

Binomial Theorem

Short Answer Type Questions

Q. 1 Find the term independent of x , where $x \neq 0$,

in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$.

Thinking Process

The general term in the expansion of $(x-a)^n$ i.e., $T_{r+1} = {}^nC_r(x)^{n-r}(-a)^r$. For the term independent of x , put $n-r=0$, then we get the value of r .

Sol. Given expansion is $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$.

Let T_{r+1} term is the general term.

$$\begin{aligned} \text{Then, } T_{r+1} &= {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r \\ &= {}^{15}C_r 3^{15-r} x^{30-2r} 2^{r-15} (-1)^r \cdot 3^{-r} \cdot x^{-r} \\ &= {}^{15}C_r (-1)^r 3^{15-2r} 2^{r-15} x^{30-3r} \end{aligned}$$

For independent of x ,

$$30 - 3r = 0$$

$$3r = 30 \Rightarrow r = 10$$

$\therefore T_{r+1} = T_{10+1} = 11\text{th term is independent of } x.$

$$\therefore T_{10+1} = {}^{15}C_{10} (-1)^{10} 3^{15-20} 2^{10-15}$$

$$= {}^{15}C_{10} 3^{-5} 2^{-5}$$

$$= {}^{15}C_{10} (6)^{-5}$$

$$= {}^{15}C_{10} \left(\frac{1}{6}\right)^5$$

Q. 2 If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k .

Sol. Given expansion is $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$.

Let T_{r+1} is the general term.

Then,

$$\begin{aligned} T_{r+1} &= {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r \\ &= {}^{10}C_r (x)^{\frac{1}{2}(10-r)} (-k)^r \cdot x^{-2r} \\ &= {}^{10}C_r x^{5-\frac{r}{2}} (-k)^r \cdot x^{-2r} \\ &= {}^{10}C_r x^{5-\frac{r}{2}-2r} (-k)^r \\ &= {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r \end{aligned}$$

For free from x , $\frac{10-5r}{2} = 0$

$\Rightarrow 10-5r = 0 \Rightarrow r = 2$

Since, $T_{2+1} = T_3$ is free from x .

$\therefore T_{2+1} = {}^{10}C_2 (-k)^2 = 405$

$\Rightarrow \frac{10 \times 9 \times 8!}{2! \times 8!} (-k)^2 = 405$

$\Rightarrow 45k^2 = 405 \Rightarrow k^2 = \frac{405}{45} = 9$

$\therefore k = \pm 3$

Q. 3 Find the coefficient of x in the expansion of $(1-3x+7x^2)(1-x)^{16}$.

Sol. Given, expansion = $(1-3x+7x^2)(1-x)^{16}$.

$$= (1-3x+7x^2)({}^{16}C_0 1^{16} - {}^{16}C_1 1^{15} x^1 + {}^{16}C_2 1^{14} x^2 + \dots + {}^{16}C_{16} x^{16})$$

$$= (1-3x+7x^2)(1-16x+120x^2+\dots)$$

$\therefore$ Coefficient of $x = -3 - 16 = -19$

Q. 4 Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$.

Thinking Process

The general term in the expansion of $(x-a)^n$ i.e., $T_{r+1} = {}^nC_r (x)^{n-r} (-a)^r$.

Sol. Given expansion is $\left(3x - \frac{2}{x^2}\right)^{15}$.

Let T_{r+1} is the general term.

$$\begin{aligned} \therefore T_{r+1} &= {}^{15}C_r (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r = {}^{15}C_r (3x)^{15-r} (-2)^r x^{-2r} \\ &= {}^{15}C_r 3^{15-r} x^{15-3r} (-2)^r \end{aligned}$$

For independent of x , $15-3r = 0 \Rightarrow r = 5$

Since, $T_{5+1} = T_6$ is independent of x .

$$\begin{aligned}\therefore T_{5+1} &= {}^{15}C_5 \cdot 3^{15-5}(-2)^5 \\ &= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10!}{5 \times 4 \times 3 \times 2 \times 1 \times 10!} \cdot 3^{10} \cdot 2^5 \\ &= -3003 \cdot 3^{10} \cdot 2^5\end{aligned}$$

Q. 5 Find the middle term (terms) in the expansion of

(i) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$

(ii) $\left(3x - \frac{x^3}{6}\right)^9$

Thinking Process

In the expansion of $(a+b)^n$, if n is even, then this expansion has only one middle term

i.e., $\left(\frac{n}{2} + 1\right)$ th term is the middle term and if n is odd, then this expansion has two middle

terms i.e., $\left(\frac{n+1}{2}\right)$ th and $\left(\frac{n+1}{2} + 1\right)$ th are two middle terms.

Sol. (i) Given expansion is $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$.

Here, the power of Binomial i.e., $n = 10$ is even.

Since, it has one middle term $\left(\frac{10}{2} + 1\right)$ th term i.e., 6th term.

$$\begin{aligned}\therefore T_6 = T_{5+1} &= {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(-\frac{a}{x}\right)^5 \\ &= -{}^{10}C_5 \left(\frac{x}{a}\right)^5 \left(\frac{a}{x}\right)^5 \\ &= -\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \left(\frac{x}{a}\right)^5 \left(\frac{a}{x}\right)^{-5} \\ &= -9 \times 4 \times 7 = -252\end{aligned}$$

(ii) Given expansion is $\left(3x - \frac{x^3}{6}\right)^9$.

Here, $n = 9$

Since, the Binomial expansion has two middle terms i.e., $\left(\frac{9+1}{2}\right)$ th and $\left(\frac{9+1}{2} + 1\right)$ th [odd]

i.e., 5th term and 6th term.

$$\begin{aligned}\therefore T_5 = T_{(4+1)} &= {}^9C_4 (3x)^{9-4} \left(-\frac{x^3}{6}\right)^4 \\ &= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!} 3^5 x^5 x^{12} 6^{-4} \\ &= \frac{7 \times 6 \times 3 \times 3^1}{2^4} x^{17} = \frac{189}{8} x^{17}\end{aligned}$$

$$\begin{aligned}
 \therefore T_6 = T_{5+1} &= {}^9C_5(3x)^{9-5}\left(-\frac{x^3}{6}\right)^5 \\
 &= -\frac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2 \times 1} \cdot 3^4 \cdot x^4 \cdot x^{15} \cdot 6^{-5} \\
 &= \frac{-21 \times 6}{3 \times 2^5} x^{19} = \frac{-21}{16} x^{19}
 \end{aligned}$$

Q. 6 Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$.

Sol. Given expansion is $(x - x^2)^{10}$.

Let the term T_{r+1} is the general term.

$$\begin{aligned}
 \therefore T_{r+1} &= {}^{10}C_r x^{10-r} (-x^2)^r \\
 &= (-1)^r \cdot {}^{10}C_r \cdot x^{10-r} \cdot x^{2r} \\
 &= (-1)^r {}^{10}C_r x^{10+r}
 \end{aligned}$$

For the coefficient of x^{15} ,

$$\begin{aligned}
 10 + r &= 15 \Rightarrow r = 5 \\
 T_{5+1} &= (-1)^5 {}^{10}C_5 x^{15}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Coefficient of } x^{15} &= -1 \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times 5!} \\
 &= -3 \times 2 \times 7 \times 6 = -252
 \end{aligned}$$

Q. 7 Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Thinking Process

In this type of questions, first of all find the general terms, in the expansion $(x-y)^n$ using the formula $T_{r+1} = {}^nC_r x^{n-r} (-y)^r$ and then put $n-r$ equal to the required power of x of which coefficient is to be find out.

Sol. Given expansion is $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Let the term T_{r+1} contains the coefficient of $\frac{1}{x^{17}}$ i.e., x^{-17} .

$$\begin{aligned}
 \therefore T_{r+1} &= {}^{15}C_r (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r \\
 &= {}^{15}C_r x^{60-4r} (-1)^r x^{-3r} \\
 &= {}^{15}C_r x^{60-7r} (-1)^r
 \end{aligned}$$

For the coefficient x^{-17} ,

$$60 - 7r = -17$$

$$\Rightarrow 7r = 77 \Rightarrow r = 11$$

$$\Rightarrow T_{11+1} = {}^{15}C_{11} x^{60-77} (-1)^{11}$$

$$\begin{aligned}
 \therefore \text{Coefficient of } x^{-17} &= \frac{-15 \times 14 \times 13 \times 12 \times 11!}{11! \times 4 \times 3 \times 2 \times 1} \\
 &= -15 \times 7 \times 13 = -1365
 \end{aligned}$$

Q. 8 Find the sixth term of the expansion $(y^{1/2} + x^{1/3})^n$, if the Binomial coefficient of the third term from the end is 45.

Sol. Given expansion is $(y^{1/2} + x^{1/3})^n$.

The sixth term of this expansion is

$$T_6 = T_{5+1} = {}^nC_5 (y^{1/2})^{n-5} (x^{1/3})^5 \quad \dots(i)$$

Now, given that the Binomial coefficient of the third term from the end is 45.

We know that, Binomial coefficient of third term from the end = Binomial coefficient of third term from the beginning = nC_2

$$\therefore {}^nC_2 = 45$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2!(n-2)!} = 45$$

$$\Rightarrow n(n-1) = 90$$

$$\Rightarrow n^2 - n - 90 = 0$$

$$\Rightarrow n^2 - 10n + 9n - 90 = 0$$

$$\Rightarrow n(n-10) + 9(n-10) = 0$$

$$\Rightarrow (n-10)(n+9) = 0$$

$$\Rightarrow (n+9) = 0 \text{ or } (n-10) = 0$$

$$\therefore n = 10$$

$$[\because n \neq -9]$$

From Eq. (i),

$$T_6 = {}^{10}C_5 y^{5/2} x^{5/3} = 252 y^{5/2} \cdot x^{5/3}$$

Q. 9 Find the value of r , if the coefficients of $(2r+4)$ th and $(r-2)$ th terms in the expansion of $(1+x)^{18}$ are equal.

Thinking Process

Coefficient of $(r+1)$ th term in the expansion of $(1+x)^n$ is nC_r . Use this formula to solve the above problem.

Sol. Given expansion is $(1+x)^{18}$.

Now, $(2r+4)$ th term i.e., T_{2r+3+1} .

$$\begin{aligned} \therefore T_{2r+3+1} &= {}^{18}C_{2r+3} (1)^{18-2r-3} (x)^{2r+3} \\ &= {}^{18}C_{2r+3} x^{2r+3} \end{aligned}$$

Now, $(r-2)$ th term i.e., T_{r-3+1} .

$$\therefore T_{r-3+1} = {}^{18}C_{r-3} x^{r-3}$$

$$\text{As, } {}^{18}C_{2r+3} = {}^{18}C_{r-3} \quad [\because {}^nC_x = {}^nC_y \Rightarrow x+y=n]$$

$$\Rightarrow 2r+3+r-3=18$$

$$\Rightarrow 3r=18$$

$$\therefore r=6$$

Q. 10 If the coefficient of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in AP, then show that $2n^2 - 9n + 7 = 0$.

Thinking Process

In the expansion of $(x+y)^n$, the coefficient of $(r+1)$ th term is nC_r . Use this formula to get the required coefficient. If a, b and c are in AP, then $2b = a + c$.

Sol. Given expansion is $(1 + x)^{2n}$.

Now, coefficient of 2nd term = ${}^{2n}C_1$

Coefficient of 3rd term = ${}^{2n}C_2$

Coefficient of 4th term = ${}^{2n}C_3$

Given that, ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in AP.

Then, $2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$$\Rightarrow 2 \left[\frac{2n(2n-1)(2n-2)!}{2 \times 1 \times (2n-2)!} \right] = \frac{2n(2n-1)!}{(2n-1)!} + \frac{2n(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!}$$

$$\Rightarrow n(2n-1) = n + \frac{n(2n-1)(2n-2)}{6}$$

$$\Rightarrow n(12n-6) = n(6+4n^2-4n-2n+2)$$

$$\Rightarrow 12n-6 = (4n^2-6n+8)$$

$$\Rightarrow 6(2n-1) = 2(2n^2-3n+4)$$

$$\Rightarrow 3(2n-1) = 2n^2-3n+4$$

$$\Rightarrow 2n^2-3n+4-6n+3=0$$

$$\Rightarrow 2n^2-9n+7=0$$

Q. 11 Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.

Sol. Given, expansion = $(1 + x + x^2 + x^3)^{11} = [(1 + x) + x^2(1 + x)]^{11}$

$$= [(1 + x)(1 + x^2)]^{11} = (1 + x)^{11} \cdot (1 + x^2)^{11}$$

Now, above expansion becomes

$$= ({}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + {}^{11}C_3x^3 + {}^{11}C_4x^4 + \dots)({}^{11}C_0 + {}^{11}C_1x^2 + {}^{11}C_2x^4 + \dots)$$

$$= (1 + 11x + 55x^2 + 165x^3 + 330x^4 + \dots)(1 + 11x^2 + 55x^4 + \dots)$$

$$\therefore \text{Coefficient of } x^4 = 55 + 605 + 330 = 990$$

Long Answer Type Questions

Q. 12 If p is a real number and the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, then find the value of p .

Sol. Given expansion is $\left(\frac{p}{2} + 2\right)^8$.

Here, $n = 8$

[even]

Since, this Binomial expansion has only one middle term i.e., $\left(\frac{8}{2} + 1\right)$ th = 5th term

$$T_5 = T_{4+1} = {}^8C_4 \left(\frac{p}{2}\right)^{8-4} \cdot 2^4$$

$$\Rightarrow 1120 = {}^8C_4 p^4 \cdot 2^{-4} 2^4$$

$$\Rightarrow 1120 = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1} p^4$$

$$\begin{aligned} \Rightarrow 1120 &= 7 \times 2 \times 5 \times p^4 \\ \Rightarrow p^4 &= \frac{1120}{70} = 16 \Rightarrow p^4 = 2^4 \\ \Rightarrow p^2 &= 4 \Rightarrow p = \pm 2 \end{aligned}$$

Q. 13 Show that the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is

$$\frac{1 \times 3 \times 5 \times \dots \times (2n-1)}{n!} \times (-2)^n.$$

Sol. Given, expansion is $\left(x - \frac{1}{x}\right)^{2n}$. This Binomial expansion has even power. So, this has one middle term.

i.e., $\left(\frac{2n}{2} + 1\right)$ th term = $(n+1)$ th term

$$\begin{aligned} T_{n+1} &= {}^{2n}C_n (x)^{2n-n} \left(-\frac{1}{x}\right)^n = {}^{2n}C_n x^n (-1)^n x^{-n} \\ &= {}^{2n}C_n (-1)^n = (-1)^n \frac{(2n)!}{n!n!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \dots (2n-1)(2n)}{n!n!} (-1)^n \\ &= \frac{1 \cdot 3 \cdot 5 \dots (2n-1) \cdot 2 \cdot 4 \cdot 6 \dots (2n)}{12 \cdot 3 \cdot \dots n(n!)} (-1)^n \\ &= \frac{1 \cdot 3 \cdot 5 \dots (2n-1) \cdot 2^n (1 \cdot 2 \cdot 3 \dots n)}{(1 \cdot 2 \cdot 3 \dots n)(n!)} (-1)^n \\ &= \frac{[1 \cdot 3 \cdot 5 \dots (2n-1)]}{n!} (-2)^n \end{aligned}$$

Hence proved.

Q. 14 Find n in the Binomial $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7th term from the beginning to the 7th term from the end is $\frac{1}{6}$.

Sol. Here, the Binomial expansion is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$.

$$\text{Now, 7th term from beginning } T_7 = T_{6+1} = {}^nC_6 (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6 \quad \dots(i)$$

and 7th term from end i.e., T_7 from the beginning of $\left(\frac{1}{\sqrt[3]{3}} + \sqrt[3]{2}\right)^n$

$$\text{i.e., } T_7 = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} (\sqrt[3]{2})^6 \quad \dots(ii)$$

$$\text{Given that, } \frac{{}^nC_6 (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6}{{}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} (\sqrt[3]{2})^6} = \frac{1}{6} \Rightarrow \frac{2^{\frac{n-6}{3}} \cdot 3^{-6/3}}{3^{-\left(\frac{n-6}{3}\right)} \cdot 2^{6/3}} = \frac{1}{6}$$

$$\Rightarrow \left(2^{\frac{n-6}{3}} \cdot 2^{\frac{-6}{3}}\right) \left(3^{\frac{-6}{3}} \cdot 3^{\frac{(n-6)}{3}}\right) = 6^{-1}$$

$$\Rightarrow \left(2^{\frac{n-6}{3}-\frac{6}{3}}\right) \cdot \left(3^{\frac{n-6}{3}-\frac{6}{3}}\right) = 6^{-1} \Rightarrow (2 \cdot 3)^{\frac{n}{3}-4} = 6^{-1}$$

$$\Rightarrow \frac{n}{3} - 4 = -1 \Rightarrow \frac{n}{3} = 3$$

$$\therefore n = 9$$

Q. 15 In the expansion of $(x+a)^n$, if the sum of odd terms is denoted by O and the sum of even term by E . Then, prove that

(i) $O^2 - E^2 = (x^2 - a^2)^n$.

(ii) $4OE = (x+a)^{2n} - (x-a)^{2n}$.

Sol. (i) Given expansion is $(x+a)^n$.

$$\therefore (x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + {}^nC_3 x^{n-3} a^3 + \dots + {}^nC_n a^n$$

Now, sum of odd terms

i.e., $O = {}^nC_0 x^n + {}^nC_2 x^{n-2} a^2 + \dots$

and sum of even terms

i.e., $E = {}^nC_1 x^{n-1} a + {}^nC_3 x^{n-3} a^3 + \dots$

$$\therefore (x+a)^n = O + E \quad \dots(i)$$

Similarly, $(x-a)^n = O - E \quad \dots(ii)$

$$\therefore (O+E)(O-E) = (x+a)^n (x-a)^n \quad [\text{on multiplying Eqs. (i) and (ii)}]$$

$$\Rightarrow O^2 - E^2 = (x^2 - a^2)^n$$

(ii) $4OE = (O+E)^2 - (O-E)^2 = [(x+a)^n]^2 - [(x-a)^n]^2 \quad [\text{from Eqs. (i) and (ii)}]$
 $= (x+a)^{2n} - (x-a)^{2n}$

Hence proved.

Q. 16 If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, then prove that its coefficient is $\frac{2n!}{3! \frac{(4n-p)!}{3!} \frac{(2n+p)!}{3!}}$.

Sol. Given expansion is $\left(x^2 + \frac{1}{x}\right)^{2n}$.

Let x^p occur in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$.

$$T_{r+1} = {}^{2n}C_r (x^2)^{2n-r} \left(\frac{1}{x}\right)^r$$

$$= {}^{2n}C_r x^{4n-2r} x^{-r} = {}^{2n}C_r x^{4n-3r}$$

Let $4n - 3r = p$

$$\Rightarrow 3r = 4n - p \Rightarrow r = \frac{4n-p}{3}$$

$$\therefore \text{Coefficient of } x^p = {}^{2n}C_r = \frac{(2n)!}{r! (2n-r)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(2n - \frac{4n-p}{3}\right)!}$$

$$= \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(\frac{6n-4n+p}{3}\right)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \left(\frac{2n+p}{3}\right)!}$$

Q. 17 Find the term independent of x in the expansion of

$$(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9.$$

Sol. Given expansion is $(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9$.

Now, consider $\left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9$

$$\begin{aligned} T_{r+1} &= {}^9C_r \left(\frac{3}{2}x^2 \right)^{9-r} \left(-\frac{1}{3x} \right)^r \\ &= {}^9C_r \left(\frac{3}{2} \right)^{9-r} x^{18-2r} \left(-\frac{1}{3} \right)^r x^{-r} = {}^9C_r \left(\frac{3}{2} \right)^{9-r} \left(-\frac{1}{3} \right)^r x^{18-3r} \end{aligned}$$

Hence, the general term in the expansion of $(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9$

$$= {}^9C_r \left(\frac{3}{2} \right)^{9-r} \left(-\frac{1}{3} \right)^r x^{18-3r} + {}^9C_r \left(\frac{3}{2} \right)^{9-r} \left(-\frac{1}{3} \right)^r x^{19-3r} + 2 \cdot {}^9C_r \left(\frac{3}{2} \right)^{9-r} \left(-\frac{1}{3} \right)^r x^{21-3r}$$

For term independent of x , putting $18 - 3r = 0$, $19 - 3r = 0$ and $21 - 3r = 0$, we get

$$r = 6, r = 19/3, r = 7$$

Since, the possible value of r are 6 and 7.

Hence, second term is not independent of x .

$\therefore$ The term independent of x is ${}^9C_6 \left(\frac{3}{2} \right)^{9-6} \left(-\frac{1}{3} \right)^6 + 2 \cdot {}^9C_7 \left(\frac{3}{2} \right)^{9-7} \left(-\frac{1}{3} \right)^7$

$$\begin{aligned} &= \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2} \cdot \frac{3^3}{2^3} \cdot \frac{1}{3^6} - 2 \cdot \frac{9 \times 8 \times 7!}{7! \times 2 \times 1} \cdot \frac{3^2}{2^2} \cdot \frac{1}{3^7} \\ &= \frac{84}{8} \cdot \frac{1}{3^3} - \frac{36}{4} \cdot \frac{2}{3^5} = \frac{7}{18} - \frac{2}{27} = \frac{21-4}{54} = \frac{17}{54} \end{aligned}$$

Objective Type Questions

Q. 18 The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

(a) 50

(b) 202

(c) 51

(d) None of these

Sol. (c) Here, $(x + a)^{100} + (x - a)^{100}$

Total number of terms is 102 in the expansion of $(x + a)^{100} + (x - a)^{100}$

50 terms of $(x + a)^{100}$ cancel out 50 terms of $(x - a)^{100}$. 51 terms of $(x + a)^{100}$ get added to the 51 terms of $(x - a)^{100}$.

Alternate Method

$$\begin{aligned} (x + a)^{100} + (x - a)^{100} &= {}^{100}C_0 x^{100} + {}^{100}C_1 x^{99}a + \dots + {}^{100}C_{100} a^{100} \\ &\quad + {}^{100}C_0 x^{100} - {}^{100}C_1 x^{99}a + \dots + {}^{100}C_{100} a^{100} \\ &= 2 \left[\underbrace{{}^{100}C_0 x^{100} + {}^{100}C_2 x^{98} a^2 + \dots + {}^{100}C_{100} a^{100}}_{51 \text{ terms}} \right] \end{aligned}$$

Q. 19 If the integers $r > 1$, $n > 2$ and coefficients of $(3r)$ th and $(r + 2)$ nd terms in the Binomial expansion of $(1 + x)^{2n}$ are equal, then

- (a) $n = 2r$ (b) $n = 3r$
 (c) $n = 2r + 1$ (d) None of these

Thinking Process

In the expansion of $(x + y)^n$, the coefficient of $(r + 1)$ th term is nC_r .

Sol. (a) Given that, $r > 1$, $n > 2$ and the coefficients of $(3r)$ th and $(r + 2)$ th term are equal in the expansion of $(1 + x)^{2n}$.

$$\text{Then, } T_{3r} = T_{3r-1+1} = {}^{2n}C_{3r-1} x^{3r-1}$$

$$\text{and } T_{r+2} = T_{r+1+1} = {}^{2n}C_{r+1} x^{r+1}$$

$$\text{Given, } {}^{2n}C_{3r-1} = {}^{2n}C_{r+1} \quad [\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$$

$$\Rightarrow 3r - 1 + r + 1 = 2n$$

$$\Rightarrow 4r = 2n \Rightarrow n = \frac{4r}{2}$$

$$\therefore n = 2r$$

Q. 20 The two successive terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

- (a) 3rd and 4th (b) 4th and 5th
 (c) 5th and 6th (d) 6th and 7th

Sol. (c) Let two successive terms in the expansion of $(1 + x)^{24}$ are $(r + 1)$ th and $(r + 2)$ th terms.

$$\therefore T_{r+1} = {}^{24}C_r x^r$$

$$\text{and } T_{r+2} = {}^{24}C_{r+1} x^{r+1}$$

$$\text{Given that, } \frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4}$$

$$\Rightarrow \frac{\frac{(24)!}{r!(24-r)!}}{(24)!} = \frac{1}{4}$$

$$\Rightarrow \frac{(r+1)!(24-r-1)!}{r!(24-r)(23-r)!} = \frac{1}{4}$$

$$\Rightarrow \frac{r+1}{24-r} = \frac{1}{4} \Rightarrow 4r + 4 = 24 - r$$

$$\Rightarrow 5r = 20 \Rightarrow r = 4$$

$$\therefore T_{4+1} = T_5 \text{ and } T_{4+2} = T_6$$

Hence, 5th and 6th terms.

Q. 21 The coefficient of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ are in the ratio

(a) 1 : 2

(b) 1 : 3

(c) 3 : 1

(d) 2 : 1

Sol. (d) $\therefore$ Coefficient of x^n in the expansion of $(1+x)^{2n} = {}^{2n}C_n$
and coefficient of x^n in the expansion of $(1+x)^{2n-1} = {}^{2n-1}C_n$

$$\begin{aligned}\therefore \frac{{}^{2n}C_n}{{}^{2n-1}C_n} &= \frac{\frac{(2n)!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}} \\ &= \frac{(2n)!n!(n-1)!}{n!n!(2n-1)!} \\ &= \frac{2n(2n-1)!n!(n-1)!}{n!n(n-1)!(2n-1)!} \\ &= \frac{2n}{n} = \frac{2}{1} = 2 : 1\end{aligned}$$

Q. 22 If the coefficients of 2nd, 3rd and the 4th terms in the expansion of $(1+x)^n$ are in AP, then the value of n is

(a) 2

(b) 7

(c) 11

(d) 14

Sol. (b) The expansion of $(1+x)^n$ is ${}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \dots + {}^nC_nx^n$

$\therefore$ Coefficient of 2nd term = nC_1 ,

Coefficient of 3rd term = nC_2 ,

and coefficient of 4th term = nC_3 .

Given that, nC_1 , nC_2 and nC_3 are in AP.

$\therefore 2 {}^nC_2 = {}^nC_1 + {}^nC_3$

$$\Rightarrow 2 \left[\frac{(n)!}{(n-2)!2!} \right] = \frac{(n)!}{(n-1)!} + \frac{(n)!}{3!(n-3)!}$$

$$\Rightarrow \frac{2 \cdot n(n-1)(n-2)!}{(n-2)!2!} = \frac{n(n-1)!}{(n-1)!} + \frac{n(n-1)(n-2)(n-3)!}{3 \cdot 2 \cdot 1(n-3)!}$$

$$\Rightarrow n(n-1) = n + \frac{n(n-1)(n-2)}{6}$$

$$\Rightarrow 6n - 6 = 6 + n^2 - 3n + 2$$

$$\Rightarrow n^2 - 9n + 14 = 0$$

$$\Rightarrow n^2 - 7n - 2n + 14 = 0$$

$$\Rightarrow n(n-7) - 2(n-7) = 0$$

$$\Rightarrow (n-7)(n-2) = 0$$

$$\therefore n = 2 \text{ or } n = 7$$

Since, $n = 2$ is not possible.

$$\therefore n = 7$$

Q. 23 If A and B are coefficient of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then $\frac{A}{B}$ equals to

- (a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{n}$

Sol. (b) Since, the coefficient of x^n in the expansion of $(1+x)^{2n}$ is ${}^{2n}C_n$.

$$\therefore A = {}^{2n}C_n$$

Now, the coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is ${}^{2n-1}C_n$.

$$\therefore B = {}^{2n-1}C_n$$

$$\text{Now, } \frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{2}{1} = 2$$

Same as solution No. 21.

Q. 24 If the middle term of $\left(\frac{1}{x} + x \sin x\right)^{10}$ is equal to $7\frac{7}{8}$, then the value of x is

- (a) $2n\pi + \frac{\pi}{6}$ (b) $n\pi + \frac{\pi}{6}$
(c) $n\pi + (-1)^n \frac{\pi}{6}$ (d) $n\pi + (-1)^n \frac{\pi}{3}$

Sol. (c) Given expansion is $\left(\frac{1}{x} + x \sin x\right)^{10}$.

Since, $n = 10$ is even, so this expansion has only one middle term i.e., 6th term.

$$\therefore T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{1}{x}\right)^{10-5} (x \sin x)^5$$

$$\Rightarrow \frac{63}{8} = {}^{10}C_5 x^{-5} x^5 \sin^5 x$$

$$\Rightarrow \frac{63}{8} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} \sin^5 x$$

$$\Rightarrow \frac{63}{8} = 2 \cdot 9 \cdot 2 \cdot 7 \cdot \sin^5 x$$

$$\Rightarrow \sin^5 x = \frac{1}{32}$$

$$\Rightarrow \sin^5 x = \left(\frac{1}{2}\right)^5$$

$$\Rightarrow \sin x = \frac{1}{2}$$

$$\therefore x = n\pi + (-1)^n \pi / 6$$

Fillers

Q. 25 The largest coefficient in the expansion of $(1+x)^{30}$ is

Thinking Process

In the expansion of $(1+x)^n$, the largest coefficient is ${}^nC_{n/2}$ (when n is even).

Sol. Largest coefficient in the expansion of $(1+x)^{30} = {}^{30}C_{30/2} = {}^{30}C_{15}$

Q. 26 The number of terms in the expansion of $(x+y+z)^n$

Sol. Given expansion is $(x+y+z)^n = [x+(y+z)]^n$.

$$[x+(y+z)]^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}(y+z) + {}^nC_2 x^{n-2}(y+z)^2 + \dots + {}^nC_n (y+z)^n$$

$\therefore$ Number of terms = $1+2+3+\dots+n+(n+1)$

$$= \frac{(n+1)(n+2)}{2}$$

Q. 27 In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the value of constant term is

Sol. Let constant be T_{r+1} .

$$\begin{aligned} \therefore T_{r+1} &= {}^{16}C_r (x^2)^{16-r} \left(-\frac{1}{x^2}\right)^r \\ &= {}^{16}C_r x^{32-2r} (-1)^r x^{-2r} \\ &= {}^{16}C_r x^{32-4r} (-1)^r \end{aligned}$$

For constant term, $32-4r=0 \Rightarrow r=8$

$$\therefore T_{8+1} = {}^{16}C_8$$

Q. 28 If the seventh term from the beginning and the end in the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ are equal, then n equals to

Sol. Given expansions is $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$.

$$\therefore T_7 = T_{6+1} = {}^nC_6 (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6 \quad \dots(i)$$

Since, T_7 from end is same as the T_7 from beginning of $\left(\frac{1}{\sqrt[3]{3}} + \sqrt[3]{2}\right)^n$.

$$\text{Then, } T_7 = {}^nC_6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} (\sqrt[3]{2})^6 \quad \dots(ii)$$

$$\text{Given that, } {}^nC_6 (2)^{\frac{n-6}{3}} (3)^{-6/3} = {}^nC_6 (3)^{-\frac{(n-6)}{3}} 2^{6/3}$$

$$\Rightarrow (2)^{\frac{n-12}{3}} = \left(\frac{1}{3^{1/3}}\right)^{n-12}$$

which is true, when $\frac{n-12}{3} = 0$.

$$\Rightarrow n-12=0 \Rightarrow n=12$$

Q. 29 The coefficient of $a^{-6}b^4$ in the expansion of $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$ is

Thinking Process

In the expansion of $(x-a)^n$, $T_{r+1} = {}^nC_r \cdot x^{n-r}(-a)^r$

Sol. Given expansion is $\left(\frac{1}{a} - \frac{2b}{3}\right)^{10}$.

Let T_{r+1} has the coefficient of $a^{-6}b^4$.

$$\therefore T_{r+1} = {}^{10}C_r \left(\frac{1}{a}\right)^{10-r} \left(-\frac{2b}{3}\right)^r$$

For coefficient of $a^{-6}b^4$, $10-r=6 \Rightarrow r=4$

$$\begin{aligned} \therefore \text{Coefficient of } a^{-6}b^4 &= {}^{10}C_4 (-2/3)^4 \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{2^4}{3^4} = \frac{1120}{27} \end{aligned}$$

Q. 30 Middle term in the expansion of $(a^3 + ba)^{28}$ is

Sol. Given expansion is $(a^3 + ba)^{28}$.

$$\therefore n = 28$$

[even]

$$\therefore \text{Middle term} = \left(\frac{28}{2} + 1\right)\text{th term} = 15\text{th term}$$

$$\begin{aligned} \therefore T_{15} &= T_{14+1} \\ &= {}^{28}C_{14} (a^3)^{28-14} (ba)^{14} \\ &= {}^{28}C_{14} a^{42} b^{14} a^{14} \\ &= {}^{28}C_{14} a^{56} b^{14} \end{aligned}$$

Q. 31 The ratio of the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ is

Sol. Given expansion is $(1+x)^{p+q}$.

$$\therefore \text{Coefficient of } x^p = {}^{p+q}C_p$$

$$\text{and coefficient of } x^q = {}^{p+q}C_q$$

$$\therefore \frac{{}^{p+q}C_p}{{}^{p+q}C_q} = \frac{{}^{p+q}C_p}{{}^{p+q}C_p} = 1:1$$

Q. 32 The position of the term independent of x in the expansion of

$$\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10} \text{ is$$

Sol. Given expansion is $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$.

Let the constant term be T_{r+1} .

Then,

$$\begin{aligned}
 T_{r+1} &= {}^{10}C_r \left(\sqrt{\frac{x}{3}} \right)^{10-r} \left(\frac{3}{2x^2} \right)^r \\
 &= {}^{10}C_r \cdot x^{\frac{10-r}{2}} \cdot 3^{\frac{-10+r}{2}} \cdot 3^r \cdot 2^{-r} \cdot x^{-2r} \\
 &= {}^{10}C_r \cdot x^{\frac{10-5r}{2}} \cdot 3^{\frac{-10+3r}{2}} \cdot 2^{-r}
 \end{aligned}$$

For constant term, $10 - 5r = 0 \Rightarrow r = 2$

Hence, third term is independent of x .

Q. 33 If 25^{15} is divided by 13, then the remainder is

Sol. Let

$$\begin{aligned}
 25^{15} &= (26 - 1)^{15} \\
 &= {}^{15}C_0 26^{15} - {}^{15}C_1 26^{14} + \dots - {}^{15}C_{15} \\
 &= {}^{15}C_0 26^{15} - {}^{15}C_1 26^{14} + \dots - 1 - 13 + 13 \\
 &= {}^{15}C_0 26^{15} - {}^{15}C_1 26^{14} + \dots - 13 + 12
 \end{aligned}$$

It is clear that, when 25^{15} is divided by 13, then remainder will be 12.

True/False

Q. 34 The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$ is $2^{19} + \frac{{}^{20}C_{10}}{2}$.

Sol. False

$$\begin{aligned}
 \text{Given series} &= \sum_{r=0}^{10} {}^{20}C_r = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \dots + {}^{20}C_{10} \\
 &= {}^{20}C_0 + {}^{20}C_1 + \dots + {}^{20}C_{10} + {}^{20}C_{11} + \dots + {}^{20}C_{20} - ({}^{20}C_{11} + \dots + {}^{20}C_{20}) \\
 &= 2^{20} - ({}^{20}C_{11} + \dots + {}^{20}C_{20})
 \end{aligned}$$

Hence, the given statement is false.

Q. 35 The expression $7^9 + 9^7$ is divisible by 64.

Sol. True

$$\begin{aligned}
 \text{Given expression} &= 7^9 + 9^7 = (1 + 8)^7 - (1 - 8)^9 \\
 &= ({}^7C_0 + {}^7C_1 8 + {}^7C_2 8^2 + \dots + {}^7C_7 8^7) - ({}^9C_0 - {}^9C_1 8 + {}^9C_2 8^2 \dots - {}^9C_9 8^9) \\
 &= (1 + 7 \times 8 + 21 \times 8^2 + \dots) - (1 - 9 \times 8 + 36 \times 8^2 + \dots - 8^9) \\
 &= (7 \times 8 + 9 \times 8) + (21 \times 8^2 - 36 \times 8^2) + \dots \\
 &= 2 \times 64 + (21 - 36)64 + \dots
 \end{aligned}$$

which is divisible by 64.

Hence, the statement is true.

Q. 36 The number of terms in the expansion of $[(2x + y^3)^4]^7$ is 8.

Sol. False

$$\text{Given expansion is } [(2x + y^3)^4]^7 = (2x + y^3)^{28}.$$

Since, this expansion has 29 terms.

So, the given statement is false.

Q. 37 The sum of coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$ is equal to ${}^{2n-1}C_n$.

Sol. False

Here, the Binomial expansion is $(1+x)^{2n-1}$.

Since, this expansion has two middle term i.e., $\left(\frac{2n-1+1}{2}\right)$ th term and $\left(\frac{2n-1+1}{2}+1\right)$ th term i.e., n th term and $(n+1)$ th term.

$$\therefore \text{Coefficient of } n\text{th term} = {}^{2n-1}C_{n-1}$$

$$\text{Coefficient of } (n+1)\text{th term} = {}^{2n-1}C_n$$

$$\begin{aligned} \text{Sum of coefficients} &= {}^{2n-1}C_{n-1} + {}^{2n-1}C_n \\ &= {}^{2n-1+1}C_n = {}^{2n}C_n \quad [\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r] \end{aligned}$$

Q. 38 The last two digits of the numbers 3^{400} are 01.

Sol. True

$$\text{Given that, } 3^{400} = 9^{200} = (10-1)^{200}$$

$$\Rightarrow (10-1)^{200} = {}^{200}C_0 10^{200} - {}^{200}C_1 10^{199} + \dots - {}^{200}C_{199} 10^1 + {}^{200}C_{200} 1^{200}$$

$$\Rightarrow (10-1)^{200} = 10^{200} - 200 \times 10^{199} + \dots - 10 \times 200 + 1$$

So, it is clear that the last two digits are 01.

Q. 39 If the expansion of $\left(x - \frac{1}{x^2}\right)^{2n}$ contains a term independent of x , then n is a multiple of 2.

Sol. False

$$\text{Given Binomial expansion is } \left(x - \frac{1}{x^2}\right)^{2n}.$$

Let T_{r+1} term is independent of x .

$$\begin{aligned} \text{Then, } T_{r+1} &= {}^{2n}C_r (x)^{2n-r} \left(-\frac{1}{x^2}\right)^r \\ &= {}^{2n}C_r x^{2n-r} (-1)^r x^{-2r} = {}^{2n}C_r x^{2n-3r} (-1)^r \end{aligned}$$

For independent of x ,

$$\begin{aligned} 2n - 3r &= 0 \\ \therefore r &= \frac{2n}{3}, \end{aligned}$$

which is not a integer.

So, the given expansion is not possible.

Q. 40 The number of terms in the expansion of $(a+b)^n$, where $n \in N$, is one less than the power n .

Sol. False

We know that, the number of terms in the expansion of $(a+b)^n$, where $n \in N$, is one more than the power n .