TALENT & OLYMPIAD

Percentage

MATHEMATICS

Introduction

Percentage is a fraction whose denominator is 100. The numerator of the such fraction is called the rate percent. For example 15 percent means $\frac{15}{100}$ and denoted by 15 %.

🍄 Percentage

a % means $\frac{a}{100}$ and simplify it. e.g $45\% = \frac{45}{100} = \frac{9}{20}$

For conversion of fraction $\frac{p}{q}$ as percentage, we simply multiply it by 100 and put the sign of % or

mathematically we can write $\frac{p}{q} = \left(\frac{p}{q} \times 100\right)\%$

Illustrative EXAMPLE

The population of a village is $4500. \left(\frac{11}{18}\right)^{th}$ of them are males and the rest are females. If 40 % of the females are married, then the number of married females is: (a) 1750 (b) 700

(a) 1750	(b) 700
(c) 750	(d) 900
(e) None of these	

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Answer: (b)

Explanation

No. of males $=\frac{11}{18} \times 4500 = 2750$ No. of females = 4500 - 2750 = 1750Thus no. of married females $= 40 \% \times 1750 = 700$

Illustrative EXAMPLE



In a class of 38 girls, 3 are absent, 20 % of the remainder have failed to do the home work. Find the number of girls who did their homework.

(a) 7	(b) 25
(c) 28	(d) 30
(e) None of these	

Solution: (c)

No. of girls present =38-3=35No. of girls who did not do home work $=20\% \times 35=7$ No. of girls who did their home work =35-7=28.

Application Based Problem on Percentage

- The following are the points to remember to solve the problem related to variation in the price of an \geq article.
- \triangleright If the price of an article increases by x % then the reduction in consumption so that expenditure remains unaffected is $\left(\frac{x}{100+x} \times 100\right)\%$

 \triangleright If the price of an article decreases by x % then the increase in consumption so that expenditure remains unaffected is

$$\left(\frac{x}{100-x} \times 100\right)\%$$

llustrative EXAMPLE

In the new budget, the price of petrol increased by 25 %. By how much % person should reduce his consumption so that his expenditure is not affected

(a) 10%	(b) 20%
(c) 25%	(d) 30%
(e) None of these	

Answer: (b)

Explanation Reduction in consumption $\left(\frac{25}{100+25} \times 100\right)\% = 20\%$

Illustrative

EXAMPLE

Due to reduction of $6\frac{1}{4}\%$ in the price of sugar, a man is able to buy 1 kg more sugar for Rs 120. The

reduced rate of sugar is:

(a) Rs 8 per kg	(b) Rs 6.5 per kg
(c) Rs 7.5 per kg	(d) Rs 9 per kg
(e) None of these	

Answer: (c)

Explanation

Suppose original rate of sugar Rs x per kg.

Reduced rate = $\left[\left(100 - \frac{25}{4} \right) \times \frac{1}{100} \times x \right] = \frac{15x}{16}$ According to question, $\frac{120}{15x} - \frac{120}{x} = 1$ $\Rightarrow \frac{128}{x} - \frac{120}{x} = 1 \Rightarrow x = 8$ Reduced rate $=\frac{15}{16} \times 8 = Rs7.5$ per kg

Problem Based on the Population of a Locality

Suppose the present population of a locality be 'A' and let it increases by x % per annum then

- > Population after y years $= A \left(1 + \frac{x}{100} \right)^y$
- > Population before y years $\frac{A}{\sqrt{2}}$

$$\left(1+\frac{x}{100}\right)^{\frac{1}{2}}$$

Illustrative EXAMPLE

The population of a town is 176400. If it increases at the rate of 5 % per annum then what was its population 2 years ago?

- (a) 166400
- (b) 154600
- (c) 160000
- (d) 166000
- (e) None of these

Answer: (c)

Explanation

According to question

Population of city before y years, if it increases at the rate of $x\% = \frac{A}{\left(1 + \frac{x}{100}\right)^{y}}$

:. The population if city
=
$$\frac{176400}{\left(1 + \frac{5}{100}\right)^2} = 176400 \left(\frac{20}{21} \times \frac{20}{21}\right) = 160000$$

Illustrative EXAMPLE

In a certain year the population of London is 200000. If it increases at the rate of 6.5 % per annum then what will be its population after 2 years?

(a) 226845

- (b) 228645
- (c) 224685
- (d) 228465
- (e) None of these

Answer: (a) Explanation Population after 2 years

$$=200000\left(1+\frac{6.5}{100}\right)^2=226845$$

Commonly Asked

UESTIONS



Match the following:

(i)	$\frac{1}{2}$ is what percent of $\frac{1}{3}$	(a)	4%
(ii)	What percent of 7 is 84?	(b)	150%
(iii)	What percent of 6.5 liter is 130 ml	(c)	2%
(iv)	5 is what percent of 125	(d)	1200%

Which one of the following options is correct?

(a) i - b, ii - d, iii - a, iv - c
(b) i - b ii - c, iii - d, iv - a
(c) i-c, ii-d, iii-b, iv-a
(d) i - b,ii - d, iii - c, iv-a
(e) None of these
Answer: (d)



Stephen's mathematics test had 85 problems, which contains 20 algebra, 30 statistics and 35 geometry problems. He answered 70 % of algebra, 40 % of the statistics and 60 % of geometry problems correctly. He did not pass the test because he got less than 60 % of the problem correct. How many more questions he would have need to answer correctly to earn 60% passing grade?

(a) 5	(b) 4
(c) 1	(d) 3
(a) Nana afthaca	

(e) None of these

Answer: (b)

Explanation

Number of questions attempted correctly = $(70 \ \% 20 + 40 \ \% \ of 30 + 60 \ \% \ of 35)$ of = (14 + 12 + 21) = 47Questions to be answered correctly to obtained 60% = 60% of 85 = 51 Required number of question = (51-47) = 4

The tax on an electronic goods is 15 % and is increased by 10 %. What the percent of total tax on the electronic goods?

- (a) 26.5 % increase
- (b) 23 % decrease
- (c) 25 % increase
- (d) 27 % decrease
- (e) None of these

Answer: (a)

If the duty on an article be reduced by 40 % of its present amount, then by how much percent must the consumption be increased in order than the revenue may remain unaltered?

(a) 50 %

(b) $166\frac{2}{3}\%$ (d) 20%

(b) 750 (d) 900

(c) 40%(e) None of these

Answer: (b)

What is the total number of candidates appear in an examination, if 31% is fail and the number of passed candidates are 247 more than the number of fail candidates?

(a) 650	
(c) 800	
(e) None of these	

Answer: (a)



- Percent means parts per hundred and we use the symbol % to represent it.
- Percent can be converted into decimals, fractions, ratio and vice versa.
- If the price of an article increases by x% then the reduction in consumption so that expenditure remains un effected is

$$\left(\frac{x}{100+x} \times 100\right)\%$$

 \diamond If the price of an article decreases by x % then the increase in consumption so that expenditure

remains un effected is $\left(\frac{x}{100-x} \times 100\right)\%$

You Must KN��W

- In the 14th century, Madhava of Sangamagrama, the founder of the so-called Kerala School of Mathematics, found the Madhava-Leibniz series and using 21 terms, computed the value of n as 3.14159265359.
- In the 12th century, Bhskara II lived in southern India and wrote extensively on all then known branches of mathematic. His work contains mathematical objects equivalent or approximately equivalent to infinitesimals, derivatives, the mean value theorem and the derivative of the sine function.

Self Evaluation



1.	A man losses $12\frac{1}{2}\%$ of his money and now he has 70% of the remainder. Which is Rs 210. Find the money									
	he had?									
	(a) Rs 342.85	(b) Rs 810.34								
	(c) Rs 450.78	(d) Rs 600								
	(e) None of these									
2.	If a man's wages is incre percent is:	eased by 10% and after	ward decreased by 10% then the total change in wage in							
	(a) 9%	(b) 10%								
	(c)1%	(d) 2%								
	(e) None of these									
3.	If an error of 5 % extra is the area of the square is	made in measurement	of the sides of a square, then the error % in the calculating							
	(a) $10\frac{1}{4}\%$	(b) 25%								
	(c) $5\frac{1}{2}\%$	(d) 10%								
	(e) None of these									
4.	If 40 % of a number is ad	ded to 42 then the resu	It is the number itself. The number is?							
	(a) 82	(b) 105								
	(c) 70	(d) 72								
	(e) None of these									
5.	If 60 is subtracted from 6	50 % of a number then t	he result is 60 %. The number is:							
	(a) 120	(d) 200								
	(c) 100 (e) None of these	(u) 200								
	(c) None of these									
6.	If duty on an article be r be increased so that the	educed by 40 % of its prevenue is increased by	price then by how much % the consumption of that article 10 %?							
	(a) 80%	(b) 60%								
	(c) 75%	(d) $83\frac{1}{3}\%$	(e) None of these							

7.	In an examination 70 % of the candidates passed in English, 65 % in mathematics and 27 % failed in both subjects. The pass percentage was:								
	(a) 37%	(b) 38%							
	(c) 62%	(d) 80%							
	(e) None of these								
8.	If the price of tea be in consumption of tea so tl	creased by 20 % then find by how much percent a householder must reduce his nat he has not to increase his expenditure?							
	(a) 20%	(b) 25%							
	(c) $16\frac{2}{3}\%$	(d) 40 %							
	(e) None of these								
9.	The petrol price is reduc that his expenditure on	ed by 10 %. Find by how much a user must increase the consumption of petrol so petrol is not decreased?							
	(a) 15%	(b) $9\frac{1}{11}\%$							
	(c) 10%	(d) $11\frac{1}{9}\%$							
	(e) None of these								
10.	A candidate must gets 3 number of marks?	3 % marks to pass. He gets 220 marks and fails by 11 marks. What is the maximum							
	(a) 500	(b) 689							
	(c) 711	(d) 700							
	(e) None of these								

Answers – Self Evaluation Test																		
1.	А	2.	С	3.	А	4.	С	5.	D	6.	D	7.	С	8.	С	9.	D	10. D

Self Evaluation Test SOLUTIONS

1. Let x be the money he had He losses $12\frac{1}{2}$ % of x - . Now money he has $x - \frac{x}{8} = \frac{7x}{8}$ According to the question $\frac{7x}{8}$ of $70\% = \text{Rs } 210 \Rightarrow \frac{7x}{8} \times \frac{70}{100} = 210 \Rightarrow \frac{7x \times 70}{800} = 210$ $\Rightarrow 70x \times 70 = 210 \times 210 \times 800 \therefore 7x = \frac{210 \times 800}{70} = 2400 \therefore x = \frac{2400}{700} = Rs342.85$

2. Let Rs 100 is its original wags then wages in the second case. $= Rs100 \times \frac{110}{100} \times \frac{90}{100} = Rs = 99$ ∴ change = (100-99)% = 1%

3. Area in the first case $= a^2$ and area in the 2nd case, $(105 \times a)^2 = \frac{21}{20} \times \frac{21}{20} \times a^2 = \frac{441}{400} a^2$ Change $= \frac{411}{400} a^2 - a^2 = \frac{41}{400} a^2$ \therefore Error $= \left[\frac{41}{400} a^2 \div a^2\right] \times 100 = \frac{41}{4} \% = 10\frac{1}{4} \%$

- 4. Let number is x then from question $x \times 40\% + 42 = x \Longrightarrow x \Longrightarrow \times \frac{40}{100} + 42 = x \Longrightarrow x = 70$
- 5. Let the number is x. According to question 60% of $\Rightarrow \frac{3x}{5} = 60 + 60 \Rightarrow x = 200$
- 6. Consumption after tax is reduced

$$=\frac{Revenue}{Tax}=\frac{110}{60\%}=\frac{110\times100}{60}=183\frac{1}{3}\%$$

: Change =
$$183\frac{1}{3}\% - 100\% = 83\frac{1}{3}\%$$

- 7. Those failed in maths = 35,%, those failed in English = 30 %, failed in both = 27 % failed one or both the subjects = (35+30-27) % = 38 % \Rightarrow Pass = 62 %
- 8. If 1st expenditure is Rs 100 then after increase, it is Rs 120, consumption of Rs 20 be reduced out of Rs 120. ∴ Reduction in Consumption

$$=\frac{20}{120}\times100=16\frac{2}{3}\%$$

9. Increase in petrol
$$=\frac{10}{100-10} \times 100 = 11\frac{1}{9}\%$$

10. Let number is *x* then 33 % of x = 220 + 11 = 231or $x = 231 \times \frac{100}{33} = 700$