SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)*	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	2(2)#	1(2)*	1(3)*	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	4(4)#	–	–	1(5)*	**5(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	1(4)	2(4)	–	–	**3(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If $A=\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then for what value of λ, A^{-1} will exist?

OR

Find the values of x for which $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$.

2. Show that the function $f: R \rightarrow R$, given by $f(x) = |x|$ is neither one-one nor onto.

3. Find the direction cosines of the perpendicular from the origin to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}-6\hat{k})=5$.

OR

Find the equation of plane passing through the point (1, 2, 3) and the direction cosines of the normal as l, m, n.

4. Check whether the matrix $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ is a symmetric matrix.

5. Evaluate : $\int\left(5x^3+2x^{-5}-7x+\frac{1}{\sqrt{x}}+\frac{5}{x}\right)dx$

OR

Evaluate : $\int\frac{\sin(x-a)}{\sin(x+a)}\,dx$

6. Find the number of bijective functions from set A to itself when A contains 106 elements.

7. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$, then find $|\vec{a}\times\vec{b}|$.

OR

Find the value of λ so that the vectors $2\hat{i}-4\hat{j}+\hat{k}$ and $4\hat{i}-8\hat{j}+\lambda\hat{k}$ are perpendicular.

8. Evaluate : $\int_{\pi/4}^{\pi/2}\cos 2x\,dx$

9. Write the direction cosines of a line parallel to z-axis.

10. Determine the order and degree of $5\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$.

OR

Find the integrating factor of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x$.

11. Let $\vec{a}$ and $\vec{b}$ are non-collinear. If $\vec{c}=(x-2)\vec{a}+\vec{b}$ and $\vec{d}=(2x+1)\vec{a}-\vec{b}$ are collinear, then find x.

12. Find the equation of the plane with intercept 2, 3 and 4 on the x,y and z-axis, respectively.

13. Let $f: R\to R$ be defined by $f(x)=\begin{cases}2x : x>3\\ x^2 : 1<x\le 3\\ 3x : x\le 1\end{cases}$. Find $f(-1)+f(2)+f(4)$.

14. The equation of a line is $5x-3=15y+7=3-10z$. Write the direction cosines of the line.

15. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$.

16. If $A=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$, then find $A+A'$, where A' is the transpose of matrix A.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Ajay wants to construct a rectangular fish tank for his new house that can hold 72 ft^3 of water. The top of the tank is open. The width of tank will be 5 ft but the length and heights are variables. Building the tank cost ₹ 10 per sq. foot for the base and ₹ 5 per sq. foot for the side.

Based on the above information, answer the following question :

(i) In order to make a least expensive fish tank, Ajay need to minimize its

(a) Volume (b) Base (c) Curved surface area (d) Cost

(ii) Total cost of tank as a function of h can be represented as

(a) $c(h) = 50\,h - 144 - 720/h$ (b) $c(h) = 50\,h - 144\,h - 720\,h^2$

(c) $c(h) = 50 + 144\,h + 720\,h^2$ (d) $c(h) = 50\,h + 144 + \dfrac{720}{h}$

(iii) Range of h is

(a) $(3, 5)$ (b) $(0, \infty)$ (c) $(0, 8)$ (d) $(0, 3)$

(iv) Value of h at which $c(h)$ is minimum, is

(a) $\sqrt{14.4}$ (b) $\sqrt{12.2}$ (c) 14.5 (d) 12.5

(v) The cost of least expensive tank is

(a) ₹ 500 (b) ₹ 502.04 (c) ₹ 523.47 (d) ₹ 600.05

18. A night before sleep, grandfather gave a puzzle to Rohan and Payal. The probability of solving this specific puzzle independently by Rohan and Payal are $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

Based on the above information answer the following :

(i) Probability that both solved the puzzle, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

(ii) Probability that puzzle is solved by Rohan but not by Payal, is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{3}{5}$ (d) $\frac{1}{3}$

(iii) Find the probability that puzzle is solved.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

(iv) Probability that exactly one of them solved the puzzle.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

(v) Probability that none of them solved the puzzle.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) None of these

PART - B

Section - III

19. Find the area enclosed by the line $y = 3x$, the x-axis, and the ordinates $x = 1$ and $x = 4$.

20. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right)$.

OR

Find the two branches other than the principal value branch of $\tan^{-1}x$.

21. A coin is tossed and then a die is thrown. Find the probability of obtaining a '6' given that head came up.

22. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$, then find A^{-1}.

23. Evaluate : $\displaystyle\int_1^3 \frac{1}{x(1+\log x)^2}\,dx$

OR

Evaluate : $\displaystyle\int_0^{2\pi} e^x \left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

24. If $y = \cot x$, then show that $\dfrac{d^2y}{dx^2} + 2y\dfrac{dy}{dx} = 0$.

25. Find the order and degree of the differential equation given by

$$\begin{vmatrix} x^3 & y^2 & 3 \\ 2x^2 & 3y\dfrac{dy}{dx} & 0 \\ 5x & 2\left(y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2\right) & 0 \end{vmatrix} = 0.$$

26. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\dfrac{2}{3}$.

27. A couple has 2 children. Find the probability that both are boys, if it is known that

(i) one of them is a boy,

(ii) the older child is a boy.

28. If A, B, C have position vectors (2, 0, 0), (0, 1, 0), (0, 0, 2), show that ΔABC is isosceles.

OR

If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$, it is being given that $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

Section - IV

29. Find the equations of tangent and normal to the curve $2x^2 + 3y^2 - 5 = 0$ at (1, 1).

30. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\dfrac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = Cx$.

31. Determine the values of a, b and c for which the function $f(x) = \begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, & x>0 \end{cases}$ may be continuous at $x = 0$.

OR

If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, then prove that $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

32. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

33. Let P be the set of all the points in a plane and the relation R in set P be defined as $R = \{(A, B) \in P \times P \mid$ distance between points A and B is less than 3 units$\}$. Show that the relation R is not an equivalence relation.

34. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

OR

Evaluate : $\int_1^3 |x^2 - 2x| \, dx$

35. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

Section - V

36. Solve the following problem graphically.

Minimize $Z = \frac{4x}{1000} + \frac{6y}{1000}$

subject to constraints :

$0.1\,x + 0.05\,y \le 50$

$0.25\,x + 0.5\,y \ge 200$

$x, y \ge 0$

OR

Solve the following problem graphically.

Minimize $Z = 150x + 200y$

subject to constraints :

$6x + 10y \ge 60$

$4x + 4y \le 32$

$x, y \ge 0$

37. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for helping others and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category.

OR

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of equations $2x + 3y + 4z = 17; x - y = 3; y + 2z = 7$.

38. Find the shortest distance between the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

OR

If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p.

SOLUTIONS

1. We know that A^{-1} exists if $|A| \neq 0$.

$\therefore \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0 \Rightarrow 2(6-5) + 1(5\lambda + 6) \neq 0$

$\Rightarrow 2 + 5\lambda + 6 \neq 0 \quad \Rightarrow \quad 5\lambda \neq -8$

$\Rightarrow \lambda \neq \dfrac{-8}{5}$

OR

We have, $3 - x^2 = 3 - 8 \Rightarrow x^2 = 8$

$\Rightarrow x = \pm 2\sqrt{2}$

2. Since, $f(1) = f(-1) = 1$, therefore f is not one-one. Also, Range $(f) = [0, \infty) \neq R$, therefore f is not onto.

3. The perpendicular from the origin to plane $\vec{r}\cdot(2\hat{i}-3\hat{j}-6\hat{k}) = 5$ is along the vector $2\hat{i}-3\hat{j}-6\hat{k}$.

$\therefore$ Its d.c's are $\dfrac{2}{\sqrt{4+9+36}}, \dfrac{-3}{\sqrt{4+9+36}}, \dfrac{-6}{\sqrt{4+9+36}}$

$= \left(\dfrac{2}{7}, \dfrac{-3}{7}, \dfrac{-6}{7}\right)$

OR

Equation of plane passing through (1, 2, 3) having direction cosines of normal as l, m, n is

$l(x-1) + m(y-2) + n(z-3) = 0$

$\Rightarrow lx + my + nz = l + 2m + 3n$

4. Since, $A' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = A$

$\therefore$ A is a symmetric matrix.

5. We have $\int\left(5x^3 + 2x^{-5} - 7x + \dfrac{1}{\sqrt{x}} + \dfrac{5}{x}\right)dx$

$= 5\int x^3 dx + 2\int x^{-5}dx - 7\int x dx + \int x^{-1/2}dx + 5\int \dfrac{1}{x}dx$

$= 5\cdot\dfrac{x^4}{4} + 2\cdot\dfrac{x^{-4}}{(-4)} - 7\cdot\dfrac{x^2}{2} + \dfrac{x^{1/2}}{(1/2)} + 5\log|x| + C$

$= \dfrac{5x^4}{4} - \dfrac{1}{2x^4} - \dfrac{7x^2}{2} + 2\sqrt{x} + 5\log|x| + C$

OR

Let $I = \int \dfrac{\sin(x-a)}{\sin(x+a)}dx$

Put $x + a = t \Rightarrow x = t - a \Rightarrow dx = dt$

$\therefore I = \int \dfrac{\sin(t-2a)}{\sin t}dt$

$= \int \dfrac{\sin t\cos 2a - \cos t \sin 2a}{\sin t}dt$

$= \int \dfrac{\sin t \cos 2a}{\sin t}dt - \int \dfrac{\cos t \sin 2a}{\sin t}dt$

$= \cos 2a \int dt - \sin 2a \int \cot t \, dt$

$= t\cos 2a - \sin 2a \log|\sin t| + C$

$= (x+a)\cos 2a - \sin 2a \log|\sin(x+a)| + C$

6. The total number of bijections from a set containing n elements to itself is $n!$

Hence, required number = (106)!

7. We have, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$

$= \hat{i}(-2-15) - (-4-9)\hat{j} + (10-3)\hat{k} = -17\hat{i} + 13\hat{j} + 7\hat{k}$

Hence, $|\vec{a}\times\vec{b}| = \sqrt{(-17)^2 + (13)^2 + (7)^2} = \sqrt{507}$

OR

The given vectors will be at right angles if their dot product vanishes, *i.e.*,

$(2\hat{i} - 4\hat{j} + \hat{k})\cdot(4\hat{i} - 8\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 8 + 32 + \lambda = 0 \Rightarrow \lambda = -40$

8. We have, $I = \int_{\pi/4}^{\pi/2} \cos 2x \, dx = \left[\dfrac{\sin 2x}{2}\right]_{\pi/4}^{\pi/2}$

$= \left[\dfrac{\sin\pi}{2} - \dfrac{\sin\frac{\pi}{2}}{2}\right] = 0 - \dfrac{1}{2} = -\dfrac{1}{2}$

9. We know that, two parallel lines have same set of direction cosines. Therefore, required direction cosines are the direction cosines of Z-axis, *i.e.*, 0, 0, 1

10. The given differential equation can be written as

$\left\{1 + \left(\dfrac{dy}{dx}\right)^2\right\}^3 = \left(5\dfrac{d^2y}{dx^2}\right)^2$

Clearly, it can be observed that order of differential equation is 2 and degree is 2.

OR

Given, $\dfrac{dy}{dx} + (\sec x)y = \tan x$,

which is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$

Here, $P = \sec x$ and $Q = \tan x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \sec x \, dx} = e^{\log|\sec x + \tan x|}$

$= \sec x + \tan x$

11. Given, $\vec{c}=(x-2)\,\vec{a}+\vec{b}$, $\vec{d}=(2x+1)\,\vec{a}-\vec{b}$ are collinear, therefore $\vec{c}=m\vec{d}$

$\Rightarrow\ (x-2)\vec{a}+\vec{b}=m\left((2x+1)\vec{a}-\vec{b}\right)$

$\Rightarrow\ m=-1$

and $m\,(2x+1)=x-2 \Rightarrow -2x-1=x-2 \Rightarrow x=\dfrac{1}{3}$

12. Here, $a = 2$, $b = 3$ and $c = 4$

$\therefore$ Required equation of plane is $\dfrac{x}{2}+\dfrac{y}{3}+\dfrac{z}{4}=1$.

$\Rightarrow\ 6x + 4y + 3z = 12,$

which is the required equation of plane.

13. Clearly $f(-1) = 3(-1) = -3$;

$f(2) = (2)^2 = 4$ and $f(4) = 2(4) = 8$

$\therefore\ f(-1) + f(2) + f(4) = -3 + 4 + 8 = 9$

14. The given equation of line can be written in standard form as

$$5\left(x-\frac{3}{5}\right)=15\left(y+\frac{7}{15}\right)=-10\left(z-\frac{3}{10}\right)$$

$$\Rightarrow\ \frac{x-\frac{3}{5}}{\frac{1}{5}}=\frac{y+\frac{7}{15}}{\frac{1}{15}}=\frac{z-\frac{3}{10}}{-\frac{1}{10}}$$

$$\Rightarrow\ \frac{x-\frac{3}{5}}{6}=\frac{y+\frac{7}{15}}{2}=\frac{z-\frac{3}{10}}{-3}$$

Thus, d.r.'s of given line are 6, 2, –3.

Hence, d.c.'s of given line are $\dfrac{6}{7},\dfrac{2}{7},\dfrac{-3}{7}$.

15. Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$.

Now, projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{(\hat{i}+3\hat{j}+7\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+(-3)^2+6^2}}=\frac{2-9+42}{\sqrt{4+9+36}}=\frac{35}{7}=5$$

16. We have, $A=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ therefore $A'=\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}$

$$\Rightarrow\ A+A'=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}+\begin{bmatrix}3 & 2\\ 4 & 3\end{bmatrix}=\begin{bmatrix}6 & 6\\ 6 & 6\end{bmatrix}$$

17. (i) (d) : In order to make least expensive tank, Ajay need to minimize its cost.

(ii) (d) : Let l be the length and h be the height of the tank. Since breadth is equal to 5 ft. (Given)

$\therefore$ Two sides will be $5h$ sq. feet and two sides will be lh sq. feet. So, the total area of the sides is $(10\,h + 2\,l\,h)\text{ft}^2$

Cost of the sides is ₹ 5 per sq. foot. So, the cost to build the sides is $(10h + 2lh) \times 5 =$ ₹ $(50h + 10lh)$)

Also, cost of base $= (5\,l) \times 10 =$ ₹ $50\,l$

Total cost of the tank in ₹ is given by

$c = 50\,h + 10\,l\,h + 50\,l$

Since, volume of tank = 72 ft^3

$\therefore\ 5\,l\,h = 72\text{ ft}^3\ \therefore\ l=\dfrac{72}{5h}$

$\therefore\ c\,(h) = 50\,h + 10\left(\dfrac{72}{5h}\right)h+50\left(\dfrac{72}{5h}\right)$

$= 50\,h + 144 + \dfrac{720}{h}$

(iii) (b) : Since, all side lengths must be positive.

$\therefore\ h > 0$ and $\dfrac{72}{5h}>0$

Since, $\dfrac{72}{5h}>0$, whenever $h > 0$

$\therefore$ Range of h is $(0, \infty)$

(iv) (a) : To minimize cost, $\dfrac{dc}{dh}=0$

$\Rightarrow\ 50-\dfrac{720}{h^2}=0$

$\Rightarrow\ 50\,h^2 = 720\ \Rightarrow\ h^2 = 14.4 \Rightarrow\ h=\pm\sqrt{14.4}$

$\Rightarrow\ h=\sqrt{14.4}$ [$\because$ height can not be negative]

(v) (c) : $\therefore$ Cost of least expensive tank is given by

$c\left(\sqrt{14.4}\right)=50\sqrt{14.4}+144+\dfrac{720}{\sqrt{14.4}}$

$= 100\times\sqrt{14.4}+144$

≈ 523.47

18. Let E_1 be the event that Rohan solved the puzzle and E_2 be the event that Payal solved the puzzle.

Then, $P(E_1) = 1/2$ and $P(E_2) = 1/3$

(i) (c) : Since, E_1 and E_2 are independent events

$\therefore\ P(\text{both solved the puzzle}) = P(E_1 \cap E_2)$

$= P(E_1)\cdot P(E_2) = \dfrac{1}{2}\times\dfrac{1}{3}=\dfrac{1}{6}$

(ii) (d) : P(puzzle is solved by Rahul but not by Payal)

$= P(\bar{E}_2)P(E_1) = \left(1-\dfrac{1}{3}\right)\cdot\dfrac{1}{2}=\dfrac{2}{3}\cdot\dfrac{1}{2}=\dfrac{1}{3}$

(iii) (c) : P(puzzle is solved) $= P(E_1$ or $E_2)$

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$=\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{6}=\dfrac{4}{6}=\dfrac{2}{3}$

(iv) (b) : P(Exactly one of them solved the puzzle)

$= P(E_1$ and $\bar{E}_2)$ or $(E_2$ and $\bar{E}_1)$

$= P\,(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)$

$= P(E_1)\times P(\bar{E}_2) + P(E_2)\times P(\bar{E}_1)$

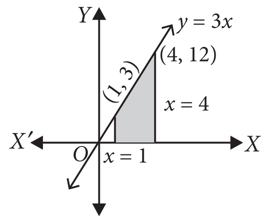
$=\dfrac{1}{2}\times\dfrac{2}{3}+\dfrac{1}{3}\times\dfrac{1}{2}$ [$\because\ P(\bar{E}_1) = 1 - P(E_1)$]

$=\dfrac{1}{3}+\dfrac{1}{6}=\dfrac{3}{6}=\dfrac{1}{2}$

(v) (b) : P(none of them solved the puzzle)

$$= P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1)\cdot P(\bar{E}_2) = \frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$$

19. Area enclosed by line $y = 3x$, x-axis, $x = 1$ and $x = 4$ is shown in figure.



$\therefore$ Required area $= \int_1^4 3x\,dx$

$$= \left[\frac{3x^2}{2}\right]_1^4 = \frac{3}{2}[16-1]$$

$$= \frac{3}{2}\times 15 = \frac{45}{2} = 22.5 \text{ sq. units}$$

20. Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$

Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $\left(\frac{-\pi}{6}\right)$

$\therefore$ Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{-1}{2}\right)$

$$= \frac{2\pi}{3}+\left(2\times\frac{-\pi}{6}\right)=\frac{2\pi}{3}-\frac{\pi}{3}=\frac{\pi}{3}$$

OR

We known that, principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

$\therefore$ Other two branches are

$\left(-\frac{\pi}{2}-\pi,\frac{\pi}{2}-\pi\right)$ and $\left(-\frac{\pi}{2}+\pi,\frac{\pi}{2}+\pi\right)$

i.e., $\left(-\frac{3\pi}{2},-\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$.

21. The sample space S associated to the given random experiment is given by

$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$ and

Let the event $B = \{(H, 6), (T, 6)\}$ and

$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

$\therefore$ Required probability $= P(B \mid A)$

$$= \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{12}}{\frac{6}{12}} = \frac{1}{6}$$

22. Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$

$\therefore$ $|A| = 10 - 9 = 1 \neq 0$

So, A^{-1} exists.

Now, adj $A = \begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj } A) = \begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}.$$

23. $I = \int_1^3 \frac{1}{x(1+\log x)^2}dx$

Put $1 + \log x = t \Rightarrow \frac{dx}{x} = dt$

When $x = 1$, $t = 1$ and when $x = 3$, $t = 1 + \log 3$

$$\therefore I = \int_1^{1+\log 3} \frac{dt}{t^2}$$

$$= \left[\frac{t^{-1}}{-1}\right]_1^{1+\log 3} = -\left[\frac{1}{1+\log 3}-1\right] = \frac{\log 3}{1+\log 3}$$

OR

$$\int_0^{2\pi} e^x\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \int_0^{2\pi}\left(\frac{x}{2}e^x+\frac{\pi}{4}e^x\right)dx$$

$$= \left[\left(\frac{x}{2}\right)(e^x)-\left(\frac{1}{2}\right)(e^x)+\frac{\pi}{4}e^x\right]_0^{2\pi}$$

$$= \pi e^{2\pi}-\frac{1}{2}e^{2\pi}+\frac{\pi}{4}e^{2\pi}+\frac{1}{2}-\frac{\pi}{4}$$

$$= e^{2\pi}\left(\frac{5\pi}{4}-\frac{1}{2}\right)+\frac{1}{2}-\frac{\pi}{4}$$

24. We have, $y = \cot x \Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$

$$\Rightarrow \frac{dy}{dx} = -(1+\cot^2 x) \Rightarrow \frac{dy}{dx} = -(1+y^2)$$

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\left(2y\frac{dy}{dx}\right) \Rightarrow \frac{d^2y}{dx^2}+2y\frac{dy}{dx} = 0$$

25. We have,

$$\begin{vmatrix} x^3 & y^2 & 3 \\ 2x^2 & 3y\frac{dy}{dx} & 0 \\ 5x & 2\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right) & 0 \end{vmatrix} = 0$$

Expanding along C_3, we get

$$3\left[4x^2\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}-15xy\frac{dy}{dx}\right] = 0$$

$$\Rightarrow 4x^2y\frac{d^2y}{dx^2}+4x^2\left(\frac{dy}{dx}\right)^2-15xy\frac{dy}{dx} = 0$$

Now, highest order derivative is $\frac{d^2y}{dx^2}$. So, its order is 2 and degree is 1.

26. Slope of tangent to the given curve at (x, y) is

$$\frac{dy}{dx} = \frac{1}{2}(4x-3)^{\frac{-1}{2}} 4 = \frac{2}{\sqrt{4x-3}}$$

The given slope is $\frac{2}{3}$.

So, $\frac{2}{\sqrt{4x-3}} = \frac{2}{3}$

Now, $y = \sqrt{4x-3} - 1$

So, when $x = 3$, $y = \sqrt{4(3)-3} - 1 = 2$

Thus, the required point is (3, 2).

27. Let $B_i (i = 1, 2)$ denote the i^{th} child is a boy and $G_i (i = 1, 2)$ denote the i^{th} child is a girl respectively.
Then sample space is,

$$S = \{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$$

Let A be the event that both are boys, B be the event that one of them is a boy and C be the event that the older child is a boy.

$A = \{B_1B_2\}$, $B = \{G_1B_2, B_1G_2, B_1B_2\}$

$C = \{B_1B_2, B_1G_2\} \Rightarrow A \cap B = \{B_1B_2\}$ and $A \cap C = \{B_1B_2\}$

(i) Required probability $= P(A|B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

(ii) Required probability $= P(A|C)$

$$= \frac{P(A \cap C)}{P(C)} = \frac{1/4}{2/4} = \frac{1}{2}$$

28. We have, $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$\Rightarrow \overrightarrow{AB} = (0\hat{i}+\hat{j}+0\hat{k})-(2\hat{i}+0\hat{j}+0\hat{k}) = -2\hat{i}+\hat{j}+0\hat{k}$

$\therefore AB = |\overrightarrow{AB}| = \sqrt{(-2)^2+1^2+0^2} = \sqrt{5}$

$\overrightarrow{BC}$ = P.V. of C – P.V. of B

$\Rightarrow \overrightarrow{BC} = (0\hat{i}+0\hat{j}+2\hat{k})-(0\hat{i}+\hat{j}+0\hat{k}) = 0\hat{i}-\hat{j}+2\hat{k}$

$\therefore BC = |\overrightarrow{BC}| = \sqrt{0^2+(-1)^2+2^2} = \sqrt{5}$

Since, $AB = BC$, therefore, ΔABC is isosceles.

OR

Given, $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$

Now, $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$= (\vec{a}\times\vec{b})-(\vec{d}\times\vec{b})-(\vec{a}\times\vec{c}) + (\vec{d}\times\vec{c})$

[By distributive law]

$= (\vec{a}\times\vec{b})+(\vec{b}\times\vec{d})-(\vec{a}\times\vec{c})-(\vec{c}\times\vec{d}) = 0$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0} \Rightarrow (\vec{a}-\vec{d}) || (\vec{b}-\vec{c})$

[Here $|\vec{a}-\vec{d}| \neq 0$ and $|\vec{b}-\vec{c}| \neq 0$ as $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$].

29. We have, $2x^2 + 3y^2 - 5 = 0$ ...(i)

Differentiating both sides w.r.t. x, we get

$$4x + 6x\frac{dy}{dx} - 0 = 0$$

$$\therefore 6y\frac{dy}{dx} = -4x \Rightarrow \frac{dy}{dx} = \frac{-2x}{3y}$$

Now, $\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{-2(1)}{3(1)} = -\frac{2}{3}$ = slope of the tangent at (1, 1).

$\therefore$ The equation of the tangent at (1, 1) is

$$y-1 = -\frac{2}{3}(x-1) \Rightarrow 3y - 3 = -2x + 2$$

$\therefore 2x + 3y - 5 = 0$

Now, the slope of the normal at (1, 1) is $\frac{3}{2}$.

$\therefore$ The equation of the normal at (1, 1) is

$$y-1 = \frac{3}{2}(x-1)$$

$\therefore 2y - 2 = 3x - 3 \Rightarrow 3x - 2y - 1 = 0$

Hence, the equations of tangent and normal are $2x + 3y - 5 = 0$ and $3x - 2y - 1 = 0$ respectively.

30. It is given that $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

It is homogeneous equation.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2vx^2} = \frac{1+v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v \Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x} \Rightarrow \int\frac{2v\,dv}{v^2-1} = -\int\frac{dx}{x}$$

$\Rightarrow \log|v^2 - 1| = -\log|x| + \log C_1$

$\Rightarrow \log|(v^2 - 1)x| = \log C_1$

$\Rightarrow (y^2 - x^2) = C_1x \Rightarrow x^2 - y^2 = Cx$, where $C = -C_1$

31. Here, $f(0) = c$

L.H.L. at $x = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to} \frac{\sin(a+1)x + \sin x}{x}$$

$$= \lim_{x\to 0^-}\left(\frac{\sin(a+1)x}{x} + \frac{\sin x}{x}\right)$$

$$= \lim_{x\to 0^-}\left\{\frac{\sin(a+1)x}{(a+1)x}\right\}\cdot(a+1) + \lim_{x\to 0^-}\frac{\sin x}{x}$$

$= (a + 1) + 1 = a + 2$

R.H.L. at $x = 0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-}\frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}} = \lim_{x\to 0^-}\frac{\sqrt{x}\{\sqrt{1+bx}-1\}}{bx\sqrt{x}}$$

$$= \lim_{x\to 0^+}\left(\frac{\sqrt{1+bx}-1}{bx}\right)\times\left(\frac{\sqrt{1+bx}+1}{\sqrt{1+bx}+1}\right)$$

$$= \lim_{x\to 0^+}\frac{1+bx-1}{bx(\sqrt{1+bx}+1)}$$

$= \lim_{x\to 0^+} \frac{1}{\sqrt{1+bx}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

Now, f is continuous at $x = 0$ if

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

i.e., if $a + 2 = c = \frac{1}{2} \Rightarrow a = -\frac{3}{2}$ and $c = \frac{1}{2}$

Hence, for $f(x)$ to be continuous at $x = 0$, we must have $a = -\frac{3}{2}$, $c = \frac{1}{2}$; b may have any real value.

OR

We have, $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$

Let $x = \sin A$, $y = \sin B$

$\therefore \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow \frac{A-B}{2} = \cot^{-1} a$

$\Rightarrow A - B = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

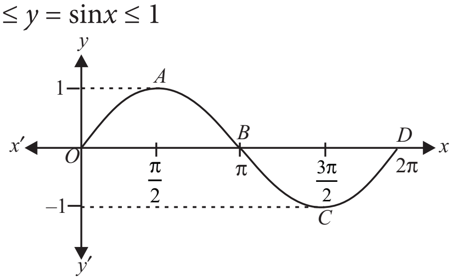
Differentiating w.r.t. x, we get

$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

32. The given curve is $y = \sin x$

x	0	$\pi/2$	3π	$3\pi/2$	2π
$y = \sin x$	0	1	0	-1	0

and $-1 \le y = \sin x \le 1$



$\therefore$ Graph between $x = 0$ and $x = \pi$, $x = \pi$ and $x = 2\pi$ has equal area above the x-axis and below the x-axis.

$\therefore \int_0^{2\pi} \sin x\,dx = 2(\text{area } OAB) = 2\int_0^{\pi} \sin x\,dx$

$= 2[-\cos x]_0^{\pi} = 2[-\cos\pi + \cos 0] = 2\,[1 + 1] = 4$ sq. units.

33. Given, $R = \{(A, B) \in P \times P \mid$ distance between points A and B is less than 3 units$\}$

For reflexivity : $(A, A) \in R$ is true as distance between points A and A is 0, which is less than 3 units for all $A \in P$. Hence, R is reflexive.

For symmetry : Let $A, B \in P$ and $(A, B) \in R \Rightarrow$ distance between points A and B is less than 3 units.

$\Rightarrow$ Distance between B and A is less than 3 units.

So, $(B, A) \in R$

Hence, R is symmetric.

For transitivity : Let points A, B and C are collinear. B is mid-point of AC such that distance between A and B is 2 units and between B and C is also 2 units, *i.e.*, $(A, B) \in R$ and $(B, C) \in R$, we notice distance between A and C is 4 units $\Rightarrow (A, C) \notin R$. Hence, R is not transitive.

Hence, R is not an equivalence relation.

34. Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ ...(i)

Then, $I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi} \frac{(x+\pi-x)\sin x}{1+\cos^2 x}dx = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$ ...(iii)

Put $z = \cos x \Rightarrow dz = -\sin x\,dx$

Also, when $x = 0$, $z = 1$ and when $x = \pi$, $z = -1$

$\therefore$ From (iii),

$I = \frac{\pi}{2}\int_1^{-1} \frac{-dz}{1+z^2} = -\frac{\pi}{2}\int_1^{-1} \frac{1}{1+z^2}dz = -\frac{\pi}{2}[\tan^{-1} z]_1^{-1}$

$= -\frac{\pi}{2}[\tan^{-1}(-1) - \tan^{-1}(1)] = -\frac{\pi}{2}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi^2}{4}$

OR

Let $I = \int_1^3 |x^2 - 2x|dx$

$\because |x^2 - 2x| = \begin{cases} -(x^2-2x), & \text{when } 1 \le x < 2 \\ (x^2-2x), & \text{when } 2 \le x \le 3 \end{cases}$

$\Rightarrow I = \int_1^2 |x^2 - 2x|dx + \int_2^3 |x^2 - 2x|dx$

$= \int_1^2 -(x^2 - 2x)dx + \int_2^3 (x^2 - 2x)dx$

$= -\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$

$= -\left(-\frac{4}{3} + \frac{2}{3}\right) + \left(\frac{4}{3}\right) = \frac{6}{3} = 2$

35. Given, $x = a(\cos 2\theta + 2\theta \sin 2\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta\cos 2\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a(4\theta\cos 2\theta)$ ...(i)

Again, $y = a(\sin 2\theta - 2\theta\cos 2\theta)$

$\Rightarrow \dfrac{dy}{d\theta} = a(2\cos 2\theta + 4\theta\sin 2\theta - 2\cos 2\theta)$

$\Rightarrow \dfrac{dy}{d\theta} = a(4\theta\sin 2\theta)$ ...(ii)

From (i) and (ii), we get

$$\frac{dy}{dx} = \frac{a(4\theta\sin 2\theta)}{a(4\theta\cos 2\theta)} = \tan 2\theta$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 2\sec^2 2\theta.\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 2\theta\cdot\frac{1}{a(4\theta\cos 2\theta)}$$

$$\therefore \left[\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\sec^2\frac{\pi}{4}\cdot\frac{1}{a\left(\frac{\pi}{2}\cos\frac{\pi}{4}\right)} = \frac{8\sqrt{2}}{\pi a}$$

36. The given problem is

Minimize $Z = \dfrac{4x}{1000} + \dfrac{6y}{1000}$

Subject to constraints:

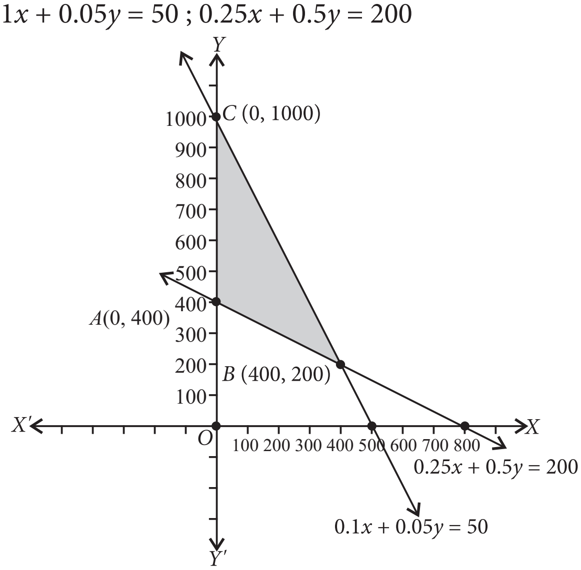
$0.1\,x + 0.05\,y \le 50$

$0.25\,x + 0.5\,y \ge 200$

$x, y \ge 0$

Convert the inequations into equations and draw the graph of lines:

$0.1x + 0.05y = 50$; $0.25x + 0.5y = 200$



As $x \ge 0, y \ge 0$ $\therefore$ Solution lies in first quadrant

Here, the shaded region is the feasible region. Now, we find the value of Z at each corner point.

Corner Points	Value of Z
A (0, 400)	2.4 ← Minimum
B (400, 200)	2.8
C (0, 1000)	6

Thus, Z has minimum value 2.4, when $x = 0$ and $y = 400$

OR

The given problem is

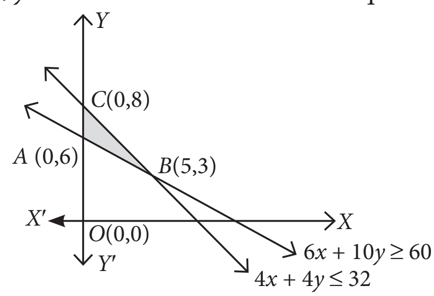
Minimize $Z = 150x + 200y$

subject to constraints

$6x + 10y \geqq 60$;

$4x + 4y \le 32$; $x \ge 0, y \ge 0$

As $x \ge 0, y \ge 0$ $\therefore$ Solution lies in first quadrant



Convert the inequations into equations and draw the graph of lines:

$6x + 10y = 60$; $4x + 4y = 32$

Here, shaded region is the feasible region.

Corner points of feasible region are $A(0, 6)$, $B(5, 3)$ and $C(0, 8)$.

Value of Z at these corner points are:

Corner Points	Value of Z
$A(9, 6)$	1200 (minimum)
$B(5, 3)$	1350
$C(0, 8)$	1600

Thus, Z has minimum value 1200 when $x = 0$ and $y = 6$.

37. According to given conditions, we have

$x + y + z = 12$, $2x + 3(y + z) = 33$, $x + z = 2y$

i.e., $x + y + z = 12$, $2x + 3y + 3z = 33$, $x - 2y + z = 0$.

The given system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{vmatrix}$

$= 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$

$= 9 + 1 - 7 = 3 \neq 0$

$\therefore$ A^{-1} exists and the solution is given by $X = A^{-1}B$.

Now, $\text{adj } A = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108-99+0 \\ 12+0+0 \\ -84+99+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \Rightarrow x = 3, y = 4 \text{ and } z = 5.$$

$\therefore$ The number of awardees for honesty is 3, for helping others is 4 and supervising the workers is 5.

OR

We have, $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$\therefore$ $|A| = 2(-2-0) - 3(2-0) + 4(1-0) = -6 \neq 0$

$\therefore$ A^{-1} exists.

Cofactors are

$A_{11} = -2, A_{12} = -2, A_{13} = 1,$

$A_{21} = -2, A_{22} = 4, A_{23} = -2,$

$A_{31} = 4, A_{32} = 4, A_{33} = -5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}' = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix} \quad \text{...(i)}$$

System of equations can be written as $AX = B$,

Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$

Now, $AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix} \quad \text{[From (i)]}$$

$$\Rightarrow X = -\frac{1}{6}\begin{bmatrix} -34-6+28 \\ -34+12+28 \\ 17-6-35 \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow x = 2, y = -1$ and $z = 4$

38. The equation of two given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{...(i)}$$

$$\text{and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad \text{...(ii)}$$

Line (i) passes through (1, 2, 3) and has direction ratios proportional to 2, 3, 4. So, its vector equation is

$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ ...(iii)

where, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Line (ii) passes through (2, 4, 5) and has direction ratios proportional to 3, 4, 5. So, its vector equation is

$\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ ...(iv)

where, $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

The shortest distance between the lines (iii) and (iv) is given by

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| \quad \text{...(v)}$$

We have,

$\vec{a}_2 - \vec{a}_1 = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$\therefore$ $|\vec{b}_1 \times \vec{b}_2| = \sqrt{1+4+1} = \sqrt{6}$

and $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (\hat{i} + 2\hat{j} + 2\hat{k})\cdot(-\hat{i} + 2\hat{j} - \hat{k})$

$= -1 + 4 - 2 = 1$

Substituting the values of $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)$ and $|\vec{b}_1 \times \vec{b}_2|$ in (v), we get $\text{S.D.} = 1/\sqrt{6}$ unit.

OR

The given plane is $\vec{r}\cdot(3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k})\cdot(3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

$\Rightarrow 3x + 4y - 12z + 13 = 0$

$$\text{Now,} \left|\frac{3(1)+4(1)-12(p)+13}{\sqrt{9+16+144}}\right| = \left|\frac{3(-3)+4(0)-12(1)+13}{\sqrt{9+16+144}}\right|$$

(Given points are equidistant from the given plane)

$\Rightarrow |20 - 12p| = |-8| \Rightarrow 20 - 12p = \pm 8$

$\Rightarrow 5 - 3p = \pm 2$

$\Rightarrow -3p = 2 - 5$ or $-3p = -2 - 5$

$\Rightarrow -3p = -3$ or $-3p = -7$

Hence, $p = 1$ or $p = \dfrac{7}{3}$

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