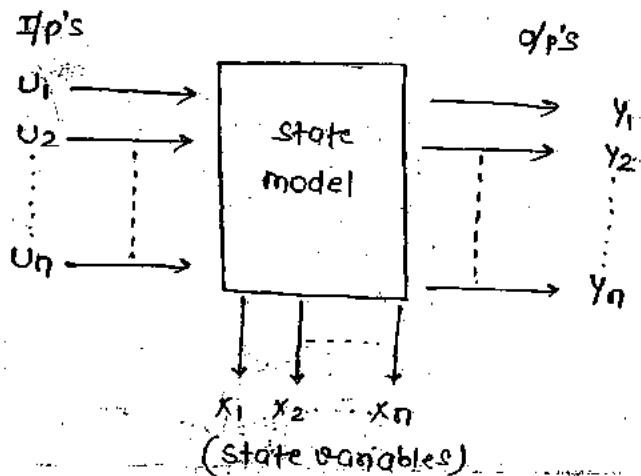


Chapter-05 State Space Variables



(1) State eqn $\rightarrow$

$$\dot{X}(t) = A X(t) + B U(t)$$

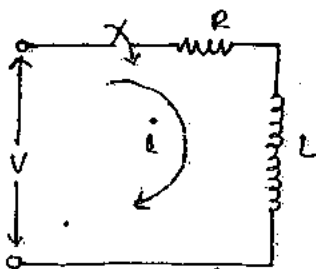
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

(2) O/p eqn $\rightarrow$

$$Y(t) = C X(t) + D U(t)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

Ex:



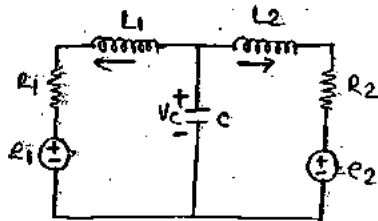
at $t = 0^-$
 $i(0^-) = i_L(0^-) = 0 \text{ Amps}$
 At $t = 0^+$
 $i_L(0^+) = \frac{1}{L} \int_{t=0}^{0^+} V dt = 0 \text{ Amps}$

$$V = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{-Ri + V}{L} \quad (\text{A/s})$$

State eqⁿ of N/w.

Contⁿ(L)
80



Loop (1)

$$L_1 \frac{di_1}{dt} + i_1 R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt} = \frac{-R_1}{L_1} i_1 + \frac{V_c}{L_1} - \frac{e_1}{L_1} \quad \text{--- (i)}$$

KCL at Node $V_c \rightarrow$

$$i_1 + i_2 + C \frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} = -\frac{i_1}{C} - \frac{i_2}{C} \quad \text{--- (ii)}$$

Loop (2)

$$L_2 \frac{di_2}{dt} + i_2 R_2 + e_2 - V_c = 0$$

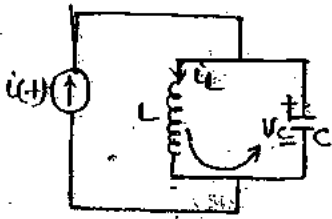
$$\frac{di_2}{dt} = \frac{-R_2}{L_2} i_2 + \frac{V_c}{L_2} - \frac{e_2}{L_2} \quad \text{--- (iii)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{-R_2}{L_2} & \frac{1}{L_2} \\ \frac{-1}{C} & \frac{-1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{-1}{L_1} & 0 \\ 0 & \frac{-1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Let o/p $y = i_1$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + 0$$

(b.)



$$i(t) = i_L + i_C$$

$$i(t) = i_L + C \frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} = \frac{-i_L}{C} + \frac{i(t)}{C} \quad \text{--- (i)}$$

$$L \frac{di_L}{dt} - v_C = 0 \Rightarrow \frac{di_L}{dt} = \frac{v_C}{L} \quad \text{--- (ii)}$$

* Type-(i) Problem $\rightarrow$

To obtain state model from differential eqn

$$\frac{d^3 y}{dt^3} + 4 \frac{d^2 y}{dt^2} + 6 \frac{dy}{dt} + 10y = 20u$$

$$[s^3 + 4s^2 + 6s + 10] Y(s) = 20 U(s)$$

$$\text{Let } y = x_1$$

$$\frac{dy}{dt} = \dot{x}_1 = x_2 ; \quad \frac{d^2 y}{dt^2} = \dot{x}_2 = x_3 ; \quad \frac{d^3 y}{dt^3} = \dot{x}_3$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = 20u - 4x_3 - 6x_2 - 10x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

BUSH/COMPANION FORM

* shortcut method for BUSH form →

Ex 80

$$\frac{Y(s)}{U(s)} = \frac{1}{s^4 + 5s^3 + 8s^2 + 6s + 3}$$

← Reverse order with rev sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

constant numerator.

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Que. → $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = \frac{2dy}{dt} + u$

Soln →

$$(s^2 + 7s + 9)Y(s) = (2s + 1)U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{(2s + 1)}{s^2 + 7s + 9}$$

Que. (8) phase variable method →

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 7s + 9} \quad (2s + 1) \leftarrow \text{only rev order} *$$

← rev order with rev sign

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ex:-

$$\frac{Y(s)}{U(s)} = \frac{10(s^2 + 2s + 4)}{s^3 + 6s^2 + 8s + 12}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

* Type (2) problem $\rightarrow$

To obtain TF from state model.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) + Du(t)$$

Applying LT on eqn (1)

$$sX(s) = AX(s) + BU(s) - x(0)$$

$$Y(s) = CX(s) + DU(s)$$

For TF $x(0) = 0$

$$sX(s) - AX(s) = BU(s)$$

$$(sI - A)X(s) = BU(s)$$

$$X(s) = (sI - A)^{-1} B \cdot U(s)$$

$$Y(s) = [C(sI - A)^{-1} B + D] U(s)$$

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D$$

(Transfer matrix)

Q(1) → $\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T X$$

Solⁿ →

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (s+1) & 0 \\ 0 & (s+2) \end{bmatrix}$$

$$\text{adj}(sI - A) = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$|(sI - A)| = (s+1)(s+2)$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & -\frac{1}{s+2} \end{bmatrix}$$

$$(sI - A)^{-1} B = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & -\frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C(sI - A)^{-1} B \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} Y(s) \\ U(s) \end{bmatrix} = \frac{1}{s+1}}$$

Q(2)

$$\dot{X}(t) = -2X(t) + 2U(t)$$

$$Y(t) = 0.5X(t)$$

Solⁿ →

$$sX(s) = -2X(s) + 2U(s)$$

$$(s+2)X(s) = 2U(s)$$

$$X(s) = \frac{2}{s+2} U(s)$$

$$Y(s) = 0.5X(s)$$

$$Y(s) = 0.5 \times \frac{2}{s+2} U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s+2}}$$

* Type(3) Problem $\rightarrow$ Stability for sm $\rightarrow$

$$\frac{Y(s)}{U(s)} = C[SI-A]^{-1}B + D$$

$$= C \frac{\text{Adj}(SI-A) \cdot B}{|SI-A|} + D$$

$$\frac{Y(s)}{U(s)} = \frac{C \cdot \text{Adj}(SI-A) \cdot B + |SI-A| \cdot D}{|SI-A|}$$

— Zeros $\rightarrow C \cdot \text{Adj}(SI-A) \cdot B + |SI-A| \cdot D$

Poles $\rightarrow (1 + G(s) \cdot H(s) = 0) = |SI-A| = 0$

Eigen values of sys. matrix $[A]$ = CL poles

Cond(2)
BO

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$C = [-17 \quad -5] X + [1] r$$

Soln $\rightarrow$

$$(SI-A) = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 20 & S+9 \end{bmatrix}$$

$$\text{Adj}(SI-A) = \begin{bmatrix} S+9 & 1 \\ -20 & S \end{bmatrix}$$

$$\text{Adj}(sI-A)B \Rightarrow \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C \text{Adj}(sI-A)B \Rightarrow [17 \quad -5] \begin{bmatrix} 1 \\ s \end{bmatrix} = -17-5s$$

$$|sI-A| = s(s+9) + 20$$

$$|sI-A| = (s^2 + 9s + 20)(1)$$

$$\text{Zeros} \Rightarrow -17-5s+s^2+9s+20=0$$

$$s^2 + 4s + 3 = 0$$

$$s = -1, -3$$

$$\text{Poles} \Rightarrow |sI-A| = 0$$

$$s^2 + 9s + 20 = 0$$

$$s = -4, -5 \quad (\text{sys is stable because of -ve eigen values})$$

Que (6)
80

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad u = [-0.5 \quad -3 \quad -5] X + v$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [-0.5 \quad -3 \quad -5] X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

The sum of principle diagonal = Sum of eigen values

Ans (a)

* Type (04) Diagonalization $\rightarrow$

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

Let $X = PZ \Rightarrow$ transformation matrix

$$P\dot{Z} = APZ + BU$$

$$Y = CPZ + DU$$

$$\boxed{\begin{aligned}\dot{Z} &= [P^{-1}AP]Z + [P^{-1}B]U \\ Y &= CPZ + DU\end{aligned}}$$

$$P = \text{Vander mode matrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \dots & \lambda_n^2 \end{bmatrix}$$

$P^{-1}AP \Rightarrow$ diagonal matrix with diagonal elements as eigen values.

Ex: $\rightarrow$ Diagonalize

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 6 & S+5 \end{bmatrix}$$

$$|SI - A| = S(S+5) + 6 = 0$$

$$S^2 + 5S + 6 = 0$$

$$S = -2, -3$$

$$P = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$P^{-1} = \frac{\text{Adj}(P)}{|P|} = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

* Type (05) Controllability & observability →

Controllability → To control the state variables

Observability → To measure the state variables

KALMAN'S Test →

$$Q_c = [B \ AB \ A^2B \ \dots \ A^{n-1}B] = |Q_c| \neq 0$$

$$Q_o = [C^T \ A^T C^T \ (A^T)^2 C^T \ \dots \ (A^T)^{n-1} C^T] = |Q_o| \neq 0$$

Q.1 → $\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1 \ 1] x$

Soln → $Q_c = [B \ AB]$

$$Q_o = [C^T \ A^T C^T]$$

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = -1$$

$$Q_o = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4$$

Q.101 $\dot{x} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ $y = [b \ 0] x$

Soln → $A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$$Q_o = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix} = 2b^2 \neq 0$$

ans (c)

* Type (c) solution of state eqn $\rightarrow$

$$\dot{X}(t) = AX(t) + Bu(t)$$

(a) Free Response $\rightarrow U(t) = 0$

$$\dot{X}(t) = AX(t)$$

$$\dot{X}(t) - AX(t) = 0 \text{ ----- (i)}$$

$$X(t) = ke^{At} \text{ ----- (ii)}$$

Applying LT to eqn (i)

$$sX(s) - X(0) - AX(s) = 0$$

$$sX(s) - AX(s) = X(0)$$

$$(sI - A) X(s) = X(0)$$

$$X(s) = (sI - A)^{-1} X(0)$$

$$\phi(s) = (sI - A)^{-1} = \text{Resolvent matrix}$$

$$X(t) = \mathcal{L}^{-1}[(sI - A)^{-1}] X(0)$$

$$X(t) = e^{At} \quad k$$

$$\boxed{\phi(t) = e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}]}$$

state transition matrix

(b) Forced Response $\rightarrow$

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$(sI - A) X(s) = X(0) + BU(s)$$

$$X(s) = (sI - A)^{-1} X(0) + (sI - A)^{-1} B U(s)$$

$$X(t) = \mathcal{L}^{-1}[(sI - A)^{-1}] X(0) + \mathcal{L}^{-1}[(sI - A)^{-1} B U(s)]$$

Que (3) $\rightarrow \dot{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} X, X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ —

Solⁿ $\rightarrow$

ZIR (zero impulse response)
 Short cut method
 At $t=0$
 $X(t) = X(0)$

$$sI - A \Rightarrow \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (s-1) & 0 \\ 0 & (s-1) \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

At $t=0$

$$e^{At} = \phi(t) = \mathcal{L}^{-1}(sI - A)^{-1} \quad X(t) = X(0), e^{A(0)} = I$$

$$s.t.m. \Rightarrow \begin{bmatrix} e^t & 0 \\ t e^t & e^t \end{bmatrix}$$

Property of s.t.m.
 At $t=0, e^{A0} = I$

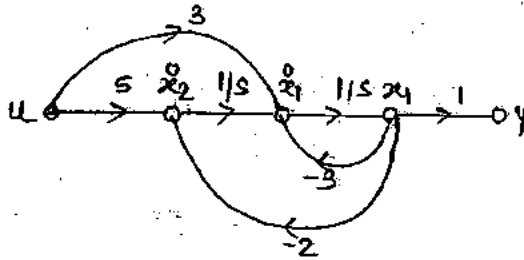
$$\bar{X}(t) = \begin{bmatrix} e^t & 0 \\ t e^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ t e^t \end{bmatrix}$$

Ans (C)

* TYPE(7) State diagrams $\rightarrow$

(1) Physical Variable Form $\rightarrow$

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2} = \frac{\frac{3}{s} + \frac{5}{s^2}}{1 - \left[\frac{-3}{s} - \frac{2}{s^2} \right]}$$



$$\dot{x}_1 = -3x_1 + x_2 + 3u$$

$$\dot{x}_2 = +2x_1 + 5u$$

$$y = x_1$$

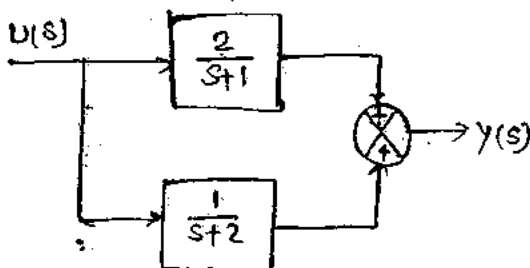
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$$

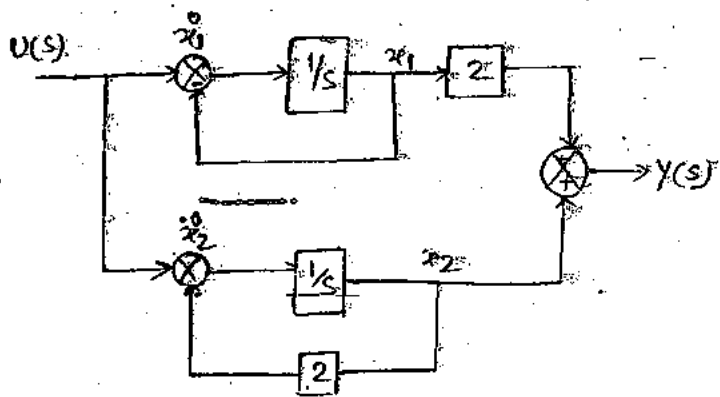
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(2) Canonical Variable Form $\rightarrow$

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{(s+1)(s+2)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$Y(s) = \frac{2U(s)}{s+1} + \frac{U(s)}{s+2}$$





$$\dot{x}_1 = -x_1 + 4$$

$$\dot{x}_2 = -2x_2 + u$$

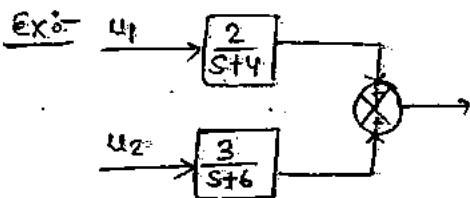
$$\dot{y} = 2x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

Unit matrix

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Residues
of PFS



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$