



Assignment

Exponential series

Basic Level

1. $1 + \frac{4^2}{3!} + \frac{4^4}{5!} + \dots \infty$
 - (a) $\frac{e^4 + e^{-4}}{4}$
 - (b) $\frac{e^4 - e^{-4}}{4}$
 - (c) $\frac{e^4 + e^{-4}}{8}$
 - (d) $\frac{e^4 - e^{-4}}{8}$
2. $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots \infty =$ [EAMCET 2003]
 - (a) e
 - (b) $2e$
 - (c) $\frac{e}{2}$
 - (d) None of these
3. $1 + \frac{(\log_e x)^2}{2!} + \frac{(\log_e x)^4}{4!} + \dots \infty$
 - (a) x
 - (b) $\frac{1}{x}$
 - (c) $\frac{1}{2}(x + x^{-1})$
 - (d) $\frac{1}{2}(e^x + e^{-x})$
4. $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots \infty =$
 - (a) e
 - (b) $2e$
 - (c) $3e$
 - (d) None of these
5. $\frac{1}{1!} + \frac{4}{2!} + \frac{7}{3!} + \frac{10}{4!} + \dots \infty =$
 - (a) $e + 4$
 - (b) $2 + e$
 - (c) $3 + e$
 - (d) e
6. $\frac{2}{1!} + \frac{2+4}{2!} + \frac{2+4+6}{3!} + \dots \infty =$ [MNR 1985]
 - (a) e
 - (b) $2e$
 - (c) $3e$
 - (d) None of these
7. $\left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots \infty\right)^2 - \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots \infty\right)^2 =$
 - (a) 0
 - (b) 1
 - (c) -1
 - (d) 2
8. $1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots \infty =$ [MP PET 1991]
 - (a) e^{-1}
 - (b) e
 - (c) $\frac{e + e^{-1}}{2}$
 - (d) $\frac{e - e^{-1}}{2}$
9. $1 + \frac{\log_e x}{1!} + \frac{(\log_e x)^2}{2!} + \frac{(\log_e x)^3}{3!} + \dots \infty =$ [Kurukshetra CEE 1998; JMI CET 2000]
 - (a) $\log_e x$
 - (b) x
 - (c) x^{-1}
 - (d) $-\log_e(1+x)$
10. $\frac{x^2 - y^2}{1!} + \frac{x^4 - y^4}{2!} + \frac{x^6 - y^6}{3!} + \dots \infty =$
 - (a) $e^x - e^y$
 - (b) $e^{x^2} - e^{y^2}$
 - (c) $2 + e^{x^2} - e^{y^2}$
 - (d) $\frac{e^x - e^y}{2}$

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11. $1 + x \log_e a + \frac{x^2}{2!} (\log_e a)^2 + \frac{x^3}{3!} (\log_e a)^3 + \dots =$ [EAMCET 2002]
 (a) a^x (b) x (c) $a^{\log_a x}$ (d) a
12. $3 + \frac{5}{1!} + \frac{7}{2!} + \frac{9}{3!} + \dots \infty =$
 (a) $3e$ (b) $5e$ (c) $5e - 1$ (d) None of these
13. $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \infty =$ [MP PET 1986]
 (a) e^x (b) e^{-x} (c) e (d) e^{x^2}
14. $\frac{2}{1!} \log_e 2 + \frac{2^2}{2!} (\log_e 2)^2 + \frac{2^3}{3!} (\log_e 2)^3 + \dots \infty =$
 (a) 2 (b) 3 (c) 4 (d) None of these
15. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8(2)!} + \frac{1}{16(3)!} + \frac{1}{32(4)!} + \dots \infty =$
 (a) e (b) $\sqrt{e}$ (c) $\frac{\sqrt{e}}{2}$ (d) None of these
16. Sum to infinity of the series $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$ is [MP PET 1994]
 (a) $\frac{e^x - e^{-x}}{2}$ (b) $\frac{e^x + e^{-x}}{2}$ (c) $\frac{e^{-x} - e^x}{2}$ (d) $\frac{-(e^x + e^{-x})}{2}$
17. Sum of the infinite series $1 + 2 + \frac{1}{2!} + \frac{2}{3!} + \frac{1}{4!} + \frac{2}{5!} + \dots$ is [Roorkee 2000]
 (a) e^2 (b) $e + e^{-1}$ (c) $\frac{e - e^{-1}}{2}$ (d) $\frac{3e - e^{-1}}{2}$
18. The value of $1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \dots$ is [MP PET 1998]
 (a) 2 (b) $\frac{1}{2}$ (c) $\log 3$ (d) None of these
19. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$ is [DCE 2002]
 (a) e (b) $e^{-\frac{1}{2}}$ (c) e^{-2} (d) None of these
20. If $S = \sum_{n=2}^{\infty} {}^nC_2 \frac{3^{n-2}}{n!}$, then $2S$ equals
 (a) $e^{3/2}$ (b) e^3 (c) $e^{-3/2}$ (d) e^{-3}
21. The coefficient of x^r in the expansion of $1 + \frac{a+bx}{1!} + \frac{(a+bx)^2}{2!} + \dots + \frac{(a+bx)^n}{n!} + \dots$ is [MP PET 1989]
 (a) $\frac{(a+b)^r}{r!}$ (b) $\frac{b^r}{r!}$ (c) $\frac{e^a b^r}{r!}$ (d) e^{a+b^r}
22. In the expansion of $\frac{a+bx}{e^x}$, the coefficient of x^r is
 (a) $\frac{a-b}{r!}$ (b) $\frac{a-br}{r!}$ (c) $(-1)^r \frac{a-br}{r!}$ (d) None of these
23. In the expansion of $(e^x - 1)(e^{-x} + 1)$, the coefficient of x^3 is
 (a) 0 (b) $1/3$ (c) $2/3$ (d) $1/6$
24. In the expansion of $\frac{a+bx+cx^2}{e^x}$, the coefficient of x^n will be
 (a) $\frac{a(-1)^n}{n!} + \frac{b(-1)^{n-1}}{(n-1)!} + \frac{c(-1)^{n-2}}{(n-2)!}$ (b) $\frac{a}{n!} + \frac{b}{(n-1)!} + \frac{c}{(n-2)!}$ (c) $\frac{(-1)^n}{n!} + \frac{(-1)^{n-1}}{(n-1)!} + \frac{(-1)^{n-2}}{(n-2)!}$ (d) None of these

25. If n is even, then in the expansion of $\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right)^2$, the coefficient of x^n is
- (a) $\frac{2^n}{n!}$ (b) $\frac{2^n - 2}{n!}$ (c) $\frac{2^{n-1} - 1}{n!}$ (d) $\frac{2^{n-1}}{n!}$
26. If $e^x = y + \sqrt{1 + y^2}$, then $y =$ [MNR 1990; UPSEAT 2000]
- (a) $\frac{e^x + e^{-x}}{2}$ (b) $\frac{e^x - e^{-x}}{2}$ (c) $e^x + e^{-x}$ (d) $e^x - e^{-x}$
27. $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{2n} =$
- (a) e (b) e^{-1} (c) e^2 (d) e^{-2}

Advance Level

28. $\frac{1^2}{2!} + \frac{2^2}{3!} + \frac{3^2}{4!} + \dots \infty =$
- (a) e (b) $e - 1$ (c) $e + 1$ (d) e^2
29. The sum of the series $1 + \frac{3}{2!} + \frac{7}{3!} + \frac{15}{4!} + \dots \infty$ is [AMU 1992; Kurukshetra CEE 1999]
- (a) $e(e + 1)$ (b) $e(1 - e)$ (c) $e(e - 1)$ (d) $3e$
30. $\left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots\right) \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots\right) =$
- (a) e^4 (b) $\frac{e^2 - 1}{e^2}$ (c) $\frac{e^4 - 1}{4e^2}$ (d) $\frac{e^4 + 1}{4e^2}$
31. $\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots \infty}{1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots \infty} =$
- (a) $\frac{e + 1}{e - 1}$ (b) $\frac{e - 1}{e + 1}$ (c) $\frac{e^2 + 1}{e^2 - 1}$ (d) $\frac{e^2 - 1}{e^2 + 1}$
32. $\frac{1 + \frac{2^2}{2!} + \frac{2^4}{3!} + \frac{2^6}{4!} + \dots \infty}{1 + \frac{1}{2!} + \frac{2}{3!} + \frac{2^2}{4!} + \dots \infty} =$
- (a) e^2 (b) $e^2 - 1$ (c) $e^{3/2}$ (d) None of these
33. $1 + \frac{2^4}{2!} + \frac{3^4}{3!} + \frac{4^4}{4!} + \dots \infty =$
- (a) $5e$ (b) e (c) $15e$ (d) $2e$
34. $(1 + 3)\log_e 3 + \frac{1 + 3^2}{2!}(\log_e 3)^2 + \frac{1 + 3^3}{3!}(\log_e 3)^3 + \dots \infty =$ [Roorkee 1989]
- (a) 28 (b) 30 (c) 25 (d) 0
35. $\frac{1}{1!} + \frac{1 + 2}{2!} + \frac{1 + 2 + 2^2}{3!} + \dots \infty =$ [AMU 1992; Kurukshetra CEE 1999; EAMCET 2002]
- (a) e^2 (b) $e^2 - 1$ (c) $e^2 - e$ (d) $e^3 - e^2$
36. The sum of the series $\sum_{n=0}^{\infty} \frac{n^2 - n + 1}{n!}$ is [AMU 1991]

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- (a) e (b) $\frac{3}{2}e$ (c) $2e$ (d) $3e$
37. $\frac{1^2 \cdot 2}{1!} + \frac{2^2 \cdot 3}{2!} + \frac{3^2 \cdot 4}{3!} + \dots =$ [UPSEAT 1999]
 (a) $6e$ (b) $7e$ (c) $8e$ (d) $9e$
38. The value of $\sqrt{e}$ will be [UPSEAT 1999]
 (a) 1.648 (b) 1.547 (c) 1.447 (d) 1.348
39. The sum of the infinite series $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots$ is [AMU 1999]
 (a) $e - 2$ (b) $\frac{2}{3}e - 1$ (c) 1 (d) $3/2$
40. The sum of $\frac{2}{1!} + \frac{6}{2!} + \frac{12}{3!} + \frac{20}{4!} + \dots$ is [UPSEAT 2000]
 (a) $\frac{3e}{2}$ (b) e (c) $2e$ (d) $3e$
41. $1 + \frac{(\log_e n)^2}{2!} + \frac{(\log_e n)^4}{4!} + \dots =$ [MP PET 1996]
 (a) n (b) $1/n$ (c) $\frac{1}{2}(n + n^{-1})$ (d) $\frac{1}{2}(e^n + e^{-n})$
42. The sum of the series $\frac{1^2}{1 \cdot 2!} + \frac{1^2 + 2^2}{2 \cdot 3!} + \frac{1^2 + 2^2 + 3^2}{3 \cdot 4!} + \dots + \frac{1^2 + 2^2 + \dots + n^2}{n \cdot (n+1)!} + \dots =$ [AMU 2002]
 (a) e^2 (b) $\frac{1}{2}(e + e^{-1})^2$ (c) $\frac{3e - 1}{6}$ (d) $\frac{4e + 1}{6}$
43. The value of $(a+b)(a-b) + \frac{1}{2!}(a+b)(a-b)(a^2 + b^2) + \frac{1}{3!}(a+b)(a-b)(a^4 + a^2b^2 + b^4) + \dots$ is
 (a) $e^{a^2} - e^{b^2}$ (b) $e^{a^2} + e^{b^2}$ (c) $e^{a^2 - b^2}$ (d) None of these
44. Sum of the series $C = 1 + \frac{\cos x}{1!} + \frac{\cos 2x}{2!} + \frac{\cos 3x}{3!} + \dots$ and $S = \frac{\sin x}{1!} + \frac{\sin 2x}{2!} + \frac{\sin 3x}{3!} + \dots$ is equal to [AMU 2001]
 (a) $\exp(ix)$ (b) $\exp[\cos(\sin x) + i \sin(\sin x)]$ (c) $\exp[\exp(ix)]$ (d) $\exp(\cos x)[\exp(ix)]$
45. The sum of the series $\frac{4}{1!} + \frac{11}{2!} + \frac{22}{3!} + \frac{37}{4!} + \frac{56}{5!} + \dots$ is [Kurukshetra CEE 2002]
 (a) $6e$ (b) $6e - 1$ (c) $5e$ (d) $5e + 1$
46. The sum of the series $\frac{1}{1 \cdot 2} + \frac{1 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \dots =$ [Kurukshetra CEE 2002]
 (a) $15e$ (b) $e^{1/2} + e$ (c) $e^{1/2} - 1$ (d) $e^{1/2} - e$
47. $\frac{9}{1!} + \frac{16}{2!} + \frac{27}{3!} + \frac{42}{4!} + \dots =$ [Roorkee 1992]
 (a) $5e$ (b) $7e$ (c) $9e$ (d) $11e - 6$
48. If S_n denotes the sum of the products of the first n natural numbers taken two at a time, then $\sum_{n=0}^{\infty} \frac{S_n}{(n+1)!} =$
 (a) $\frac{11e}{24}$ (b) $\frac{11e}{12}$ (c) $\frac{13e}{24}$ (d) None of these
49. The sum of the series $1 + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + \frac{1^2 + 2^2 + 3^2 + 4^2}{4!} + \dots$ is
 (a) $3e$ (b) $\frac{17}{6}e$ (c) $\frac{13}{6}e$ (d) $\frac{19}{6}e$

50. The sum of the series $\frac{9}{1!} + \frac{19}{2!} + \frac{35}{3!} + \frac{57}{4!} + \frac{85}{5!} + \dots$ is
 (a) $12e - 7$ (b) $12e - 5$ (c) $12e - 11$ (d) None of these
51. The sum of the series $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots$ is
 (a) $27e$ (b) $24e$ (c) $28e$ (d) None of these
52. If $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$ and $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is
 (a) 1 (b) 0 (c) -1 (d) -2

Logarithmic series

Basic Level

53. $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \dots \infty =$
 (a) $\frac{x}{1+x} - \log_e(1-x)$ (b) $\frac{x}{1+x} + \log_e(1-x)$ (c) $\frac{x}{1-x} - \log_e(1-x)$ (d) $\frac{x}{1-x} + \log_e(1-x)$
54. $1 + \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^4} + \frac{1}{7 \cdot 2^6} + \dots \infty =$
 (a) $\log_e 3$ (b) $2\log_e 3$ (c) $\frac{1}{2}\log_e 3$ (d) None of these
55. $\frac{1}{2} + \frac{3}{2} \cdot \frac{1}{4} + \frac{5}{3} \cdot \frac{1}{8} + \frac{7}{4} \cdot \frac{1}{16} + \dots \infty =$
 (a) $2 - \log_e 2$ (b) $2 + \log_e 2$ (c) $\log_e 4$ (d) None of these
56. $\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \dots \infty =$
 (a) $\log_e \frac{x-1}{x}$ (b) $\log_e \frac{x+1}{x}$ (c) $\log_e \frac{1}{x}$ (d) None of these
57. $\left(\frac{a-b}{a}\right) + \frac{1}{2}\left(\frac{a-b}{a}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a}\right)^3 + \dots =$ [MNR 1979; MP PET 1990; UPSEAT 2001, 02]
 (a) $\log_e(a-b)$ (b) $\log_e\left(\frac{a}{b}\right)$ (c) $\log_e\left(\frac{b}{a}\right)$ (d) $e^{\left(\frac{a-b}{a}\right)}$
58. $\frac{1}{5} + \frac{1}{2} \cdot \frac{1}{5^2} + \frac{1}{3} \cdot \frac{1}{5^3} + \dots \infty =$
 (a) $\log_e \frac{4}{5}$ (b) $\log_e \frac{\sqrt{5}}{2}$ (c) $2\log_e \frac{\sqrt{5}}{2}$ (d) None of these
59. The sum of the series $\frac{1}{2 \cdot 3} + \frac{1}{4 \cdot 5} + \frac{1}{6 \cdot 7} + \dots =$ [MP PET 1998]
 (a) $\log \frac{2}{e}$ (b) $\log \frac{e}{2}$ (c) $\frac{2}{e}$ (d) $\frac{e}{2}$
60. $\frac{1}{3} + \frac{1}{2 \cdot 3^2} + \frac{1}{3 \cdot 3^3} + \frac{1}{4 \cdot 3^4} + \dots \infty =$ [MNR 1975]
 (a) $\log_e 2 - \log_e 3$ (b) $\log_e 3 - \log_e 2$ (c) $\log_e 6$ (d) None of these
61. $1 + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \dots \infty =$
 (a) $\log_e 3$ (b) $\log_e 4$ (c) $\log_e\left(\frac{e}{2}\right)$ (d) $\log_e\left(\frac{2}{3}\right)$

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62. $\log_e \frac{4}{5} + \frac{1}{4} - \frac{1}{2} \left(\frac{1}{4} \right)^2 + \frac{1}{3} \left(\frac{1}{4} \right)^3 - \dots$
 (a) $2 \log_e \frac{4}{5}$ (b) $\log_e \frac{5}{4}$ (c) 1 (d) 0
63. $\frac{1}{n^2} + \frac{1}{2n^4} + \frac{1}{3n^6} + \dots \infty =$
 (a) $\log_e \left(\frac{n^2}{n^2+1} \right)$ (b) $\log_e \left(\frac{n^2+1}{n^2} \right)$ (c) $\log_e \left(\frac{n^2}{n^2-1} \right)$ (d) None of these
64. The sum of $\frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2^3} + \frac{1}{5} \cdot \frac{1}{2^5} + \dots \infty$ is [MP PET 1991]
 (a) $\log_e \sqrt{\frac{3}{2}}$ (b) $\log_e \sqrt{3}$ (c) $\log_e \sqrt{\frac{1}{2}}$ (d) $\log_e 3$
65. If $0 < y < 2^{1/3}$ and $x(y^3 - 1) = 1$, then $\frac{2}{x} + \frac{2}{3x^3} + \frac{2}{5x^5} + \dots =$ [EAMCET 2003]
 (a) $\log \left[\frac{y^3}{y^3-2} \right]$ (b) $\log \left[\frac{y^3}{1-y^3} \right]$ (c) $\log \left[\frac{2y^3}{1-y^3} \right]$ (d) $\log \left[\frac{y^3}{1-2y^3} \right]$
66. The sum to infinity of the given series $\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \frac{1}{4n^4} + \dots$ is [MP PET 1994]
 (a) $\log_e \left(\frac{n+1}{n} \right)$ (b) $\log_e \left(\frac{n}{n+1} \right)$ (c) $\log_e \left(\frac{n-1}{n} \right)$ (d) $\log_e \left(\frac{n}{n-1} \right)$
67. $e^{\left(x - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots \right)}$ is equal to [DCE 2001]
 (a) $\log x$ (b) $\log(x-1)$ (c) x (d) None of these
68. If the sum of $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \dots$ to n terms is S , then S is equal to [Kerala (Engg.) 2002]
 (a) $\frac{n(n+3)}{4}$ (b) $\frac{n(n+2)}{4}$ (c) $\frac{n(n+1)(n+2)}{6}$ (d) n^2
69. The sum of the series $2 \{ 7^{-1} + 3^{-1} \cdot 7^{-3} + 5^{-1} \cdot 7^{-5} + \dots \}$ is
 (a) $\log_e \left(\frac{4}{3} \right)$ (b) $\log_e \left(\frac{3}{4} \right)$ (c) $2 \log_e \left(\frac{3}{4} \right)$ (d) $2 \log_e \left(\frac{4}{3} \right)$
70. $\log_e \sqrt{\frac{1+x}{1-x}} =$
 (a) $x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty$ (b) $2 \left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right]$ (c) $2 \left[x^2 + \frac{x^4}{4} + \frac{x^6}{6} + \dots \infty \right]$ (d) None of these
71. If α, β are the roots of the equation $x^2 - px + q = 0$, then $\log_e(1 + px + qx^2) =$
 (a) $(\alpha + \beta)x - \frac{\alpha^2 + \beta^2}{2}x^2 + \frac{\alpha^3 + \beta^3}{3}x^3 - \dots \infty$ (b) $(\alpha + \beta)x - \frac{(\alpha + \beta)^2}{2}x^2 + \frac{(\alpha + \beta)^3}{3}x^3 - \dots \infty$
 (c) $(\alpha + \beta)x + \frac{\alpha^2 + \beta^2}{2}x^2 + \frac{\alpha^3 + \beta^3}{3}x^3 + \dots \infty$ (d) None of these
72. $\log_e(x+1) - \log_e(x-1) =$
 (a) $2 \left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right]$ (b) $\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right]$ (c) $2 \left[\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \dots \infty \right]$ (d) $\left[\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \dots \infty \right]$
73. $\log_e \left[(1+x)^{1+x} (1-x)^{1-x} \right] =$
 (a) $\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots \infty$

(b) $\frac{x^2}{1.2} + \frac{x^4}{3.4} + \frac{x^6}{5.6} + \dots \infty$

(c) $2 \left[\frac{x^2}{1.2} + \frac{x^4}{3.4} + \frac{x^6}{5.6} + \dots \infty \right]$

(d) None of these

74. If m, n are the roots of the equation $x^2 - x - 1 = 0$, then the value of

$$\frac{\left(1 + m \log_e 3 + \frac{(m \log_e 3)^2}{2!} + \dots \infty\right) \left(1 + n \log_e 3 + \frac{(n \log_e 3)^2}{2!} + \dots \infty\right)}{\left(1 + mn \log_e 3 + \frac{(mn \log_e 3)^2}{2!} + \dots \infty\right)}$$
 is

(a) 9

(b) 3

(c) 0

(d) 1

75. The value of $\log_e \left(1 + ax^2 + a^2 + \frac{a}{x^2}\right)$ is

(a) $a \left(x^2 - \frac{1}{x^2}\right) - \frac{a^2}{2} \left(x^4 - \frac{1}{x^4}\right) + \frac{a^3}{3} \left(x^6 - \frac{1}{x^6}\right) - \dots$

(b) $a \left(x^2 + \frac{1}{x^2}\right) - \frac{a^2}{2} \left(x^4 + \frac{1}{x^4}\right) + \frac{a^3}{3} \left(x^6 + \frac{1}{x^6}\right) - \dots$

(c) $a \left(x^2 + \frac{1}{x^2}\right) + \frac{a^2}{2} \left(x^4 + \frac{1}{x^4}\right) + \frac{a^3}{3} \left(x^6 + \frac{1}{x^6}\right) + \dots$

(d) $a \left(x^2 - \frac{1}{x^2}\right) + \frac{a^2}{2} \left(x^4 - \frac{1}{x^4}\right) + \frac{a^3}{3} \left(x^6 - \frac{1}{x^6}\right) + \dots$

76. If $\log_e \left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$, then relation between a and b will be

[UPSEAT 1999]

(a) $a = b$

(b) $a = 2b$

(c) $2a = b$

(d) $a = \frac{b}{3}$

77. The solutions of the equation $x^{\frac{1}{2}(\log_2 x - 2)} = 16$ are

[AMU 1999]

(a) $\pm 2\sqrt{2}$

(b) 4, -2

(c) $16, \frac{1}{4}$

(d) $4, \frac{1}{16}$

78. The solution of $\log_x (\log_2 (\log_7 x)) = 0$ is

[AMU 2002]

(a) 7^2

(b) π^2

(c) 2^2

(d) None of these

79. If $x = \log_b a, y = \log_c b, z = \log_a c$; then xyz is

[MP PET 2002; UPSEAT 2003]

(a) 1

(b) 0

(c) 3

(d) None of these

Advance Level

80. $\frac{x-1}{(x+1)} + \frac{1}{2} \cdot \frac{x^2-1}{(x+1)^2} + \frac{1}{3} \cdot \frac{x^3-1}{(x+1)^3} + \dots \infty =$

(a) $\log_e x$

(b) $\log_e (1+x)$

(c) $\log_e (1-x)$

(d) $\log_e \frac{x}{1+x}$

81. $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + \dots$ is equal to

[Karnataka CET 1997]

(a) $\log \frac{8}{e}$

(b) $\log \frac{e}{8}$

(c) $\log 8e$

(d) None of these

82. $\frac{1}{x+1} + \frac{1}{2(x+1)^2} + \frac{1}{3(x+1)^3} + \dots \infty =$

(a) $\log_e \left(1 + \frac{1}{x}\right)$

(b) $\log_e \left(1 - \frac{1}{x}\right)$

(c) $\log_e \left(\frac{x}{x+1}\right)$

(d) None of these

83.
$$\frac{(a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \dots \infty}{(b-1) - \frac{(b-1)^2}{2} + \frac{(b-1)^3}{3} - \dots \infty} =$$

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- (a) $\log_b a$ (b) $\log_a b$ (c) $\log_e a - \log_e b$ (d) $\log_e a + \log_e b$
84. $1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots \infty =$
- (a) $\log_e(2\sqrt{3})$ (b) $2\log_e 2$ (c) $\log_e 2$ (d) $\log_e\left(\frac{2}{\sqrt{3}}\right)$
85. $1 + \frac{2}{1.2.3} + \frac{2}{3.4.5} + \frac{2}{5.6.7} + \dots \infty =$ [Roorkee 1980]
- (a) $\log_e 2$ (b) $\log_e \sqrt{2}$ (c) $\log_e 4$ (d) None of these
86. $\frac{4}{1.3} - \frac{6}{2.4} + \frac{12}{5.7} - \frac{14}{6.8} + \dots \infty =$
- (a) $\log_e 3$ (b) $\log_e 2$ (c) $2\log_e 2$ (d) None of these
87. $\frac{m-n}{m+n} + \frac{1}{3}\left(\frac{m-n}{m+n}\right)^3 + \frac{1}{5}\left(\frac{m-n}{m+n}\right)^5 + \dots \infty =$ [CET 1996]
- (a) $\log_e\left(\frac{m}{n}\right)$ (b) $\log_e\left(\frac{n}{m}\right)$ (c) $\log_e\left(\frac{m-n}{m+n}\right)$ (d) $\frac{1}{2}\log_e\left(\frac{m}{n}\right)$
88. If $n = (1999)!$, then $\sum_{x=1}^{1999} \log_n x$ is equal to [AMU 2002]
- (a) 1 (b) 0 (c) $\sqrt[1999]{1999}$ (d) -1
89. If $\log(1-x+x^2) = a_1x + a_2x^2 + a_3x^3 + \dots$, then $a_3 + a_6 + a_9 + \dots$ is equal to [Kurukshetra CEET 2002]
- (a) $\log 2$ (b) $\frac{2}{3}\log 2$ (c) $\frac{1}{3}\log 2$ (d) $2\log 2$
90. The sum of $1 + \frac{2}{1.2.3} + \frac{2}{3.4.5} + \frac{2}{5.6.7} + \dots$ is [Roorkee 1980; MP PET 2002, 03]
- (a) $2\log_e 2$ (b) $\log_e 2$ (c) $3\log_e 3$ (d) $3\log_e 2$
91. The sum of the series $\frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{2}{3}\left(\frac{1}{5}\right)^3 + \frac{3}{4}\left(\frac{1}{5}\right)^4 + \dots$ is
- (a) $1/4 + \log(4/5)$ (b) $1/3 + \log(2/3)$ (c) $1/2 + \log(3/2)$ (d) None of these
92. The sum of the series $\frac{x}{1+x^2} + \frac{1}{3}\left(\frac{x}{1+x^2}\right)^3 + \frac{1}{5}\left(\frac{x}{1+x^2}\right)^5 + \dots$ is
- (a) $\frac{1}{2}\log(1+x+x^2)$ (b) $\frac{1}{2}\log\left(\frac{1+x^2+x}{1+x^2-x}\right)$ (c) $\log(1-x+x^2)$ (d) None of these
93. $\log_a x$ is defined for ($a > 0$) [Roorkee 1990]
- (a) All real x (b) All negative (-) real $x \neq 1$
(c) All positive (+) real $x \neq 0$ (d) $a \geq e$
94. If $7^{\log_7(x^2-4x+5)} = x-1$, then x can have the values [Roorkee 1990; DCE 2001]
- (a) (2, 3) (b) 7 (c) (-2, -3) (d) (2, -3)
95. $\log_e(1+x) = \sum_{i=1}^{\infty} \left[\frac{(-1)^{i+1} x^i}{i} \right]$ is defined for [Roorkee 1990]
- (a) $x \in (-1, 1)$ (b) Any positive (+) real x
(c) $x \in (-1, 1]$ (d) Any positive (+) real $x (x \neq 1)$
96. If $2^x \cdot 3^{x+4} = 7^x$, then $x =$ [MP PET 1991]
- (a) $\frac{4\log_e 3}{\log_e 7 - \log_e 6}$ (b) $\frac{4\log_e 3}{\log_e 6 - \log_e 7}$ (c) $\frac{2\log_e 4}{\log_e 7 + \log_e 6}$ (d) $\frac{2\log_e 4}{\log_e 7 + \log_e 6}$
97. If $x = 1 + \log_a(bc)$, $y = 1 + \log_b(ca)$ and $z = 1 + \log_c(ab)$, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$ [MP PET 1991]

- (a) 0 (b) 1 (c) 3 (d) xyz
98. The value of x obtained from equation $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$ will be [UPSEAT 1999]
 (a) 10 (b) 100 (c) 5 (d) 2
99. In the expansion of $2 \log_e x - \log_e(x+1) - \log_e(x-1)$, the coefficient of x^{-4} is
 (a) $\frac{1}{2}$ (b) -1 (c) 1 (d) None of these
100. If $|x| < 1$, then the coefficient of x^5 in the expansion of $(1-x) \cdot \log_e(1-x)$ is
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{20}$ (d) $\frac{1}{10}$
101. In the expansion of $\log_e \frac{1}{1-x-x^2+x^3}$, the coefficient of x is
 (a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$
102. If n is not a multiple of 3, then the coefficient of x^n in the expansion of $\log_e(1+x+x^2)$ is [Roorkee 1992, Kurukshetra CEE]
 (a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $-\frac{1}{n}$ (d) $-\frac{2}{n}$
103. If n is a multiple of 3, then the coefficient of x^n in the expansion of $\log_e(1+x+x^2)$ is [Roorkee 1994]
 (a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $-\frac{1}{n}$ (d) $-\frac{2}{n}$

Miscellaneous Problems

Basic Level

104. If $y = 2x^2 - 1$, then $\left[\frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \dots \right]$ is equal to
 (a) $\frac{1}{2} \left[\frac{1}{x^2} - \frac{1}{2x^4} + \frac{1}{3x^6} - \dots \right]$ (b) $\frac{1}{2} \left[\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \dots \right]$
 (c) $\frac{1}{2} \left[\frac{1}{x^2} + \frac{1}{3x^6} + \frac{1}{5x^{10}} + \dots \right]$ (d) $\frac{1}{2} \left[\frac{1}{x^2} - \frac{1}{3x^6} + \frac{1}{5x^{10}} - \dots \right]$
105. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty$, then $x =$ [MNR 1973]
 (a) $y - \frac{y^2}{2} + \frac{y^3}{3} - \dots \infty$, (b) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots \infty$ (c) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (d) None of these

Advance Level

106. If $y = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$, then $x =$
 (a) $\log_e(1-y)$ (b) $\frac{1}{\log_e(1-y)}$ (c) $\log_e \frac{1}{(1-y)}$ (d) $\log_e(1+y)$
107. If $b = a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \dots$, then $b + \frac{b^2}{2!} + \frac{b^3}{3!} + \frac{b^4}{4!} + \dots \infty =$
 (a) $\log_e a$ (b) $\log_e b$ (c) a (d) e^a
108. If $4 \left[x^2 + \frac{x^6}{3} + \frac{x^{10}}{5} + \dots \right] = y^2 + \frac{y^4}{2} + \frac{y^6}{3} + \dots$, then
 (a) $x^2 y = 2x - y$ (b) $x^2 y = 2x + y$ (c) $x = 2y^2 - 1$ (d) $x^2 y = 2x + y^2$

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109. If $t_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1, 2, 3, \dots$, then $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \dots + \frac{1}{t_{2003}} =$ **[EAMCET 2003]**

(a) $\frac{4006}{3006}$

(b) $\frac{4003}{3007}$

(c) $\frac{4006}{3008}$

(d) $\frac{4006}{3009}$



Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	c	c	b	b	c	b	d	b	b	a	b	b	b	c	b	d	b	d	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	c	b	a	d	b	d	b	c	c	b	b	c	a	c	c	b	a	c	d
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	a	c	b	c	d	a	b	b	a	a	d	a	a	b	b	c	b	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
b	d	c	b	a	a	d	a	a	a	a	c	c	a	b	a	c	a	a	a
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
a	a	a	a	c	b	d	a	b	a	a	b	c	a	c	a	b	a	a	c
101	102	103	104	105	106	107	108	109											
b	a	d	b	b	c	c	a	d											