 0, 0 1, 1. The carrier phase during the reception of the first two message bits is π and 0. The carrier phase for the remaining five message bits is

(A) π, 0, π, π, π (B) 0, π, 0, 0, 0
(C) π, 0, π, 0, π (D) 0, π, π, 0, 0

Solution

In DPCM, to send symbol 0, we phase advance the current signal waveform by 180°.

To send symbol '1', we leave the phase of the current signal waveform unchanged.

bk	1	0	1	0	0	1	1
DPSK Phase	π	0	0	π	0	0	0

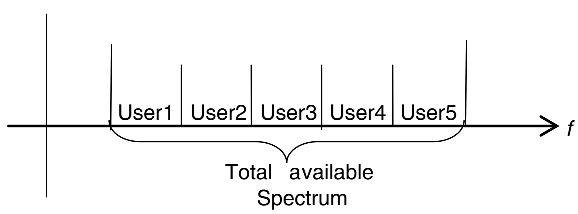
Multiple Access Techniques

Multiple access techniques are used to access the same physical channel by different users. There exists three different types of multiple accesses:

1. Frequency division multiple access (FDMA)
2. Time division multiple access (TDMA)
3. Code division multiple access (CDMA)

FDMA

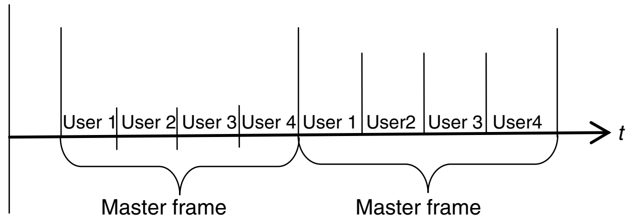
In this technique, frequency range of physical channel is divided into different parts and each part of the frequency range is allocated to a different user.



All the users can transmit parallelly by using different carrier frequencies in FDMA.

TDMA

In this technique, time available to transmit is divided into frames and each frame is allotted to a different user. All the users use the same carrier frequency and the same spectrum for transmission.



PAM signal or any pulse digital modulated signal can be used for TDMA.

Global Systems For Mobile (GSM)

The multiple access technique used in this standard is TDMA and FDMA. The available bandwidth is divided into 200 kHz channels by using FDMA. Each 200 kHz channel is shared by eight different users by using TDMA.

Table 1 *Summary of GSM Parameters*

Parameter	Specification
Uplink frequency	890–915 MHz
Downlink frequency	935–960 MHz
Number of carriers	125
Carrier bandwidth	200 kHz
Number of users per carrier	8
Multiple access method	TDMA
Data rate per carrier	270.8 kbps
Speech coding rate	13 kbps
Speech encoder	Residual pulse exited linear predictive coding (RPE LPC)
Coded speech code (after channel coding)	22.8 kbps
Modulation	GMSK (Gaussian minimum shift keying) with BT = 0.3
Demodulation	Matched filter and equalization
Frequency hopping rate	217 hopes/s

In GSM, the time domain is divided into 4.6 ms master frame. Each master frame is further divided into eight frames of each 0.577 μs and each frame is allotted to a different user.

Frequency reuse factor: in GSM, available spectrum is divided into N equal parts and each part is allocated to a cell such that two adjacent cells do not have same frequency band. The frequency reuse factor is typically $\frac{1}{7}$ or $\frac{1}{8}$ depending on cell structure.

If the total bandwidth available in GSM is 10 MHz and if the frequency reuse factor is $\frac{1}{5}$, then $10\times\frac{1}{5}=$ 2 MHz if the bandwidth available for each cell.

IS-95 CDMA Cellular System

The multiple access technique used in this standard is CDMA and FDMA. The available bandwidth is divided into 1.25 MHz channels by using FDMA. Each 1.25 MHz channel is shared by 64 different users by using Walsh–Hadamard orthogonal codes.

Table 2 *Summary of IS-95 Parameters*

Parameter	Specification
Uplink frequency	824–849 MHz
Down link frequency	869–894 MHz
Number of carriers	20
Carrier bandwidth	1.25 MHz
Multiple access method	CDMA
Data rate per carrier (chip rate)	1.228 Mbps
Speech coding rate	9,600/4,800/2,400/1,200 bps
Speech coder	Code excited linear predictive coding
Channel coding	1/2 convolution code for downlink 1/3 convolution code for uplink
Modulation	QPSK
Demodulation	RAKE receiver with matched filters
Signature sequence for multiplexing	64-bit Hadamard– Walsh codes
PN sequence length	$N = 2^{15} - 1$ for spreading $N = 2^{42} - 1$ for long code

Advantage of IS-95 Over GSM

1. Frequency reuse factor is unity in IS-95. Therefore, same frequency can be used in adjusted cells in IS-95.
2. IS-95 is more immune to narrow band noise and fading.
3. Soft handoff is possible in IS-95. In GSM, only hard handoff is possible.

Code Division Multiple Access

In CDMA, narrow band data sequence is converted into a noise-like wide band sequence by using a noise-like spreading code called pseudorandom code or pseudo-noise sequence. In CDMA, the multiple accesses are achieved by allocating different pseudo-noise sequences to each user.

Pseudo-Noise (PN) Sequence

PN sequences are periodic binary sequences with a noise-like waveforms. PN sequences are generated by feedback shift registers. If the length of PN sequence produces by a linear feedback shift register with memory M is $2^M - 1$, then the sequence is called maximum length sequence.

Properties of Maximum Length Sequences

1. **Balance property:** The number of 1s are one greater than the number of 0s. Therefore, if the length of ML sequence is 127, there will be 64 number of 1s and 63 number of 0s in the sequence.
2. **Run property:** Among the runs of 1s and 0s in each period of the sequence, one half of the runs of each kind are of length one, one-fourth are of length 2, one-eighth are of length three, etc., as long as the fraction of run represents a meaningful number ('run' means a sequence of identical symbols). The total number of runs in the maximum length sequences is 2^{n-1}.
3. The autocorrelation function of a maximum length sequence is periodic and binary valued.

$$R_c(\tau) = 1 - \frac{N+1}{N}.|\tau| \qquad \text{if } |\tau| \le T_c$$

$$= \frac{-1}{N} \qquad \text{if } |\tau| \ge T_c \text{ and within } NT_c \text{ time.}$$

Processing Gain DS CDMA

In direct sequence CDMA receivers,

$$(\text{SNR})_0 = \frac{2T_b}{T_c}(\text{SNR})_I$$

$\frac{2T_b}{T_c} = 2N$ is the gain of the receiver in separating noise.

$\frac{T_b}{T_c} = N =$ Spreading factor is called processing gain of DS CDMA system.

Probability of bit error in CDMA against the jamming power J is given by (by assuming BPSK modulation)

$$P_C = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right).$$

Jamming Margin of DS CDMA

$$\text{Jamming margin} = \frac{J}{P} = \frac{\text{Processing gain}}{E_b/No}.$$

where J is the jamming power and P is the power off the DS CDMA signal.

Example 8

Four independent messages have bandwidths of 200 Hz, 200 Hz, 300 Hz, and 400 Hz, respectively. Each signal is transmitted at Nyquist rate and the samples are time division multiplexed and transmitted. The transmitted sample rate is

(A) 6,400 samples/s (B) 2,200 samples/s
(C) 1,600 samples/s (D) 3,200 samples/s

Solution

The Nyquist sample rates for the four message signals is 400, 400, 600, and 800 samples/s, respectively.
Number of samples required to transmit per second by using TDM is 400 + 400 + 600 + 800 = 2,200 samples/s.

Example 9

A message signal is having frequency range from 0 Hz to 5 kHz. The message signals are required to transmit through a coaxial cable by using SSB modulation and FDM. A dot band of 1 kHz is required in between two adjacent channels. The number of message signals, which can be transmitted simultaneously if the coaxial cable offers a bandwidth of 150 kHz is

(A) 25. (B) 30. (C) 15. (D) 12.

Solution

BW required for SSB modulation = W = 5 kHz
1 kHz dot band is required to be kept between two adjacent channels.
BW required for each message signal

$$= 5 \text{ kHz} + 1 \text{ kHz} = 6 \text{ kHz}$$

$$\text{Number of channels } = \frac{150\,\text{KHz}}{6\,\text{KHz}} = 25$$

Therefore, 25 message signals can be transmitted simultaneously.

Example 10

In a GSM system, eight channels can coexist in 200 kHz bandwidth using TDMA. A GSM-based cellular operator is allocated 12 MHz bandwidth. Assuming a frequency reuse factor of $\frac{1}{6}$, the maximum number of simultaneous speech channels that can exist at one cell at a time is

(A) 480 (B) 10 (C) 80 (D) 60

Solution

Frequency reuse factor = $\frac{1}{6}$ means six cells share the available bandwidth equally.
In this problem, each cell will have a bandwidth of $\frac{12\,\text{KHz}}{6}$ = 2 MHz

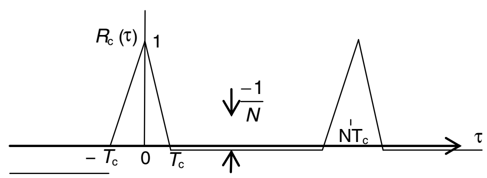
$$\text{Number of 200 kHz channels} = \frac{2\,\text{MHz}}{200\,\text{K}} = 10$$

Each 200 kHz can support eight speech channels.
Maximum number of simultaneous speech channels
$= 10 \times 8 = 80$

Example 11

In a DS CDMA system, the chip rate = 1.2288×10^6 chips/s. If the processing gain desires is at least 10, the data rate can be

(A) 1.2288×10^5 (B) 1.2288×10^7
(C) 1.2288×10^4 (D) 1.2288×10^8



Solution

$$\text{Processing gain} = \frac{\textit{chip rate}}{\textit{data rate}}$$

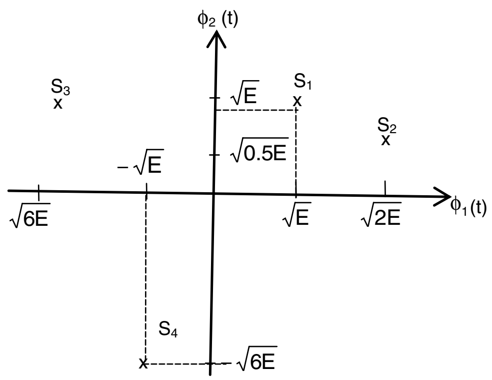
$$\text{Data rate} = \frac{1.2288\times10^6}{10} = 1.2288 \times 10^5 \text{ bits/s}$$

Exercises

Practice Problems I

Direction for questions 1 to 17: Select the correct alternative from the given choices.

1. The signal space diagram for a pass-band modulation is given as follows.

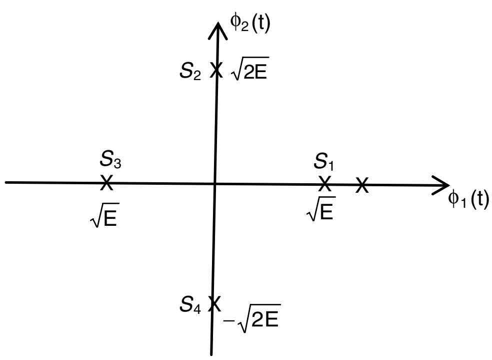


The approximate value of probability of symbol error if all the four symbols are transmitted with equal probability is (assume AWGN with PSD $N_0/2$).

(A) $\frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\right)$ (B) $\frac{1}{2}erfc\left(\sqrt{\frac{E}{2N_0}}\right)$

(C) $\frac{1}{4}erfc\left(\frac{\sqrt{E}}{2\sqrt{N_0}}\right)$ (D) $\frac{1}{4}erfc\left(\frac{\sqrt{E}}{4\sqrt{N_0}}\right)$

2. The signal space diagram of a pass-band modulation scheme is given by the following diagram.



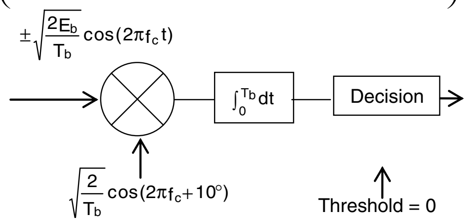
If the symbols S_1, S_2, S_3, and S_4 are transmitted with probabilities 0.2, 0.4, 0.2, and 0.2, respectively. The average energy transmitted per symbol in this modulation scheme is

(A) E (B) $2E$ (C) $1.5E$ (D) $1.6E$

3. A binary source is emitting a data of 100 kbps. The bandwidth required to transmit this data by using 256-ary PSK modulation is

(A) 25 kHz (B) 50 kHz
(C) 75 kHz (D) 12.5 kHz

4. The following is the demodulator for PSK. If there exists a phase error 10° in the local oscillator of demodulator, the probability of symbol error is given by $\left(\text{assume AWGN with special density } \frac{N_0}{2}\right)$.



(A) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_c}}\right)$

(B) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}\cos 10^\circ}\right)$

(C) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\cos 10^\circ\right)$

(D) $\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}\sin 10^\circ}\right)$

5. MSK modulation is used to transmit a binary data of 100 kbps. If f_1 = 1 MHz, the value of f_2 is

(A) 1.1 MHz. (B) 1.05 MHz.
(C) 1.2 MHz. (D) 2.00 MHz.

6. If Gray coding is used in a 64-ary PSK modulation, the approximate value of symbol error and bit error if the 64-ary symbols are transmitted in AWGN environment with power special density $\frac{N_0}{2}$

(A) $P_S = \frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\sin\frac{\pi}{64}\right)$

$P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\sin\frac{\pi}{64}\right)$

(B) $P_S = \frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{120}\right)\right)$

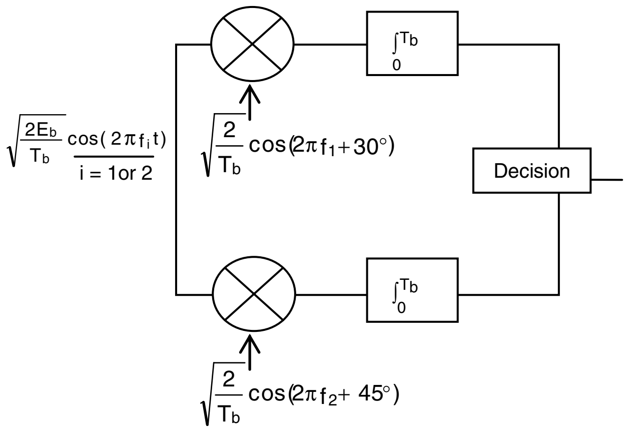
$P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{120}\right)\right)$

(C) $P_S = \frac{1}{2}erfc\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{64}\right)\right)$

$P_e = \frac{1}{12}erfc\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{64}\right)\right)$

(D) None of the above

7. A coherent FSK demodulator is given as follows:



Assuming AWGN environment, probability of bit error in abovementioned demodulator is

(A) $\frac{1}{2}erfc\left(\sqrt{\frac{E_d}{N_0}}\right)$ (B) $\frac{1}{2}erfc\left(\sqrt{\frac{E_d}{2N_0}}\right)$

(C) $\frac{1}{2}erfc\left(\sqrt{\frac{E_d}{N_0}}\sqrt{\frac{3}{2}}\right)$ (D) $\frac{1}{2}erfc\left(\sqrt{\frac{E_d}{N_0}}\sqrt{\frac{5}{16}}\right)$

8. In a QPSK demodulator, the probability of bit error in I channel is 0.01 and probability of error in channel is 0.05. The probability of symbol error in this QPSK demodulator is

(A) 0.01. (B) 0.05. (C) 0.06. (D) 0.03.

9. DPSK modulation is used with $\frac{E_b}{N_0}$ = 13 dB. The probability of bit error in this scenario is

(A) $\frac{1}{2}erfc\left(\sqrt{20}\right)$ (B) $\frac{1}{2}erfc\left(\sqrt{10}\right)$

(C) $\frac{1}{2}\exp(-20)$ (D) $\frac{1}{2}\exp(-10)$

10. BPSK modulation is used to transmit a computer data of 10 kbps. If the symbols used to represent 1 and 0 are $\pm$ 5 cos ($2\pi\times 10^6 t$), the modulation is used to transmit in an AWGN channel with N_o = 1 mW. P_e in this scenario is

(A) $\frac{1}{2}erfc(2)$. (B) $\frac{1}{2}erfc(1.6)$.

(C) $\frac{1}{2}erfc(1.2)$. (D) $\frac{1}{2}erfc(1.8)$.

11. If P_1 is the probability of error by using DPSK with $\frac{E_b}{N_0}$ of 12 dB. If the same probability of error p_1 is required to maintain by non-coherent FSK modulator, the value of $\frac{E_b}{N_0}$ should be

(A) 12 dB. (B) 15 dB. (C) 24 dB. (D) 9 dB.

12. If 20 kbps is required to transmit by using 8-ary PSK and 8-ary FSK, the bandwidth required in both the cases, respectively, is

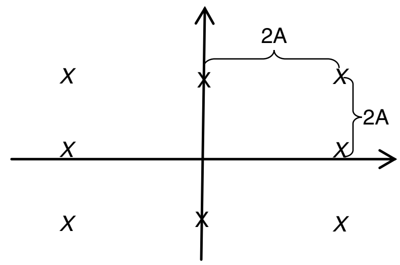
(A) 20 kHz, 20 kHz. (B) 13.3 kHz, 26.6 kHz.
(C) 13.3 kHz, 20 kHz. (D) 13.3 kHz, 33.3 kHz.

13. Given f_1 = 2 MHz and T = 1 μs, the frequency which is not in orthogonal with f_1 over the interval T is

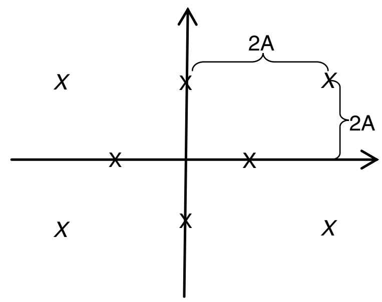
(A) 4 MHz. (B) 5 MHz.
(C) 2.5 MHz. (D) 1.0 MHz.

14. Consider the two 8-point constellations shown in the following figure. The minimum distance between adjacent points is 2 A. The ratio of transmitted energy in the first constellation to the second constellation is

(i)



(ii)



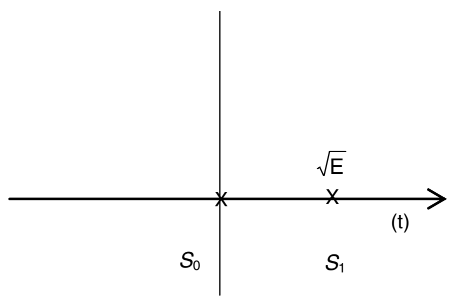
(A) 1 (B) $\frac{8}{7}$ (C) $\frac{6}{7}$ (D) $\frac{4}{5}$

15. Orthonormal basis function pair for T_b = 20 ms is

(A) 10 cos ($2\pi\times 10^3 t$)
10 sin ($2\pi\times 10^3 t$)
(B) 20 cos ($2\pi\times 10^3 t$)
20 sin ($2\pi\times 10^3 t$)
(C) 10 cos ($2\pi\times 10^3 t$)
20 sin ($2\pi\times 10^3 t$)
(D) 10 cos (120 πt)
10 sin (120 πt)

Direction for question 16:

16. The signal space diagram of a specific binary modulation scheme is given in the following figure. The probability of error in this modulation for coherent detection by assuming AWGN channel is



(A) $\frac{1}{2} erfc\left(\sqrt{\frac{E}{4N_0}}\right)$ (B) $\frac{1}{2} erfc\left(\sqrt{\frac{E}{2N_0}}\right)$

(C) $\frac{1}{2} erfc\left(\sqrt{\frac{2E}{N_0}}\right)$ (D) erfc $\left(\sqrt{\frac{E}{N_0}}\right)$

17. The probability of error in the abovementioned transmission model, if the detection is of non-coherent type is

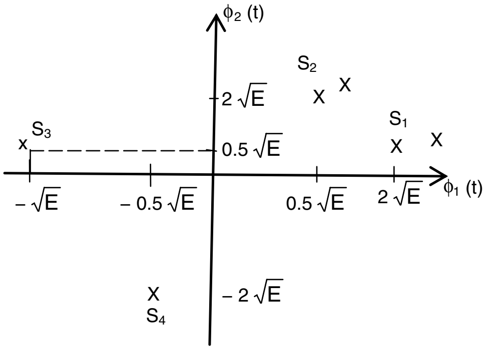
(A) $P_e = \frac{1}{2}\exp\left(\frac{-E}{N_0}\right)$ (B) $P_e = \frac{1}{2}\exp\left(\frac{-2E}{N_0}\right)$

(C) $P_e = \frac{1}{2}\exp\left(-\sqrt{\frac{E}{N_0}}\right)$ (D) $P_e = \frac{1}{2}\exp\left(\frac{-E}{2N_0}\right)$

Practice Problems 2

Direction for questions 1 to 17: Select the correct alternative from the given choices.

1. Signal space diagram for a pass-band communication mode is given in the following figure.



If all the symbols are transmitted with equal probability, the average energy transmitted per symbol is

(A) 2 E. (B) 2.5 E. (C) 4 E. (D) 3.5 E.

2. Number of orthonormal basis functions required to represent signal space diagram of 8-ary PSK and 8-ary FSK, respectively, are

(A) 2, 2. (B) 2, 4. (C) 4, 4. (D) 2, 8.

3. If 10 W power is required to transmit to get a P_e of 10^{-5} by using BPSK modulation in AWGN channel, the power required to transmit to get probability of bit error 2.5×10^{-6} by using 16 PSK with Gray coding is

(A) 10 W. (B) 40 W. (C) 62.5 W. (D) 50 W.

4. A message bit sequence input to a DPSK modulator is 101101. The carrier phase during the reception of first two message bits is 0, π. The carrier phase for the remaining four message bits is

(A) 0, 0, π, π. (B) π, π, 0, 0.
(C) 0, π, 0, π. (D) π, 0, π, 0.

5. In a satellite communication channel, 20 kHz bandwidth is available. Maximum data rate that can be transmitted by using BFSK, BPSK, and QPSK, respectively, is

(A) 10 kbps, 10 kbps, 20 kbps.
(B) 10 kbps, 20 kbps, 40 kbps.
(C) 20 kbps, 10 kbps, 20 kbps.
(D) 20 kbps, 20 kbps, 40 kbps.

6. Which of the following pair of frequencies can be used for MSK modulation with data rate of 100 kbps?

(A) 1 MHz, 1.1 MHz (B) 1 MHz, 1.01 MHz
(C) 1 MHz, 1.05 MHz (D) 1 MHz, 1.02 MHz

7. A 100 kbps data is required to be transmitted by using 16-ary PSK modulation. Which of the following can be used as orthonormal basis function?

(A) 1,000 $\cos(2\pi \times 10^6 t)$ (B) 100 $\cos(2\pi \times 10^6 t)$
(C) 224 $\cos(2\pi \times 10^6 t)$ (D) 448 $\cos(2\pi \times 10^6 t)$

8. If $\frac{E_b}{N_0}$ = 10 dB, which of the following modulation scheme performance is more reliable (probability of bit error is less)?

(A) Coherent BPSK
(B) Coherent QPSK
(C) MSK
(D) All of the above perform almost equally

9. The output communication channel by using BPSK modulation is given by

$$x(t) = \pm\sqrt{E_b}\phi_1(t) + |w(t)|$$

where $w(t)$ is AWGN with power spectral density $\frac{N_0}{2}$.

If 1 is represented by $\sqrt{E_b}\phi(t)$,

0 is represented by $-\sqrt{E_b}\phi_1(t)$

$|w(t)|$ is the modules of $w(t)$ and coherent detector is used at receiver with the threshold value $\lambda = 0$.

The probability of bit error, if symbol 1 is transmitted is

(A) Zero. (B) $erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$.

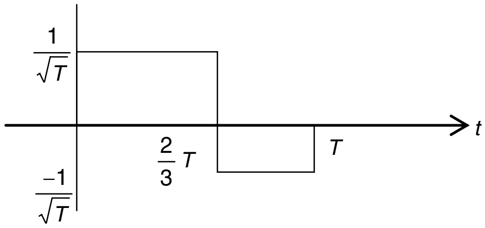
(C) $\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$. (D) $\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$.

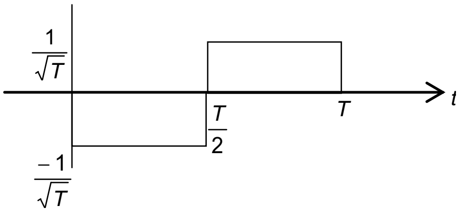
10. In problem 9, the probability of error if symbol '0' is transmitted, then it is given by

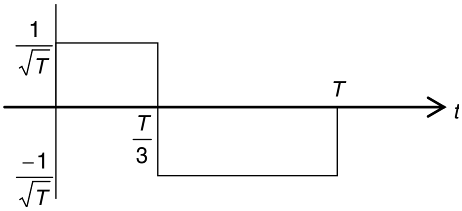
(A) Zero. (B) $\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$.

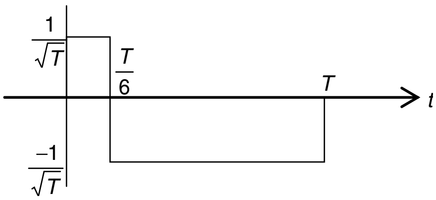
(C) $erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$. (D) $\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$.

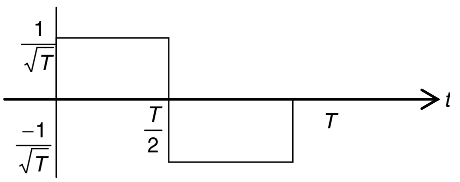
11. The function which is orthogonal to $g_1(t)$ is $g_1(t)$



(A)


(B)


(C)


(D)


12. If $erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-2}$, the value of $erf\left(\sqrt{\frac{E_b}{N_0}}\right) =$

(A) 0.01 (B) 0.99 (C) 0.02 (D) 0.98

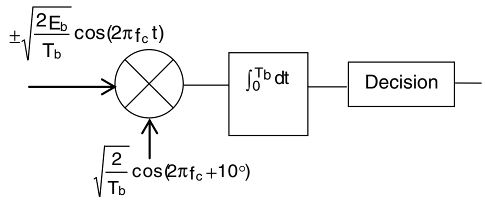
13. If $\text{erfc}(x) = 10^{-3}$, the value of the integral $\frac{1}{\sqrt{\pi}} \int_{-x}^{\infty} e^{-w^2} dw$ is

(A) 0.999. (B) 0.99875.
(C) 0.99925. (D) 0.002.

14. If the coherent PSK requires a $\frac{E_b}{N_0}$ of 6 dB to maintain required probability of bit error, $\frac{E_b}{N_0}$ required by using coherent FSK to maintain same probability of bit error is

(A) 6 dB. (B) 9 dB. (C) 3 dB. (D) 12 dB.

15. A demodulator for BPSK is as mentioned in the following figure.



Let the local oscillator is generating the carrier frequency with a phase error of 10° in the coherent detector. To compensate this phase error, the amount of power the modulator should increase so as to get the same probability of error without phase error is

(A) cos 10°. (B) $\cos^2 10°$.
(C) sec 10°. (D) $\sec^2 10°$.

16. A transmission model uses 64-ary FSK modulation and the bandwidth efficiency in this modulation is

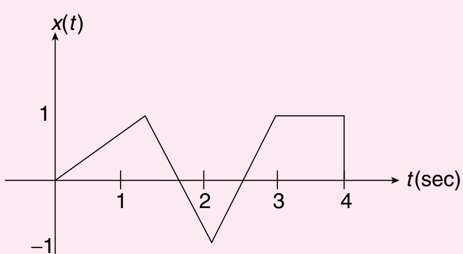
(A) 0.5 (B) 0.25 (C) 0.36 (D) 0.18

17. A 200 kbps data is required to transmit over a 100 MHz carrier by using a PSK modulation. The BW range required is

(A) 99.9 MHz to 100.1 MHz
(B) 99.8 MHz to 100.2 MHz
(C) 99.6 MHz to 10,041 MHz
(D) 99.9 MHz to 100.2 MHz

Previous Years' Questions

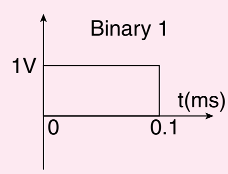
1. Consider the signal $x(t)$ shown in the figure. Let $h(t)$ denote the impulse response of the filter matched to $x(t)$ with $h(t)$ being non-zero only in the interval 0 to 4 s. The slope of $h(t)$ in the interval $3 < t < 4$ s is **[2004]**

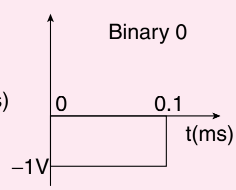


(A) $\frac{1}{2}\text{s}^{-1}$ (B) -1s^{-1}

(C) $-\frac{1}{2}\text{s}^{-1}$ (D) $1\ \text{s}^{-1}$

2. A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in the figure.





The source output is transmitted using two modulation schemes, namely binary PSK(BPSK) and quadrature PSK(QPSK). Let B_1 and B_2 be the bandwidth requirements of BPSK and QPSK, respectively. Assuming that the bandwidth of the abovementioned rectangular pulses is 10 kHz, B_1, and B_2 are **[2004]**

(A) $B_1 = 20$ kHz, $B_2 = 20$ kHz
(B) $B_1 = 10$ kHz, $B_2 = 20$ kHz
(C) $B_1 = 20$ kHz, $B_2 = 10$ kHz
(D) $B_1 = 10$ kHz, $B_2 = 10$ kHz

3. In a GSM system, eight channels can co-exist in 200 kHz bandwidth using TDMA. A GSM-based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of $\frac{1}{5}$, that is, a five-cell repeat pattern, the maximum number of simultaneous channels that can exist in one cell is **[2007]**

(A) 200 (B) 40 (C) 25 (D) 5

4. In a direct sequence CDMA system, the chip rate is 1.2288×10^6 chips/s. If the processing gain is desired to be at least 100, the data rate **[2007]**

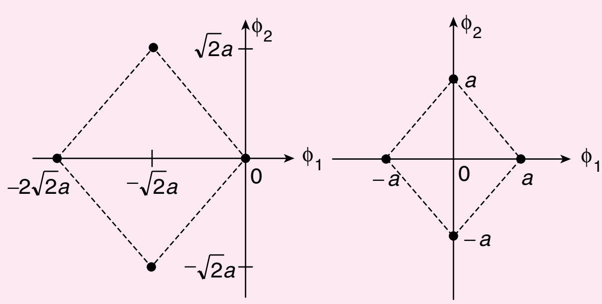
(A) must be less than or equal to 12.288×10^3 bits/s
(B) must be greater than 12.288×10^3 bits/s
(C) must be exactly equal to 12.288×10^3 bits/s
(D) can take any value less than 122.88×10^3 bits/s

5. The raised cosine pulse $p(t)$ is used for zero ISI in digital communications. The expression for $p(t)$ with unity roll-off factor is given by $p(t) = \dfrac{\sin 4\pi\varpi t}{4\pi\varpi t(1-16\varpi^2 t^2)}$ and the value of $P(t)$ at $t = \dfrac{1}{4\varpi}$ is **[2007]**

(A) –0.5 (B) 0 (C) 0.5 (D)

Direction for questions 6 and 7:

Two 4-ray signal constellations are shown in the figure. It is given that ϕ_1 and ϕ_2 constitute an orthonormal basis for the two constellations. Assume that the four symbols in both the constellations are equiprobable. Let $\frac{N_0}{2}$ denote the power spectral density of white Gaussian noise



Constellation 1 Constellation 2

6. The ratio of the average energy of constellation 1 to the average energy of constellation 2 is **[2007]**

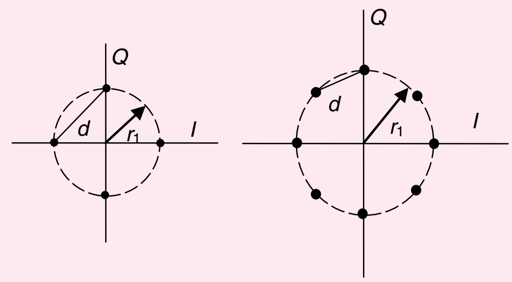
(A) $4a^2$ (B) 4 (C) 2 (D) 8

7. If these constellations are used for digital communications over an AWGN channel, then which of the following statements is true? **[2007]**

(A) Probability of symbol error for constellation 1 is lower.
(B) Probability of symbol error for constellation 1 is higher.
(C) Probability of symbol error is equal for both the constellations.
(D) The value of N_0 will determine which of the two constellations has a lower probability of symbol error.

Direction for questions 8 and 9:

A four-phase and an eight-phase signal constellation are shown in the following figure.



8. For the constraint that the minimum distance between pairs of signal points be d for both constellations, the radii r_1 and r_2 of the circles are **[2011]**

(A) $r_1 = 0.707d$, $r_2 = 2.782d$
(B) $r_1 = 0.707d$, $r_2 = 1.932d$
(C) $r_1 = 0.707d$, $r_2 = 1.545d$
(D) $r_1 = 0.707d$, $r_2 = 1.307d$

9. Assuming high SNR and that all signals are equally probable, the additional average transmitted signal energy required by the 8-PSK signal to achieve the same error probability as the 4-PSK signal is **[2011]**

(A) 11.90 dB (B) 8.73 dB
(C) 6.79 dB (D) 5.33 dB

10. A BPSK scheme operating over an AWGN channel with noise power spectral density of $N_0/2$ uses equiprobable signals $S_1(t) = \sqrt{\frac{2E}{T}}\sin(\omega_c t)$ and $S_2(t) = -\sqrt{\frac{2E}{T}}\sin(\omega_c t)$over the symbol interval $(0, T)$. If the local oscillator in a coherent receiver is a head in phase by 45° with respect to the received signal, the probability of error in the resulting system is **[2012]**

(A) $Q\left[\sqrt{\frac{2E}{N_0}}\right]$ (B) $Q\left[\sqrt{\frac{E}{N_0}}\right]$

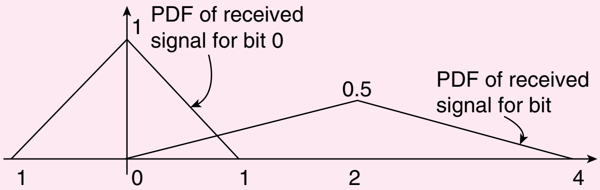
(C) $Q\left[\sqrt{\frac{E}{2N_0}}\right]$ D) $Q\left[\sqrt{\frac{E}{4N_0}}\right]$

11. The bit rate of a digital communication system is R kbits/s. The modulation used is 32-QAM. The minimum bandwidth required for ISI-free transmission is **[2013]**

(A) $R/10$ Hz (B) $R/10$ kHz
(C) $R/5$ Hz (D) $R/5$ kHz

Direction for questions 12 and 13:

Bits 1 and 0 are transmitted with equal probability. At the receiver, the PDF of the respective received signals for both bits are as shown in the following figure.



12. If the detection threshold is 1, the BER will be **[2013]**

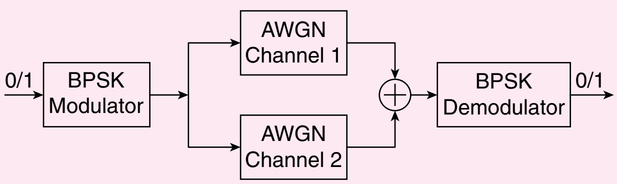
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

13. The optimum threshold to achieve minimum bit error rate (BER) is **[2013]**

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$ (C) 1 (D) $\frac{3}{2}$

14. In a code-division multiple access (CDMA) system with $N = 8$ chips, the maximum number of users who can be assigned mutually orthogonal signature sequences is ________ **[2014]**

15. Let $Q\left(\sqrt{\gamma}\right)$ be the BER of a BPSK system over an AWGN channel with two-sided noise power spectral density $N_0/2$. The parameter γ is a function of bit energy and noise power spectral density.
A system with two independent and identical AWGN channels with noise power spectral density $N_0/2$ is shown in the figure. The BPSK demodulator receives the sum of outputs of both the channels.



If the BER of this system is $Q\left(b\sqrt{\gamma}\right)$, then the value of b is ________. **[2014]**

16. Coherent orthogonal binary FSK modulation is used to transmit two equiprobable symbol waveforms $s_1(t) = \alpha \cos 2\pi f_1 t$ and $s_2(t) = \alpha \cos 2\pi f_2 t$, where α = 4 mV. Assume an AWGN channel with two-sided noise power spectral density $\frac{N_0}{2} = 0.5\times10^{-12}$ W / HZ. Using an optimal receiver and the relation $Q(v) = \frac{1}{\sqrt{2\pi}}\int_v^{\infty} e^{-u/2}du$, the bit error probability for a data rate of 500 kbps is **[2014]**

(A) $Q(2)$ (B) $Q(2\sqrt{2})$
(C) $Q(4)$ (D) $Q(4\sqrt{2})$

17. An M-level PSK modulation scheme is used to transmit independent binary digits over a band-pass channel with bandwidth 100 kHz. The bit rate is 200 kbps and the system characteristic is a raised cosine spectrum with 100% excess bandwidth. The minimum value of M is ________ **[2014]**

18. Consider a discrete-time channel $Y = X + Z$, where the additive noise Z is signal-dependent. In particular, given the transmitted symbol $X \,\varepsilon\, \{-a, +a\}$ at any instant, the noise sample Z is chosen independently from a Gaussian distribution with mean αX and unit variance. Assume a threshold detector with zero threshold at the receiver.
When $\alpha = 0$, the BER was found to be $Q(a) = 1 \times 10^{-8}$
$Q(v) = \frac{1}{\sqrt{2\pi}}\int_v^{\infty} e^{-u^2/2}du$, and for $v > 1$, use $Q(v) \approx e^{-v^2/2}$)

When $\alpha = -0.3$, the BER is closest to **[2014]**

(A) 10^{-7} (B) 10^{-6} (C) 10^{-4} (D) 10^{-2}

19. The signal $\cos\left[10\pi t + \frac{\pi}{4}\right]$ is ideally sampled at a sampling frequency of 15 Hz. The sampled signal is passed through a filter with impulse response $\left(\frac{\sin(\pi t)}{\pi t}\right) \cos\left(40\pi t - \frac{\pi}{2}\right)$. The filter output is **[2015]**

(A) $\frac{15}{2}\cos\left(40\pi t - \frac{\pi}{4}\right)$

(B) $\frac{15}{2}\left(\frac{\sin(\pi t)}{\pi t}\right)\cos\left(10\pi t + \frac{\pi}{4}\right)$

(C) $\frac{15}{2}\cos\left(10\pi t - \frac{\pi}{4}\right)$

(D) $\frac{15}{2}\left(\frac{\sin(\pi t)}{\pi t}\right)\cos\left(40\pi t - \frac{\pi}{2}\right)$

20. The modulation scheme commonly used for transmission from GSM mobile terminals is **[2015]**

(A) 4-QAM
(B) 16-PSK
(C) Walsh–Hadamard orthogonal codes
(D) Gaussian minimum shift keying (GMSK)

21. An ideal band pass channel 500Hz – 2000Hz is deployed for communication. A modem is designed to transmit bits at the rate of 4800 bits/s using 16 – QAM. The roll off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band is _______. **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** D	**3.** A	**4.** C	**5.** B	**6.** C	**7.** D	**8.** C	**9.** C	**10.** B
11. B	**12.** B	**13.** C	**14.** B	**15.** A	**16.** A	**17.** D			

Practice Problems 2

1. D	**2.** D	**3.** C	**4.** B	**5.** C	**6.** C	**7.** C	**8.** D	**9.** A	**10.** C
11. C	**12.** B	**13.** C	**14.** B	**15.** D	**16.** C	**17.** D			

Previous Years' Questions

1. B	**2.** C	**3.** B	**4.** A	**5.** C	**6.** C	**7.** B	**8.** D	**9.** D	**10.** B
11. B	**12.** D	**13.** B	**14.** 7.99 to 8.01		**15.** 1.4 to 1.42		**16.** C	**17.** 16	**18.** C
19. A	**20.** D	**21.** 0.25							