MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 23

Total Marks : 31

Max. Time : 35 min.

Topics : Continuity & Derivability, Function, Limits, Quadratic Equation, Trigonometric Ratio

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2	(3 marks, 3 min.)	[6, 6]
Multiple choice objective (no negative marking) Q.3	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.4,5,6,7,8	(4 marks, 5 min.)	[20, 25]

1. If $f(x) = \frac{a\cos x - \cos bx}{x^2}$, $x \neq 0$ and $f(0) = 4$ is continuous at $x = 0$, then the ordered pair (a, b) is

(A) $(\pm 1, 3)$ (B) $(1, \pm 3)$ (C) $(-1, -3)$ (D) $(-1, 3)$

2. Let $A = \{9, 10, 11, 12, 13\}$ and $f : A \to N$ be a function defined as f(x) = Highest prime factor of x. Then number of elements in the range of f(x) is :-

(A) 5 (B) 4 (C) 3 (D) None of these

3. Which of the statements(s) is/are INCORRECT ?

(A) If f + g is continuous at x = a, then f and g are continuous at x = a.

(B) If $\lim_{x\to a}$ (f g) exists, then $\lim_{x\to a}$ f and $\lim_{x\to a}$ g both exists.

(C) Discontinuity at x = a $\Rightarrow$ non existences of limit

(D) All functions defined on a closed interval attain maximum or a minimum value in its interval.

4. Evaluate ७

(i) $\lim_{x\to 0} \frac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$ (ii) $\lim_{x\to 0} (\cos ax)^{\cos ec^2 bx}$

5. Evaluate :

(i) $\lim_{x\to 2a^+} \frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ (ii) $\lim_{x\to 0^+} \left(\frac{e^{x\ln(2^x-1)} - (2^x-1)^x \sin x}{e^{x\ln x}}\right)^{1/x}$

6. Find the sum of an infinite geometric progression whose first term is the limiting value of the function

$f(x) = \frac{\sin\left(x - \frac{\pi}{6}\right)}{\sqrt{3} - 2\cos x}$ at $x = \frac{\pi}{6}$ and whose common ratio is the limiting value of the function

$g(x) = \frac{\sin(x)^{1/3}\,\ell n\,(1+3x)}{(\arctan\sqrt{x})^2\left(e^{5\cdot x^{1/3}} - 1\right)}$ as $x \to 0^+$.

7. Find the exact value of the expression $\frac{\tan 70^\circ - \tan 20^\circ - 2\tan 40^\circ}{\tan 10^\circ}$.

8. Find all values of a for which the inequality $(a - 3)x^2 - 2ax + 3a - 6 > 0$ is satisfied for all values of x.

Answers Key

1. (B) **2.** (B) **3.** (A)(B)(C)(D)

4. (i) – 2 (ii) $e^{-\frac{a^2}{2b^2}}$

5. (i) $\frac{1}{2\sqrt{a}}$ (ii) $\frac{1}{e}\ell n\ 2$ **6.** $a = 1, r = \frac{3}{5}, S_\infty = \frac{5}{2}$

7. 4 **8.** $a \in (6, \infty)$