CUBES AND CUBE ROOTS

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CHAPTER

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CUBES

A cube is a solid figure which has all its sides equal. If side of a cube is 1 cm then 27 such cubes can make a big cube of side 3 cm.

So, no. 1, 8, 27, 64, …. are called perfect cube numbers.

Table–1

Number	***Cube***
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 = 125$
6	$6^3 = 216$
7	$7^3 = 343$
8	$8^3 = 512$
9	$9^3 = 729$
10	$10^3 = 1000$

There are only ten perfect cubes from 1 to 1000 and four from 1 to 100 .

Following are the cubes of the numbers from 11 to 20.

Table–2

Number	***Cube***
11	*1331*
12	*1728*
13	*2197*
14	*2744*
15	*3375*
16	*4096*
17	*4913*
18	*5832*
19	*6859*
20	*8000*

Results :

1. Cube of even number is also an even number.
2. Cube of an odd number is also an odd number.
3. Unit place of cube of a number whose unit digit is 2, 3, 7, 8 is 8, 7, 3, 2 respectively

SOME INTERESTING PATTERNS

1. Adding consecutive odd numbers :

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

Ex.1 How many consecutive odd numbers will be needed to obtain the sum as 10^3 ?

Sol. 10 (91, 93, 95, 97, 99, 101, 103, 105, 107, 109)

2. **Prime factors of perfect cube :**

Each prime number appears three or multiple of 3 times in its cube.

Eg. $8 = 2 \times 2 \times 2$

Eg. $64 = (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

Eg $125 = (5 \times 5 \times 5) = 5^3$ = perfect cube number

$\therefore$ a^3 is a perfect cube number.

Ex.2 Is 128 a perfect cube ?

Sol. $128 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times 2$

$= 2^7$

Θ power of 2 is not a multiple of 3.

$\therefore$ it is not a perfect cube.

Ex.3 Find the cubes of the following numbers:

(a) 2 (b) 3 (c) 7 (d) 0.9

(e) (–5) (f) – 0.1

Sol. (a) $2 \times 2 \times 2 = 8 \quad \Rightarrow 2^3 = 8$

(b) $3 \times 3 \times 3 = 27 \quad \Rightarrow 3^3 = 27$

(c) $7 \times 7 \times 7 = 343 \quad \Rightarrow 7^3 = 343$

(d) $0.9 \times 0.9 \times 0.9 = 0.729 \quad \Rightarrow (0.9)^3 = 0.729$

(e) $(-5) \times (-5) \times (-5) = -125 \Rightarrow (-5)^3 = -125$

(f) $(-0.1) \times (-0.1) \times (-0.1) = -0.001$

$\Rightarrow (-0.1)^3 = -0.001$

A natural number is said to be a perfect cube if it is the cube of another natural number.

We know that when odd number of negative factors are multiplied, the product is always negative, so cube can be negative also.

➤ CUBE ROOT

If $2^2 = 4$, then the square root of 4, i.e., $\sqrt{4} = 2$. Similarly, if $2^3 = 8$, then the cube root of 8 is 2. It is written as $\sqrt[3]{8} = 2$. If $3^3 = 27$, then the cube root of 27 is 3. Thus, $\sqrt[3]{27} = 3$.

Note that the symbol $\sqrt{\ }$ implied square root. For our convenience, we omit 2 from $\sqrt[2]{\ }$. But for a cube root, we should use the symbol $\sqrt[3]{\ }$, and it cannot be omitted also we can use $(\ )^{1/3}$ for cube root.

Prime Factorisation Method for Finding the Cube Root

Let us take some examples here

Ex.4 Find the cube root of 1728.

Sol. $\sqrt[3]{1728} = (1728)^{1/3}$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Step : 1 First factorise the given number into its prime factors

$\sqrt[3]{1728} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

Step : 2 Then group the factors in 3s.

$\sqrt[3]{1728} = \sqrt[3]{2^3 \times 2^3 \times 3^3}$

Step : 3 Take one prime factor from each group.

$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

$\therefore \sqrt[3]{1728} = 12$

Ex.5 Find the value of $\sqrt[3]{216}$

Sol. $\sqrt[3]{216} = (216)^{1/3}$

2	216
2	108
2	54
3	27
3	9
	3

Step-1: Factorise the given number into its prime factors.

$\sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

Step-2: Group the factors in 3s.

$\sqrt[3]{216} = \sqrt[3]{2^3 \times 3^3}$

Step-3 : Take one prime factor from each group.

$\sqrt[3]{216} = 2 \times 3 = 6$

$\therefore \sqrt[3]{216} = 2 \times 3 = 6$

Observe $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125, \ldots$

All cubes of even numbers are even and cubes of odd numbers are odd. Cubes of negative numbers are negative.

Ex.6 Find the cube root of 46656.

Sol. (i) The unit digit of the number is 6, so the cube root will also have 6 in the unit digit.

(ii) Separate the number as 46 656. 46 is greater than 3^3 but less than 4^3, so the tens digit is 3.

(iii) The required number is 36.

Ex.7 Find the cube root of 195112.

Sol. (i) Unit digit of the given number is 2, so the required number has unit digit 8.

(ii) 195 112, so $195 > 5^3$ but $< 6^3$.

So, required number is 58.

Note : Above method works for perfect cube numbers called cube root by approximation.

➢ DIGITS IN CUBE ROOT OF A NUMBER

Use dots on digit of given number starting from unit digit & leaving 2 next digits, now digits in cube root is same as the sum of dots.

Ex.8 Find the digits in cube root of the following numbers.

(i) 1728 (ii) 175616

(iii) 8 (iv) 97336

(v) 9261 (vi) 68921000

Sol. $\dot{1}72\dot{8}$ two dots $\therefore$ 2 digits in cube root

$17\dot{5}61\dot{6}$ two dots $\therefore$ 2 digits in cube root

$\dot{8}$ Only one dot $\therefore$ 1 digit in cube root

$\dot{9}26\dot{1}$ Two dots $\therefore$ 2 digit in cube root

$6\dot{8}92\dot{1}00\dot{0}$ Three dots $\therefore$ 3 digit in cube root

➢ SUM OF NUMBERS

The sum of first '*n*' natural numbers.

$$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$$

Ex.9 Find sum of first 6 natural numbers.

Sol. $n = 6 \therefore$ Sum $= \frac{6(6+1)}{2} = 3 \times 7 = 21$

Ex.10 Find sum of 10 + 11 + ... + 20.

Sol. Θ Sum of 1 to 20 is

$$\frac{20(20+1)}{2} = 10 \times 21 = 210$$

and sum of 1 to 9 is $\frac{9(9+1)}{2} = \frac{9 \times 10}{2} = 45$

$\therefore 10 + 11 + \ldots\ldots + 20 = 210 - 45 = 165$

The sum of Square of first '*n*' natural numbers.

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex.11 Find sum of squares of first five natural numbers.

Sol. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 \therefore n = 5$

$$\therefore \text{sum} = \frac{5(5+1)(10+1)}{6} = 55$$

The sum of cube of first 'n' natural numbers.

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots\ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Ex.12 Find sum of cube of first five natural numbers.

Sol. $1^3 + 2^3 + \ldots\ldots\ldots + 5^3 \quad \therefore n = 5$

$$\therefore \text{sum} = \left[\frac{5(5+1)}{2}\right]^2 = (5 \times 3)^2 = 225$$

EXERCISE

Q.1 Find the number of digits in the cube root if number of digits in perfect cube numbers as follows.
(i) 6 (ii) 5 (iii) 4 (iv) 3
(v) 2 (vi) 1 (vii) 7

Q.2 Find the value of $\sqrt{117+\sqrt[3]{19683}}$.

Q.3 Which of the following are perfect cube ?
(i) 10 (ii) 100 (iii) 1000 (iv) 10^4
(v) 10^5 (vi) 10^6

Q.4 Find the value of $\frac{(2)^3+(10)^3}{\sqrt{1016064}}$

Q.5 Find the sum of cubes of first 10 natural numbers.

Q.6 Find the value of
$(1^3 + 2^3 + 3^3 + 4^4 + ...+ 15^3) – (1^2 + 2^2 + 3^2 + ...+ 10^2)$

Q.7 Find the cube root of the following numbers by inspection.
(i) 12167 (ii) 46.656 (iii) 6859 (iv) 912673
(v) 29791

Q.8 Find cube root of $[\,5\sqrt{100} + \sqrt{49} + (79507)^{1/3}]$.

Q.9 Find cube root by prime factorisation
(i) 4913 (ii) 13824 (iii) 175616 (iv) 456533

Q.10 Find the least number by which when multiply the following numbers, such that the number become perfect cube.
(i) 2048 (ii) 1029 (iii) 45 (iv) 5832

Q.11 Find the least number by which when divide the following numbers. The number become perfect cube also find cube root of new number
(i) 4394 (ii) 8575 (iii) 7986 (iv) 28672

ANSWER KEY

1. (i) 2 ; (ii) 2 ; (iii) 2 ; (iv) 1 ; (v) 1 ; (vi) 1; (vii) 3
2. $\sqrt{117+27} = 12$
3. (iii), (vi)
4. 1
5. 3025
6. 14015
7. (i) 23 ; (ii) 3.6; (iii) 19; (iv) 97; (v) 31
8. 10
9. (i) 17;(ii)24; (iii) 56; (iv) 77
10. (i) 2; (ii) 9; (iii) 75; (iv) 1
11. (i) 2, 13; (ii) 25, 7; (iii) 6, 11; (iv) 7, 16