5. Probability

Exercise 5.1

1. Question

A probability experiment was conducted. Which of these cannot be considered as a probability of an outcome?

(i) 1/3 (ii) -1/5 (iii) 0.80 (iv) -0.78

(v) 0 (vi) 1.45 (vii) 1 (viii) 33%

(ix) 112%

Answer

Probability to its very basics is the ratio of 'No. of favorable outcomes' and 'Total number of outcomes.'

i.e., $P = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible of outcomes}}$

∴ It cannot be negative as both numerator and denominator are always positive.

Also, it cannot be greater than 1 (∵ number of favorable outcomes is always a subset of total outcome)

Now in this question the values of

(ii) $\frac{-1}{5}$ (iv)- 0.78 (vi) 1.45 (ix) 112% are negative or greater than 1.

∴ they are not probability of any event.

2. Question

Define: (i) experiment (ii) deterministic experiment (iii) random experiment

(iv) sample space (v) event (vi) trial

Answer

(i) Experiment

In probability an experiment is defined as the procedure which can be repeated infinite times and have a well-defined set of any number of possible outcomes.

(ii) Deterministic Experiment

A deterministic experiment is the subset of Experiments in which there is only one possible outcome. i.e., we know before the experiment what will be the result as there is only one possibility.

(iii) Random Experiment

A random experiment is the subset of experiments in which there are two or more outcomes. i.e., it can never pe predicted with 100% surety as it always has more than one possible outcomes, it can also be said as the opposite of Deterministic Experiment.

(iv) Sample Space

Sample Space is the set of all possible outcomes of that experiment. We can also say that the sum of probability of all the events in a sample space is always 1, as it consists of every possibility of any experiment.

(v) Event

An event is the set of outcomes of an experiment which are assigned different probabilities based on possibility of each event. An event is the subset of Sample Space and sum of probabilities of all event of an experiment is 1.

(vi) Trial

A trial can be defined as the number of steps we are taking to complete an experiment.

Consider a die which is rolled four times and we are supposed to find the probability of getting six in all four attempts then this complete experiment requires four steps of rolling dice and the number 4 is known as the no. of trial of this experiment.

3. Question

Define empirical probability.

Answer

Empirical Probability is the probability in more actual sense than the theoretical sense. It may seem quite difficult to understand but it is not.

An empirical probability tends to change with every trial, according to the result of a trial.

Okay, now we'll try to understand it through an example.

Consider a company X selling its product Y (which is a hair oil) to a set of consumers having different ages, different geographical background and different hair color. Now at our first day of sell of product we have an estimate probability of how many customers of the total customers the company approaches will buy this product.

Now after the first day of sell we have a difference between our estimated and actual data of sell, this data helps us to estimate a more accurate probability of sell on day 2.

Here, the sell on Day 1 was our trial and the as per the result of trial we got a probability of sell on Day 2. This type of Probability is known as empirical probability.

4. Question

During the last 20 basketball games, Sangeeth has made 65 and missed 35 free throws. What is the empirical probability if a ball was selected at random that Sangeeth make a foul shot?

Answer

Okay, Here according to the question

When Sangeeth has the ball either he can pass it through the basket or he may make a foul shot, i.e., there are only two possible outcomes.

So, we have the data of 20 matches and in these matches, he made a basket 65 times and foul shot 35 times.

⇒ The total number of attempts is 65 + 35 = 100

Out of which no. of foul shot = 35

∴ the empirical probability that he would make a foul shot from the data will be (number of foul shots)/(total number of attempts) = 35

5. Question

The record of a weather station shows that out of the past 300 consecutive days, its weather was forecasted correctly 195 times. What is the probability that on a given day selected at random, (i) it was correct (ii) it was not correct.

Answer

Data

Total number days – 300

Number of days when prediction was correct – 195

Probability based on above data that the prediction will be correct.

$$= \frac{195}{300}$$

$$= \frac{13}{20}$$

Now there are only two possible outcomes either the prediction will be correct, or it will be incorrect

∴ Probability that the prediction will be incorrect

$= 1 - \frac{13}{20}$ (∵ the total probability of all events is always 1)

$= \frac{7}{20}$

6. Question

Gowri asked 25 people if they liked the taste of a new health drink. The responses are,

Responses	Like	Dislike	Undecided
No. of people	15	8	2

Find the probability that a person selected at random

(i) likes the taste (ii) dislikes the taste (iii) undecided about the taste

Answer

Total number of people Gowri approached is

= 15 + 8 + 2

= 25

(i) Now as per the data 15 out of 25 people liked the drink

So, the probability as per the data provided will be,

$$P(A) = \frac{15}{25}$$

$$P(A) = \frac{3}{5}$$

(ii) Number of people who disliked it as per the data of 25 people is 8

So, the probability that the person will dislike the drink

$$\Rightarrow P_2 = \frac{8}{25}$$

(iii) Number of people who were undecided as per the data of 25 people is 2

So, the probability that the person will also be undecided about the drink

$$\Rightarrow P_3 = \frac{2}{25}$$

It can be noted that sum of probability is 1 as it included all the possible outcomes.

7. Question

In the sample of 50 people, 21 has type "O" blood, 22 has type "A" blood, 5 has type "B" blood and 2 has type "AB" blood. If a person is selected at random find the probability that

(i) the person has type "O" blood (ii) the person does not have type "B" blood

(iii) the person has type "A" blood (iv) the person does not have type "AB" blood.

Answer

Sample Space consists of 50 different people where 1 person can only have 1 blood group.

∵ number of people with "O" blood is 21

$\therefore P_o = \frac{21}{50}$ ---- (1)

Similarly,

Probability of getting Blood "B", "A", "AB" respectively is

$P_B = \frac{5}{50} = \frac{1}{10}$ ---- (2)

$P_A = \frac{22}{50} = \frac{11}{25}$ ---- (3)

$P_{AB} = \frac{2}{50} = \frac{1}{25}$ ---- (4)

By (1)

(i) Probability that random person among those people has "O" group = $\frac{21}{50}$ (by (1))

(ii) Probability that random person among those people does not has "B" group = $1 - \frac{1}{10} = \frac{9}{10}$ (by (2))

(iii) Probability that random person among those people has "A" group = $\frac{11}{50}$ (by (3))

(iv) Probability that random person among those people does not has "B" group = $1 - \frac{1}{25} = \frac{24}{25}$ (by (4))

8. Question

A die is rolled 500 times. The following table shows that the outcomes of the die.

Outcomes	1	2	3	4	5	6
Frequencies	80	75	90	75	85	95

Find the probability of getting an outcome (i) less than 4

(ii) less than 2

(iii) greater than 2 (iv) getting 6 (v) not getting 6.

Answer

(i) Outcomes less than 4 includes the outcomes of getting 3, 2, 1.

Total times = 500

Favorable outcomes out of 500 is 80 + 75 + 90 (favorable outcomes of 1,2 and 3 respectively)

$$P_i = \frac{80 + 75 + 90}{500} = \frac{49}{100}$$

(ii) Outcomes less than 2 includes the outcomes of getting 1.

Total times = 500

Favorable outcomes out of 500 is 80 (favorable outcomes of getting a 1)

$$P_{ii} = \frac{80}{500} = \frac{4}{25}$$

(iii) Outcomes greater than 2 includes the outcomes of getting 3, 4, 5, 6.

Total times = 500

Favorable outcomes out of 500 is 90,75,85,90 (favorable outcomes of getting 3,4,5 and 6 respectively)

$$P_{iii} = \frac{90 + 75 + 85 + 90}{500} = \frac{69}{100}$$

(iv) Outcomes of getting a 6.

Total times = 500

Favorable outcomes out of 500 is 95 (favorable outcomes of getting 6)

$$P_{iv} = \frac{95}{500} = \frac{19}{100}$$

(v) Outcomes not getting a 6 includes the outcomes of getting 1,2,3,4,5.

Total times = 500

Favorable outcomes out of 500 is 80 + 75 + 90 + 75 + 90 (favorable outcomes of 1,2 and 3 respectively)

$$P_v = \frac{80 + 75 + 90 + 75 + 90}{500} = \frac{81}{100}$$

9. Question

2000 families with 2 children were selected randomly, and the following data were recorded.

Number of girls in a family	2	1	0
Number of families	624	900	476

Find the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) no girl

Answer

Let having 2 girls be Event E1

Having 1 girl be event E2

Having no girl be Event E3

(i) $P(E1) = \frac{624}{2000}$ (∵ no. of families with 2 girls is 624 among a total of 2000 families)

$\Rightarrow P(E1) = \frac{39}{125}$

(ii) $P(E2) = \frac{900}{2000}$ (∵ no. of families with 1 girl is 900 among a total of 2000 families)

$\Rightarrow P(E2) = \frac{9}{20}$

(iii) $P(E3) = \frac{476}{2000}$ (∵ no. of families with 0 girl is 476 among a total of 2000 families)

$\Rightarrow P(E3) = \frac{119}{500}$

10. Question

The following table gives the lifetime of 500 CFL lamps.

Life time (months)	9	10	11	12	13	14	More than 14
Number of Lamps	26	71	82	102	89	77	53

A bulb is selected at random. Find the probability that the life time of the selected bulb is

(i) less than 12 months (ii) more than 14 months

(iii) at most 12 months (iv) at least 13 months

Answer

(i) less than 12 months includes the life of 9,10,11 months

$\Rightarrow P(E1) = \frac{26+71+82}{500}$ (∵ the life of 26, 71, 82 bulbs are 9, 10, 11 among a total of 500 bulbs)

$\Rightarrow P(E1) = \frac{179}{500}$

(ii) more than 14 months

$\Rightarrow P(E2) = \frac{53}{500}$ (∵ the life of 53 bulbs is more than 14 months as per the data given)

(iii) At most 12 months

$\Rightarrow P(E3) = \frac{26+71+82+102}{500}$ (∵ the life of 26, 71, 82, 102 bulbs are 9, 10, 11, 12 among a total of 500 bulbs)

$\Rightarrow P(E3s) = \frac{281}{500}$

(iv) At least 13 months

$\Rightarrow P(E4) = \frac{89+77+53}{500}$ (∵ the life of 89, 77, 83 bulbs are 13, 14 and more than 14 months respectively among a total of 500 bulbs)

$\Rightarrow P(E2) = \frac{219}{500}$

11. Question

On a busy road in a city the number of persons sitting in the cars passing by were observed during a particular interval of time. Data of 60 such cars is given in the following table.

No. of persons in the car	1	2	3	4	5
No. of Cars	22	16	12	6	4

Suppose another car passes by after this time interval. Find the probability that it has

(i) only 2 persons sitting in it (ii) less than 3 persons in it

(iii) more than 2 persons in it (iv) at least 4 persons in it

Answer

According to the Data of 60 cars we need to find the probability of an event for a car passing after the interval.

Here, we'll be finding out empirical probability.

(i) Only two persons sitting in it

No. of cars with 2 persons in the interval = 16

Total number of cars = 60

$$\Rightarrow P(A) = \frac{16}{60}$$

(ii) Less than 3 persons in it

This is satisfied when the number of persons is 1 and 2 in a car.

No. of cars with 1 persons in the interval = 22

No. of cars with 2 persons in the interval = 16

Total number of cars = 60

$$\Rightarrow P(B) = \frac{22+16}{60}$$

$$\Rightarrow P(B) = \frac{19}{30}$$

(iii) More than 2 persons in it

This is satisfied when the number of persons is 3, 4 and 5

No. of cars with 3 persons in the interval = 12

No. of cars with 4 persons in the interval = 6

No. of cars with 5 persons in the interval = 4

Total number of cars = 60

$$\Rightarrow P(C) = \frac{12+6+4}{60}$$

$$\Rightarrow P(C) = \frac{11}{30}$$

(iv) at least 4 persons in it

This is satisfied when the number of persons is 3, 4 and 5

No. of cars with 4 persons in the interval = 6

No. of cars with 5 persons in the interval = 4

Total number of cars = 60

$$\Rightarrow P(D) = \frac{6+4}{60}$$

$$\Rightarrow P(D) = \frac{1}{6}$$

12. Question

Marks obtained by Insuvai in Mathematics in ten unit tests are listed below.

Unit Test	I	II	III	IV	V	VI	VII	VIII	IX	X
Marks obtained (%)	89	93	98	99	98	97	96	90	98	99

Based on this data find the probability that in a unit test Insuvai get

(i) more than 95% (ii) less than 95% (iii) more than 98%

Answer

(i) More than 95%

No. of unit test in which she scored more than 95% = 7

Total number of unit test = 10

$$P(A) = \frac{7}{10}$$

(ii) Less than 95%

No. of unit test in which she scored less than 95% = 3

Total number of unit test = 10

$$P(A) = \frac{3}{10}$$

(iii) More than 98%

No. of unit test in which she scored more than 98% = 2

Total number of unit test = 10

$$P(A) = \frac{2}{10}$$

$$\Rightarrow P(A) = \frac{1}{5}$$

13. Question

The table below shows the status of twenty residents in an apartment

Status / Gender	College Students	Employees
Male	5	3
Female	4	8

If one of the residents is chosen at random, find the probability that the chosen resident

will be (i) a female (ii) a college student (iii) a female student (iv) a male employee

Answer

(i) Total female residents = 4 + 8 = 12

Total male residents = 5 + 3 = 8

Total Residents = 12 + 8 = 20

$$P(A) = \frac{12}{20}$$

$$P(A) = \frac{3}{5}$$

(ii) Total College Students = 5 + 4 = 9

Total Employees = 8 + 3 = 11

Total Residents = 9 + 11 = 20

$$P(B) = \frac{9}{20}$$

(iii) Total Female student = 4

Total Residents = 20

$$P(C) = \frac{4}{20}$$

$$P(C) = \frac{1}{5}$$

(iv) Total male employees = 3

Total Residents = 20

$$P(D) = \frac{3}{20}$$

14. Question

The following table shows the results of a survey of thousand customers who bought a new or used cars of a certain model

Type \ Satisfaction level	Satisfied	Not Satisfied
New	300	100
Used	450	150

If a customer is selected at random, what is the probability that the customer

(i) bought a new car (ii) was satisfied (iii) bought an used car but not satisfied

Answer

(i) Customers who bought new car = 300 + 100(satisfied and unsatisfied) = 400

Total customers = 1000

$$P(A) = \frac{400}{1000}$$

$$P(A) = \frac{2}{5}$$

(ii) Customers who were satisfied = 300 + 450 = 750

Total customers = 1000

$$P(B) = \frac{750}{1000}$$

$$P(B) = \frac{3}{4}$$

(i) Customers who bought a used car but not satisfied = 150

Total customers = 1000

$$P(A) = \frac{150}{1000}$$

$$P(A) = \frac{3}{20}$$

15. Question

A randomly selected sample of 1,000 individuals were asked whether they were planning to buy a new cellphone in the next 12 months. A year later the same persons were interviewed again to find out whether they actually bought a new cellphone. The response of both interviews is given below

	Buyers	Non-buyers
Plan to buy	200	50
No plan to buy	100	650

If a person was selected at random, what is the probability that he/she

(i) had a plan to buy (ii) had a plan to buy but a non-buyer

(iii) had no plan to buy but a buyer.

Answer

(i) No. of individuals who had plan to buy = 200 + 50 (Buyers and Non-Buyers) = 250

Total Number of individuals = 1000

$$P(A) = \frac{250}{1000}$$

(ii) No. of individuals who had plans to buy but didn't bought = 50

Total number of individuals = 1000

$$P(B) = \frac{50}{1000}$$

$$P(B) = \frac{1}{20}$$

(iii) No. of individuals who didn't had plans to buy but bought = 100

Total number of individuals = 1000

$$P(C) = \frac{100}{1000}$$

$$P(C) = \frac{1}{10}$$

16. Question

The survey has been undertaken to determine whether there is a relationship between the place of residence and ownership of an automobile. A random sample of car owners, 200 from large cities, 150 from suburbs and 150 from rural areas were selected and tabulated as follow

Type of Area / Car ownership	Large city	Suburb	Rural
Own a foreign car	90	60	25
Do not own a foreign car	110	90	125

If a car owner was selected at random, what is the probability that he/she

(i) owns a foreign car.

(ii) owns a foreign car and lives in a suburb.

(iii) lives in a large city and does not own a foreign car.

(iv) lives in large city and owns a foreign car.

(v) neither lives in a rural area nor owns a foreign car.

Answer

(i) Total car owners = 500

Owners of foreign cars = 90 + 60 + 25 = 175

$$P(A) = \frac{175}{500}$$

$$\Rightarrow P(A) = \frac{7}{20}$$

(ii) Total car owners = 500

Owners of foreign cars living in suburb = 60

$$P(B) = \frac{60}{500}$$

$$\Rightarrow P(B) = \frac{3}{25}$$

(iii) Total car owners = 500

Owners of cars which is not foreign and live in large city = 110

$$P(C) = \frac{110}{500}$$

$$\Rightarrow P(C) = \frac{11}{50}$$

(iv) Total car owners = 500

Owners of foreign cars living in large cities = 90

$$P(D) = \frac{90}{500}$$

$$\Rightarrow P(D) = \frac{9}{20}$$

(v) Total car owners = 500

Owners not living in rural areas belongs to 4 different categories

(a) foreign car-Large City

(b) foreign car-Suburb

(c) non-foreign car-Large City

(d) non-foreign car-Suburb

Now, according to the question, among these four categories the owners should also not own a foreign car.

∴ we now have 2 categories left

(c) non-foreign car-Large City

(d) non-foreign car-Suburb

No. of owners in category (c) = 110

No. of owners in category (d) = 90

$$P(E) = \frac{200}{500}$$

$$P(E) = \frac{2}{5}$$

17. Question

The educational qualifications of 100 teachers of a Government higher secondary school are tabulated below

Education / Age	M.Phil	Master Degree Only	Bachelor Degree Only
below 30	5	10	10
30 - 40	15	20	15
above 40	5	5	15

If a teacher is selected at random what is the probability that the chosen teacher has

(i) master degree only (ii) M.Phil and age below 30 (iii) only a bachelor degree and age above 40(iv) only a master degree and in age 30-40 (v) M.Phil and age above 40

Answer

(i) Total teachers = 100

Teachers with master's degree only = 10 + 20 + 5 = 35

$$P(A) = \frac{35}{100}$$

$$\Rightarrow P(A) = \frac{7}{20}$$

(ii) Total teachers = 100

Teachers with M.Phil. and below 30 = 5

$$P(B) = \frac{5}{100}$$

$$\Rightarrow P(B) = \frac{1}{20}$$

(iii) Total teachers = 100

Teachers with bachelor's degree only above 40 = 15

$$P(C) = \frac{15}{100}$$

$$\Rightarrow P(C) = \frac{3}{20}$$

(iv) Total teachers = 100

Teachers with master's degree only in the age group 30-40 = 20

$$P(C) = \frac{20}{100}$$

$$\Rightarrow P(C) = \frac{1}{5}$$

(v) Total teachers = 100

Teachers with M.Phil. above age 40 = 5

$$P(D) = \frac{5}{100}$$

$$\Rightarrow P(D) = \frac{1}{20}$$

18. Question

A random sample of 1,000 men was selected and each individual was asked to indicate his age and his favorite sport. The results were as follows.

Sports / Age	Volleyball	Basket ball	Hockey	Football
Below 20	26	47	41	36
20 - 29	38	84	80	48
30 - 39	72	68	38	22
40 - 49	96	48	30	26
50 and above	134	44	18	4

If a respondent is selected at random, what is the probability that

(i) he prefers Volleyball (ii) he is between 20 - 29 years old

(iii) he is between 20 and 29 years old and prefers Basketball

(iv) he doesn't prefer Hockey (v) he is at most 49 of age and prefers Football.

Answer

(i) Total men = 1000

No. of men with volleyball as their favorite sport = 26 + 38 + 72 + 96 + 134 = 366

$$P(A) = \frac{366}{1000}$$

$$P(A) = \frac{183}{500}$$

(ii) Total men = 1000

No. of men between 20-29 years = 38 + 84 + 80 + 48 = 250

$$P(B) = \frac{250}{1000}$$

$$P(B) = \frac{1}{4}$$

(iii) Total men = 1000

No. of men between 20-29 years preferring basketball = 84

$$P(C) = \frac{84}{1000}$$

$$P(C) = \frac{21}{250}$$

(iv) Total men = 1000

No. of men not preferring Hockey = 1000-(41 + 80 + 38 + 30 + 18) = 793

$$P(D) = \frac{793}{1000}$$

(v) Total men = 1000

No. of men at most 49 years preferring football = 36 + 48 + 22 + 26 = 132

$$P(E) = \frac{132}{1000}$$

$$P(E) = \frac{33}{250}$$

19. Question

On one Sunday Muhil observed the vehicles at a Tollgate in the NH-45 for his science project about air pollution from 7 am. to 7 pm. The number of vehicles crossed are tabulated below.

Time interval / Vehicles	7 a.m. to 11 a.m.	11 a.m. to 3 p.m.	3 p.m. to 7 p.m.
Bus	300	120	400
Car	200	130	250
Two Wheeler	500	250	350

A vehicle is selected at random. Find the probability that the vehicle chosen is a

(i) a bus at the time interval 7 a.m. to 11 a.m.

(ii) a car at the time interval 11 a.m. to 7 p.m.

(iii) a bus at the time interval 7 a.m. to 3 p.m.

(iv) a car at the time interval 7 a.m. to 7 p. m.

(v) not a two wheeler at the time interval 7 a.m. to 7 p.m.

Answer

(i) Total number of vehicle = 2500

No. of buses in the interval 7 am to 11 am = 300

$$P(A) = \frac{300}{2500}$$

$$P(A) = \frac{3}{25}$$

(ii) Total number of vehicle = 2500

No. of cars in the interval 11 am to 7 pm = 130 + 250 = 380

$$P(B) = \frac{380}{2500}$$

$$P(B) = \frac{19}{125}$$

(iii) Total number of vehicle = 2500

No. of buses in the interval 7 am to 3 pm = 300 + 120 = 420

$$P(C) = \frac{420}{2500}$$

$$P(C) = \frac{21}{25}$$

(iv) Total number of vehicle = 2500

No. of cars in the interval 7 am to 7 pm = 200 + 130 + 250 = 580

$$P(D) = \frac{580}{2500}$$

$$P(D) = \frac{29}{25}$$

(v) Total number of vehicle = 2500

No. of non-two-wheelers in the interval 7 am to 7 pm = 2500-(500 + 250 + 350) = 1400

$$P(E) = \frac{1400}{2500}$$

$$P(E) = \frac{14}{25}$$

Exercise 5.2

1. Question

Probability of sure event is

A. 1

B. 0

C. $\frac{1}{2}$

D. 2

Answer

Probability of a sure event is always 1 as there is only one possible outcome.

2. Question

Which one can represent a probability of an event

A. $\frac{7}{4}$

B. - 1

C. $-\frac{2}{3}$

D. $\frac{2}{3}$

Answer

Probability can never be negative and can never be greater than 1.

Here (A) is greater than 1, (B) and (C) are negative.

But (D) is neither negative nor greater than 1.

3. Question

Probability of impossible event is

A. 1

B. 0

C. $\frac{1}{2}$

D. – 1

Answer

Probability of a impossible event is 0 as it has 0 favorable chances and probability is the ratio of favorable chances and total chances.

4. Question

Probability of any event x lies

A. $0 < x < 1$

B. $0 \leq x < 1$

C. $0 \leq x \leq 1$

D. $1 < x < 2$

Answer

Probability always lies in between 0 and 1. This is because it is the ratio of (a fraction of a part of a whole) and (the whole itself)

If an outcome has no favorable chances, then its probability is 0 and if the outcome is the only outcome in the experiment than it has a 100% certainty and Probability as 1.

Therefore, Probability ϵ [0,1]

5. Question

P(E') is

A. 1 - P(E)

B. P(E) - 1

C. 1

D. 0

Answer

Notation P(E') signifies the probability when the event E must not occur.

The Probability that E will occur is P(E), and since only 2 outcomes are possible i.e., E will occur, and E will not occur.

$\therefore$ P(E) + P(E') = 1 ($\because$ Total probability is 1)

$\Rightarrow$ P(E') = 1 – P(E)