16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. **Inlet** : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.
   Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.
2. (*i*) If a pipe can fill a tank in x hours, then :

   part filled in 1 hour $= \frac{1}{x}$.

   (*ii*) If a pipe can empty a full tank in y hours, then :

   part emptied in 1 hour $= \frac{1}{y}$.

   (*iii*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour $= \left(\frac{1}{x} - \frac{1}{y}\right)$.

   (*iv*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour $= \left(\frac{1}{y} - \frac{1}{x}\right)$.

SOLVED EXAMPLES

Ex. 1. ***Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?***

Sol. Part filled by A in 1 hour $= \frac{1}{36}$; Part filled by B in 1 hour $= \frac{1}{45}$.

Part filled by (A + B) in 1 hour $= \left(\frac{1}{36} + \frac{1}{45}\right) = \frac{9}{180} = \frac{1}{20}$.

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2. ***Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?***

Sol. Net part filled in 1 hour $= \left(\frac{1}{10} + \frac{1}{12} - \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$.

$\therefore$ The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 3. ***If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?***

Sol. Let the reservoir be filled by first pipe in x hours.
Then, second pipe will fill it in $(x + 10)$ hours.

$\therefore \quad \frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12} \quad \Leftrightarrow \quad \frac{x+10+x}{x(x+10)} = \frac{1}{12}$

$\Leftrightarrow \quad x^2 - 14x - 120 = 0 \quad \Leftrightarrow \quad (x-20)(x+6) = 0$

$\Leftrightarrow \quad x = 20.$ [*neglecting the – ve value of x*]

So, the second pipe will take (20 + 10) hrs *i.e.*, 30 hrs to fill the reservoir.

Ex. 4. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol. Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10}$$ [*– ve sign means emptying*]

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

Ex. 5. An electric pump can fill a tank in 3 hours. Because of a leak in the tank, it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Sol. Work done by the leak in 1 hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}.$

$\therefore$ The leak will empty the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Sol. Work done by the two pipes in 1 hour $= \left(\frac{1}{14} + \frac{1}{16}\right) = \frac{15}{112}.$

$\therefore$ Time taken by these pipes to fill the tank $= \frac{112}{15}$ hrs $= 7$ hrs 28 min.

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

$\therefore$ Work done by (two pipes + leak) in 1 hour $= \frac{1}{8}.$

Work done by the leak in 1 hour $= \left(\frac{15}{112} - \frac{1}{8}\right) = \frac{1}{112}.$

$\therefore$ Leak will empty the full cistern in 112 hours.

Ex. 7. Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?

Sol. Part filled in 7 min. $= 7\left(\frac{1}{36} + \frac{1}{45}\right) = \frac{7}{20}.$

Remaining part $= \left(1 - \frac{7}{20}\right) = \frac{13}{20}.$

Net part filled in 1 min. when A, B and C are opened $= \left(\frac{1}{36} + \frac{1}{45} - \frac{1}{30}\right) = \frac{1}{60}$.

Now, $\frac{1}{60}$ part is filled in 1 min.

$\frac{13}{20}$ part is filled in $\left(60 \times \frac{13}{20}\right) = 39$ min.

Total time taken to fill the tank = (39 + 7) min. = 46 min.

Ex. 8. ***Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes*** ?

Sol. Let B be closed after x minutes. Then,

part filled by (A + B) in x min. + part filled by A in $(18 - x)$ min. = 1

$\therefore \quad x\left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1 \quad \Leftrightarrow \quad \frac{7x}{96} + \frac{18 - x}{24} = 1$

$\Leftrightarrow \quad 7x + 4(18 - x) = 96 \quad \Leftrightarrow \quad x = 8.$

Hence, B must be closed after 8 minutes.

EXERCISE 16A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer*** :

1. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank ? **(M.A.T. 2003)**

   (*a*) 12 min (*b*) 15 min (*c*) 25 min (*d*) 50 min

2. A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the cistern get filled ? **(Hotel Management, 1997)**

   (*a*) 4.5 hrs (*b*) 5 hrs (*c*) 6.5 hrs (*d*) 7.2 hrs

3. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely ?

   (*a*) 3 hrs 15 min (*b*) 3 hrs 45 min (*c*) 4 hrs (*d*) 4 hrs 15 min **(S.S.C. 2003)**

4. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely ? **(Bank P.O. 1999)**

   (*a*) 6 min. to empty (*b*) 6 min. to fill (*c*) 9 min. to empty

   (*d*) 9 min. to fill (*e*) None of these

5. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled ? **(C.B.I. 1997)**

   (*a*) 2 (*b*) 2.5 (*c*) 3 (*d*) 3.5

6. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in :

   (*a*) $1\frac{13}{17}$ hours (*b*) $2\frac{8}{11}$ hours (*c*) $3\frac{9}{17}$ hours (*d*) $4\frac{1}{2}$ hours **(Bank P.O. 2002)**

7. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes ? **(D.M.R.C. 2003)**

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$ (c) $\frac{7}{11}$ (d) $\frac{8}{11}$

8. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time, the third pipe alone can empty the cistern ? **(S.S.C. 2003)**

(a) 90 min (b) 100 min (c) 110 min (d) 120 min

9. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in : **(S.S.C. 2002)**

(a) $4\frac{1}{3}$ hrs (b) 7 hrs (c) 8 hrs (d) 14 hrs

10. Two taps A and B can fill a tank in 5 hours and 20 hours respectively. If both the taps are open then due to a leakage, it took 30 minutes more to fill the tank. If the tank is full, how long will it take for the leakage alone to empty the tank ?

(a) $4\frac{1}{2}$ hrs (b) 9 hrs (c) 18 hrs (d) 36 hrs

11. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately ? **(NABARD, 2001)**

(a) 1 hr (b) 2 hrs (c) 6 hrs (d) 8 hrs.

12. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in : **(C.B.I. 2003)**

(a) 81 min (b) 108 min (c) 144 min (d) 192 min

13. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank ?

(a) 20 hrs (b) 25 hrs (c) 35 hrs
(d) Cannot be determined (e) None of these **(Bank P.O. 2003)**

14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is :

(a) 6 hrs (b) 10 hrs (c) 15 hrs (d) 30 hrs **(M.B.A. 2002)**

15. 12 buckets of water fill a tank when the capacity of each tank is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres ?

(a) 8 (b) 15 (c) 16 (d) 18

16. Bucket P has thrice the capacity as bucket Q. It takes 60 turns for bucket P to fill the empty drum. How many turns it will take for both the buckets P and Q, having each turn together to fill the empty drum ?

(a) 30 (b) 40 (c) 45 (d) 90

17. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively. If both the taps are opened simultaneously, and the tap A is closed after 3 minutes, then how much more time will it take to fill the tank by tap B ?

(a) 7 min 15 sec (b) 7 min 45 sec (c) 8 min 5 sec (d) 8 min 15 sec

18. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank ? **(U.P.S.C. 2002)**
(*a*) 10 min 20 sec (*b*) 11 min 45 sec (*c*) 12 min 30 sec (*d*) 14 min 40 sec

19. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. In how much time, will the tank be full ?
(*a*) 12 hrs (*b*) 13 hrs (*c*) 16 hrs (*d*) 18 hrs

20. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half ?
(*a*) 15 min (*b*) 20 min (*c*) 27.5 min (*d*) 30 min
(D.M.R.C. 2003)

21. Two pipes A and B can fill a cistern in 12 minutes and 15 minutes respectively while a third pipe C can empty the full tank in 6 minutes. A and B are kept open for 5 minutes in the beginning and then C is also opened. In what time is the cistern emptied ?
(*a*) 30 min (*b*) 33 min (*c*) $37\frac{1}{2}$ min (*d*) 45 min

22. Two pipes A and B can fill a tank in 6 hours and 4 hours respectively. If they are opened on alternate hours and if pipe A is opened first, in how many hours, the tank shall be full ?
(*a*) 4 (*b*) $4\frac{1}{2}$ (*c*) 5 (*d*) $5\frac{1}{2}$

23. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in : **(S.S.C. 1999)**
(*a*) 6 hrs (*b*) $6\frac{2}{3}$ hrs (*c*) 5 (*d*) $7\frac{1}{2}$ hrs

24. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump ?
(*a*) 50 m^3/min (*b*) 60 m^3/min (*c*) 72 m^3/min (*d*) None of these

25. A leak in the bottom of a tank can empty the full tank in 8 hours. An inlet pipe fills water at the rate of 6 litres a minute. When the tank is full, the inlet is opened and due to the leak, the tank is empty in 12 hours. How many litres does the cistern hold ?
(*a*) 7580 (*b*) 7960 (*c*) 8290 (*d*) 8640

26. Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is : **(Bank P.O. 2001)**
(*a*) 60 gallons (*b*) 100 gallons (*c*) 120 gallons (*d*) 180 gallons

27. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the pipe B is turned off after : **(S.S.C. 2004)**
(*a*) 5 min (*b*) 9 min (*c*) 10 min (*d*) 15 min

28. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is : **(L.I.C.A.A.O. 2003)**
(*a*) 10 (*b*) 12 (*c*) 14 (*d*) 16

ANSWERS

1. (a)	2. (d)	3. (b)	4. (a)	5. (c)	6. (c)	7. (b)	8. (b)
9. (d)	10. (d)	11. (c)	12. (c)	13. (c)	14. (c)	15. (d)	16. (c)
17. (d)	18. (d)	19. (a)	20. (d)	21. (d)	22. (c)	23. (c)	24. (a)
25. (d)	26. (c)	27. (b)	28. (c)				

SOLUTIONS

1. Part filled by A in 1 min. $= \frac{1}{20}$; Part filled by B in 1 min. $= \frac{1}{30}$.

   Part filled by (A + B) in 1 min. $= \left(\frac{1}{20} + \frac{1}{30}\right) = \frac{1}{12}$.

   $\therefore$ Both the pipes can fill the tank in 12 minutes.

2. Net part filled in 1 hour $= \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}$.

   $\therefore$ The cistern will be filled in $\frac{36}{5}$ hrs *i.e.*, 7.2 hrs.

3. Time taken by one tap to fill half the tank = 3 hrs.

   Part filled by the four taps in 1 hour $= \left(4 \times \frac{1}{6}\right) = \frac{2}{3}$.

   Remaining part $= \left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

   $\therefore \quad \frac{2}{3} : \frac{1}{2} :: 1 : x \quad$ or $\quad x = \left(\frac{1}{2} \times 1 \times \frac{3}{2}\right) = \frac{3}{4}$ hrs *i.e.*, 45 mins.

   So, total time taken = 3 hrs 45 min.

4. Clearly, pipe B is faster than pipe A and so, the tank will be emptied.

   Part to be emptied $= \frac{2}{5}$.

   Part emptied by (A + B) in 1 minute $= \left(\frac{1}{6} - \frac{1}{10}\right) = \frac{1}{15}$.

   $\therefore \quad \frac{1}{15} : \frac{2}{5} :: 1 : x \quad$ or $\quad x = \left(\frac{2}{5} \times 1 \times 15\right) = 6$ min.

   So, the tank will be emptied in 6 min.

5. Part filled by (A + B + C) in 1 hour $= \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{30}\right) = \frac{1}{3}$.

   $\therefore$ All the three pipes together will fill the tank in 3 hours.

6. Net part filled in 1 hour $= \left(\frac{1}{5} + \frac{1}{6} - \frac{1}{12}\right) = \frac{17}{60}$.

   $\therefore$ The tank will be full in $\frac{60}{17}$ hrs *i.e.*, $3\frac{9}{17}$ hrs.

7. Part filled by (A + B + C) in 3 minutes $= 3\left(\frac{1}{30} + \frac{1}{20} + \frac{1}{10}\right) = \left(3 \times \frac{11}{60}\right) = \frac{11}{20}$.

Part filled by C in 3 minutes $= \frac{3}{10}$.

$\therefore$ Required ratio $= \left(\frac{3}{10} \times \frac{20}{11}\right) = \frac{6}{11}$.

8. Work done by the third pipe in 1 min.

$$-\frac{1}{50} - \left(\frac{1}{60} + \frac{1}{75}\right) = \left(\frac{1}{50} - \frac{3}{100}\right) = -\frac{1}{100}. \quad [-\text{ ve sign means emptying}]$$

$\therefore$ The third pipe alone can empty the cistern in 100 min.

9. Work done by the leak in 1 hour $= \left(\frac{1}{2} - \frac{3}{7}\right) = \frac{1}{14}$.

$\therefore$ Leak will empty the tank in 14 hrs.

10. Part filled by (A + B) in 1 hour $= \left(\frac{1}{5} + \frac{1}{20}\right) = \frac{1}{4}$.

So, A and B together can fill the tank in 4 hours.

Work done by the leak in 1 hour $= \left(\frac{1}{4} - \frac{2}{9}\right) = \frac{1}{36}$.

$\therefore$ Leak will empty the tank in 36 hrs.

11. Let the cistern be filled by pipe A alone in x hours.

Then, pipe B will fill it in $(x + 6)$ hours.

$$\therefore \quad \frac{1}{x} + \frac{1}{(x+6)} = \frac{1}{4} \quad \Leftrightarrow \quad \frac{x+6+x}{x(x+6)} = \frac{1}{4}$$

$$\Leftrightarrow \quad x^2 - 2x - 24 = 0 \quad \Leftrightarrow \quad (x-6)(x+4) = 0$$

$\Leftrightarrow \quad x = 6.$ [*neglecting the – ve value of x*]

12. Let the slower pipe alone fill the tank in x minutes.

Then, faster pipe will fill it in $\frac{x}{3}$ minutes.

$$\therefore \quad \frac{1}{x} + \frac{3}{x} = \frac{1}{36} \quad \Leftrightarrow \quad \frac{4}{x} = \frac{1}{36} \quad \Leftrightarrow \quad x = 144 \text{ min.}$$

13. Suppose pipe A alone takes x hours to fill the tank.

Then, pipes B and C will take $\frac{x}{2}$ and $\frac{x}{4}$ hours respectively to fill the tank.

$$\therefore \quad \frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5} \quad \Leftrightarrow \quad \frac{7}{x} = \frac{1}{5} \quad \Leftrightarrow \quad x = 35 \text{ hrs.}$$

14. Suppose, first pipe alone takes x hours to fill the tank. Then, second and third pipes will take $(x - 5)$ and $(x - 9)$ hours respectively to fill the tank.

$$\therefore \quad \frac{1}{x} + \frac{1}{(x-5)} = \frac{1}{(x-9)} \quad \Leftrightarrow \quad \frac{x-5+x}{x(x-5)} = \frac{1}{(x-9)}$$

$$\Leftrightarrow \quad (2x-5)(x-9) = x(x-5) \quad \Leftrightarrow \quad x^2 - 18x + 45 = 0$$

$\Leftrightarrow \quad (x-15)(x-3) = 0 \quad \Leftrightarrow \quad x = 15.$ [*neglecting $x = 3$*]

15. Capacity of the tank $= (12 \times 13.5)$ litres $= 162$ litres.

Capacity of each bucket $= 9$ litres.

Number of buckets needed $= \left(\frac{162}{9}\right) = 18$.

16. Let capacity of P be x litres. Then, capacity of Q $= \frac{x}{3}$ litres.

Capacity of the drum $= 60x$ litres.

Required number of turns $= \frac{60x}{\left(x+\frac{x}{3}\right)} = \left(60x \times \frac{3}{4x}\right) = 45.$

17. Part filled in 3 min. $= 3\left(\frac{1}{12}+\frac{1}{15}\right) = \left(3 \times \frac{9}{60}\right) = \frac{9}{20}.$

Remaining part $= \left(1-\frac{9}{20}\right) = \frac{11}{20}.$

Part filled by B in 1 min. $= \frac{1}{15}.$

$\frac{1}{15} : \frac{11}{20} :: 1 : x$ or $x = \left(\frac{11}{20} \times 1 \times 15\right) = 8\frac{1}{4}$ min. $= 8$ min. 15 sec.

$\therefore$ Remaining part is filled by B in 8 min. 15 sec.

18. Part filled in 4 minutes $= 4\left(\frac{1}{15}+\frac{1}{20}\right) = \frac{7}{15}.$

Remaining part $= \left(1-\frac{7}{15}\right) = \frac{8}{15}.$

Part filled by B in 1 minute $= \frac{1}{20}.$

$\frac{1}{20} : \frac{8}{15} :: 1 : x$ or $x = \left(\frac{8}{15} \times 1 \times 20\right) = 10\frac{2}{3}$ min. $= 10$ min. 40 sec.

$\therefore$ The tank will be full in (4 min. + 10 min. 40 sec) = 14 min. 40 sec.

19. Part filled in 10 hours $= 10\left(\frac{1}{15}+\frac{1}{20}-\frac{1}{25}\right) = \frac{23}{30}.$

Remaining part $= \left(1-\frac{23}{30}\right) = \frac{7}{30}.$

(A + B)'s 1 hour's work $= \left(\frac{1}{15}+\frac{1}{20}\right) = \frac{7}{60}.$

$\frac{7}{60} : \frac{7}{30} :: 1 : x$ or $x = \left(\frac{7}{30} \times 1 \times \frac{60}{7}\right) = 2$ hours.

$\therefore$ The tank will be full in (10 + 2) hrs = 12 hrs.

20. Part filled by (A + B) in 1 minute $= \left(\frac{1}{60}+\frac{1}{40}\right) = \frac{1}{24}.$

Suppose the tank is filled in x minutes.

Then, $\frac{x}{2}\left(\frac{1}{24}+\frac{1}{40}\right) = 1 \Leftrightarrow \frac{x}{2} \times \frac{1}{15} = 1 \Leftrightarrow x = 30$ min.

21. Part filled in 5 min. $= 5\left(\frac{1}{12}+\frac{1}{15}\right) = \left(5 \times \frac{9}{60}\right) = \frac{3}{4}.$

Part emptied in 1 min. when all the pipes are opened

$$= \frac{1}{6} - \left(\frac{1}{12}+\frac{1}{15}\right) = \left(\frac{1}{6}-\frac{3}{20}\right) = \frac{1}{60}.$$

Now, $\frac{1}{60}$ part is emptied in 1 min.

$\therefore$ $\frac{3}{4}$ part will be emptied in $\left(60 \times \frac{3}{4}\right) = 45$ min.

22. A's work in 1 hour $= \frac{1}{6}$, B's work in 1 hour $= \frac{1}{4}$.

(A + B)'s 2 hour's work when opened alternately $= \left(\frac{1}{6} + \frac{1}{4}\right) = \frac{5}{12}$.

(A + B)'s 4 hour's work when opened alternately $= \frac{10}{12} = \frac{5}{6}$.

Remaining part $= \left(1 - \frac{5}{6}\right) = \frac{1}{6}$.

Now, it is A's turn and $\frac{1}{6}$ part is filled by A in 1 hour.

$\therefore$ Total time taken to fill the tank = (4 + 1) hrs = 5 hrs.

23. (A + B)'s 1 hour's work $= \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{9}{60} = \frac{3}{20}$.

(A + C)'s 1 hour's work $= \left(\frac{1}{12} + \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$.

Part filled in 2 hrs $= \left(\frac{3}{20} + \frac{2}{15}\right) = \frac{17}{60}$; Part filled in 6 hrs $= \left(3 \times \frac{17}{60}\right) = \frac{17}{20}$.

Remaining part $= \left(1 - \frac{17}{20}\right) = \frac{3}{20}$.

Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.

$\therefore$ Total time taken to fill the tank = (6 + 1) hrs = 7 hrs.

24. Let the filling capacity of the pump be x m^3 / min.

Then, emptying capacity of the pump = $(x + 10)$ m^3 / min.

So, $\frac{2400}{x} - \frac{2400}{(x+10)} = 8 \quad \Leftrightarrow \quad x^2 + 10x - 3000 = 0$

$\Leftrightarrow \quad (x - 50)(x + 60) = 0 \quad \Leftrightarrow \quad x = 50.$ *[neglecting the – ve value of x]*

25. Work done by the inlet in 1 hour $= \left(\frac{1}{8} - \frac{1}{12}\right) = \frac{1}{24}$.

Work done by the inlet in 1 min. $= \left(\frac{1}{24} \times \frac{1}{60}\right) = \frac{1}{1440}$.

$\therefore$ Volume of $\frac{1}{1440}$ part = 6 litres.

$\therefore$ Volume of whole = (1440 × 6) litres = 8640 litres.

26. Work done by the waste pipe in 1 minute

$$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) - \left(\frac{1}{15} - \frac{11}{120}\right) = -\frac{1}{40}.$$ *[– ve sign means emptying]*

$\therefore$ Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3 × 40) gallons = 120 gallons.

27. Let B be turned off after x minutes. Then,
Part filled by (A + B) in x min. + Part filled by A in $(30 - x)$ min. = 1.

$$\therefore \quad x\left(\frac{2}{75}+\frac{1}{45}\right)+(30-x)\frac{2}{75}=1$$

$$\Leftrightarrow \quad \frac{11x}{225}+\frac{(60-2x)}{75}=1 \quad \Leftrightarrow \quad 11x+180-6x=225 \quad \Leftrightarrow \quad x=9.$$

28. Part filled in 2 hours $= \frac{2}{6}=\frac{1}{3}$, Remaining part $= \left(1-\frac{1}{3}\right)=\frac{2}{3}$.

$\therefore$ (A + B)'s 7 hour's work $= \frac{2}{3}$; (A + B)'s 1 hour's work $= \frac{2}{21}$.

$\therefore$ C's 1 hour's work = [(A + B + C)'s 1 hour's work − (A + B)'s 1 hour's work]

$$=\left(\frac{1}{6}-\frac{2}{21}\right)=\frac{1}{14}.$$

$\therefore$ C alone can fill the tank in 14 hours.

EXERCISE 16B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 4*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. How long will it take to empty the tank if both the inlet pipe A and the outlet pipe B are opened simultaneously ?
   I. A can fill the tank in 16 minutes.
   II. B can empty the full tank in 8 minutes.
2. Two taps A and B, when opened together, can fill a tank in 6 hours. How long will it take for the pipe A alone to fill the tank ?
   I. B alone takes 5 hours more than A to fill the tank.
   II. The ratio of the time taken by A to that taken by B to fill the tank is 2 : 3.
3. A tank is fitted with two inlet pipes A and B. Both the pipes are kept open for 10 minutes so that the tank is two-thirds full and then pipe A is closed. How much time will B take to fill the remaining part of the tank ?
   I. Pipe A is thrice as fast as pipe B.
   II. Pipe B alone can fill the tank in 60 minutes.
4. How much time will the leak take to empty the full cistern ?
   I. The cistern is normally filled in 9 hours.
   II. It takes one hour more than the usual time to fill the cistern because of a leak in the bottom.

Directions (*Questions 5-6*) : ***Each of the questions below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question*** :

5. A tank is fitted with two taps A and B. In how much time will the tank be full if both the taps are opened together ?

I. A is 50% more efficient than B.

II. A alone takes 16 hours to fill the tank.

III. B alone takes 24 hours to fill the tank.

(*a*) II and III only (*b*) All I, II and III

(*c*) I and II only (*d*) I and III only

(*e*) Any two of the three

6. If both the pipes are opened, how many hours will be taken to fill the tank ?

I. The capacity of the tank is 400 litres.

II. The pipe A fills the tank in 4 hours.

III. The pipe B fills the tank in 6 hours. **(R.B.I. 2003)**

(*a*) Only I and II (*b*) Only II and III

(*c*) All I, II and III (*d*) Any two of the three

(*e*) Even with all the three statements, answer cannot be given.

ANSWERS

1. (*e*) **2.** (*c*) **3.** (*c*) **4.** (*c*) **5.** (*e*) **6.** (*b*)

SOLUTIONS

1. I. A's 1 minute's filling work $= \frac{1}{16}$.

II. B's 1 minute's emptying work $= \frac{1}{8}$.

(A + B)'s 1 minute's emptying work $= \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{1}{16}$.

$\therefore$ Tank will be emptied in 16 minutes.

Thus, both I and II are necessary to answer the question.

$\therefore$ Correct answer is (*e*).

2. (A + B)'s 1 hour filling work $= \frac{1}{6}$.

I. Suppose A takes x hours to fill the tank.

Then, B takes $(x + 5)$ hours to fill the tank.

$\therefore$ (A's 1 hour work) + (B's 1 hour work) = (A + B)'s 1 hour work

$\Leftrightarrow \quad \frac{1}{x} + \frac{1}{(x+5)} = \frac{1}{6} \qquad \Leftrightarrow \quad \frac{(x+5)+x}{x(x+5)} = \frac{1}{6}$

$\Leftrightarrow \quad x^2 - 5x = 12x + 30 \qquad \Leftrightarrow \quad x^2 - 7x - 30 = 0$

$\Leftrightarrow \quad x^2 - 10x + 3x - 30 = 0 \qquad \Leftrightarrow \quad x(x - 10) + 3(x - 10) = 0$

$\Leftrightarrow \quad (x - 10)(x + 3) = 0 \qquad \Leftrightarrow \quad x = 10.$

So, A alone takes 10 hours to fill the tank.

II. Suppose A takes $2x$ hours and B takes $3x$ hours to fill the tank. Then,

$$\frac{1}{2x}+\frac{1}{3x}=\frac{1}{6} \Leftrightarrow \left(\frac{1}{2}+\frac{1}{3}\right)\cdot\frac{1}{x}=\frac{1}{6} \Leftrightarrow \frac{5}{6x}=\frac{1}{6} \Leftrightarrow x=5.$$

So, A alone takes $(2 \times 5) = 10$ hours to fill the tank.

Thus, each one of I and II gives the answer.

$\therefore$ Correct answer is (*c*).

3. I. Let B's 1 min. work $= \frac{1}{x}$. Then, A's 1 min. work $= \frac{3}{x}$.

(A + B)'s 1 min. work $= \left(\frac{1}{x}+\frac{3}{x}\right)=\frac{4}{x}$.

(A + B)'s 10 min. work $= \left(\frac{4}{x}\times 10\right)=\frac{40}{x}$.

$\therefore \quad \frac{40}{x}=\frac{2}{3} \Leftrightarrow x=60.$

$\therefore$ B's 1 min. work $= \frac{1}{60}$.

$\frac{1}{60}$ part is filled by B in 1 min.

$\frac{1}{3}$ part is filled by B in $\left(60\times\frac{1}{3}\right)$ min. = 20 min.

II. B's 1 min. work $= \frac{1}{60}$.

$\frac{1}{60}$ part is filled by B in 1 min.

$\frac{1}{3}$ part is filled by B in $\left(60\times\frac{1}{3}\right)$ min. = 20 min.

Hence, the correct answer is (*c*).

4. I. Time taken to fill the cistern without leak = 9 hours.

Part of cistern filled without leak in 1 hour $= \frac{1}{9}$.

II. Time taken to fill the cistern in presence of leak = 10 hours.

Net filling in 1 hour $= \frac{1}{10}$.

Work done by leak in 1 hour $= \left(\frac{1}{9}-\frac{1}{10}\right)=\frac{1}{90}$.

$\therefore$ Leak will empty the full cistern in 90 hours.

Clearly, both I and II are necessary to answer the question.

$\therefore$ Correct answer is (*e*).

5. II. A's 1 hour work $= \frac{1}{16}$.

Suppose B fills the tank in x hours. Then, B's 1 hour work $= \frac{1}{x}$.

I. Work done by A in 1 hour = 150% of $\frac{1}{x} = \left(\frac{1}{x}\times\frac{150}{100}\right)=\frac{3}{2x}$.

$\therefore \quad \frac{3}{2x} = \frac{1}{16} \Leftrightarrow x = 24.$

So, B can fill the tank in 24 hours.

(A + B)'s 1 hour work $= \left(\frac{1}{16} + \frac{1}{24}\right) = \frac{5}{48}$.

$\therefore$ (A + B) can fill the tank in $\frac{48}{5}$ hrs.

Thus, I & II give the answer.

III. Work done by B in 1 hour $= \frac{1}{24}$.

From II & III, we get the same answer.

From III & I, we get :

A's 1 hour work = 150% of $\frac{1}{24} = \left(\frac{1}{24} \times \frac{150}{100}\right) = \frac{1}{16}$.

Thus, from III & I, we get the same answer.

$\therefore$ Correct answer is (*e*).

6. II. Part of the tank filled by A in 1 hour $= \frac{1}{4}$.

III. Part of the tank filled by B in 1 hour $= \frac{1}{6}$.

(A + B)'s 1 hour's work $= \left(\frac{1}{4} + \frac{1}{6}\right) = \frac{5}{12}$.

$\therefore$ When both A and B are opened together, they will fill the tank in $\frac{12}{5}$ hrs = 2 hrs 24 min.

So, II and III are needed.

$\therefore$ Correct answer is (*b*).