OBJECTIVE - I

1. Let $\vec{A}$ be a unit vector along the axis of rotation of a purely rotating body and $\vec{B}$ be a unit vector along the velocity of a particle P of the body away from the axis. The value of $\vec{A}.\vec{B}$ is

(A) 1 (B) – 1 (C*) 0 (D) none of these

Sol. *C*

$\vec{A}$, unit vector along the radial direction + $\vec{B}$, unit vector along the tangential direction

angle between $\vec{A}$ & $\vec{B}$ is 90º.

So $\vec{A}\,.\,\vec{B}$ = AB cos q = AB cos 90º = 0

2. A body is uniformly rotating about an axis fixed in an inertial frame of reference. Let $\vec{A}$ be a unit vector along the axis of rotation and $\vec{B}$ be the unit vector along the resultant force on a particle P of the body away from the axis. The value of $\vec{A}.\vec{B}$ is -

(A) 1 (B) – 1 (C*) 0 (D) none of these

Sol. *C*

$\vec{A}$, unit vector along the radial direction + $\vec{B}$, unit vector along the away from the axis.

angle between $\vec{A}$ & $\vec{B}$ is 90º.

So $\vec{A}\,.\,\vec{B}$ = |A| |B| cos q = 0

3. A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin

(A) is zero (B*) remains constant

(C) goes on increasing (D) goes on decreasing

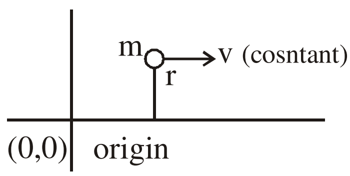
Sol. *B*

Angular momentum w.r.t. origin

$= m\left(\vec{r}\times\vec{v}\right)$

$= mvr \otimes$

= Constant



4. A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity w of the body are related as w = $\frac{v}{r}$. Thus

(A) w µ $\frac{1}{r}$ (B) w µ r (C) w = 0

(D*) w is independent of r.

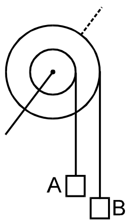
Sol. *'w' is independent of r but velocity is dependent upon 'r'.*

$w = \frac{v}{r}$

v = wr

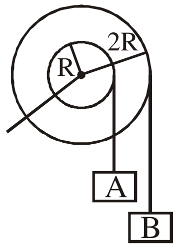
\ v µ r

5. Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distances travelled by A and B in the same time interval, then



(A) $x = 2y$ (B) $x = y$ (C*) $y = 2x$ (D) none of these

Sol. Angular velocity 'w' is same for both the wheel.



$v_A = wR$

$v_B = w2R$

$x = v_A t = wRt$ .... (1)

$y = v_B t = w(2R)t$ .... (2)

From equation (1) & (2) we get $\Rightarrow$

$y = 2x$

6. A body is rotating uniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is -

(A) vertical (B) horizontal and skew with the axis

(C*) horizontal and intersecting the axis (D) none of these

Sol. *C*

The resultant force on a particle is in the vertical direction not in horizontal or intersecting the axis. Because body is rotating uniformaly along the vertical axis in an inertial frame.

7. A body is rotating nonuniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is -

(A) vertical (B*) horizontal and skew with the axis

(C) horizontal and intersecting the axis (D) none of these

Sol. B

Body is rotating non uniformaly along the vertical axis is horizontal and skew with the axis.

8. Let $\vec{F}$ be a force acting on a particle having position vector $\vec{r}$. Let $\vec{\Gamma}$ be the torque of this force about the origin, then

(A*) $\vec{r}\,.\,\vec{\Gamma} = 0$ and $\vec{F}\,.\,\vec{\Gamma} = 0$ (B) $\vec{r}\,.\,\vec{\Gamma} = 0$ but $\vec{F}\,.\,\vec{\Gamma} \neq 0$

(C) $\vec{r}\,.\,\vec{\Gamma} \neq 0$ but $\vec{F}\,.\,\vec{\Gamma} \neq 0$ (D) $\vec{r}\,.\,\vec{\Gamma} \neq 0$ and $\vec{F}\,.\,\vec{\Gamma} \neq 0$

Sol. A

$$\vec{\Gamma} = \vec{r} \times \vec{F}$$

$$= r\,F \sin q$$

$\vec{F}$ is along the position vector $\vec{r}$ so angle between r & F is 0.

$$\vec{\Gamma} = rF \sin 0^\circ = 0$$

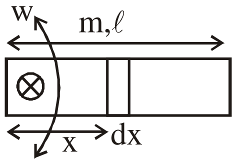
$\Rightarrow$ $\vec{r}.\vec{\Gamma} = 0$ and $\vec{F}.\vec{\Gamma} = 0$

9. One end of a uniform rod of mass m and length ℓ is clamped. The rod lies on a smooth horizontal surface and rotates on it about the clamped end at a uniform angular velocity w. The force exerted by the clamp on the rod has a horizontal component

(A) $mw^2\ell$ (B) zero (C) mg (D*) $\frac{1}{2}mw^2\ell$

Sol. D

$dm = \frac{m}{\ell}dx$



Centripetal force is Þ $\int_0^{\ell}\frac{m}{\ell}w^2x\,dx$

$= \frac{m}{\ell}w^2\frac{x^2}{2}\Big|_0^{\ell}$

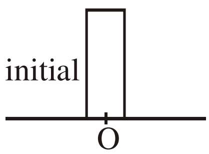
$= \frac{1}{2}mw^2l$

10. A uniform rod is kept vertically on a hroizontal smooth surface at a point O. If it is rotated slightly and released, it falls down in the horizontal surface. The lower end will remain

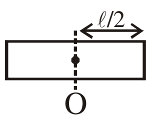
(A) at O (B) at a distance less than $\ell/2$ from O

(C*) at a distance $\ell/2$ from O (D) at a distance larger than $\ell/2$ from O.

Sol. C



Cantre of mass of the rod remain constant along the y-axis.



The lower end will remain at a distance l/2 from O.

11. A circular disc A of radius r is made from an iron plate of thickness t and another circular disc b of radius 4r is made from an iron plate of thickness t/4. The relation between the moments of inertia I_A and I_B is

(A) $I_A > I_B$ (B) $I_A = I_B$ (C*) $I_A < I_B$

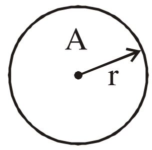
(D) depends on the actual values of t and r.

Sol. C

Thickness 't' {Q I fordisc is $\frac{mr^2}{2}$ }

$m_A = rpr^2t$

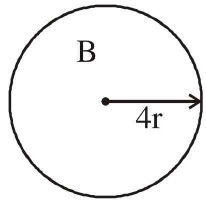
$I_A = \frac{m_A r^2}{2} = \frac{\rho\pi r^2 t.r^2}{2} = \frac{\rho\pi r^4 t}{2}$ .... (1)



Thickness 't/4'

$m_B = rp(4r)^2 t/4 = 4rpr^2 t$

$I_B = \frac{m_B (4r)^2}{2} = \frac{64\rho\pi r^4 t}{2}$ .... (2)



from (1) & (2) we get

$I_B > I_A$

12. Equal torques act on the disc A and B of the previous problem, initially both being at rest. At a later instant, the linear speeds of a point on the rim of A and another point on the rim of B are v_A and v_B respectively. We have

(A*) $v_A > v_B$ (B) $v_A = v_B$ (C) $v_A < v_B$

(D) the relation depends on the actual magnitude of the torques.

Sol. *A*

$t_A = t_B$ (\t = I µ)

$I_A \mu_A = I_B \mu_B$ I Þ moments of inertial

\ $I_B > I_A$ µ Þ angular acceleration

\ $v_A > v_B$ t Þ Torque

13. A closed cylindrical tube containing some water (not filling the entire tube) lies in a horizontal plane. If the tube is rotated about a perpendicular bisector, the moment of inertia of water about the axis

(A*) increases (B) decreases (C) remains constant

(D) increases if the rotation is clockwise and decreases if it is anticlockwise.

Sol. A

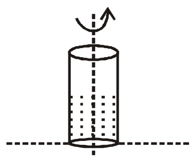
Moment of inertia

$I = mr^2$

distance of the particle of the water is increase.

$I = \mu r^2$

So I is increase.



14. The moment of inertia of a uniform semicircular wire of mass M and radius r about a line perpendicular to the plane of the wire through the centre is -

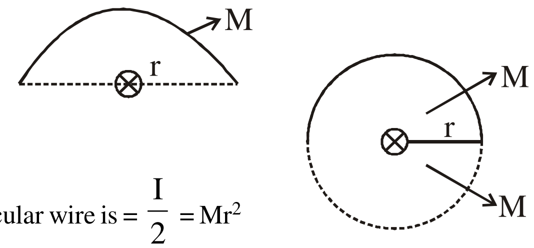
(A*) Mr^2 (B) $\frac{1}{2}Mr^2$ (C) $\frac{1}{4}Mr^2$ (D) $\frac{2}{5}Mr^2$.

Sol. *A*

Use the ymmetricily candition

$I = M_T r^2$

$I = 2Mr^2$



So the moment of inertia of uniform semicircular wire is = $\frac{I}{2} = Mr^2$

15. Let I_1 and I_2 be the moments of inertia of two bodies of indentical geometrical shape, the first made of aluminium and the second of iron.

(A*) $I_1 < I_2$ (B) $I_1 = I_2$ (C) $I_1 > I_2$

(D) relation between I_1 and I_2 depends on the actual shapes of the bodies

Sol. A

$I = mr^2$

density of Iron > density of aluminium

So mass if Iron > mass of aluminium

$I_2 > I_1$

16. A body having its centre of mass at the origin has three of its particles at (a, 0, 0), (0, a, 0), (0, 0, a). The moments of inertia of the body about the X and Y axes are 0.20 kg-m² each. The moment of inertia about the z-axis -

(A) is 0.20 kg - m² (B) is 0.40 kg-m² (C) is $0.20\sqrt{2}$ kg-m²

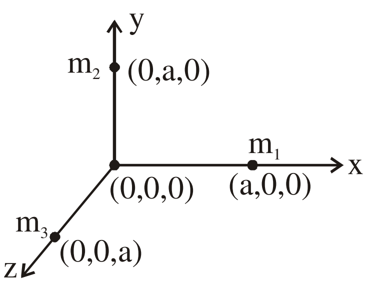
(D*) cannot be deduced with this information

Sol. *D*

$I_x = m_2a^2 + m_3a^2 = 0.20$ .... (1)

$I_y = m_1a^2 + m_3a^2 = 0.20$ .... (2)

$I_z = m_1a^2 + m_2a^2$ .... (3)



I_z cannot be deduced with this information or solving equation (1) & (2).

17. A cubical block of mass M and edge a slides down a rough inclined plane of inclination q with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude -

(A) zero (B) Mga (C) Mga sin q (D*) $\frac{1}{2}$ Mga sinq

Sol. N = Mg cos q

Block move with uniform velocity

f = Mg sin q

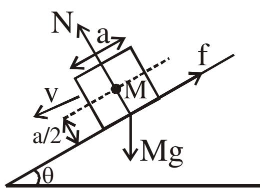
Net torque on the block is zero.

$\tau_{N^{\otimes}} + \tau_{f^{\odot}} = 0$

$t_N = t_f$

= Mg sin q . a/2

= 1/2 Mg a sin q



18. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed w. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become

(A) $\frac{\omega M}{M+m}$ (B*) $\frac{\omega M}{M+2m}$ (C) $\frac{\omega (M-2m)}{M+2m}$ (D) $\frac{\omega (M+2m)}{M}$

Sol. B

By angular momentum conservation

initial angular momentum = final angular momentum

$$Iw = I'w'$$

$$Mr^2w = (Mr^2 + 2mr^2)\, w'$$

$$w' = \frac{wM}{M+2m}$$

Here I is the moment of inertia of circular ring.

I' is the moment of inertia of system (circular ring + two particle)

Here moment of inertia of each particle is 'mr^2' about the cantre of the circular ring.

19. A person sitting firmly over a rotating stool has his arms stretched. If the folds his arms, his angular momentum about the axis of rotation

(A) increases (B) decreases (C*) remains unchanged (D) doubles

Sol. C

Angular momentum about the axis of rotation is remain unchanged.

$$I_1w_1 = I_2w_2$$

If the stretched his arms, I is increase, because of distance of some mass of body increase ($I = mr^2$). That causes angular velocity is decrease.

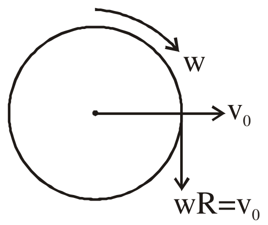
If he folds his arms, I isdecreases & angular velocity is increase.

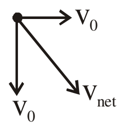
20. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(A) zero (B) v_0 (C*) $\sqrt{2}v_0$ (D) $2v_0$

Sol. *C*

The velocity diagram of the particle which is same level of centre of wheel is





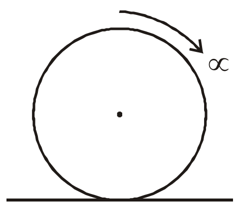
$v_{net} = \sqrt{2}\, v_0$

So speed of the particle is $\sqrt{2}\, v_0$.

21. A wheel of radius 20 cm is pushed to move it on a rough horizontal surface. It is found to move through a distance of 60 cm on the road during the time it completes one revolution about the centre. Assume that the linear and the angular accelerations are uniform. The frictional force acting on the wheel by the surface is -

(A*) along the velocity of the wheel (B) opposite to the velocity of the wheel

(C) perpendicular to the velocity of the wheel (D) zero



Sol. A

Causes of friction force wheel is move along the surface.

So we can say that frictional force acting on the wheel by the surface is along the velocity of the wheel.

22. The angular velocity of the engine (and hence of the wheel) of a scooter is proportional to the petrol input per second. The scooter is moving on a frictionless road with uniform velocity. If the petrol input is increased by 10%, the linear velocity of the scooter is increased by

(A) 50% (B) 10% (C) 20% (D*) 0%

Sol. D

w µ petrol input / second

"on a friction less road" Þ angular velocity of the engine w = 0, It move with uniform velocity.

The linear velocity of the scooter is remain same.

23. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of a smooth incline and released. Least time will be taken in reaching the bottom by -

(A) the solid sphere (B) the hollow sphere

(C) the disc (D*) all will take same time

Sol. *D*

A solid sphere, a hollow sphere and a disc is placed at the top of smooth incline. At friction less surface angular velocity of there is zero and acceleration at the incline plane is same equal to g sin q. So we can say that all will take same time to reaches the bottom of inclined plane.

24. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by

(A) the solid sphere (B) the hollow sphere

(C) the disc (D*) all will take same time.

Sol. D

Since linear acceleration is same for all (a = Mg sinq – m Mg cosq) as they have same mass 'M' and same 'm'

Hence, all will reach the bottom simultaneously.

Hence (D)

25. In the previous question, the smallest kinetic energy at the bottom of the incline will be achieved by

(A) the solid sphere (B*) the hollow sphere

(C) the disc (D) all will achieve same kinetic energy.

Sol. B

For all the bodies, torque is same .

Hence, angular momentum (L) is also same.

Now, $K.E. = \frac{1}{2}mv^2 + \frac{L^2}{2I}$

Linear velocity 'v' is same for all as same force acts on them.

Therefore more value of moment of inertia implies lesser kinetic energy.

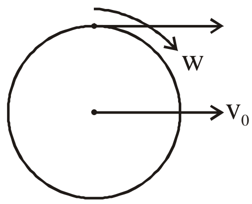
Among all, the hollow sphere has the maximum moment of inertia $I = \left(\frac{2}{3}MR^2\right)$.

Hence (B).

26. A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A man standing at a distance ℓ from the cylinder holds on end of the string and pulls the cylinder towards him (figure). There is no slipping anywhere. The length of the string passed through the hand of the man while the cylinder reaches his hands is

(A) ℓ (B*) 2ℓ (C) 3ℓ (D) 4ℓ

Sol. *B*



$v_0 + wR = 2v_0$

\ $1 = v_0t$

Length passes through the Hand = $2v_0t = 2l$

OBJECTIVE - II

1. The axis of rotation of a purely rotating body

(A) must pass through the centre of mass (B*) may pass through the centre of mass

(C) must pass through a particle of the body (D*) may pass through a particle of the body.

Sol. BD

The axis of rotation of a purely rotating body may pass through the centre of mass or may pass through a particle of the body.

2. Consider the following two equations

(a) $L = Iw$ (b) $\frac{dL}{dt} = \Gamma$

In non inertial frames

(A) both A and B are true (B*) A is true but B is false

(C) B is true but A is false (D) both A and B are false

Sol. *B*

Angular momentum $L = Iw$

or $\frac{d\vec{L}}{dt} = \vec{\Gamma}_{ext}$

where $\vec{\Gamma}_{ext}$ is the total torque onthe system due to all the external forces acting on the system.

3. A particle moves on a straight line with a uniform velocity. Its angular momentum

(A) is always zero

(B*) is zero about a point on the straight line

(C*) is not zero about a point away from the straight line

(D*) about any given point remains constant

Sol. *BCD*

Angular mometum $= m(\vec{r} \times \vec{v})$

about P is zero because $\vec{r} = 0$

about Q is non zero = mvl

4. If there is no external force acting on a nonrigid body, which of the following quantities must remain constant ?

(A*) angular momentum (B*) linear momentum

(C) kinetic energy (D) moment of inertia

Sol. *AB*

$\vec{\Gamma}_{ext} = \frac{d\vec{L}}{dt}$

$0 = \frac{d\vec{L}}{dt}$ {\Fext=0}

$\vec{L}$ (Angular momentum) is remain constant.

$F_{ext} = 0$,

$Fext = \frac{d\vec{P}}{dt} = 0$

$\vec{P}$ (Linear momentum) is remain constant.

5. Let I_A and I_B be moments of inertia of a body about two axes A and B respectively. The axis A passes through the centre of mass of the body but B does not.

(A) $I_A < I_B$

(B) $I_A < I_B$, the axes are parallel

(C*) if the axes are parallel, $I_A < I_B$

(D) if the axes are not parallel, I_A ³ I_B

Sol. *C*

By parallel axis theorem

$I_B = I_A + Id^2$

$I_B > I_A$

6. A sphere is rotating about a diameter.

(A) the particles on the surface of the sphere do not have any linear acceleration

(B*) the particles on the diameter mentioned above do not have any linear acceleration

(C) different particles on the surface have different angular speeds

(D) all the particles on the surface have same linear speed

Sol. B

If sphere is rotating about a diameter, the particle on the diameter mentioned above do not have any linear acceleration.

7. The density of a rod gradually decreases from one end to the other. It is pivoted at an end so that it can move about a vertical axis through the pivot. A horizontal force F is applied on the free end in a direction perpendicular to the rod. The quantities, that do not depend on which end of the rod is pivoted, are

(A) angular acceleration

(B) angular velocity when the rod completes on rotation

(C) angular momentum when the rod completes one rotation

(D*) torque of the applied force

Sol. D

Torque = $\vec{r} \times \vec{F}$ (Torque is depend on force & length of rod)

$= lF \odot$ (upwards direction)

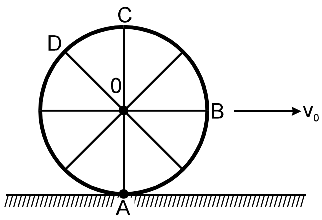
t = I µ

µ = t/I (µ ® angular acceleration)

If piroted end is change then the position of moment of inertia is shift along vertical axis.

angular momentum = Iw

8. Consider a wheel of a bicycle rolling on a level road at a linear speed v_0 (figure)



(A*) the speed of the particle A is zero

(B) the speed of B, C and D are all equal to v_0

(C*) the speed of C is $2v_0$

(D*) the speed of B is greater than the speed of O.

Sol. ACD

The speed of 'O' is v_0 pure rolling v_0 = wR

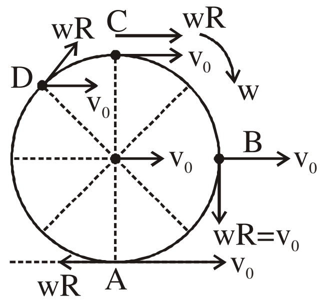
So speed of the particle A is zero.

The speed of C is v_0 + wR = 2 v_0 m/s

The speed of C is $\sqrt{(v_0)^2 + (wR)^2 + 2v_0(wR)\cos 45°}$

The speed of D is $\sqrt{2{v_0}^2 + 2{v_0}^2 \times \frac{1}{\sqrt{2}}}$

v_0 = $\sqrt{2+\sqrt{2}}$ m/s



9. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the sphere roll without slipping,

(A) the heavier sphere reaches the bottom first

(B) the bigger sphere reaches the bottom first

(C*) the two spheres reach the bottom together

(D) the information given is not sufficient to tell which sphere will reach the bottom first.

Sol. C

Acceleration of both sphere on the cline plane is

$a_{com} = \frac{g\sin\theta}{1 + I_{com}/MR^2}$

I_{com} for first solid sphere is = $\frac{2}{5}$ $M_1{R_1}^2$

So $a_{com} = \frac{g\sin\theta}{1+\frac{2/5M_1{R_1}^2}{M_1{R_1}^2}} = \frac{5}{7}$ g sin q

The acceleration of Both the sphere is same. So we can say that both sphere will reach to bottom together.

10. A hollow sphere and a solid sphere having same mass and same radii are rolled down a rough inclined plane.

(A) the hollow sphere reaches the bottom first

(B*) the solid sphere reaches the bottom with greater speed

(C) the solid sphere reaches the bottom with greater kinetic energy

(D) the two spheres will reach the bottom with same linear momentum

Sol. B

Acceleration on the inclined plane is

$a_{com} = \frac{g\sin\theta}{1+I_{com}MR^2}$

I_{com} for hollow shpere = $\frac{2}{3}$ MR^2

a_{com} of hollow sphere = $\frac{g\sin\theta}{1+2/3} = \frac{3}{5}$ gsinq

I_{com} for hollow shpere = $\frac{2}{5}$ MR^2

a_{com} of hollow sphere = $\frac{g\sin\theta}{1+2/5} = \frac{5}{7}$ gsinq

a_{com} of solid sphere > a_{com} of hollow sphere

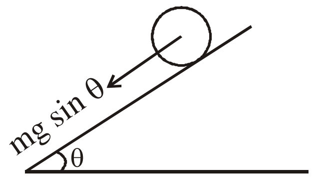
The solid sphere reaches the bottom with greater speed.

11. A sphere cannot roll on

(A) a smooth horizontal surface (B*) a smooth inclined surface

(C) a rough horizontal surface (D) a rough inclined surface

Sol. B



A sphere cannot roll on a smooth inclined surface.

12. In rear-wheel drive cars, the engine rotates the rear wheels and the front wheels rotate only because the car moves. If such a car acceleration on a horizontal road, the friction

(A*) on the rear wheels is in the forward direction

(B*) on the front wheels is in the backward direction

(C*) on the rear wheels has larger magnitude than the friction on the front wheels

(D) on the car is in the backward direction

Sol. ABC

Engin force apply on the rear wheels in back ward direction so friction force oppose it that causes friction force on the rear wheels is in the forward direction.

Friction force oppose the motion of the particle, Here front wheel freely rotated in forward direction so friction force on the front wheel is in the backward direction.

Due to friction force 'car' is move, so we can say that friction force on the front wheels has larger magnitude than the friction on the front wheels.

13. A sphere can roll on a surface inclined at an angle q if the friction coefficient is more than $\frac{2}{7}$ g tanq . Suppose the friction coefficient is $\frac{1}{7}$ g tan q. If a sphere is released from rest on the incline,

(A) it will stay at rest

(B) it will make pure translational motion

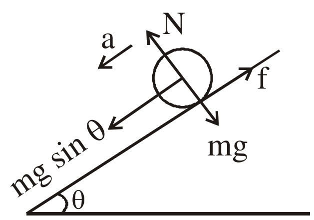
(C*) it will translate and rotate about the centre

(D) the angular momentum of the sphere about its centre will remain constant

Sol. C

Acceleration of the sphere down the plane is 'a'.

$f_r = I\mu$ $\quad$ (Q $\mu = \frac{a}{r}$)



$f_r = \frac{2}{5} mr^2 . \left(\frac{a}{r}\right)$

$f = \frac{2}{5} ma$ .... (1)

mg sinq – f = ma .... (2)

from (1) & (2)

$a = \frac{5}{7}$ gsin q

$f = \frac{2}{7}$ mgsin q .... (3)

here mg sin q > f .... (4)

The normal force is equal to mg cos q, as there is no acceleration perpendicular to the incline. The maximum friction that can act is, therefore mmg cosq, where m is the coefficient od static friction. Thus, for pure rolling

$$\text{mmg cos q} > \frac{2}{7}\ \text{mg sin q}$$

$$\text{m} > \frac{2}{7}\ \text{tan q} \qquad \text{.........(5)}$$

From equation (4) & (5) we conclude that shpere will translate and rotate about the centre.

14. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tries to

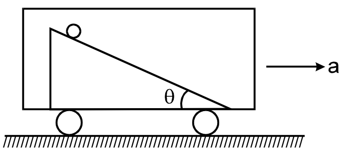
(A*) decrease the linear velocity (B*) increase the angular velocity

(C) increase the linear momentum (D) decrease the angular velocity

Sol. AB

The force of friction tries to decrease the linear velocity & increases the angular velocity.

15. Figure shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angel of incline q is related to the acceleration a of the car as a = g tanq. If the sphere is set in pure rolling on the incline



(A*) it will continue pure rolling (B) it will slip down the plane

(C) its linear velocity will increase (D) its linear velocity will decrease

Sol. A

a = g tan q (Given)

Component of pseudo force in inclined plane is = ma cos q

= mg tan q cos q

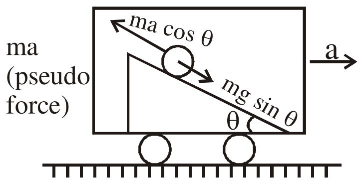
= mg sin q

Net force on the inclined plane direction is

= mg sin q – ma cos q

= mg sin q – mg sin q

= 0



So we can say that

If the sphere is set in pure rolling on the incline, it will continue pure rolling.